Transportation Research Part B 94 (2016) 121–149

Contents lists available at ScienceDirect

Transportation Research Part B

journal homepage: www.elsevier.com/locate/trb

Designing a supply chain resilient to major disruptions and supply/demand interruptions

Armin Jabbarzadeh[a,b], Behnam Fahimnia[a,1], Jiuh-Biing Sheu[c,2,*], Hani Shahmoradi Moghadam[b]

[a] Institute of Transport and Logistics Studies, The University of Sydney, Darlington, NSW 2008, Australia
[b] Department of Industrial Engineering, Iran University of Science and Technology, Tehran 1684613114, Iran
[c] Department of Business Administration, National Taiwan University, No. 1, Sec. 4, Roosevelt Road, Raipei 10016, Taiwan, R.O.C.

ARTICLE INFO

Article history:
Received 2 October 2014
Revised 4 June 2016
Accepted 12 September 2016
Available online 29 September 2016

Keywords:
Supply chain network design
Risk
Disruption
Interruption
Uncertainty
Reliability
Robust optimization
Stochastic optimization
Monte Carlo simulation

ABSTRACT

Global supply chains are more than ever under threat of major *disruptions* caused by devastating natural and man-made disasters as well as recurrent *interruptions* caused by variations in supply and demand. This paper presents a hybrid robust-stochastic optimization model and a Lagrangian relaxation solution method for designing a supply chain resilient to (1) supply/demand interruptions and (2) facility disruptions whose risk of occurrence and magnitude of impact can be mitigated through fortification investments. We study a realistic problem where a disruption can cause either a complete facility shutdown or a reduced supply capacity. The probability of disruption occurrence is expressed as a function of facility fortification investment for hedging against potential disruptions in the presence of certain budgetary constraints. Computational experiments and thorough sensitivity analyses are completed using some of the existing widely-used datasets. The performance of the proposed model is also examined using a Monte Carlo simulation method. To explore the practical application of the proposed model and methodology, a real world case example is discussed which addresses mitigating the risk of facility fires in an actual oil production company. Our analysis and investigation focuses on exploring the extent to which supply chain design decisions are influenced by factors such as facility fortification strategies, a decision maker's conservatism degree, demand fluctuations, supply capacity variations, and budgetary constraints.

1. Introduction

Supply chain network design decisions form the backbone of supply chain management with direct impact on a firm's return on investment and its overall performance (Farahani et al., 2014; Zokaee et al., 2014). It concerns strategic decisions on supply chain configuration which includes determining the number, location and capacity of facilities in order to serve a predetermined, but possibly evolving, customer base (Rezaee et al., 2016). Since these decisions are by nature costly and difficult to reverse, supply chain networks are designed to last for several years and hence need to be robust to cope with future uncertainties (Jabbarzadeh et al., 2014; Snyder et al., 2007). Tang (2006) defines two types

* Corresponding author.
E-mail addresses: armin.jabbarzadeh@sydney.edu.au, arminj@iust.ac.ir (A. Jabbarzadeh), behnam.fahimnia@sydney.edu.au (B. Fahimnia), jbsheu@ntu.edu.tw (J.-B. Sheu), hani.shahmoradi.ie@gmail.com (H.S. Moghadam).
[1] Fax: +61 2 9114 1863.
[2] Fax: +886-2-2362-5379.

http://dx.doi.org/10.1016/j.trb.2016.09.004

of risks facing supply chains: operational risks and disruption risks. Operational risks are caused by inherent interruptions such as uncertain customer demand, uncertain supply capacity, and uncertain procurement costs. Disruption risks are caused by major incidents such as natural and man-made disasters (e.g., earthquakes, floods, terrorist attacks, fires, etc.). Esmaeilikia et al. (2014a, 2014b) provide a similar definition and classifies supply chain risks into those posed by major *disruptions* (rare events, but devastating impacts) and supply/demand *interruptions* (frequent occurrence, but less detrimental).

In most cases, the impact of disruptions on business performance is much larger than that of operational risks (Tang, 2006). For today's supply chains, the primary causes of increased exposure to disruptions are the lean and relentless cost-minimization practices, global reach of supply chains, and shorter product life cycles. Recent examples of natural disasters which have disrupted the performance of several supply chains include the tsunamis in the Indian Ocean (2004) and Japan (2011), the earthquakes in China (2008) and Chile (2011 and 2015), and Typhoon Haiyan in the Philippines (2013) (Fahimnia et al., 2015; Klibi et al., 2010). We have no direct control over the probability of occurrence of such disasters, and the only way to reduce the impact of these disasters is to consider the *location* of facilities or suppliers; that is, avoidance of flood-prone areas, earthquake zones, and areas exposed to high sea level rise and storm surges.

There are however disasters whose probability of occurrence and magnitude of impact can be mitigated by greater facility fortification investments. Some examples of such disasters include bushfire where prescribed burning (backburning) will prevent the occurrence, and creating firebreaks will minimize property damage. Fire within factory facilities can be prevented by maintenance of electrical wiring and appliances, education on basic electrical safety principles, and investment in low risk fire appliances. Also, installation of fire detection and sprinkler systems will reduce the impact of fire should it occur. Another example could be cyber-attacks which can be prevented by advanced firewalls and cyber security systems. It is the disruptions caused by these disasters that this paper seeks to address. Our case study analysis and discussions in this paper will focus on an actual fire disaster.

According to a recent survey by the insurance company Zurich Financial Services Australia Ltd, 85% of Australian-based companies experienced at least one supply chain disruption during 2011. Supply chain disruptions can have substantial impacts on the both short-term and long-term performance of firms (Hendricks et al., 2009; Peng et al., 2011). Hendricks and Singhal (2005) reported that companies suffering from even smaller-scale supply chain disruptions experienced 33–40% lower stock returns relative to their industry benchmarks. These illustrations and statistics reinforce the need to consider hedging against disruption risks when designing supply chain networks, a highly complex task due to the many influencing factors including budget availability (capital investment), decision maker's risk attitude, type of network under consideration, and the probability of disruption occurrence. Given that disruptions tend to be rare events, a primary complexity in designing resilient supply chains is the lack of historical data available from past disasters. The interaction between operational risks, more importantly demand variation risks, and disruption risks can add to this complexity. For instance, under demand uncertainty, it may be more beneficial for a company to run fewer number of larger facilities taking advantage of economies of scale in purchasing (Daskin et al., 2002), while it may be more worthwhile to operate more number of smaller facilities to minimize the impact of a disruption in one facility on the overall supply chain performance (Jabbarzadeh et al., 2015 ; Snyder et al., 2006).

To address these challenges, we present a hybrid robust optimization model (applying a robust optimization approach to a stochastic model) for designing a supply chain resilient to supply/demand variations and major disruptions whose risk of occurrence and magnitude of impact can be mitigated through facility fortification investments. The objective of the proposed model is to minimize the total cost of establishing the network while maximizing the supply chain resilience. Disruption occurrence probability is expressed as a function of capital investment for facility fortification. Facilities established at lower costs receive a higher probability of failure (less reliable facilities) and those with greater capital investment are assigned a smaller disruption probability value (more reliable facilities). Obviously, for situations when the probability of a disruption is not a function of investment level, one can simply set equal probabilities for different fortification levels. The ultimate goal of the proposed model is to determine the supply chain design decisions including the number, location and type of facilities (reliable or unreliable facilities) in the presence of certain budgetary constraints. We will investigate how the proposed model is able to capture the decision-makers' risk attitude to develop tradeoff between the supply chain design costs and disruption risks.

The remainder of the paper is organized as follows. Section 2 briefly reviews the related modeling efforts in the existing literature. Section 3 starts with a background of robust optimization followed by the formulation of the resilient supply chain network design model. Section 4 presents a Lagrangian relaxation solution method to tackle large-scale problems (we use GAMS to solve smaller-scale problems). Computational results are presented in Section 5 along with the practical and managerial insights obtained from the numerical results. Conclusions and directions for further research in this area are presented in Section 6.

2. Literature review

Reviews of facility location modeling efforts have been completed by Snyder (2006); ReVelle et al. (2008), and Melo et al. (2009). The more recent review of Snyder et al. (2016) indicates that a research focus on the design of 'resilient supply chains' has only been a recent occurrence; becoming only about 10 years old in 2015. Snyder and Daskin (2005) were among the first to incorporate disruption risks into classical facility location problems. They present reliability models based on a P-median problem and an uncapacitated fixed-charge location problem in which facilities are subject to disruptions. Their model aims to minimize facility location costs while taking into account the expected transportation cost when an unexpected disruption occurs. Aryanezhad et al. (2010) include inventory decisions to this model and present integer programming models minimizing the sum of facility construction costs, expected inventory costs and expected customer costs under normal and disruption situations. Chen et al. (2011) propose a Lagrangian relaxation method to solve this model.

The above studies assume equal disruption probabilities in all facilities, an assumption that has been relaxed in some of the more recent works by Berman et al. (2007); Li and Ouyang (2010); Shen et al. (2011), and Cui et al. (2010). Berman et al. (2007) presented a nonlinear integer programming model where facilities face independent disruptions with different probabilities. Due to model intractability, a heuristic algorithm was developed to solve the problem. O' Hanley et al. (2013) proposed an efficient technique for linearizing the facility location problem with site-dependent failure probabilities to tackle the intractability issue. Cui et al. (2010) presented an exact linear formulation for this problem to consider heterogeneous facility failure probabilities utilizing the linearization method of Sherali and Alameddine (1992).

Lim et al. (2010) incorporate the facility fortification concept into a facility location model to hedge against the risk of facility disruptions. They assume that if a serving facility fails, the associated demand point is immediately assigned to its backup. The problem is formulated as a mixed integer programming model for which a Lagrangian relaxation algorithm is proposed as a solution method. Li et al. (2013) extend this model by incorporating the rate of return for fortification investment and compare the results with that of alternative investment opportunities. For instance, a firm may choose to invest in network fortification only if the rate of return exceeds a minimum acceptable rate of return. The problem is further extended and investigated by Li and Savachkin (2013) where a facility can be fortified to a certain reliability level (a partial fortification strategy). All of these studies assume unlimited facility capacity.

The aforementioned models assume that a disrupted facility is completely out of service and hence disregard the probability that the performance of a facility can only be partially affected. Jabbarzadeh et al. (2012) present a supply chain design model for a situation where a facility may be partly disrupted, but may yet be able to fulfill a fraction of the initially assigned demand. Two solution methods based on Lagrangian relaxation and genetic algorithms are developed to solve the model. Liberatore et al. (2012) study the problem of optimally protecting a capacitated median where disasters may result in partial or complete shutdown of facilities. The proposed model optimizes protection plans when facing large area disruptions (i.e., disruptions that affect regions rather than single elements of the system). An algorithm is designed to solve the model optimally and is tested on a set of data from 2009 L'Aquila earthquake. Azad et al. (2013) formulate a capacitated location allocation model that accounts for partial disruptions considering deterministic supply chain demand. Benders decomposition is utilized to solve this computationally intractable model.

All the above models assume a risk-neutral decision maker who wishes to optimize the expected value of the objective function. Some of the most recent studies focus on risk aversion decision making through bi-level model formulation and optimizing worst-case objectives (Hernandez et al., 2014; Liberatore et al., 2011; Losada et al., 2012; Medal et al., 2014). Medal et al. (2014) investigate the minimax facility location and hardening problem seeking to minimize the maximum distance from a demand point to its closest located facility after facility disruptions. A decision maker in this case is interested in mitigation against a facility disruption scenario with the largest consequence. Likewise, Hernandez et al. (2014) apply a worst-case approach to hedge against disruptions. Using a multi-objective optimization approach, their model provides a decision maker with an option to tradeoff total weighted travelling distance before and after disruptions in a facility location problem. It allows investigating the impact that the opening of additional facilities can have on total distance travelled. Losada et al. (2012) present a bi-level mixed integer linear program for protecting an uncapacitated median type facility network against worst-case losses, taking into account the role of facility recovery time on system performance and the possibility of multiple disruptions over time. Their model differs from a typical facility protection model in that protection is not assumed to always successfully avoid facility failure, but rather to speed up recovery time post disruptions. One limitation of the worst-case approaches is that it can be highly over conservative in practical cases as the probability at which uncertain parameters reach their worst values may be very low (Snyder, 2006).

There are also studies that consider the risk preference of decision makers using scenario-based models. Peng et al. (2011) present a scenario-based modeling approach in which each scenario includes a set of facilities that can fail simultaneously. Their model aims at minimizing the total cost under normal circumstances while reducing the disruption risk using the *p*-robustness criterion (bounding the cost in disruption scenarios and allowing capturing risk aversion). A genetic algorithm approach is used to solve the model. Similarly, Baghalian et al. (2013) develop a scenario-based model for designing a supply chain whose objective is to maximize profit under the risks of disruption. To address the risk-aversion attitude of a decision maker, the variance of total profit is incorporated into the model. The model is formulated using mixed integer nonlinear programming and approximated using multiple linear regressions. The limitation of a scenario-based approach is that solving such models becomes more difficult as the number of scenarios increases (Peng et al., 2011).

Table 1 summarizes the characteristics of the published supply chain design models that incorporate disruption risks (more comprehensive reviews can be found in Snyder et al. (2006); Snyder et al. (2007) and (2016)). Our study contributes to this literature in the following ways. First, unlike the published models, our model is able to tackle multiple types of risks,

Table 1
Characteristics of published supply chain network design models that consider facility disruption risks.

References	Uncertain parameters			Modeling approach					Disaster severity		Fortification options		Key constraints	
	Demand	Probability of Disruption occurrence	Capacity	Expected value	Worst-case	*p*-robustness	Scenario-based robust optimization	Interval robust optimization	Complete shutdown	Facility partially affected	Full	Partial	Capacity	Budget
Aryanezhad et al. (2010)	*			*					*					
Azad et al. (2013)			*	*					*	*	*	*	*	
Baghalian et al. (2013)	*						*		*				*	*
Berman et al. (2007)				*					*					
Chen et al. (2011)	*			*										
Cui et al. (2010)				*					*					
Jabbarzadeh et al. (2012)	*			*					*	*				
O'Hanley et al. (2013)				*							*			
Li and Savachkin (2013)				*					*		*	*		
Liberatore et al. (2012)					*				*		*		*	
Li et al. (2013)				*					*		*	*		
Lim et al. (2010)				*					*		*			
Losada et al. (2012)					*				*		*			
Li and Ouyang (2010)				*					*					
Hernandez et al. (2014)					*				*					
Medal et al. (2014)					*				*		*			
Peng et al. (2011)			*			*			*	*			*	
Shen et al. (2011)				*					*					
Qi et al. (2010)				*					*					
Shishebori et al. (2014)				*					*					*
Snyder and Daskin (2005)				*					*					
This study	*	*	*					*	*	*	*	*	*	*

including strategic disruption risks and operational supply/demand uncertainties. This allows the effective design of supply chain networks where historical risk data is limited or nonexistent. Second, we present a hybrid robust-stochastic method (i.e., applying a robust optimization approach to a stochastic model) that overcomes the limitations of the scenario-based methods (the computational overhead for managing a large number of scenarios) and the worst-case approaches (over-conservative attitude in practical cases). The approach has the flexibility of adjusting the conservativeness level of solutions while preserving the computational complexity of the nominal problem. In addition, the hybrid nature of the presented formulation facilitates the modeling of a complex situation where even the probability of random disruptions is uncertain. Third, our modeling effort takes into consideration a realistic range of assumptions (e.g., partial or complete shutdown of facilities when disruptions occur), variables (e.g., both partial and full facility fortification options) and constraints (e.g., budget and capacity constraints); representing a more realistic situation than those studied in the past (see the review of Snyder et al. (2016) for a more comprehensive review of the existing literature).

3. Model formulation

We first present a background of robust optimization to better inform the mathematical formulation of the resilient supply chain network design. A stochastic model is then developed for supply chain network design considering the risk of disruptions. This model is then extended to incorporate uncertainties in demand, probability of disruption occurrence and capacity of facilities into the model; forming a hybrid robust-stochastic optimization formulation. The latter model aims to design a resilient supply chain network, a supply chain that is resilient to disruptions and supply/demand interruptions.

3.1. Background of robust optimization

Although stochastic programming methods are powerful in modeling uncertain factors (Birge and Louveaux, 2011), they usually require the availability of probability distributions of random variables (Klibi et al., 2010). Robust optimization methods have been used to tackle this drawback when there is the lack of historical data to estimate the actual distribution of uncertain parameters. They are also capable of incorporating decision-makers' risk attitude (Bental et al., 2009). Here, we explain the framework of the robust formulation introduced by Bertsimas and Sim (2003, 2004) which has been extensively adopted in the past to address supply chain uncertainty issues (Gabrel et al., 2014).

Let us consider a linear mathematical programming model as:

$$Min \quad c'_j x_j \tag{1}$$

Subject to:

$$\sum_j a_{ij} x_j \le b_i \qquad \forall\, i = 1, 2, 3, \ldots, m. \tag{2}$$

$$x_j \in \{0, 1\} \tag{3}$$

Here, a_{ij} denotes uncertain parameters andJ_i is the set of uncertain parameters in ith constraint. Bertsimas and Sim (2003, 2004) assume that each uncertain parameter a_{ij} is a random variable which takes values in interval $[\bar{a}_{ij} - \hat{a}_{ij}, \bar{a}_{ij} + \hat{a}_{ij}]$. Where $\bar{a}_{ij}$ represents the nominal value of the uncertain parameter and $\hat{a}_{ij}$ is the deviation of the nominal value. Using duality theory, Bertsimas and Sim (2003, 2004) prove that the robust counterpart of the uncertain linear programming model (1) to (3) can be written as:

$$Min\, c'_j x_j \tag{4}$$

Subject to:

$$\sum_{j \in J} \bar{a}_{ij} x_j + Z_i \Gamma_i + \sum_{j \in J} p_{ij} \le b_i \qquad \forall i \tag{5}$$

$$Z_i + p_{ij} \ge \hat{a}_{ij} x_j \qquad \forall j \in J \tag{6}$$

$$Z_i \ge 0 \qquad \forall i \tag{7}$$

$$p_{ij} \ge 0 \qquad \forall i, j \tag{8}$$

$$x_j \in \{0, 1\} \tag{9}$$

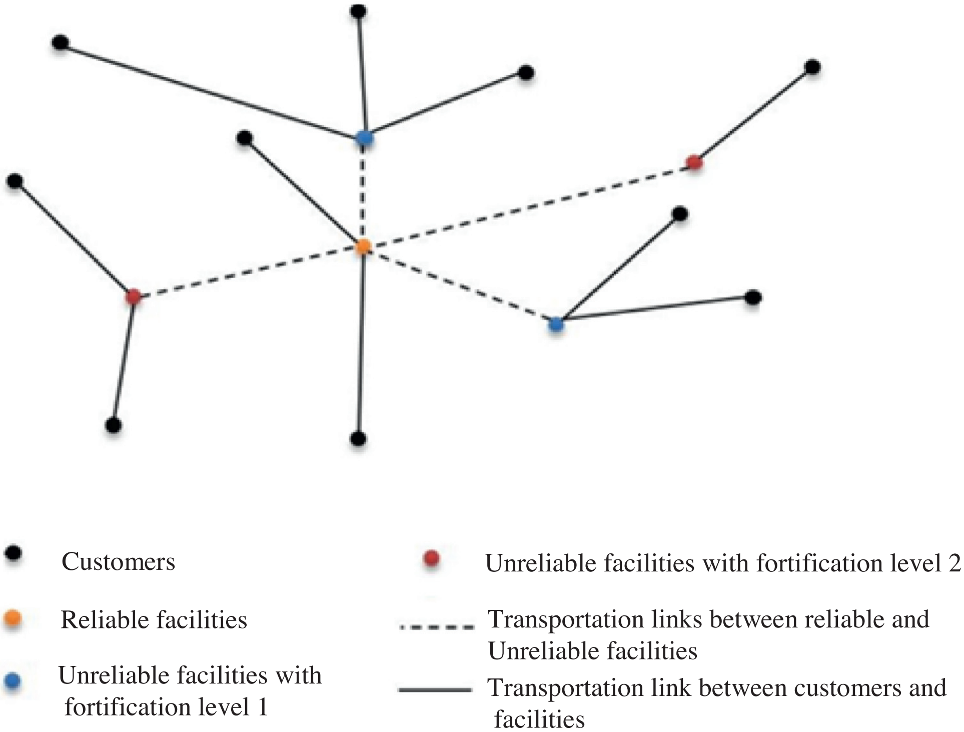

Fig. 1. The assignment of customers to facilities and the shipment of products between facilities and between facilities and customers.

Z_i and p_{ij} are auxiliary variables and Γ_i is a parameter called *"uncertainty budget"*. The parameter $\Gamma_i$adjusts the uncertainty level in each row varying in interval of $[0,|J_i|]$. In other words, the robust formulation aims to protect against all cases that up to Γ_i of uncertain parameters $a_{ij}$are allowed to change. When Γ_i is set equal to zero, the constraints are equivalent to that of the nominal problem. Likewise, when Γ_i is set to$|J_i|$, the robust model acts with the highest level of conservatism. The role of Γ_i is thus to adjust the conservatism level of the robust formulation. Further details about robust formulation can be found in Bertsimas and Sim (2003, 2004).

3.2. Formulation of the base stochastic model

We now formulate a stochastic network design model for a supply chain under random disruptions, assuming no demand and supply uncertainties. We consider a generic supply chain network in which facilities fulfil market demands at customer locations. A disruption at any facility can cause either a complete shutdown or a reduced supply capacity. Disruption probabilities in different facilities are assumed to be independent and location specific. The facilities can be either partially or fully fortified requiring capital investments corresponding to the degree of fortification. An example of such investments is the acquisition, installation and implementation of infection control measures to contain and prevent disease from disabling a workface. Another example is the acquisition and installation of advanced fire protection systems to mitigate the risk of factory fires. Therefore, we assume that the probability and magnitude of a disruption in a facility can be expressed as a function of fortification degree in that facility (as is the case in many disruptions). Compliance with a full fortification degree will make a facility *reliable* (resilient to major disruptions). Partially fortified facilities still remain *unreliable* and may be affected by disruptions with a given probability. When affected by a disruption, an unreliable facility can be supplied by other reliable facilities to compensate for the reduced supply capacity so that the assigned demands can still be satisfied. For a hypothetical example with two fortification levels, Fig. 1 illustrates the assignment of customers to facilities and the shipment of products between these nodes.

The objective is to minimize the total cost of the supply chain in a way that customer demands are satisfied even in disruptions. In the presence of certain budgetary constraints, the proposed model aims to determine (1) the number of facilities to open, (2) the location of facilities, (3) the allocation of facilities to customers, (4) the required fortification degree of each facility, (5) the quantity of products shipped between reliable and unreliable facilities when a disruption occurs. Modeling indices, parameters and decision variables are defined below.

Sets:

K: Set of customers
J: Set of potential locations for unreliable facilities
M: Set of potential locations for reliable facilities
I: Set of potential locations for facilities ($I = J \cup M$)
N: Set of fortification levels for unreliable facilities

Parameters:

D_k: Demand of customer k ($\forall k \in K$)
B: Budget available for establishing facilities
f_{jn}^U: Fixed cost of locating an unreliable facility at location j with fortification level n
f_m^R: Fixed cost of locating a reliable facility at location m ($\forall m \in M$)
o_{jk}: Unit transportation cost from unreliable facility at location j to customer k ($\forall j \in J$, $\forall k \in K$)
l_{mk}: Unit transportation cost from reliable facility at location m to customer k ($\forall m \in M$, $\forall k \in K$)
C_{mj}: Unit transportation cost from reliable facility at location m to unreliable facility at location j ($\forall m \in M$, $\forall j \in J$)
CU_j: Capacity of unreliable facility at location j under normal circumstances($\forall j \in J$)
CR_m: Capacity of reliable facility at location m ($\forall m \in M$)
q_{jn}: Disruption probability in unreliable facility at location j with fortification level n($\forall j \in J$, $\forall n \in N$)
a_{jn}: Percentage of total capacity lose when a disruption occurs in unreliable facility at location j with fortification level n

Decision variables:
T_{mj}: Quantity of products shipped from reliable facility at location m to unreliable facility at location j ($\forall m \in M$, $\forall j \in J$)

$$Y_{jn} = \begin{cases} 1 & \text{If unreliable facility } j \text{ is opened with fortification level } n \qquad (\forall j \in J, \forall n \in N) \\ 0 & \text{Otherwise} \end{cases}$$

$$X_m = \begin{cases} 1 & \text{If reliable facility } m \text{ is opened} \qquad (\forall m \in M) \\ 0 & \text{Otherwise} \end{cases}$$

$$U_{jk} = \begin{cases} 1 & \text{If customer } k \text{ is assigned to unreliable facility } j \qquad (\forall j \in J, \forall k \in K) \\ 0 & Otherwise \end{cases}$$

$$R_{mk} = \begin{cases} 1 & \text{If customer } k \text{ is assigned to reliable facility } m \qquad (\forall m \in M, \forall k \in K) \\ 0 & \text{Otherwise} \end{cases}$$

The stochastic supply chain network design model can now be developed by incorporating the impact of fortification levels of facilities on the probability of disruptions as well as the associated capacity and budget constraints into the model of Azad et al. (2013). The model is formulated as follows (note: uncertainties in demand, probability of disruption occurrence and capacity of facilities will be incorporated into the model in a later stage):

$$Min : \sum_{j \in J} \sum_{n \in N} f_{jn}^U Y_{jn} + \sum_{m \in M} f_m^R X_m + \sum_{j \in J} \sum_{k \in K} o_{jk} D_k U_{jk} + \sum_{m \in M} \sum_{k \in K} l_{mk} D_k R_{mk} + \sum_{j \in J} \sum_{n \in N} q_{jn} Y_{jn} \left(\sum_{m \in M} T_{mj} C_{mj} \right) \tag{10}$$

Subject to:

$$\sum_{j \in J} \sum_{n \in N} f_{jn}^U Y_{jn} + \sum_{m \in M} f_m^R X_m \le B \tag{11}$$

$$\sum_{m \in M} X_m \ge 1 \tag{12}$$

$$X_i + \sum_{n \in N} Y_{in} \le 1 \qquad \forall i \in I \tag{13}$$

$$R_{mk} \le X_m \qquad \forall m \in M, \ k \in K \tag{14}$$

$$\sum_{k \in K} D_k U_{jk} \le \sum_{n \in N} CU_j Y_{jn} \qquad \forall j \in J \tag{15}$$

$$\sum_{m\in M} T_{mj} + \left(1 - \sum_{n\in N} a_{jn} Y_{jn}\right) CU_j \geq \sum_{k\in K} D_k U_{jk} \qquad \forall j \in J \tag{16}$$

$$\sum_{j\in J} T_{mj} + \sum_{k\in K} D_k R_{mk} \leq CR_m X_m \qquad \forall m \in M \tag{17}$$

$$\sum_{j\in J} U_{jk} + \sum_{m\in M} R_{mk} = 1 \qquad \forall k \in K \tag{18}$$

$$X_m \in \{0, 1\} \qquad \forall m \in M \tag{19}$$

$$Y_{jn} \in \{0, 1\} \qquad \forall j \in J, \forall n \in N \tag{20}$$

$$R_{mk} \in \{0, 1\} \qquad \forall m \in M, \ \forall k \in K \tag{21}$$

$$U_{jk} \in \{0, 1\} \qquad \forall j \in J, \ \forall k \in K \tag{22}$$

$$T_{mj} \geq 0 \qquad \forall m \in M, \ \forall j \in J \tag{23}$$

The objective function (10) minimizes the expected total cost including the costs of locating reliable and unreliable facilities with different fortification levels, transportation costs for shipment of products from facilities to customers, and expected transportation costs for shipment of products from reliable facilities to unreliable facilities when disruptions occur. Constraint (11) expresses the total budget limitation. Constraint (12) enforces that at least one reliable facility must be opened to guarantee demand satisfaction when all unreliable facilities are disrupted. Constraint (13) ensures that only one facility can be opened at each location. For this constraint, we set $X_i = 0$ for $i \notin M$ and $Y_{in} = 0$ for $i \notin J$. Constraint (14) ensures that customers can only be assigned to open facilities. Constraint (15) expresses the capacity restriction of unreliable facilities. Constraint (16) guaranties that demand assigned to each unreliable facility is satisfied. Constraint (17) expresses the capacity limit of reliable facilities. Constraint (18) enforces that each customer is assigned to a facility. Constraints (19–23) define the domains of the decisions variables.

The model formulation (10–23) is nonlinear by the term $\sum_{j\in J} \sum_{n\in N} q_{jn} Y_{jn} \left(\sum_{m\in M} T_{mj} C_{mj} \right)$ in objective function (10). This formulation can be linearized using a new auxiliary variable named H_{jnm} and a new constraint (25) as follows.

$$Min: \sum_{j\in J} \sum_{n\in N} f^U_{jn} Y_{jn} + \sum_{m\in M} f^R_m X_m + \sum_{j\in J} \sum_{k\in K} o_{jk} D_k U_{jk} + \sum_{m\in M} \sum_{k\in K} l_{mk} D_k R_{mk} + \sum_{j\in J} \sum_{m\in M} \sum_{n\in N} q_{jn} C_{mj} H_{jnm} \tag{24}$$

Subject to:
Constraints (11–23)

$$H_{jnm} \geq T_{mj} + M(Y_{jn} - 1) \qquad \forall\, j \in J, m \in M, n \in N \tag{25}$$

$$H_{jnm} \geq 0 \qquad \forall j \in J, \forall k \in K \tag{26}$$

Where M is a big number and the auxiliary H_{jnm} is defined as follows.

$$H_{jnm} = Y_{jn} T_{mj} \qquad \forall m \in M, \ j \in J, n \in N \tag{27}$$

Constraint (25) ensures that products cannot be transported from a reliable facility to an unreliable facility that is not yet established. Bringing the objective function (24) into the constraints and defining a new variableλ, the above model can be rewritten as:

$$Min: \quad \lambda \tag{28}$$

Subject to:
Constraints (11–23) and (25) and (26)

$$\begin{aligned} &\sum_{j\in J} \sum_{n\in N} f^U_{jn} Y_{jn} + \sum_{m\in M} f^R_m X_m + \sum_{j\in J} \sum_{k\in K} o_{jk} D_k U_{jk} + \sum_{m\in M} \sum_{k\in K} l_{mk} D_k R_{mk} \\ &\quad + \sum_{j\in J} \sum_{m\in M} \sum_{n\in N} q_{jn} C_{mj} H_{jnm} \leq \lambda \end{aligned} \tag{29}$$

$$\lambda \geq 0 \tag{30}$$

3.3. Formulation of the hybrid robust-stochastic model

We now extend the stochastic model presented in Section 3.2 to include uncertainties in demand, supply capacity and probability of disruption occurrence, forming a robust-stochastic optimization model. We first look at demand uncertainty in Section 3.3.1 and will then incorporate uncertainty in the likelihood of disruption occurrence and uncertainty in capacity of facilities in Section 3.3.2.

3.3.1. Formulating demand uncertainty

We utilize the robust optimization approach discussed in Section 2 to formulate demand uncertainty. The uncertain parameter D_k takes the values within the range of $[\overline{D}_k - \hat{D}_k, \overline{D}_k + \hat{D}_k]$ corresponding to all customers. Also, budget uncertainty Γ^D (conservatism degree) is considered for customer demands taking values between zero and the number of customers. As discussed in Section 2, the robust model can be written as follows.

$$Min: \quad \lambda \tag{31}$$

Subject to:
Constraints (11–14), (18–23), (25) and (26)

$$\begin{aligned}&\sum_{j\in J}\sum_{n\in N} f^U_{jn} Y_{jn} + \sum_{m\in M} f^R_m X_m + \sum_{j\in J}\sum_{k\in K} o_{jk}\bar{D}_k U_{jk} + \sum_{m\in M}\sum_{k\in K} l_{mk}\bar{D}_k R_{mk} + \sum_{j\in J}\sum_{m\in M}\sum_{n\in N} q_{jn} C_{mj} H_{jnm}\\ &\sum_{k\in K} p^1_k + \sum_{k\in K} p^2_k + Z^0\Gamma^D + Z^1\Gamma^D \le \lambda\end{aligned} \tag{32}$$

$$Z^0 + p^1_k \ge o_{jk}\hat{D}_k U_{jk} \qquad \forall j \in J,\ k \in K \tag{33}$$

$$Z^1 + p^2_k \ge l_{mk}\hat{D}_k R_{mk} \qquad \forall j \in J,\ k \in K \tag{34}$$

$$\sum_{k\in K} \bar{D}_k U_{jk} + \sum_{k\in K} p^3_k + Z^2_j\Gamma^D \le \sum_{n\in N} CU_j Y_{jn} \qquad \forall j \in J \tag{35}$$

$$Z^2_j + p^3_k \ge \hat{D}_k U_{jk} \qquad \forall j \in J,\ k \in K \tag{36}$$

$$\sum_{m\in M} T_{mj} + \left(1 - \sum_{n\in N} a_{jn} Y_{jn}\right) CU_j \ge \sum_{k\in K} \bar{D}_k U_{jk} + \sum_{k\in K} p^4_k + Z^3_j\Gamma^D \qquad \forall j \in J \tag{37}$$

$$p^4_k + Z^3_j \ge \hat{D}_k U_{jk} \qquad \forall j \in J,\ k \in K \tag{38}$$

$$\sum_{j\in J} T_{mj} + \sum_{k\in K} \bar{D}_k R_{mk} + \sum_{k\in K} p^5_k + Z^4_j\Gamma^D \le CR_m X_m \qquad \forall m \in M \tag{39}$$

$$p^5_k + Z^4_j \ge \hat{D}_k U_{jk} \qquad \forall j \in J,\ k \in K \tag{40}$$

$$p^1_k, p^2_k, p^3_k, p^4_k, p^5_k, Z^0_j, Z^1_j, Z^2_j, Z^3_j, Z^4_j \ge 0 \qquad \forall j \in J,\ k \in K. \tag{41}$$

Where variables $p^1_k, p^2_k, p^3_k, p^4_k, p^5_k, Z^0_j, Z^1_j, Z^2_j, Z^3_j, Z^4_j$ are auxiliary variables.

3.3.2. Formulating supply uncertainty

We now formulate uncertainty in supply capacity of facilities (a_{jn}) and uncertainty in the probability of disruption occurrence (q_{jn}). Consider uncertain parameters q_{jn} and a_{jn} that can take values within intervals $[\bar{q}_{jn} - \hat{q}_{jn}, \bar{q}_{jn} + \hat{q}_{jn}]$ and $[\bar{a}_{jn} - \hat{a}_{jn}, \bar{a}_{jn} + \hat{a}_{jn}]$, respectively. Here, Γ^q denotes the uncertainty budget for the probability of disruption occurrence ranging between zero and the number of facilities multiplied by number of fortification levels. Also, the uncertainty budget for capacity of facilities is denoted by Γ^a which takes values between zero and the number of fortification levels. Therefore, the robust optimization model including the uncertainties in demand, probability of a disruption occurrence and capacity of facilities can be formulated as follows.

$$Min: \quad \lambda \tag{42}$$

Subject to:

Constraints (11–14), (18–23), (25), (26), (33–36), and (38–41)

$$\sum_{j\in J}\sum_{n\in N} f^U_{jn} Y_{jn} + \sum_{m\in M} f^R_m X_m + \sum_{j\in J}\sum_{k\in K} o_{jk}\bar{D}_k U_{jk} + \sum_{m\in M}\sum_{k\in K} l_{mk}\bar{D}_k R_{mk} + \sum_{j\in J}\sum_{m\in M}\sum_{n\in N}\bar{q}_{jn} C_{mj} H_{jnm} + \sum_{j\in J}\sum_{n\in N} p^6_{jn} + \sum_{k\in K} p^1_k + \sum_{k\in K} p^2_k + Z^0\Gamma^D + Z^1\Gamma^D + Z^5\Gamma^q \le \lambda \tag{43}$$

$$Z^5 + p^6_{jn} \ge \hat{q}_{jn} H_{jnm} \qquad \forall j \in J,\ n \in N, m \in M \tag{44}$$

$$\sum_{m\in M} T_{mj} + \left(1 - \sum_{n\in N}\bar{a}_{jn} Y_{jn}\right) CU_j \ge \sum_{k\in K}\bar{D}_k U_{jk} + \sum_{n\in N} p^7_{jn} + \sum_{k\in K} p^4_k + Z^3_j\Gamma^D + Z^6_j\Gamma^a \qquad \forall j \in J \tag{45}$$

$$Z^6_j + p^7_{jn} \ge \hat{a}_{jn} CU_j Y_{jn} \qquad \forall j \in J, \forall n \in N \tag{46}$$

$$Z^5,\ Z^6_j,\ p^6_{jn},\ p^7_{jn} \ge 0 \qquad \forall j \in J, \forall n \in N \tag{47}$$

Where variables Z^5, Z^6_j, p^6_{jn}, p^7_{jn} are axillary variables.
Considering constraint (43), the above model can be rewritten as follows:

$$Min: \sum_{j\in J}\sum_{n\in N} f^U_{jn} Y_{jn} + \sum_{m\in M} f^R_m X_m + \sum_{j\in J}\sum_{k\in K} o_{jk}\bar{D}_k U_{jk} + \sum_{m\in M}\sum_{k\in K} l_{mk}\bar{D}_k R_{mk} + \sum_{j\in J}\sum_{m\in M}\sum_{n\in N}\bar{q}_{jn} C_{mj} H_{jnm} + \sum_{j\in J}\sum_{n\in N} p^6_{jn} + \sum_{k\in K} p^1_k + \sum_{k\in K} p^2_k + Z^0\Gamma^D + Z^1\Gamma^D + Z^5\Gamma^q \tag{48}$$

Subject to:
Constraints (11–14), (18–23), (25), (26), (33–36), (38–41), and (44–47).

4. A Lagrangian relaxation solution method

The model presented in Section 3 could be solved using commercial optimization packages like GAMS and CPLEX. However, the model runtime may become excessive long as the problem size increases. A Lagrangian relaxation method is able to find quality solutions to large problem instances within a reasonable length of time. Lagrangian relaxation is a powerful solution approach with demonstrated successful application in solving a range of supply chain design problems (see for instance, Chen et al. (2011); Daskin et al. (2002); Diabat et al. (2015); Li et al. (2013); Ozsen et al. (2008); Qi et al. (2010); Snyder and Daskin (2005); Snyder et al. (2007)). In essence, the method provides upper and lower bounds of an optimal objective value allowing a decision maker to realize how far the best found feasible solution is from the optimality (Fisher, 2004). The process is completed in three steps: (1) finding a lower bound for optimal solutions, (2) finding an upper bound for optimal solutions, and (3) updating the upper and lower bounds. These steps are repeated until the lower and upper bounds reach a certain closeness. The following subsections discuss these steps for solving the model presented in Section 3.

4.1. Obtaining a lower bound

The first step of a Lagrangian relaxation approach involves relaxing one or more constraints to form Lagrangian Dual problems. Solving the resulting Lagrangian Dual problem can provide a lower bound for the original optimization problem (Fisher, 2004). We relax assignment constraints (18) with Lagrange multipliers π_k to obtain the following Lagrangian Dual problem:

$$\underset{\pi}{Max}\ Min: \sum_{j\in J}\sum_{n\in N} f^U_{jn} Y_{jn} + \sum_{m\in M} f^R_m X_m + \sum_{j\in J}\sum_{k\in K} o_{jk}\bar{D}_k U_{jk} + \sum_{m\in M}\sum_{k\in K} l_{mk}\bar{D}_k R_{mk} + \sum_{j\in J}\sum_{m\in M}\sum_{n\in N}\bar{q}_{jn} C_{mj} H_{jnm} + \sum_{j\in J}\sum_{n\in N} p^6_{jn} + \sum_{k\in K} p^1_k + \sum_{k\in K} p^2_k + Z^0\Gamma^D + Z^1\Gamma^D + Z^5\Gamma^q + \sum_{k\in K}\pi_k\left(1 - \sum_{j\in J} U_{jk} + \sum_{m\in M} R_{mk}\right) \tag{49}$$

Subject to:
Constraints (11–14), (19–23), (25), (26), (33–36), (38–41), and (44–47)

For fixed values of the Lagrange multipliers,π_k, we aim to minimize Eq. (49) over decision variables. Optimal objective value of the Lagrangian Dual problem (49) provides a lower bound for the model (48).

4.2. Obtaining an upper bound

At each iteration of the Lagrangian process, an upper bound is obtained as follows. If the solution from solving Lagrangian Dual problem (49) is feasible, then it provides an upper bound as well. In this case, the algorithm terminates, since the upper and lower bounds are equal and an optimal solution is reached. If the solution from solving Lagrangian Dual problem (49) is infeasible, the infeasibility is fixed and a feasible solution is sought. The possible infeasibility of a solution is clearly due to relaxing constraints (18). From a managerial perspective, there may exist customers unassigned to any of the facilities. Therefore, to fix the infeasibility each customer needs to be allocated to one (or more) opened reliable facility (i.e., the facilities for which $X_m = 1$). A customer is assigned to the nearest possible reliable facility. While doing the assignments, one must also ensure that the capacity of a reliable facility is sufficient for a new assignment. If there is not enough capacity to serve a customer, a new reliable facility is opened in the decreasing order of $\frac{CR_m}{f_m^R}$. That is, a facility with higher ratio of 'capacity to location cost' is prioritized. The opening of new facilities continues until the sum of the capacities of the reliable facilities is equal or greater than the sum of all demands. The resulting feasible solution provides an upper bound for the model (48).

4.3. Updating lower and upper bounds

At each iteration of the Lagrangian procedure, the Lagrange multipliers π_k are updated and new lower and upper bounds are obtained. We use the subgradient optimization approach introduced by Fisher (2004) and Daskin (1995) to update the values of the Lagrange multipliers at each iteration v:

$$\pi_k^{v+1} \leftarrow \pi_k^v + \delta^v \left(1 - \sum_{j \in J} U_{jk} + \sum_{m \in M} R_{mk}\right) \tag{50}$$

In Eq. (50), δ^v is the step size of the algorithm and is computed from:

$$\delta^v = \frac{\beta^v (UB - LB^v)}{\sum_{k \in K} \left(1 - \sum_{j \in J} U_{jk} + \sum_{m \in M} R_{mk}\right)^2} \tag{51}$$

Where UB is the best found upper bound and LB^v is the lower bound obtained at iteration v. We initially set $\beta = 2$ and if no improvement in LB is achieved for four consecutive iterations, then β is halved. This process continues until a feasible solution within the desired optimality tolerance is obtained or the minimum value of the step-size is reached.

5. Computational experiments

In this section, first, we complete sensitivity analysis experiments using some well-known datasets in the literature of supply chain network. Then, the application of the developed model is investigated utilizing real data collected from a base oil supply chain in Iran.

5.1. Experimental design

The application of the proposed model is investigated for 21-node, 32-node and 49-node datasets presented in Daskin (1995). For the 21-node and 32-node datasets, the nodes represent the state capitals of the lower 21 and 32 United States. The 49-node dataset consists of the 48 state capitals of the United States plus Washington, DC. The same datasets have been used in some other studies of Snyder and Daskin (2005); Snyder et al. (2007); Aryanezhad et al. (2010); Qi et al. (2010) and Jabbarzadeh et al. (2012). The computational experiments for these data sets are completed using a branch and bound algorithm coded in GAMS 24.1 on a laptop with Intel Core i2 CPU, 2.53 GHz and 3GB of RAM. We also need larger datasets to evaluate the performance of the proposed Lagrangian relaxation method. For this purpose, we develop and adopt three larger datasets: 88-node, 100-node and 150-node datasets. The 88-node dataset includes the 49-node dataset, plus the 50 largest cities in the United States, minus duplicates. The 150-node dataset includes the 150 largest cities in the United States based on 1990 census data (Daskin, 1995). The 100-node dataset is comprised of random data, adopted from Snyder and Daskin (2005).

For the 100-node dataset, the values of all parameters are obtained similar to Snyder and Daskin (2005). For the other datasets, the nominal demand is obtained by dividing the population data given in Daskin (1995) by 1000. Three levels of fortification—full, moderate and low—are considered for facilities, denoted as FF, FM and FL, respectively. The fixed cost of establishing a reliable (i.e., fully fortified facilities) facility is obtained by dividing the fixed facility cost by 10. The fixed costs of locating unreliable facilities with moderate and low fortification levels are set equal to 32% and 20% of establishing a reliable facility. Unit transportation cost from facilities to customers is assumed to be 50% of the great-circle distance between facilities and customers. Unit shipment cost from reliable facilities to unreliable facilities is equal to 10% of the great-circle distance between them. The available budget for establishing all facilities is $200,000. The values of the other parameters

Table 2
Input parameters for all datasets.

	Capacity under normal circumstance			Probability of disruption occurrence		Percentage disrupted capacity	
	FF	FM	FL	FM	FL	FM	FL
21-Node	1600	1400	1400	0.85	0.95	0.55	0.75
32-Node	2000	1500	1500	0.85	0.90	0.3	0.60
49-Node	2600	1900	1900	0.9	0.95	0.4	0.75
88-Node	2000	1500	1500	0.9	0.95	0.25	0.5
100-Node	2600	1900	1900	0.85	0.95	0.3	0.7
150-Node	2600	1900	1900	0.9	0.95	0.3	0.6

Table 3
Initial model outputs for the 21-node dataset.

Conservatism Degrees			Location of Facilities			Number of Assigned Customers		
Γ^D	Γ^q	Γ^a	FF	FM	FL	FF	FM	FL
0	0	0	7	5,6	3,4,15	2	11	8
5	10	0	7	5, 6	3, 4, 9, 10, 15	2	9	10
10	20	1	5	3,6	4, 7, 9, 10, 15	5	8	8
15	30	1	5	6,17	4, 7, 9, 10,15	5	7	9
18	36	2	5	6,17	4, 7, 9, 10, 12, 15	5	7	9
21	42	2	5	4,6,17	4,7, 9, 12, 15	5	11	5

Table 4
Initial model outputs for the 32-node dataset.

Conservatism Degrees			Location of Facilities			Number of Assigned Customers		
Γ^D	Γ^q	Γ^a	FF	FM	FL	FF	FM	FL
0	0	0	7	5	3, 4, 9,18 ,28 ,30 ,32	3	4	25
5	10	0	7	5	3, 4, 9,18 ,28 ,30 ,32	3	5	24
8	16	0	5	–	3, 4,7, 9,18 ,28 ,30 ,32	5	–	27
12	24	1	5	–	3, 4,7, 9,18 ,28 ,30 ,32	5	–	27
17	34	1	5	9	3, 4, 7, 18, 28, 30	5	5	22
21	42	1	5	17	4, 7,9, 18, 28, 30, 32	5	3	21
25	50	2	5	17	4,7,9,10,18,28,30,32	5	3	21
29	58	2	5	6	4,7,9,12,17,18,28,30	2	4	26
32	64	2	5	9,17	4,7,12,15,18,28	2	8	22

Table 5
Initial model outputs for the 49-node dataset.

Conservatism Degrees			Location of Facilities			Number of Assigned Customers		
Γ^D	Γ^q	Γ^a	FF	FM	FL	FF	FM	FL
0	0	0	7	5,46	3,4,15,18 ,29,30,33	7	13	22
5	10	0	7	5,46	3,4,15,18,29,30,33	7	13	22
10	20	0	7	5,46	3,4,15,18,29,30,33	8	13	21
20	40	1	7	5,46	3,4,15,18,29,30,33	7	13	22
25	50	1	7	5,46	3,4,12,15,18,29,30,33	5	11	33
30	60	1	7	5,46	3,4,12,15,18,29,30,33	7	10	32
40	80	2	5	46	3,4,7,15,28,30,33,34	6	8	35
45	90	2	5	46	3,4,7,12,15,28,30,33,34	3	8	38
49	98	2	5	46	3,4,7,12,15,28,30,33,34	2	8	39

are given in Table 2. Computational experiments are conducted considering 5% variability in uncertain parameters from the nominal values.

5.2. Model implementation and initial observations

Initial numerical results are shown in Tables 3–5 providing the optimal location of facilities as well as the optimal assignment of customers to facilities corresponding to different conservatism degrees for the three datasets. From these initial findings, one can see that the optimal location of facilities, especially the reliable facilities, is almost analogous at different conservatism degrees. Reliable facilities tend to be opened at sites 5 and 7 regardless of the conservatism degree chosen. One possible reason for this can be the more convenient proximity of these facilities to customers and other facilities re-

Table 6
Supply chain cost and model runtime at various conservatism degrees for the 21-node dataset.

Γ^D	Γ^q	Γ^a	Total Cost ($)	Cost Difference (%)	Runtime (Seconds)
0	0	0	266,459	0.0	3.2
5	10	0	290,078	8.9	10
10	20	1	292,757	0.9	17
15	30	1	298,763	2.1	23
18	36	2	301,572	0.9	11
21	42	2	307,879	2.1	10

Table 7
Supply chain cost and model runtime at various conservatism degrees for the 32-node dataset.

Γ^D	Γ^q	Γ^a	Total Cost ($)	Cost Difference (%)	Runtime (Seconds)
0	0	0	289,838	0.0	2
5	10	0	312,003	7.6	50
8	16	0	316,459	1.4	56
12	24	1	319,550	1.0	62
17	34	1	320,947	0.4	67
21	42	1	321,988	0.3	78
25	50	2	324,640	1.0	69
29	58	2	332,697	2.5	62
32	64	2	338,583	1.8	78

Table 8
Supply chain cost and model runtime at various conservatism degrees for the 49-node dataset.

Γ^D	Γ^q	Γ^a	Total Cost ($)	Cost Difference (%)	Runtime (Seconds)
0	0	0	314,381	0.0	7
5	10	0	319,798	2.0	33
10	20	0	327,184	2.0	38
20	40	1	330,139	0.9	42
25	50	1	331,896	0.5	67
30	60	1	335,101	1.0	72
40	80	2	336,124	1.0	78
45	90	2	342,110	1.8	265
49	98	2	348,829	2.0	394

sulting in a lower transportation cost between nodes. Likewise, in all instances, site 4 is a preferred location to open a facility with low fortification level. An important insight from these observations can be that small changes in supply chain topology (changes in location of a small fraction of facilities) can help protecting the network against some of the potential risks.

A careful comparison between the impacts that the number of customers and demand scale can have on facility location decisions can provide additional insights. Tables 3–5 indicates that the location of reliable facilities is less sensitive to changes in the number of customers served (comparing the location results for the three datasets). In other words, adjustment in location of unreliable facilities is used to deal with different demand sizes. The model is clearly taking advantage of the lower cost of opening unreliable facilities to cope with variations in the number of customers served. This is a good example of a situation where various facility fortification strategies can be used for effective demand fulfillment in different network sizes.

5.3. Analysis on the impact of a decision maker's conservatism degree

We now complete an experiment to investigate how the choice of conservatism degree can influence the overall supply chain cost and model runtime. The results are shown in Tables 6–8 for the concerned datasets. Not surprisingly, a greater conservatism degree results in a higher total supply chain cost to hedge the network against the potential risks and uncertainties. What is interesting is that in no occasion does the cost increase by more than 9%, indicating that considerable resilience improvements can be achieved with only insignificant increases in costs.

The total cost and cost difference values in Tables 6–8 show that the supply chain cost is not linearly increased as conservatism degree gets larger. For example, from Table 6, a 8.9% cost increase occurs to improve the supply resilience from the conservative level of $\Gamma^D = 0$ and $\Gamma^q = 0$ to $\Gamma^D = 5$ and $\Gamma^q = 10$; while only 0.9% cost difference is enough to move from $\Gamma^D = 5$ and $\Gamma^q = 10$ to $\Gamma^D = 10$ and $\Gamma^q = 20$. Another interesting observation is that in all datasets the greatest cost

Table 9
Supply chain cost at various conservatism degrees when facing demand and supply variations.

Demand Uncertainty			Supply Uncertainty			
Conservatism Degree	Total Cost		Conservatism Degree		Total Cost	
Γ^D	5% Demand Variability	10% Demand Variability	Γ^q	Γ^a	5% Supply Variability	10% Supply Variability
0	309,270	309,270	0	0	309,270	309,270
5	319,006	330,153	10	0	309,797	310,060
10	322,863	352,176	20	0	309,797	310,060
20	325,791	357,042	40	1	312,663	319,319
25	330,941	376,643	50	1	312,663	319,319
30	334,773	Infeasible	60	1	312,663	319,319
40	336,124	Infeasible	80	2	314,010	332,212
45	342,110	Infeasible	90	2	314,010	332,212
49	348,829	Infeasible	98	2	314,010	332,212

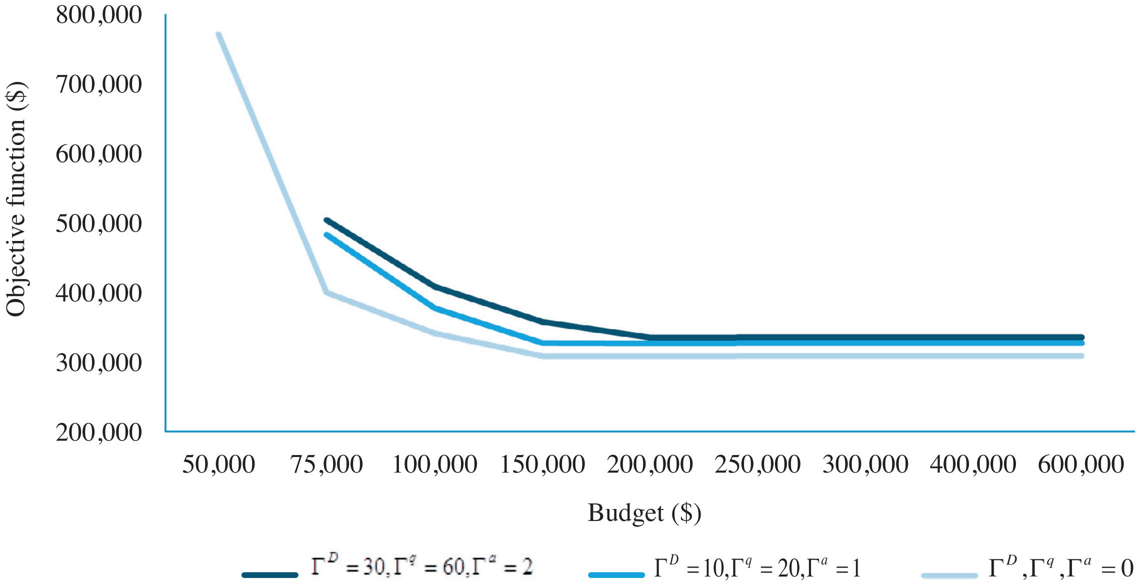

Fig. 2. The impact of initial budget on the total supply chain cost.

increase occurs in the second row, implying that the initial efforts to build resilience into the supply chain network are more costly. Note that the total cost at $\Gamma^D = 0$, $\Gamma^q = 0$ and $\Gamma^a = 0$ is obtained from the objective value of the stochastic model disregarding the supply chain resilience in the face of supply and demand variations as described in Section 3. The last columns of Tables 6–8 provide the model runtimes.

5.4. Analysis on the impacts of demand and supply uncertainties

For the 49-node dataset, Table 9 shows how demand and supply variations can influence the total supply chain cost at different conservatism degrees. What is obvious from this data is that demand variation can a have greater impact on the strategic supply chain cost when compared to supply uncertainty. In some scenarios when $\Gamma^D \geq 30$, demand variations can even result in infeasibility implying failure to satisfy customer demand and hence product shortage and lost sales. A practical implication from this finding would be for the risk managers to place the primary focus on developing more accurate demand forecasts, rather than a focus on capacity adjustments, to avoid stockout and potential reputational damage.

5.5. Analysis on the impact of budgetary constraints

For the largest dataset, Fig. 2 illustrates how the total cost is influenced by the budget availability at different conservatism degrees. Interesting insights can be obtained from this graph. First, regardless of the decision maker's conservatism degree, initial budget availability results in significant total cost reductions. This is evidenced by the steepness of all three curves at the left end. However, greater cost responses to the initial budget injections can be observed at the higher conservatism degrees (a steeper curve for a higher conservatism degree). Second, the minimum required budget for the design

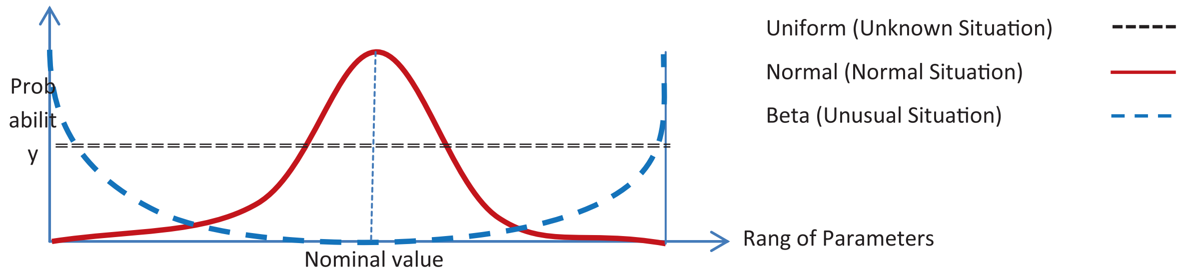

Fig. 3. Different distribution functions for uncertain parameters.

of the supply chain is set higher at a larger conservatism degree. The required supply chain design budget is $50,000 at the conservatism level $\Gamma^D = 0$, $\Gamma^q = 0$ and $\Gamma^a = 0$; while at-least $75,000 of initial budget is required to design a network at higher conservatism levels. Third, while these results confirm that budget availability can play a key role in building resilience into a supply chain network, excessive budget injections do not necessarily result in reduced total costs. That is to say that the total cost remains unchanged (i.e., no additional improvements) after certain budget injections. As could be expected, this budget unresponsiveness is reached at smaller dollar values for lower conservatism degrees.

5.6. Model performance evaluation

We use a Monte Carlo simulation method to examine the performance of the proposed model. Monte Carlo technique is an effective and popular computational algorithm that relies on tedious random sampling to obtain numerical results (Baraldi and Zio, 2008; Pouillot et al., 2004). Typically, a number of simulations are run over in order to obtain the distribution of an unknown probabilistic entity (Kalos and Whitlock, 2008). Our Monte Carlo simulation includes the following steps. First, the robust model is solved and optimal decisions for the problem are obtained for different conservatism levels. To evaluate the quality of the solutions found, we generate some random values for q_{jn}, a_{jn} and D_k within their valid domains using different distribution functions. For each of the three datasets (i.e., 21-node, 32-node and 49-node datasets), 3000 random numbers are generated: 1000 random number using a uniform distribution, 1000 numbers using normal distribution, and 1000 numbers using a beta distribution (see Fig. 3). That is, a total of 9000 random numbers are generated for the three datasets. To generate random number using a uniform distribution function, the parameters of the uniform function are set equal to the upper and lower bounds of the uncertain parameters (q_{jn}, a_{jn} and D_k). Random numbers using a normal distribution function are generated by setting the mean and variance of the normal functions equal to the nominal values of the uncertain parameters. To use a beta function for random number generation, the shape parameters of the beta function are set equal to 0.5. More information about these distribution functions can be found at Banks et al. (2010) and Ross (2006).

The rationale for considering different distribution functions is to provide an opportunity to study different situations of parameter uncertainty. To be more pellucid, the uniform, normal and beta distributions are used to model the situations in which the uncertain data is unknown, normal and unusual – represented by uniform, normal and beta distribution functions, respectively. An unknown situation occurs when we only know the range of the uncertain parameters. That is, the uncertain parameters may take any values within this range with equal probabilities of occurrence. Under normal situations, the values of the random parameters tend to be near the nominal values. Unusual situations occur when the uncertain parameters are likely to take the highest and lowest possible values. Fig. 3 illustrates the shapes of these distribution functions.

Using these generated values and the obtained optimal solutions, three following measures are calculated to evaluate the quality of solutions: mean total cost, standard deviation and percentage of infeasibility (i.e., the proportion of situations resulting in stockout or product shortage). Obviously, the lower the values for these measures are, the higher will be the quality of solutions. Tables 10–12 show the results obtained from the Monte Carlo simulation experiments for the three concerned datasets.

As could be expected, these results show that the unusual situations display the highest solution infeasibility in comparison with the normal and unknown situations. Whilst the maximum percentage infeasibilities in a normal situation are equal to 53%, 69% and 38% for the three datasets; in an unusual situation they rise substantially to 94%, 90% and 64%, in that order. This is due to the tendency of uncertain parameters to locate towards either ends of the uncertain intervals in the latter situation. The lowest infeasibility quantities are obtained under a normal situation because the random parameters tend to take values near the nominal values. Regardless of the uncertainty situation occurring (i.e., unknown, normal, and unusual), solution feasibility is highly reliant on how conservative a decision maker is.

A low conservatism degree leads to higher percentage of infeasibilities and thus increased product shortage, deteriorated service level and potential lost sales. At the worst case scenario, when choosing a zero value for all conservatism degrees (i.e., when $\Gamma^D = 0$, $\Gamma^q = 0$ and $\Gamma^a = 0$, representing the basic stochastic model presented in Section 3.2) in an unusual situation, the highest percentage infeasibilities of 94%, 90% and 64% occur for the 21-node, 32-nod and 49-node datasets, respectively. A practical insight from this observation is that failing to build resilience (in some form) into a supply chain

Table 10
Monte Carlo simulation results for 21-node dataset.

Uncertainty Situation	Γ^D	Γ^q	Γ^a	Mean Cost ($)	Mean Cost Difference (%)	Standard Deviation	Standard Deviation Difference (%)	Infeasibility (%)
Unknown	0	0	0	267,099	0.0%	5494	0.0%	64%
	5	10	0	274,765	2.9%	5015	−8.7%	4%
	10	20	1	283,420	3.1%	823	−83.6%	0%
	15	30	1	288,058	1.6%	857	4.2%	0%
	18	36	2	291,687	1.3%	857	0.0%	0%
	21	42	2	296,852	1.8%	869	1.3%	0%
Normal	0	0	0	265,369	0.0%	3131	0.0%	53%
	5	10	0	273,373	3.1%	1375	−56.1%	2%
	10	20	1	283,372	3.2%	270	−80.4%	0%
	15	30	1	288,124	1.7%	284	5.2%	0%
	18	36	2	291,613	1.2%	271	−4.6%	0%
	21	42	2	296,782	1.8%	316	16.6%	0%
Unusual	0	0	0	265,369	0.0%	4917	0.0%	94%
	5	10	0	273,373	3.0%	6653	35.3%	2%
	10	20	1	283,372	3.7%	1193	−82.1%	0%
	15	30	1	288,124	1.7%	1233	3.4%	0%
	18	36	2	291,613	1.2%	1212	−1.7%	0%
	21	42	2	296,782	1.8%	1238	2.1%	0%

Table 11
Monte Carlo simulation results for 32-node dataset.

Uncertainty Situation	Γ^D	Γ^q	Γ^a	Mean Cost ($)	Mean Cost Difference (%)	Standard Deviation	Standard Deviation Difference (%)	Infeasibility (%)
Unknown	0	0	0	290,273	0.0%	2682	0.0%	78%
	5	10	0	297,383	2.4%	4305	60.5%	2%
	8	16	0	303,545	2.1%	864	−79.9%	0%
	12	24	1	305,607	0.7%	1065	23.2%	0%
	17	34	1	304,398	0.6%	1165	9.4%	0%
	21	42	1	308,828	0.5%	899	−22.8%	0%
	25	50	2	312,241	1.0%	901	1.0%	0%
	29	58	2	320,102	2.5%	851	−5.6%	0%
	32	64	2	322,890	0.9%	1208	41.8%	0%
Normal	0	0	0	289,787	0%	848	0%	69%
	5	10	0	296,934	2.5%	1338	57.8%	1%
	8	16	0	303,409	2.2%	309	−76.9%	0%
	12	24	1	305,402	0.7%	348	12.6%	0%
	17	34	1	304,357	−0.3%	438	25.9%	0%
	21	42	1	308,675	1.4%	306	−30.1%	0%
	25	50	2	312,082	1.1%	291	−4.9%	0%
	29	58	2	322,911	3.5%	315	8.2%	0%
	32	64	2	324,806	0.6%	320	1.6%	0%
Unusual	0	0	0	289,156	0%	3737	0%	90%
	5	10	0	296,168	2.4%	5750	53.9%	4%
	8	16	0	303,549	2.5%	1149	−80.0%	0%
	12	24	1	305,497	0.6%	1271	10.6%	0%
	17	34	1	306,127	0.2%	2001	57.4%	0%
	21	42	1	308,732	0.9%	1080	−46.0%	0%
	25	50	2	312,139	1.1%	1092	1.1%	0%
	29	58	2	324,822	4.1%	1317	20.6%	0%
	32	64	2	326,705	0.6%	1930	46.5%	0%

network can significantly increase the chance of stockout and prospective reputational damage. Interestingly, at only small increases in the total supply chain cost (when moving from row 1 to row 2), the supply chain resilience is improved dramatically to only face 4%, 2% and 4% probability of stockout, respectively for the small, medium and large datasets. This cost impact can be as small as 0.5% for the largest dataset under unusual situation with a rewarding improved resilience of 60%. This finding has an important managerial implication for risk analysts and supply chain practitioners showing how a supply chain can be more protected against disruptions and supply/demand uncertainties at a reasonably minor cost increase. Whilst a similar finding and insight was reached by Snyder and Daskin (2005), our observation here reinforces that this finding holds also in situations when a supply chain faces uncertainties in supply and demand, and a disruption can cause either partial or complete facility shutdown.

Table 12
Monte Carlo simulation results for 49-node dataset.

Uncertainty Situation	Γ^D	Γ^q	Γ^a	Mean Cost ($)	Mean Cost Difference (%)	Standard Deviation	Standard Deviation Difference (%)	Infeasibility (%)
Unknown	0	0	0	309,655	0.0%	1716	0.0%	48%
	5	10	0	311,297	0.5%	2170	26.5%	4%
	10	20	0	311,698	0.1%	2389	10.1%	2%
	20	40	1	312,819	0.4%	2380	−0.4%	0%
	25	50	1	316,317	1.1%	1196	−49.7%	0%
	30	60	1	319,639	1.1%	930	−22.2%	0%
	40	80	2	321,291	1.0%	942	1.0%	0%
	45	90	2	327,476	1.9%	934	−0.8%	0%
	49	98	2	327,502	0.0%	950	1.7%	0%
Normal	0	0	0	309,200	0.0%	470	0%	38%
	5	10	0	310,899	0.5%	632	34.5%	2%
	10	20	0	311,451	0.2%	640	1.3%	0%
	20	40	1	312,371	0.3%	696	8.8%	0%
	25	50	1	317,935	1.8%	329	−52.7%	0%
	30	60	1	319,460	0.5%	262	−20.4%	0%
	40	80	2	320,796	0.4%	263	0.4%	0%
	45	90	2	327,231	2.0%	261	−0.8%	0%
	49	98	2	332,886	1.7%	116	−55.6%	0%
Unusual	0	0	0	309,001	0.0%	2092	0%	64%
	5	10	0	310,616	0.5%	2715	29.8%	4%
	10	20	0	311,104	0.2%	2708	−0.3%	0%
	20	40	1	312,024	0.3%	2955	9.1%	0%
	25	50	1	320,881	2.8%	1519	−48.6%	0%
	30	60	1	321,050	0.1%	1130	−25.6%	0%
	40	80	2	322,850	0.6%	1103	−2.4%	0%
	45	90	2	327,274	1.4%	1107	0.4%	0%
	49	98	2	332,927	1.7%	133	−88.0%	0%

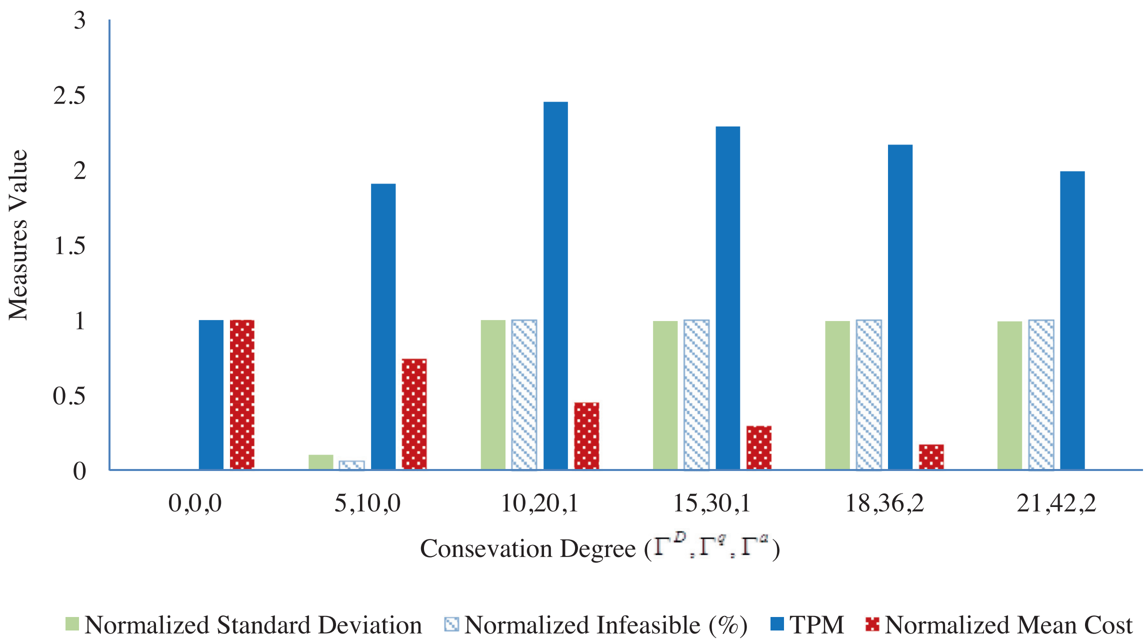

Fig. 4. Model performance at different conservatism degrees for the 21-node dataset under the unknown situation.

From Tables 10–12, one can realize that different values of conservatism degrees may cause one measure improve, while other measures may be negatively affected. For example, supply chain resilience may be improved at the cost of amplified standard deviation and increased supply chain cost. In such situations, choosing the most appropriate value for the conservatism degree is not as straightforward. To address this challenge, we introduce a unified performance measure called *Total*

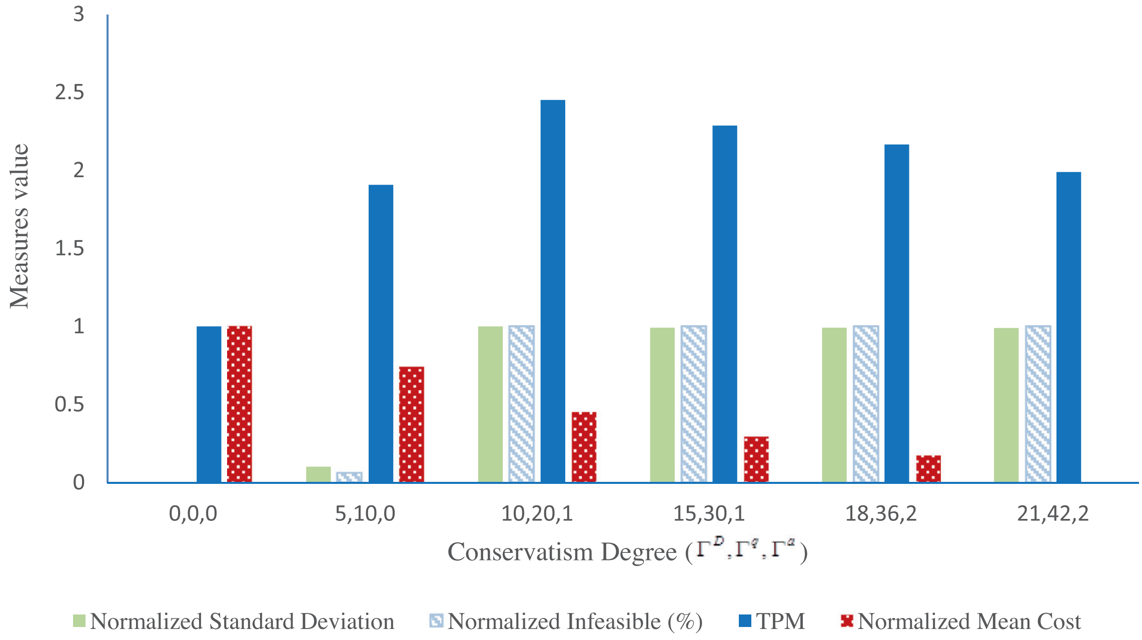

Fig. 5. Model performance at different conservatism degrees for the 21-node dataset under the normal situation.

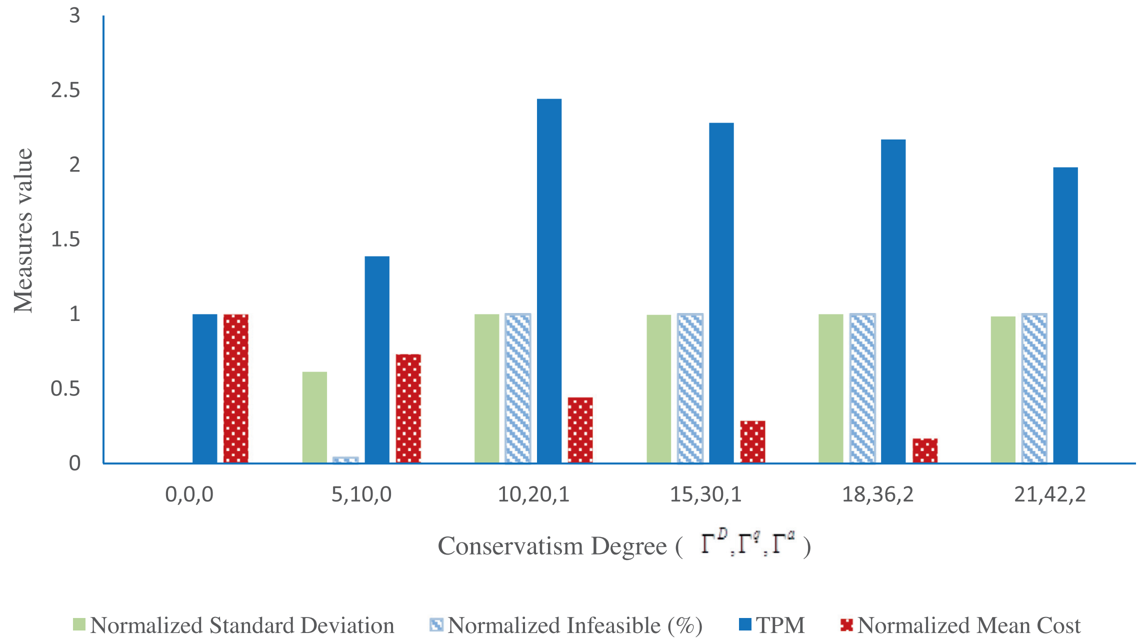

Fig. 6. Model performance at different conservatism degrees for the 21-node dataset under the unusual situation.

Performance Measure (TPM) that aims to aggregate the three measures into one single measure.

$$TPM = \omega_1[\text{Normalized mean value of cost}] + \omega_2[\text{Normalized standard deviation of cost}] \\ +\omega_3[\text{Normalized percent of infeasible solutions}] \tag{52}$$

In this equation, ω_1, ω_2 and ω_3 are the weights assigned to the measures of mean total cost, standard deviation and percentage of infeasibility, respectively. To normalize each of the measures, we respectively assign 0 and 1 to the worst and the best observed values of each measure. A number between 0 and 1 is assigned to other observed values proportional to their magnitude to the best and worst observations. The TPM formula can then be used to assist with choosing the appropriate values of conservatism degrees. For the three datasets, Figs. 4–12 use TPM as well as its three constituting measures to illus-

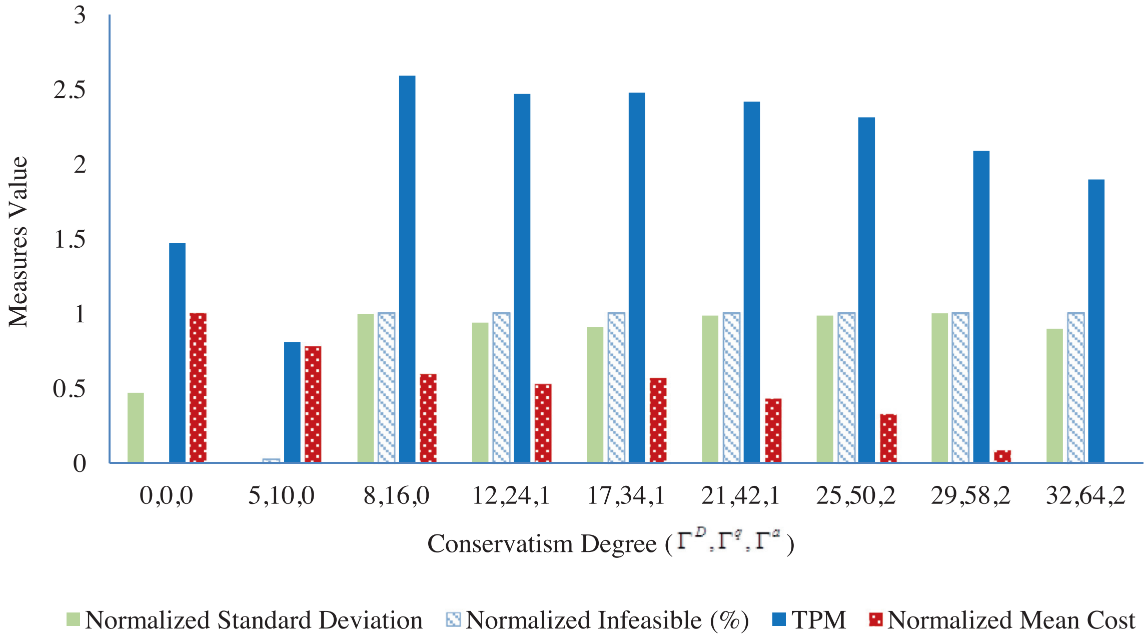

Fig. 7. Model performance at different conservatism degrees for the 32-node dataset under the unknown situation.

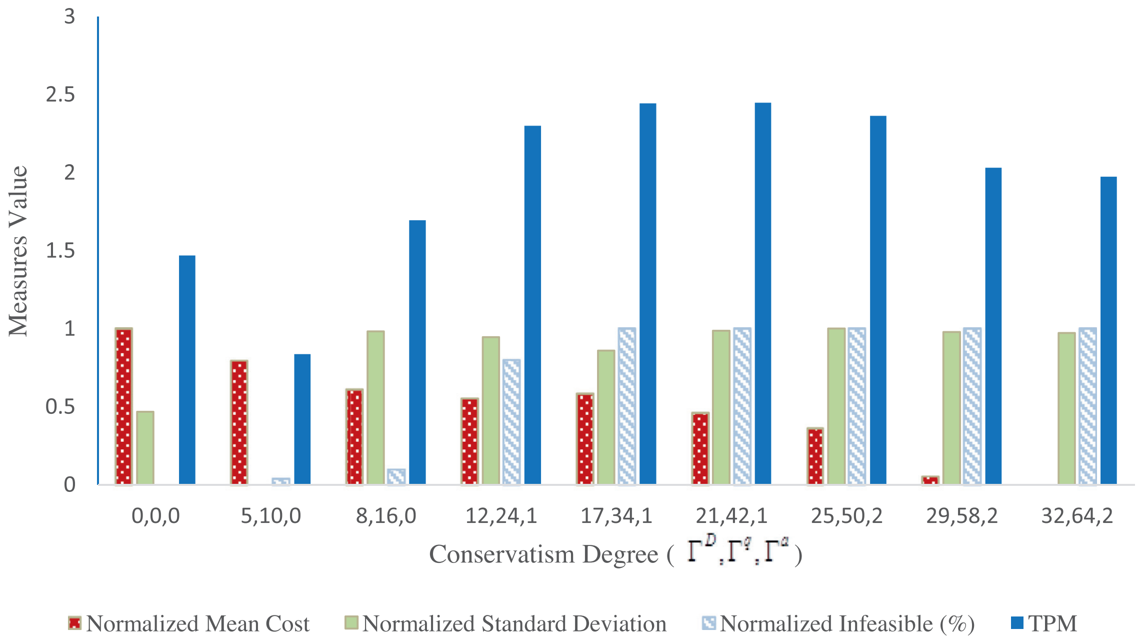

Fig. 8. Model performance at different conservatism degrees for the 32-node dataset under the normal situation.

trate the supply chain performance at different conservatism degrees at $\omega_1,\ \omega_2,\ \omega_3 = 1$. The most appropriate conservatism degree can be chosen for each dataset. For example, from Fig. 4, using TPM as the sole measure, the best performance can be obtained at $\Gamma^D = 10,\ \Gamma^q = 20$ and $\Gamma^a = 1$ for the 21-node dataset in the unknown situation.

Comparison of the results in Figs. 4–12 shows that the conservatism degrees do not vary significantly with changes in the uncertainty situation. For example, considering TPM as the sole measure, the best performance for the 21-node dataset under unknown, normal and unusual situations are obtained at $\Gamma^D = 10,\ \Gamma^q = 20$ and $\Gamma^a = 1$ (see Figs. 4–6). Likewise, the most desirable conservatism degree for the 32-node dataset, obtained at $\Gamma^D = 17,\ \Gamma^q = 34$ and $\Gamma^a = 1$, is analogous for the three situations. The best performance for the 49-node dataset is obtained at $\Gamma^D = 25,\ \Gamma^q = 50$ and $\Gamma^a = 1$ for the unknown

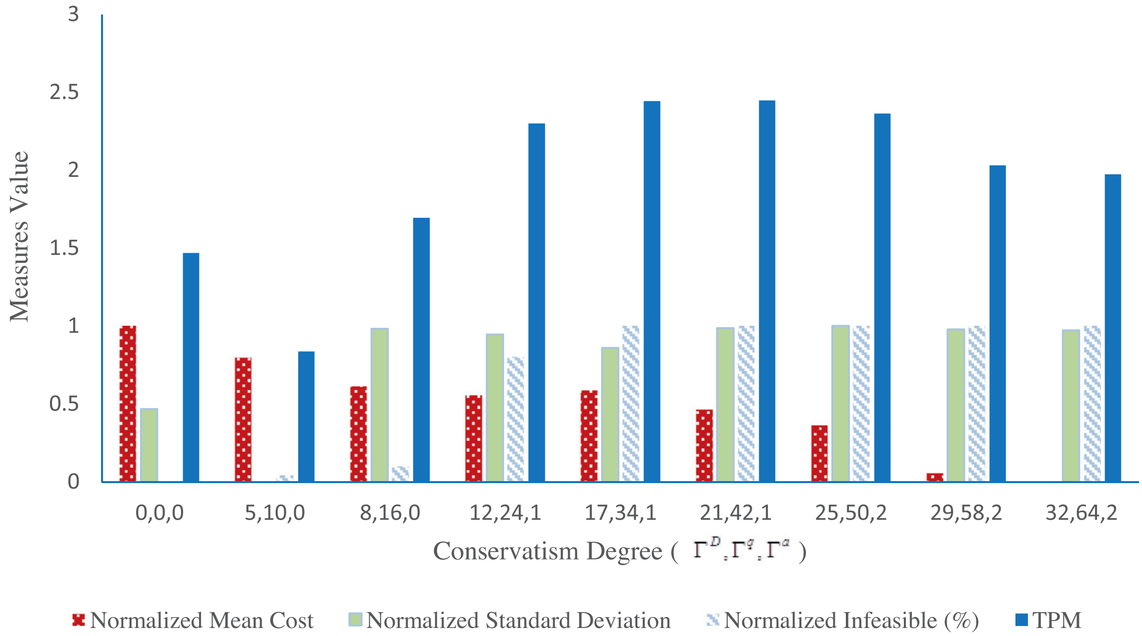

Fig. 9. Model performance at different conservatism degrees for the 32-node dataset under the unusual situation.

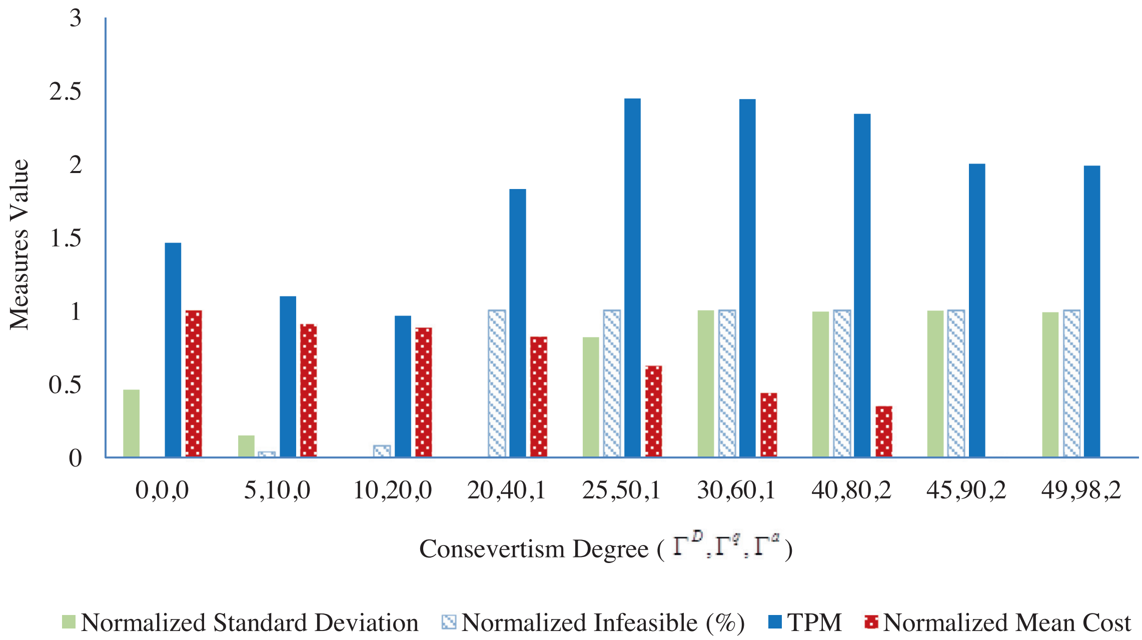

Fig. 10. Model performance at different conservatism degrees for the 49-node dataset under the unknown situation.

and normal situations; while for the unusual situation, the most desirable performance is achieved at $\Gamma^D = 40,\ \Gamma^q = 80$ and $\Gamma^a = 2$.

5.7. Performance evaluation of the Lagrangian relaxation solution method

For the 88-node, 100-node, and 150-node datasets, Table 13 presents the numerical results obtained from the Lagrangian relaxation method at different conservatism degrees. The optimality tolerance was set at 0.1%. The columns named “UB” and “LB” present respectively the upper bound and lower bound values obtained. The column labelled “Gap” is the percentage difference between the upper bound and lower bound, calculated from $\frac{UB-LB}{UB} \times 100$. For the sake of comparison, the same

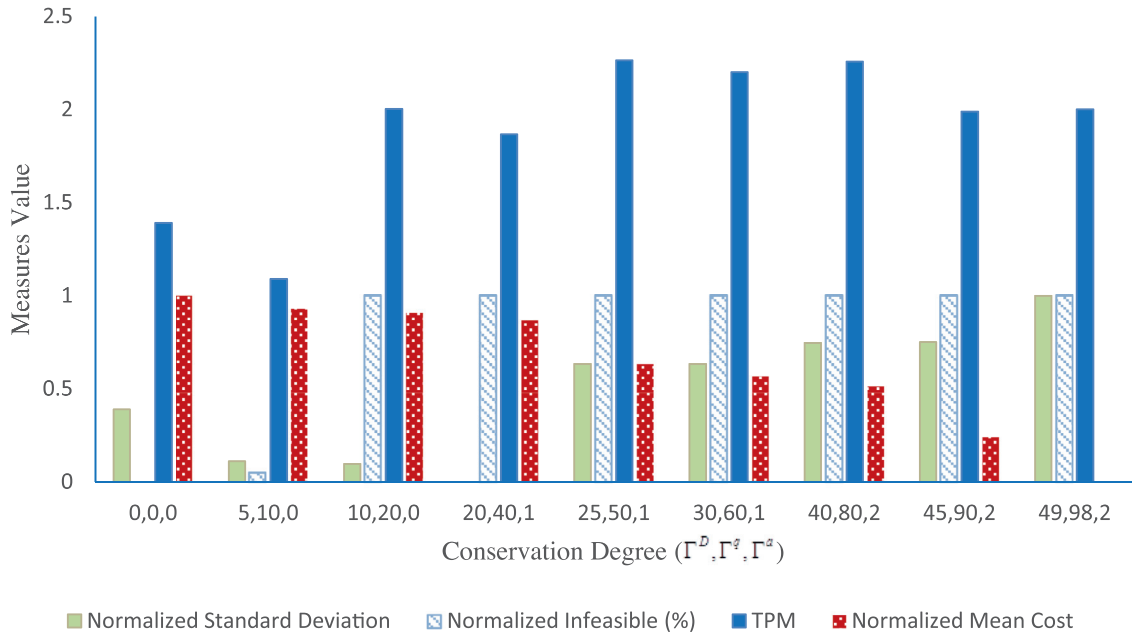

Fig. 11. Model performance at different conservatism degrees for the 49-node dataset under the normal situation.

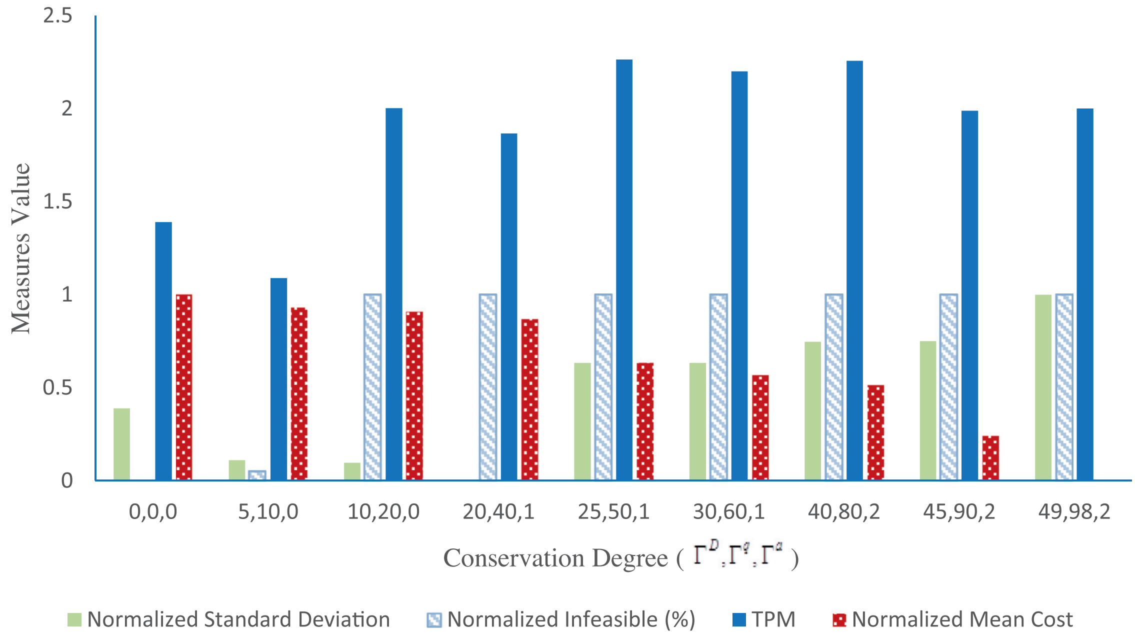

Fig. 12. Model performance at different conservatism degrees for the 49-node dataset under the unusual situation.

problems are solved with GAMS, setting the termination condition at 15,000 seconds. Model runtime using both Lagrangian relaxation and GAMS are given in the las two columns.

The results show that the Lagrangian relaxation approach is capable of finding quality solutions in all instances, evidenced by the small gaps of less than 0.1% between the upper bound and lower bound values of the objective function. The larger the problem size is, the more pronounced would be the superiority of the Lagrangian relaxation approach over GAMS in terms of solution time and quality. Overall, the approach is capable of finding quality solutions to larger problems within a reasonable length of time given the strategic nature of supply chain design problems.

Table 13
Algorithm performance for the three large datasets.

Dataset	Γ^D	Γ^q	Γ^a	LB	UP	GAP (%)		CPU Time (S)	
						Lagrangian relaxation	GAMS	Lagrangian relaxation	GAMS
88-node	0	0	0	311,637	311,948	0.10	0	11	47
	10	20	0	323,928	324,252	0.10	0	59	801
	20	40	0	326,786	327,113	0.10	0	53	966
	30	60	1	328,864	329,193	0.10	0	74	1074
	40	80	1	331,228	334,209	0.89	0	82	1451
	50	100	1	332,936	333,269	0.10	0	85	1674
	60	120	1	356,843	357,200	0.10	0	97	2105
	70	140	2	396,081	396,437	0.09	0	83	2039
	80	160	2	397,418	397,736	0.08	0	87	1767
	88	176	2	417,317	417,700	0.09	0	35	1457
100-node	0	0	0	325,586	325,910	0.10	0	31	1075
	20	40	0	337,875	338,213	0.10	0	75	1852
	30	60	0	342,812	343,121	0.09	0	64	2298
	40	80	1	345,794	346,140	0.10	0	85	3547
	50	100	1	346,845	347,192	0.10	0	128	8351
	60	120	1	362,394	362,756	0.10	0	133	9231
	70	140	1	401,275	401,676	0.10	0	135	10,258
	80	160	2	401,616	402,018	0.10	0	136	11,789
	90	180	2	403,008	403,371	0.09	0	132	12,334
	100	200	2	423,911	424,335	0.10	0	142	13,487
150-node	0	0	0	354,610	354,929	0.09	0	89	8875
	10	20	0	365,877	366,242	0.10	0	128	12,381
	25	50	0	375,968	376,306	0.09	18	165	> 15,000
	45	90	0	392,391	392,783	0.10	21	171	> 15,000
	65	130	1	402,473	402,875	0.10	22	175	> 15,000
	85	170	1	424,895	425,277	0.09	26	188	> 15,000
	105	210	1	429,162	429,591	0.10	31	217	> 15,000
	120	240	2	437,855	438,293	0.10	34	289	> 15,000
	140	280	2	441,523	441,920	0.09	35	311	> 15,000
	150	300	2	451,342	451,793	0.10	38	364	> 15,000

6. A practical case example

Sepahan Oil Company[1] (SOC) is the largest and the most modern base oil refinery in the Middle East, which began its activities in 1992 in Isfahan, Iran. The company was initially part of Esfahan Refinery, as the major domestic provider of engine oil, and was further expanded and renamed to SOC in 2002. With a production capacity of 400,000 metric tons of base stocks, SOC supplies approximately half of the oil requirement of the country. Fig. 13 illustrates the location of domestic markets for SOC, and Table 14 provides the demand of each region in Iran provided by the marketing division of SOC.

SOC has traditionally fulfilled each market's demand from a single storage unit in Esfahan. This is where bottle oils are filled, packed and labeled. Fig. 14 summarizes the main processes carried out in the storage unit.

On 28th of May 2014, a huge fire disaster occurred at SOC's storage unit[2] resulting in 16 injuries, 50 to 100 percent disruption in the operations of various machines, and a total financial loss of approximately $115 M This was only one of the few fire disasters at SOC, which motivated the management to contemplate the design of a more resilient supply chain that supports a decentralized distribution network using multiple storage units scattered throughout the country. The capital city of each province was chosen as potential locations for the establishment of storage units.

Further, to mitigate the risk of future fire disasters, SOC budgeted the installation of more advanced fire prevention/protection systems such as intelligent fire alarm, cooling systems, water storage tanks, and Hypoxic air technology (oxygen reduction system) .[3] Such initiatives can potentially mitigate both the probability of a fire occurrence as well as the magnitude of impact of different facilities. For instance, firewater and cooling systems can considerably reduce the impact of a fire disaster. Unlike traditional fire systems that extinguish the fire post detection, hypoxic air technology is able to mitigate the occurrence probability by lowering the partial oxygen pressure (Brooks, 2004; Nilsson and van Hees, 2014)

Significant capital investment is needed to locate fully fortified storage units in all capital cities, which is not a feasible option due to the rather tight budgetary constraints of SOC. A fully fortified storage unit is one equipped with the most advanced fire-prevention and protection systems available in the market. As a result, three fortification levels are considered for locating the storage units: full, moderate and low fortification levels, denoted as FF, FM and FL, respectively. The storage units with full fortification level are obviously the most expensive to establish, but their probabilities of fire occur-

[1] http://www.sepahanoil.com
[2] http://en.trend.az/iran/2279033.html, http://persiantahlil.com/?cid=CMSContent&content=62455 (in Persian)
[3] Existing fire prevention and protection systems at SOC: http://www.sepahanoil.com/pages//140

Fig. 13. Geographical dispersion of domestic markets of SOC.

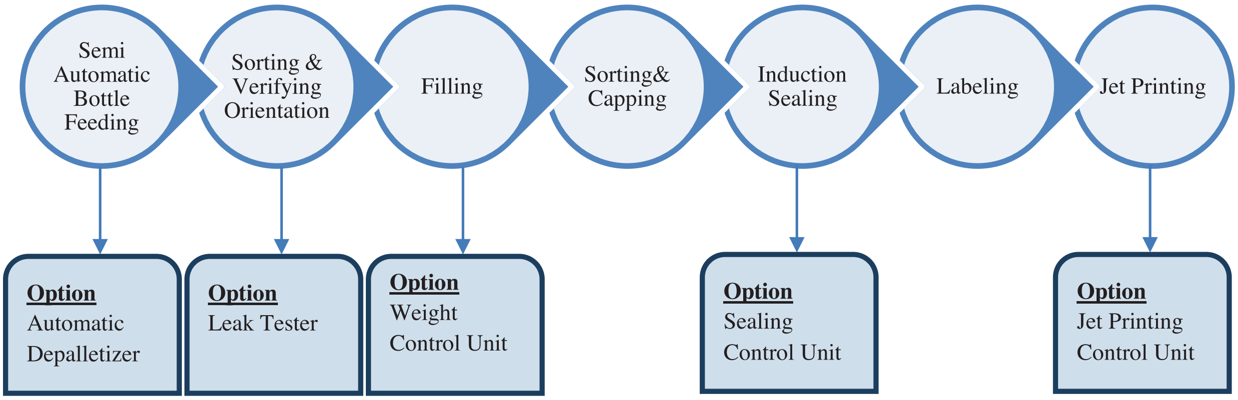

Fig. 14. The processes carried on SOC's storage unit.

rence are close to zero, thanks to the preventive-fire systems such as hypoxic air technology. Thus, they are referred to as reliable facilities in our investigation. The least expensive facilities with low fortification level are equipped with only traditional protection systems, hence are less reliable when it comes fire risks. Storage units with moderate fortification level lie between the two in terms of safety and facility location costs.

The model presented in this paper was first implemented to determine optimal locations of facilities with different fortification levels and optimal assignment of customers to these facilities at different conservatism degrees. Table 15 shows the extent to which the choice of conservatism degree can influence the overall supply chain cost for SOC.

From Table 15, a higher conservatism degree leads to a larger total supply chain cost to hedge the SOC's network against the potential fire risks and uncertainties. It can also be seen that the supply chain cost is not linearly increased as conservatism degree gets larger. These results are consistent with our findings in Section 5.3. What is interesting is the insignificant

Table 14
Demand of each domestic market for SOC (based on 2014 and 2015 demand data).

Market Name	Range of Demand (Tons)	
	Lower Bound	Upper Bound
Azarbayjan-e-sharghi	2621	2897
Azarbayjan-e-gharbi	924	1021
Ardebil	750	829
Esfahan	22,497	24,865
Alborz	3006	3322
Ilam	97	107
Bushehr	264	292
Tehran	20,510	22,669
Chaharmahal-e-Bakhtiari	1713	1893
Khorasan-e-Razavi	5818	6430
Khorasan-e-Shomali	553	611
Khorasan-e-Jonoobi	329	363
Khuzestan	2547	2815
Zanjan	262	290
Semnan	551	608
Sistan o Baloochestan	569	629
Fars	1720	1901
Qazvin	1225	1354
Qom	746	824
Kordestan	660	730
Kerman	5561	6147
Kermanshah	328	363
Kohkiluye o Boyer Ahmad	117	130
Golestan	1370	1514
Gilan	2228	2463
Lorestan	457	505
Mazandaran	3672	4059
Markazi	1040	1149
Hormozgan	1631	1803
Hamedan	1067	1179
Yazd	263	290

Table 15
Supply chain cost at various conservatism degrees for locating SOC, storage units.

Γ^D	Γ^q	Γ^a	Total Cost ($)	Cost Difference (%)
0	0	0	1187,402	0.0
5	10	0	1238,460	4.3
8	16	0	1264,468	2.1
12	24	1	1270,790	0.5
17	34	1	1286,040	1.2
21	42	1	1296,328	0.8
25	50	2	1324,847	2.2
29	58	2	1342,070	1.3
31	62	2	1379,648	2.8

changes in supply chain cost (no more than 4.3% in all instances), which indicates that significant resilience enhancements can be achieved at only small cost increases.

The Monte Carlo simulation method presented in Section 5.6 was then utilized to examine the performance of the proposed model for the data provided by SOC. The results are presented in Table 16. Three different measures are calculated to evaluate the quality of solutions: mean total cost, standard deviation, and percentage of infeasibility. Recall that the uniform, normal and beta distributions were utilized to model the situations where the data is respectively unknown, normal and unusual.

Not surprisingly, the results show that a low conservatism degree results in higher percentage of infeasibilities, increased product shortage, deteriorated service level, and potential lost sales for SOC. At the worst case scenario, when all conservatism degrees are set equal to zero (i.e., when$\Gamma^D = 0$, $\Gamma^q = 0$ and $\Gamma^a = 0$), the highest percentage infeasibilities of 81%, 67% and 93% occur for the unknown, normal and unusual situations, respectively. An important insight is that at only small increases in the total supply chain cost, when moving from row 1 to row 2 in Table 16, the resilience of the SOC's supply chain is improved considerably to only face 4%, 7% and 12% probability of stockout for the unknown, normal and unusual

Table 16
Monte Carlo simulation results for SOC data.

Uncertainty Situation	Γ^D	Γ^q	Γ^a	Mean Cost ($)	Mean Cost Difference (%)	Standard Deviation	Standard Deviation Difference (%)	Infeasibility (%)
Unknown	0	0	0	1,189,658	0%	10,421	0%	81%
	5	10	0	1,205,124	1.31%	14,935	43.31%	4%
	8	16	0	1,224,406	1.67%	2310	−84.53%	0%
	12	24	1	1,234,201	0.86%	2227	−3.60%	0%
	17	34	1	1,240,372	0.54%	2458	10.37%	0%
	21	42	1	1,244,093	0.33%	1748	−28.90%	0%
	25	50	2	1,255,290	0.91%	1526	−12.70%	0%
	29	58	2	1,269,098	1.18%	1437	−5.80%	0%
	31	62	2	1,271,636	0.23%	1483	3.20%	0%
Normal	0	0	0	1,189,064	0%	2397	0%	67%
	5	10	0	1,208,089	1.69%	3174	32.41%	7%
	8	16	0	1,211,714	0.34%	1544	−51.35%	0%
	12	24	1	1,216,561	0.47%	1345	−12.88%	0%
	17	34	1	1,218,994	0.24%	1442	7.21%	0%
	21	42	1	1,228,746	0.83%	1104	−23.47%	0%
	25	50	2	1,244,719	1.36%	1134	2.78%	0%
	29	58	2	1,250,943	0.57%	1190	4.93%	0%
	31	62	2	1,252,194	0.18%	1174	−1.37%	0%
Unusual	0	0	0	1,192,632	0%	13,100	0%	93%
	5	10	0	1,209,329	1.44%	22,470	71.53%	12%
	8	16	0	1,220,213	0.93%	12,781	−43.12%	0%
	12	24	1	1,222,653	0.24%	8159	−36.16%	0%
	17	34	1	1,229,989	0.68%	7117	−12.78%	0%
	21	42	1	1,233,679	0.39%	6571	−7.67%	0%
	25	50	2	1,247,250	1.11%	6679	1.65%	0%
	29	58	2	1,273,442	2.12%	6185	−7.4%	0%
	31	62	2	1,283,630	0.83%	6276	1.47%	0%

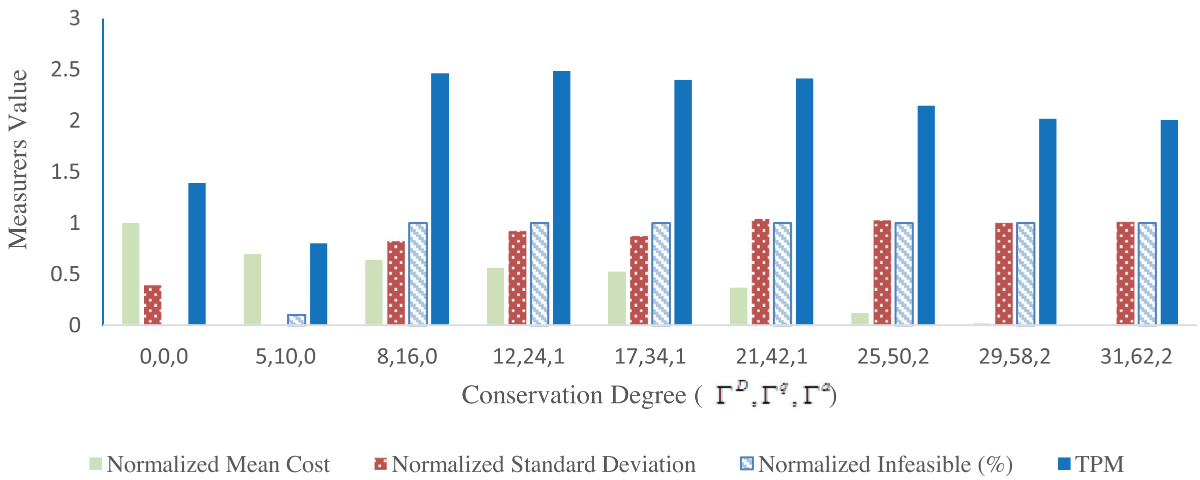

Fig. 15. Model performance for SOC data at different conservatism degrees under the unknown situation.

situations, respectively. For SOC management, this analysis helps identify the opportunities in which a supply chain can be protected against fire risks and supply/demand uncertainties at a reasonably minor supply chain cost increase.

Figs. 15–17 use TPM (see Eq. 52) as well as its three constituting measures to illustrate the SOC's performance at different situations (unknown, normal, and unusual) and conservatism degrees. These figures can help a decision maker find the most appropriate conservatism degree by selecting the scenario that generates more desirable values for the preferred measures. For example, from Fig. 15, using TPM as the sole measure, the best performance for SOC can be obtained at $\Gamma^D = 12,\ \Gamma^q = 4$ and $\Gamma^a = 1$ in the unknown situation. At these conservatism degrees, Fig. 18 displays optimal location and allocation decisions for SOC's supply chain. It can be seen that an optimal solution requires opening seven storage units as follows: two facilities with full fortification level in Tehran and Esfahan, two facilities with moderate fortification level in Alborz and Khorasan-e Razavi and three facilities with moderate fortification level in Kerman, Fars and Khuzestan. The facility in Esfahan can then continue operating as a storage unit by serving not only the neighboring markets but also the three storage facilities with low fortification level.

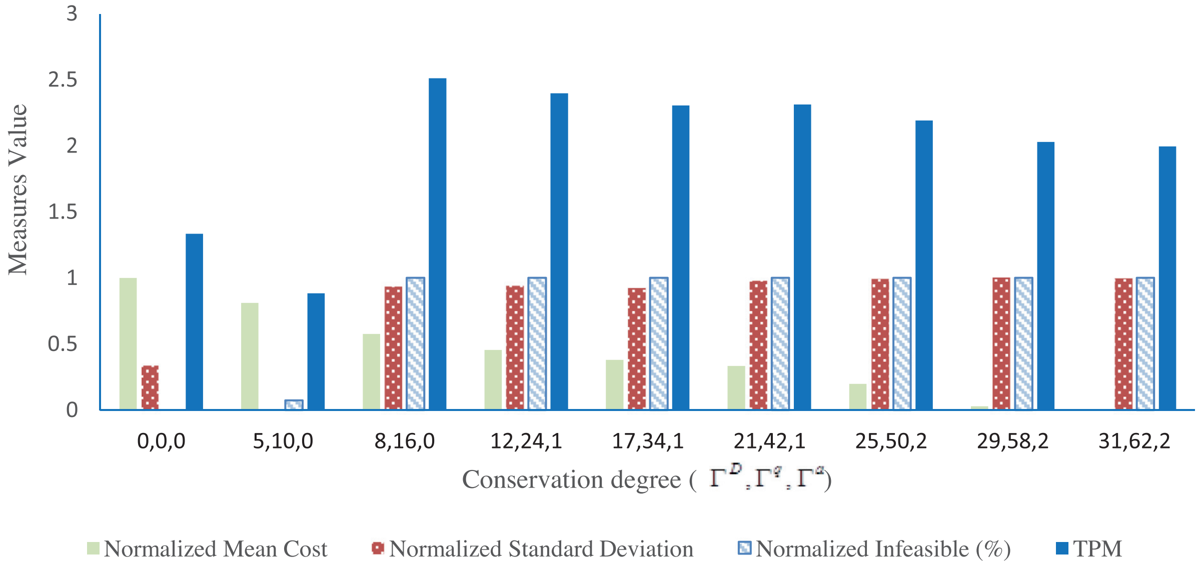

Fig. 16. Model performance for SOC data at different conservatism degrees under the normal situation.

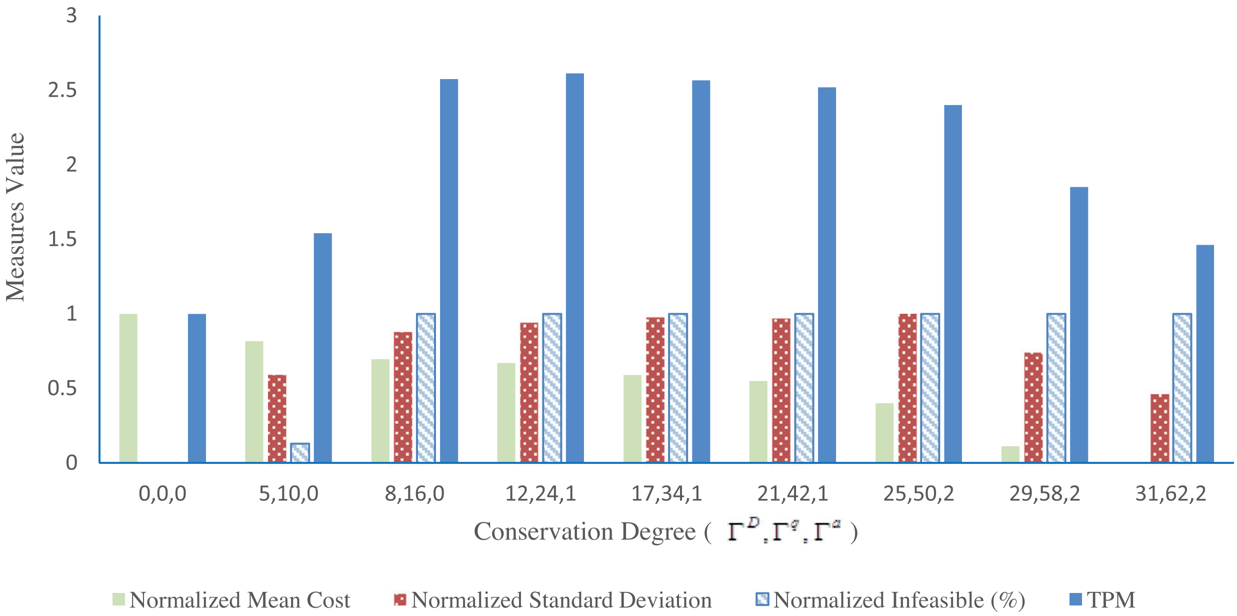

Fig. 17. Model performance for SOC data at different conservatism degrees under the unusual situation.

7. Conclusions

Today's supply chains are more difficult to design and manage. The increasing frequency and intensity of natural and man-made disasters from one hand, and systemic volatilities such as demand fluctuations and supply uncertainties from the other hand pose serious risks to global supply chains. Supply chain resilience is hence more critical to supply chain profitability and competitiveness than ever before. This paper presented an optimization model that can be used to design a supply chain resilient to (1) supply/demand interruptions and (2) facility disruptions whose risk of occurrence and magnitude of impact can be mitigated through fortification investments. The proposed robust-stochastic optimization model can also be utilized for reconfiguration of existing supply chains by assessing the affected operations and injecting more resilience into the network.

Our interpretation of the numerical results obtained from several experiments as well as a real-world case example arrived at some interesting practical implications and managerial insights. For example, from multiple viewpoints we found that supply chain resilience can be enhanced to a large extent by only slight changes in supply chain configuration and

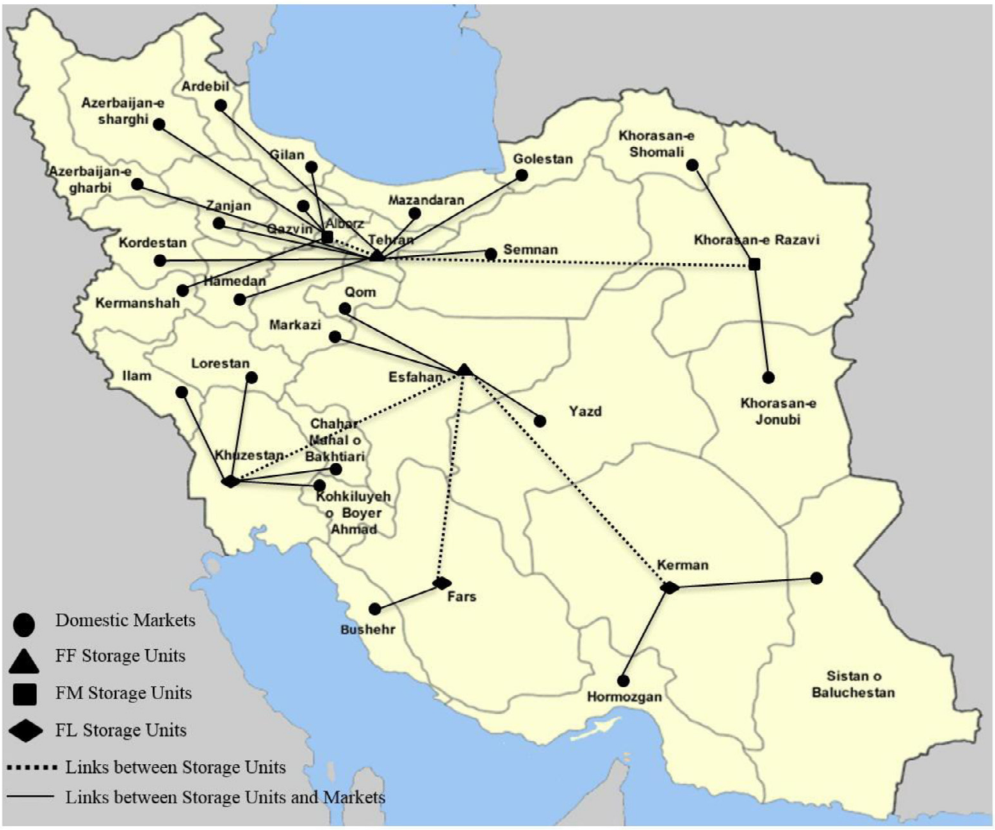

Fig. 18. Location and allocation of storage units with different fortification levels for SOC data.

only minor increase in supply chain costs. Whilst a similar finding was reached by some past studies, our observation here reinforces that this finding holds also in situations when a supply chain faces uncertainties in supply and demand, and a disruption can cause either partial or complete facility shutdown. Our analyses also showed in what ways facility fortification strategies can help address demand fluctuations. Another interesting finding is that initial capital investment plays a key role in developing a resilient supply chain and reducing the strategic supply chain costs, whilst excessive budget injections may not necessarily result in conforming supply chain cost reductions.

The investigation of the influence of disruptions and interruptions on supply chain design decisions is gaining increasing importance. The development and availability of new decision tools and risk mitigation strategies can help address many of these concerns facing supply chain practitioners. Despite the important contributions we made to this knowledge area, our study is not without limitations. The proposed model can be extended to incorporate the interdependency between supply chain disruptions/interruptions in different facilities and their impacts on supply chain decisions. Furthermore, the incorporation of critical factors such as multi-level assignment of unreliable facilities when they can back up each other may generate additional insights and practical implications. Another direction for future research can be the inclusion and analysis of customer responsiveness and agility elements such as service time and delivery lead-time which are the critical performance metrics in fast-paced business environments. The decision maker's conservatism degree can also be expressed as decision variable or in the form of a fuzzy parameter to enable examining the impacts on supply chain behavior when disruptions and interruptions occur at various supply chain levels. Given the multiple contributions of this work, we set the stage for additional and important future modeling efforts and practical investigations in this critical research area.

Acknowledgement

The authors are grateful to the managerial team of Sepahan Oil Company for providing the related data for our case example. Special thanks to Dr. Yan Dan Goh for her invaluable proofreading contributions, and the anonymous reviewers and the Editor in Chief for their insightful comments that helped us improve the presentation and contents of this paper during the revision rounds. This work was supported by grants NSC 102-2410-H-002-166-MY3 and MOST 104-2410-H-002-137-MY3 from Ministry of Science and Technology of Taiwan, R.O.C.

References

Aryanezhad, M.B., Jalali, S.G., Jabbarzadeh, A., 2010. An integrated supply chain design model with random disruptions consideration. African J. Bus. Manag. 4, 2393–2401.

Azad, N., Saharidis, G.K., Davoudpour, H., Malekly, H., Yektamaram, S.A., 2013. Strategies for protecting supply chain networks against facility and transportation disruptions: an improved Benders decomposition approach. Ann. Oper. Res. 210, 125–163.

Baghalian, A., Rezapour, S., Farahani, R.Z., 2013. Robust supply chain network design with service level against disruptions and demand uncertainties: a real-life case. Eur. J. Oper. Res. 227, 199–215.

Banks, J., Carson II, J.S., Nelson, B.L., Nicol, D.M., 2010. Discrete-Event System Simulation, 5th ed. Prentice Hall, USA.

Baraldi, P., Zio, E., 2008. A combined Monte Carlo and possibilistic approach to uncertainty propagation in event tree analysis. Risk. Anal. 28, 1309–1326.

Bental, A., Ghaoui, L.E., Nemirovski, A., 2009. Robust Optimization. Princeton Press.

Berman, O., Krass, D., Menezes, M.B., 2007. Facility reliability issues in network p-median problems: strategic centralization and co-location effects. Oper. Res. 55, 332–350.

Bertsimas, D., Sim, M., 2003. Robust discrete optimization and network flows. Math. Program. 98, 49–71.

Bertsimas, D., Sim, M., 2004. The price of robustness. Oper. Res. 52, 35–53.

Birge, J.R., Louveaux, F., 2011. Introduction to Stochastic Programming. Springer.

Brooks, J., 2004. Aircraft Cargo Fire Suppression Using Low Pressure Dual Fluid Water Mist and Hypoxic Air. International Aero Technologies LLC, Burlington, WA, pp. 1–17.

Chen, Q., Li, X., Ouyang, Y., 2011. Joint inventory-location problem under the risk of probabilistic facility disruptions. Transp. Res. Part B 45, 991–1003.

Cui, T., Ouyang, Y., Shen, Z.J.M., 2010. Reliable facility location design under the risk of disruptions. Oper. Res. 58, 998–1011.

Daskin, M.S., 1995. Network and Discrete Location: Models, Algorithms, and Applications. JohnWiley & Sons, New York, NY, USA.

Daskin, M.S., Coullard, C.R., Shen, Z.J.M., 2002. An inventory-location model: formulation, solution algorithm and computational results. Ann. Oper. Res. 110, 83–106.

Diabat, A., Battaïa, O., Nazzal, D., 2015. An improved lagrangian relaxation-based heuristic for a joint location-inventory problem. Comput. Oper. Res. 61, 170–178.

Esmaeilikia, M., Fahimnia, B., Sarkis, J., Govindan, K., Kumar, A., Mo, J., 2014a. A tactical supply chain planning model with multiple flexibility options: an empirical evaluation. Ann. Oper. Res. 1–26.

Esmaeilikia, M., Fahimnia, B., Sarkis, J., Govindan, K., Kumar, A., Mo, J., 2014b. Tactical supply chain planning models with inherent flexibility: definition and review. Ann. Oper. Res. 1–21.

Fahimnia, B., Tang, C., H., D., Sarkis, J., 2015. Quantitative models for managing supply chain risks: a review. Eur. J. Oper. Res. 247 (1), 1–15.

Farahani, R.Z., Hekmatfar, M., Fahimnia, B., Kazemzadeh, N., 2014. Hierarchical facility location problem: models, classifications, techniques, and applications. Comput. Indus. Eng. 68, 104–117.

Fisher, M.L., 2004. The Lagrangian relaxation method for solving integer programming problems. Manag. Sci. 50, 1861–1871.

Gabrel, V., Murat, C., Thiele, A., 2014. Recent advances in robust optimization: an overview. Eur. J. Oper. Res. 235, 471–483.

Hendricks, K.B., Singhal, V.R., 2005. An empirical analysis of the effect of supply chain disruptions on long-run stock price performance and equity risk of the firm. Prod. Oper. Manag. 14, 35–52.

Hendricks, K.B., Singhal, V.R., Zhang, R., 2009. The effect of operational slack, diversification, and vertical relatedness on the stock market reaction to supply chain disruptions. J. Oper. Manag. 27, 233–246.

Hernandez, I., Emmanuel Ramirez-Marquez, J., Rainwater, C., Pohl, E., Medal, H., 2014. Robust facility location: hedging against failures. Reliab. Eng. Syst. Safety 123, 73–80.

Jabbarzadeh, A., Fahimnia, B., Seuring, S., 2014. Dynamic supply chain network design for the supply of blood in disasters: a robust model with real world application. Transp. Res. Part E 70, 225–244.

Jabbarzadeh, A., Fahimnia, B., Sheu, J., 2015. An enhanced robustness approach for managing supply and demand uncertainties. Int. J. Prod. Econ. doi:10.1016/j.ijpe.2015.06.009i.

Jabbarzadeh, A., Jalali Naini, S.G., Davoudpour, H., Azad, N., 2012. Designing a supply chain network under the risk of disruptions. Math. Prob. Eng. 2012, 1–23.

Kalos, M.H., Whitlock, P.A., 2008. In: Whitlock, P.A. (Ed.), Monte Carlo Methods. John Wiley & Sons.

Klibi, W., Martel, A., Guitouni, A., 2010. The design of robust value-creating supply chain networks: a critical review. Eur. J. Oper. Res. 203, 283–293.

Li, Q., Savachkin, A., 2013. A heuristic approach to the design of fortified distribution networks. Transp. Res. Part E 50, 138–148.

Li, Q., Zeng, B., Savachkin, A., 2013. Reliable facility location design under disruptions. Comput. Oper. Res. 40, 901–909.

Li, X., Ouyang, Y., 2010. A continuum approximation approach to reliable facility location design under correlated probabilistic disruptions. Transp. Res. Part B 44, 535–548.

Liberatore, F., Scaparra, M.P., Daskin, M.S., 2011. Analysis of facility protection strategies against an uncertain number of attacks: the stochastic R-interdiction median problem with fortification. Comput. Oper. Res. 38, 357–366.

Liberatore, F., Scaparra, M.P., Daskin, M.S., 2012. Hedging against disruptions with ripple effects in location analysis. Omega 40, 21–30.

Lim, M., Daskin, M.S., Bassamboo, A., Chopra, S., 2010. A facility reliability problem: formulation, properties, and algorithm. Nav. Res. Logist. (NRL) 57, 58–70.

Losada, C., Scaparra, M.P., O'Hanley, J.R., 2012. Optimizing system resilience: a facility protection model with recovery time. Eur. J. Oper. Res. 217, 519–530.

Medal, H.R., Pohl, E.A., Rossetti, M.D., 2014. A multi-objective integrated facility location-hardening model: analyzing the pre- and post-disruption tradeoff. Eur. J. Oper. Res. 237, 257–270.

Melo, M.T., Nickel, S., Saldanha-da-Gama, F., 2009. Facility location and supply chain management – A review. Eur. J. Oper. Res. 196, 401–412.

Nilsson, M., van Hees, P., 2014. Advantages and challenges with using hypoxic air venting as fire protection. Fire Mater. 38, 559–575.

O'Hanley, J.R., Scaparra, M.P., García, S., 2013. Probability chains: a general linearization technique for modeling reliability in facility location and related problems. Eur. J. Oper. Res. 230, 63–75.

Ozsen, L., Coullard, C.R., Daskin, M.S., 2008. Capacitated warehouse location model with risk pooling. Nav. Res. Logist. 55, 295–312.

Peng, P., Snyder, L.V., Lim, A., Liu, Z., 2011. Reliable logistics networks design with facility disruptions. Transp. Res. Part B 45, 1190–1211.

Pouillot, R., Beaudeau, P., Denis, J.-B., Derouin, F., Group, A.C.S., 2004. A quantitative risk assessment of waterborne cryptosporidiosis in France using second-order Monte Carlo simulation. Risk. Anal. 24, 1–17.

Qi, L., Shen, Z.J.M., Snyder, L.V., 2010. The effect of supply disruptions on supply chain design decisions. Transp. Sci. 44, 274–289.

ReVelle, C.S., Eiselt, H.A., Daskin, M.S., 2008. A bibliography for some fundamental problem categories in discrete location science. Eur. J. Oper. Res. 184, 817–848.

Rezaee, A., Dehghanian, F., Fahimnia, B., Beamon, B., 2016. Green supply chain network design with stochastic demand and carbon price. Ann. Oper. Res. doi:10.1007/s10479-015-1936-z.

Ross, S.M., 2006. Introduction to Probability Models, 10 ed. Academic Press.

Shen, Z.-J.M., Zhan, R.L., Zhang, J., 2011. The reliable facility location problem: formulations, heuristics, and approximation algorithms. Informs J. Comput. 23, 470–482.

Sherali, H., Alameddine, A., 1992. A new reformulation-linearization technique for bilinear programming problems. J. Global Optim. 2, 379–410.

Shishebori, D., Jabalameli, M.S., Jabbarzadeh, A., 2014. Facility location-network design problem: reliability and investment budget constraint. J. Urban Plann. Dev. 140.

Snyder, L.V., 2006. Facility location under uncertainty: a review. IIE Trans. 38, 547–564.

Snyder, L.V., Atan, Z., Peng, P., Rong, Y., Schmitt, A.J., Sinsoysal, B., 2016. OR/MS models for supply chain disruptions: a review social science research network. IIE Trans 48 (2), 89–109.
Snyder, L.V., Daskin, M.S., 2005. Reliability models for facility location: the expected failure cost case. Transp. Sci. 39, 400–416.
Snyder, L.V., Daskin, M.S., Teo, C.P., 2007. The stochastic location model with risk pooling. Eur. J. Oper. Res. 179, 1221–1238.
Snyder, L.V., Scaparra, M.P., Daskin, M.S., Church, R.L., 2006. Planning for disruptions in supply chain networks. *Tut. Oper. Res*. Informs 234–257.
Tang, C.S., 2006. Perspectives in supply chain risk management. Int. J. Prod. Econ. 103, 451–488.
Zokaee, S., Jabbarzadeh, A., Fahimnia, B., Sadjadi, S., 2014. Robust supply chain network design: an optimization model with real world application. Ann. Oper. Res. 1–30.