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Optimization of incentive polices for plug-in electric vehicles

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ABSTRACT

High purchase prices and the lack of supporting infrastructure are major hurdles to the adoption of plug-in electric vehicles (PEVs). It is widely recognized that the government could help break these barriers through incentive policies, such as offering rebates to PEV buyers or funding charging stations. The objective of this paper is to propose a modeling framework that can optimize the design of such incentive policies. The proposed model characterizes the impact of the incentives on the dynamic evolution of PEV market penetration over a discrete set of time intervals, by integrating a simplified consumer vehicle choice model and a macroscopic travel and charging model. The optimization problem is formulated as a nonlinear and non-convex mathematical program and solved by a specialized steepest descent direction algorithm. We show that, under mild regularity conditions, the KKT conditions of the proposed model are necessary for local optimum. Results of numerical experiments indicate that the proposed algorithm is able to obtain satisfactory local optimal policies quickly. These optimal policies consistently outperform the alternative policies that mimic the state-of-the-practice by a large margin, in terms of both the total savings in social costs and the market share of PEVs. Importantly, the optimal policy always sets the investment priority on building charging stations. In contrast, providing purchase rebates, which is widely used in current practice, is found to be less effective.

1. Introduction

The growing concerns about energy security and global climate change have stimulated the transition to alternative fuel vehicles (AFV), widely considered an important ingredient in sustainable transportation (NRS, 2010). Of the many competing technologies, plug-in electric vehicles (PEV) have received much attention thanks to their high energy efficiency (Eberhard and Tarpenning, 2006), the ability to substitute electricity for petroleum and the potential to reduce carbon footprint (Crist, 2012). However, the adoption of PEVs is hurdled by several barriers: high retail prices, the limited range of batteries, and the lack of supporting infrastructure, especially charging stations (Hidrue et al., 2011). In the US, policy makers have created various incentive programs aiming to overcome these barriers. The American Recovery and Reinvestment Act of 2009 (ARRA) signed into law a provision that will offer up to $7500 of tax credit for each new PEV purchase starting from 2010. The state governments in the US have also implemented various policies to encourage the ownership of PEVs and installation of

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charging stations. In Illinois, for example, Electric Vehicle Initiatives provide a rebate up to $49,000 toward the installation of Level II charging stations and a rebate up to $4000 for the purchase of a new alternative fuel vehicle.[1] It is clear that the public investment on these incentive programs is a scarce resource that should be carefully allocated to maximize its impact. The objective of this paper is to propose an optimization model that supports these macroscopic decisions about resource allocation. When fully implemented, the proposed model may help the policy makers decide when and how much money should be invested on what incentive programs in order to achieve a desired goal, e.g. reduced greenhouse gas emissions or reduced dependence on petroleum.

Understanding how incentive programs might affect the adoption of PEVs calls for a behavioral model that can predict consumers' vehicle choice. There is a vast literature devoted to building such models with various discrete and discrete-continuous choice modeling approaches, see Bhat et al. (2009) for a review. Vehicle choice may be characterized using the number of vehicles owned by the household (Bhat and Pulugurta, 1998; Dargay and Vythoulkas, 1999; Golob and Burns, 1978), type of each vehicle owned (body type, fuel type, vintage and powertrain technology) (Ahn et al., 2008; Brownstone et al., 2000; Bunch et al., 1993; Dagsvik et al., 2002; Hensher and Greene, 2001; Lave and Train, 1979; Mabit et al., 2015; Mannering and Mahmassani, 1985; Mannering et al., 2002; Mohammadian and Miller, 2003; Yavuz et al., 2015), and the number of miles driven by each vehicle (Ahn et al., 2008; Bhat and Sen, 2006; Bhat et al., 2009; Fang, 2008; Train and Lohrer, 1982). Previous studies (Bhat et al., 2009; Lave and Train, 1979; Mohammadian and Miller, 2003) have also identified many factors that influence the choice of conventional vehicles, ranging from demographic characteristics (such as income, household size, number of children), vehicle attributes (price, operating cost, fuel efficiency), fuel price, driver personality and built environment characteristics. For alternative fuel vehicles, empirical evidence (Bunch et al., 1993; Dagsvik et al., 2002) identified the purchase price and the range between refueling as important attributes, in addition to those associated with conventional vehicles. A number of recent studies attempt to predict the evolution of market penetration of AFV by simulation. Many of these studies simulate the vehicle choice behavior of agents using classical multinomial logit (Shafiei et al., 2012) or nested logit model (Lin and Greene, 2010; NRC, 2013). A different decision process is proposed in Eppstein et al. (2011) to account for spatial and social effects, as well as media influences. Simulation studies are often employed to evaluate the impact of energy policy (Lin and Greene, 2010), fuel prices (Eppstein et al., 2011; Shafiei et al., 2012), and availability of infrastructure (Lin and Greene, 2011; Lin et al., 2014) on the future market share. While they are useful for the evaluation purpose, the simulation studies are not designed for optimizing interventions. The vehicle choice model employed in this paper is developed along the line of those used in the simulation studies by Lin and Greene (2010) and NRC (2013), with a few additional simplifying assumptions.

Numerous recent studies have examined the planning of charging infrastructure for PEVs (see e.g. Dashora et al., 2010; Frade et al., 2011; Chen et al., 2013; Sathaye and Kelley, 2013; Nie and Ghamami, 2013; He et al., 2013; Mak et al., 2013; Ghamami et al., 2014; Lim and Rong, 2014; Bhatti et al., 2015; Gnann and Plotz, 2015). These studies consider the optimal configuration of charging stations (e.g., location, charging power) either within city (intracity) or between cities (intercity), but typically assume the demand for such facilities as given. In other words, the interactions between planning decisions for charging stations and the long-term adoption of PEVs are not modeled. Another line of recent work in this field is concerned with more detailed routing and recharging decisions of potential PEV users (see e.g., Adler et al., 2014; De Weerdt et al., 2013; Fontana, 2013; He et al., 2014; Chen and Nie, 2015). Such problems address the range anxiety issue by balancing the need for minimizing travel cost and fulfilling relay requirements. Because the focus of this paper is on "sketchy decisions" at a highly aggregated level, we postulate that the outcomes are relatively insensitive to specific details of infrastructure planning and/or individual travel behaviors.[2] Consequently, the representation of charging infrastructure and travel behaviors is simplified in this study in order to highlight the tradeoff in the bigger picture. Specifically, the densities of public charging stations within and beyond a city limit are employed as the main surrogate for charging availability, which determines the probability of finding charging facilities.

The rest of this paper is organized as follows. The next section presents the problem and main assumptions. Section 3 first describes the travel and vehicle choice models, and then introduces the main optimization model that is formulated as a nonlinear program. Section 4 proposes a solution algorithm and Section 5 reports the results of numerical experiments. Conclusions and directions for future research are given in Section 6.

2. Problem statement and main assumptions

We consider the PEV adoption over a fixed analysis period. Each year, a certain number of consumers need to purchase a new vehicle from a discrete set of vehicles, of which a subset is PEVs. The choice of vehicles is affected by, among other things, the purchase price and charging availability, which affects the operating cost of PEVs. The government offers two incentives over the entire analysis period: purchase rebates and publicly funded charging stations, with the objective of promoting PEVs and/or minimizing social cost of travel (potentially including the environmental impacts). The question

[1] https://www2.illinois.gov/gov/green/Pages/ElectricVehicleInitiatives.aspx.

[2] This is not to dispute the value of detailed planning of charging infrastructure. Rather, the point is that a sketchy model can help the decision makers better understand the fundamental tradeoff and hence come up with simple guiding principles. The detailed design can always be performed on the basis of the blueprints obtained from the sketchy model.

addressed in this paper is how these resources should be allocated to each of the incentives over the analysis period to optimize the intended objective.

Several simplifying assumptions about vehicle choice are made here in order to reduce unnecessary complexities involved in developing an optimization model and the corresponding solution techniques.

Assumption 1. Individual consumers, instead of households, are modeled.[3]

Assumption 2. Each vehicle must be replaced and retired after it reaches certain age, which endogenizes the evolution of vehicle stock.[4]

Assumption 3. Consumers with average demographic and personal attributes (e.g. income, education) are considered and are only distinguished by the average travel pattern (i.e. distribution of daily travel distances).[5]

Assumption 4. Average consumers narrow their choice to one of the three categories: conventional gasoline engine vehicles (CGV), plug-in hybrid electric vehicles (PHEV), or battery electric vehicles (BEV). These vehicles are otherwise similar in all attributes except fuel type, fuel efficiency, range and purchase price.[6]

3. Modeling framework

The market evolution of vehicle sales and stock is modeled in a fixed period of $y = \{0, 1, 2, \ldots, Y\}$ years for a discrete set of consumers $\{1, \ldots, I\}$, who would choose a vehicle from a discrete set $\{1, \ldots, J\}$. Each consumer class i has a known base population in year 0, denoted as Q_i^0, and a newly added population in each year y, denoted as q_i^y. Consumer class i is identified by its travel pattern, which is characterized by the distribution of the daily trip distance. Our definition of travel patterns follows the driver type definition proposed in MA3T (Lin and Greene, 2010) and LAVE-Trans (NRC, 2013), which classifies drivers into modest drivers, average drivers, and frequent drivers based on 2001 National Household Travel Survey (NHTS) data. Each of the driver types corresponds to a unique set of calibrated parameters that define the shape of the trip distance distribution.

As per Assumption 4, we focus on three types of vehicles for simplicity: conventional gasoline vehicles (CGV, $j = 1$), plug-in hybrid electric vehicles (PHEV $j = 2$), and battery electric vehicle (BEV, $j = 3$). The difference between PHEV and BEV is that PHEV has both electrical and gasoline engines, so it can drive on either electricity or gasoline. Each vehicle is identified by the retail price p_j^y, home-based electric range r_j^y, life expectance (l_j^y), purchase subsidy s_j^y, and terminal (resale) value (ξ_j^y). Note that all of the above properties may change over time. Note that the home-based electric range r_j^y may depend on both the type of consumers' residency and the type of vehicles (PHEV or BEV). In this paper, however, we assume all consumers have access to home-based charging for simplicity.

Denote the number of vehicle j bought by consumer i in year y as V_{ij}^y. Accordingly, the total number of consumers i who are driving vehicle j in year y can be computed as follows.

$$Q_{ij}^y = Q_{ij}^{y-1} - V_{ij}^{y-l_j} + V_{ij}^y, y = 1, \ldots, Y \tag{1}$$

We assume that Q_{ij}^0 and $V_{ij}^y, \forall y = -l_j, \ldots$ are given, and note that the latter may be simply estimated as Q_{ij}^0 / l_j. V_{ij}^y is estimated based on

$$V_{ij}^y = \left(q_i^y + \sum_j V_{ij}^{y-l_j+1} \right) P_{ij}^y, y = 1, \ldots, Y, \tag{2}$$

where q_i^y is the number of newly added customers i and $\sum_j V_{ij}^{y-l_j+1}$ is the total number of customs i whose vehicles j have reached their life limit and hence they must acquire a new vehicle. P_{ij}^y is the probability that consumers class i chooses vehicle j in year y, which will be determined by a vehicle choice model, as discussed later.

3.1. Travel and charging model

Travel is represented using a simple model as illustrated in Fig. 1. The radius of the inner circles represents the electric range of a given vehicle (denoted as r_j) and the outer circle describes the boundary of an *imaginary city*, with a diameter of

[3] This assumption is commonly used in the literature, see e.g. Lin and Greene (2010), Eppstein et al. (2011), and NRC (2013).

[4] We note that vehicle stock data may be directly available from other sources, such as American Energy Outlook (AEO) published by the US Department of Energy (DOE, 2014). The proposed model can be revised to take these data as inputs.

[5] The simulation model developed by Oak Ridge National Laboratory, known as the Market Acceptance of Advanced Automotive Technologies (MA3T, Lin and Greene, 2010), divides the US consumers into 1458 segments based on region, residency, charging availability, attitude toward AFVs and travel pattern. We only consider travel pattern in this paper because the daily travel distance has a significant impact on the choice of ranged-limited AFVs and their operating costs. However, the optimization framework proposed herein applies to any number of consumer segments provided that their vehicle choice probabilities are available in a closed form.

[6] See Eppstein et al. (2011) for example. Again, extending to more refined vehicle types, such as those considered by Lin and Greene (2010), is straightforward within the proposed optimization framework.

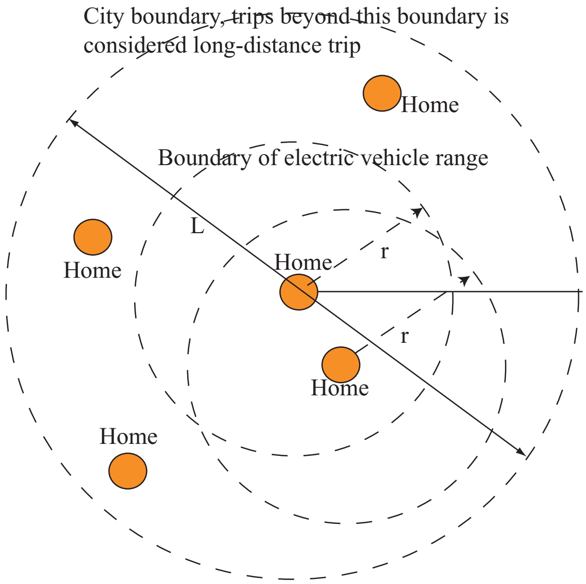


Fig. 1. Illustration of the proposed travel model.

L. Because our focus is to examine the policy tradeoff at a macroscopic level, the following assumptions are introduced to simplify the travel and charging behaviors.

Assumption 5. All consumers reside uniformly in one of the imaginary cities, each portrayed as a circle with a diameter of L (see Fig. 1). The number of cities required to hold a population of N is thus

$$n_c = \left\lceil \frac{4N}{\pi L^2 \xi} \right\rceil, \tag{3}$$

where ξ is the average population density, and $\lceil a \rceil$ denotes the minimum integer larger than or equal to a. As N changes in the analysis period, ξ, instead of n_c, is changed accordingly.

Assumption 6. Each consumer's daily total trip distance is modeled as a random variable ω, independent of time (i.e. model year y). If a realization of ω is shorter than L, the consumer is assumed to travel within city (intracity trips); otherwise it corresponds to an intercity trip. No travel details other than daily trip distance are considered in this study.

Assumption 7. All EV users will start their daily travel with a fully charged battery.[7] For intracity travel, all EV users are assumed to charge the battery after it is fully depleted, and the charging time will not be considered as a penalty.[8] For intercity travel, charging would only be considered by BEV users, as the inconvenience of charging is assumed to outweigh its benefits for PHEV users.

Assumption 8. The probability that an EV user can find public charging facilities depends on the density of such facilities, which is represented as the ratio between the number of existing charging stations to the number of charging stations required to achieve a full coverage. Charging stations within cities are assumed to distribute uniformly in the circle that represents each city. Charging stations outside cities are assumed to uniformly distribute along imaginary linear highway corridors that support intercity trips.

Assumption 9. All charging stations have enough capacity to accommodate all EV users at any given time. In other words, EV users incur no queuing delay when seeking to charge their vehicles.

As per Assumption 6, we use f_i and F_i to denote the probability density function (PDF) and cumulative probability function (CDF) of ω_i, the random daily travel distance of consumer i. Note that f_i and F_i are assumed to be time-independent for simplicity. For any given consumer i and vehicle j, we need to distinguish the trips within the inner circle ($\omega_i \le r_j$), longer than r_j but shorter than the diameter of the city ($r_j < \omega_i \le L$) and beyond L ($\omega_i > L$, or intercity trips). The average daily

[7] This assumption, which implies the universal availability of home charging, can be easily relaxed.

[8] It is widely held that intracity charging facilities should be placed where the PEV users are likely to stay for an extensive period of time (e.g., workplace, shopping malls or recreational facilities, Pound, 2012; Ghamami et al., 2014).

distance of consumer i is

$$\begin{aligned} R_i &= \int_0^{\infty} f_i(\omega)\omega d\omega \\ &= \int_0^{r_j} f_i(\omega)\omega d\omega + \int_{r_j}^{L} f_i(\omega)\omega d\omega + \int_L^{\infty} f_i(\omega)\omega d\omega \\ &\equiv R_{ij}^0 + R_{ij}^1 + R_i^2. \end{aligned} \tag{4}$$

R_{ij}^0 is the portion of the average distance contributed by trips shorter than r_j—for PEVs, this distance is traveled on electricity initially charged at home. For users who drive CGV, $R_{ij}^0 = 0$ since $r_j = 0$. R_{ij}^1 is the portion of the average distance contributed by trips longer than r_j and shorter than L. The portion of average distance traveled using energy other than electricity charged at home (either gasoline or public charging) for intracity trips, denoted as S_{ij}^1, is

$$S_{ij}^1 = \int_{r_j}^{L} f_i(\omega)(\omega - r_j)d\omega = R_{ij}^1 - r_j(F_i(L) - F_i(r_j)). \tag{5}$$

Similarly, the portion of average distance traveled using energy other than electricity charged at home for intercity trips

$$S_{ij}^2 = \int_L^{\infty} f_i(\omega)(\omega - r_j)d\omega = R_i^2 - r_j(1 - F_i(r_j)). \tag{6}$$

For all EV users, the total daily distance traveled on electricity depends on the charging facility. Let $\lambda_l^y \in [0, 1], l = 1, 2$ be the probability of finding charging for intracity ($l = 1$) and intercity trips ($l = 2$) in year y, and u_l^y be the number of chargers newly placed in year y. The cumulative number of chargers at location l in year y is

$$x_l^y = x_l^{y-1} + u_l^y. \tag{7}$$

According to Assumption 8, the probability of successfully finding public charging is

$$\lambda_l^y = \frac{x_l^y}{\kappa_l}, \tag{8}$$

where κ_l is the maximum number of charging stations at location $l = 1, 2$ that provides full charging accessibility, which is defined as being similar to the level of refueling accessibility experienced by conventional gasoline vehicles. Moreover, for EV users, the portions of the average distance that cannot use electricity are

$$A_{ij}^{ly} = S_{ij}^l(1 - \lambda_l^y), l = 1, 2 \tag{9}$$

Based on Assumption 7, PHEV users will only seek charging for intracity trips. Thus, $A_{ij}^{ly} = S_{ij}^l$ for $l = 2$ and $j = 2$. BEV users, on the other hand, may not be able to complete their trip when the following two events occur simultaneously: (1) the trip distance is larger than r_j, and (2) extending the range through public charging is impossible. The joint probability of these two events is $\mu_{ij}^1(1 - \lambda_1^y) + \mu_i^2(1 - \lambda_2^y)$, where[9]

$$\mu_{ij}^1 = \int_{r_j}^{L} f_i(\omega)d\omega; \quad \mu_{ij}^2 = \int_L^{\infty} f_i(\omega)d\omega; \quad j = 2, 3. \tag{10}$$

When it happens, we assume that alternative transportation (e.g. alternative vehicles, transit) will be used to fulfill the travel need on that day.

We proceed to estimate the parameters $\kappa_l, l = 1, 2$. Let us define the total population in the base year as $N = \sum_i Q_i^0$. Assumption 5 dictates that the population is uniformly distributed in imaginary cities of circular shape with an identical radius of $L/2$. For a given number of charging station n uniformly located in the city, the expected travel distance from home to a station is roughly (see Module 5, Daganzo, 2010)

$$d = 0.5\sqrt{\frac{\pi L^2}{4n}}.$$

Denoting the expected home-station distance that gives full accessibility as d_m (which is treated as an exogenous parameter), we can estimate

$$\kappa_1 = n_c \frac{\pi L^2}{16 d_m^2}. \tag{11}$$

[9] The formula assumes $r_j < L$. If $r_j > L$, $\mu_{ij}^1 = 0$ and $\mu_{ij}^2 = \int_{r_j}^{\infty} f_i(\omega)d\omega$.

For trips outside of city, we estimate the number of needed charging station based on per-capital length of state highway, denoted as σ (e.g. US currently has about 160,000 miles highway for a population of about 300 million). Let s_m be the average spacing between stations that achieves full accessibility. We can then estimate

$$\kappa_2 = \frac{\sigma N}{s_m}.$$

Clearly, it is reasonable to require

$$x_l^y \le \kappa_l, \quad l = 1, 2. \tag{12}$$

3.2. Vehicle choice model

We assume that consumer i's utility of choosing vehicle j can be estimated as

$$U_{i,j}^y = \beta_j^0 + \beta_i^p \left(\frac{p_j^y - s_j^y - \xi_j^y}{w_i^y} \right) + \beta_i^g \frac{\hat{g}_{ij}^y}{w_i^y} + \beta_i^t \frac{\hat{t}_{ij}^y}{w_i^y} + \beta_i^c \frac{\hat{c}_{ij}^y}{w_i^y} + \sum_l \beta_{jl}^d \lambda_l^y, \tag{13}$$

where p_j^y is the purchase price of vehicle j (with a subsidy s_j^y and a terminal value ξ_j^y), $w_i^y = 40 \times 52 \times \bar{w}_i^y$ is the annual average income for consumer i (with $\bar{w}_i^y$ being hourly wage rate), $\hat{g}_{ij}^y$ is the income-weighted aggregate fuel cost of using vehicle j by consumer i in model year y, $\hat{t}_{ij}^y$ is the income-weighted estimated refueling time cost of using vehicle j by consumer i, and $\hat{c}_{i,j}^y$ is the income-weighted carbon footprint cost of using vehicle j by consumer i, and finally $\beta_j^d \lambda_l^y$ measures the utility of charging density. We note that β_i^p, β_i^g, β_i^t and β_i^c should all have a negative sign, and β_{jl}^d should have a positive sign. β_j^0 is a vehicle-specific constant that encapsulate all "hidden" attributes. This constant will be calibrated against the observed market share, as detailed in Section 5.1.

Accordingly, the probability of choosing vehicle j using the logit formula (assuming Independence of Irrelevant Alternatives, or IIA, see e.g. Ben-Akiva and Lerman, 1985) is estimated by

$$P_{ij}^y = \frac{e^{U_{ij}^y}}{\sum_{j'} e^{U_{ij'}^y}}. \tag{14}$$

Now we proceed to define $\hat{g}_{ij}^y$, $\hat{t}_{ij}^y$ and $\hat{c}_{ij}^y$ in (13). Note that

$$\hat{g}_{ij}^y = \sum_{y'=y}^{y+l_j} \hat{g}_{ij}^{yy'}; \quad \hat{t}_{ij}^y = \sum_{y'=y}^{y+l_j} \hat{t}_{ij}^{yy'}; \quad \hat{c}_{ij}^y = \sum_{y'=y}^{y+l_j} \hat{c}_{ij}^{yy'},$$

where $\hat{g}_{ij}^{yy'}$ is the estimated fuel cost in year y' for the vehicle purchased in year y. Note that we assume consumers have knowledge of future price of vehicles, but have to rely on the currently available charging facility to estimate the vehicle's total electricity mileage.

$$\hat{g}_{ij}^{yy'} = \begin{cases} R_i h_o^{y'} & j=1 \\ (A_{ij}^{1y} + S_{ij}^2) h_o^{y'} + (R_i - A_{ij}^{1y} - S_{ij}^2) h_e^{y'} & j=2 \\ h_b(\mu_{ij}^1 (1-\lambda_1^y) + \mu_i^2 (1-\lambda_2^y)) + (R_i - A_{ij}^1 - A_{ij}^2) h_e^{y'} & j=3 \end{cases} \tag{15}$$

In contrast, the true fuel cost of vehicle j in year y is

$$g_{ij}^y = \begin{cases} R_i h_o^y & j=1 \\ (A_{ij}^{1y} + S_{ij}^2) h_o^y + (R_i - A_{ij}^{1y} - S_{ij}^2) h_e^y & j=2 \\ h_b(\mu_{ij}^1 (1-\lambda_1^y) + \mu_i^2 (1-\lambda_2^y)) + (R_i - A_{ij}^{1y} - A_{ij}^{2y}) h_e^y & j=3 \end{cases} \tag{16}$$

where h_e^y is the electricity price in year y, h_o^y is price of gasoline in year y, and h_b is the cost of finding a backup vehicle to complete the trip (estimated for each day). Similarly

$$\hat{t}_{ij}^{yy'} = \begin{cases} 0, & j=1,2 \\ S_{ij}^2 \lambda_2^y \alpha_i^{y'} \bar{w}_i^{y'} & j=3 \end{cases}; \quad t_{ij}^y = \begin{cases} 0, & j=1,2 \\ S_{ij}^2 \lambda_2^y \alpha_i^y \bar{w}_i^y & j=3 \end{cases}; \tag{17}$$

where α_i^y is the charging time per unit distance and $\bar{w}_i^y$is the average wage rate of consumer i in year y. Note that per Assumption 7, the charging time only applies to the distance traveled beyond L by BEV drivers. Finally, the carbon footprint is

$$\hat{c}_{ij}^{yy'} = \begin{cases} R_i h_c^{y'} & j=1 \\ (A_{ij}^{1y} + S_{ij}^2) h_c^{y'} & j=2 \\ (A_{ij}^{1y} + A_{ij}^{2y}) h_c^{y'} & j=3 \end{cases}, \quad c_{ij}^y = \begin{cases} R_i h_c^y & j=1 \\ (A_{ij}^{1y} + S_{ij}^2) h_c^y & j=2 \\ (A_{ij}^{1y} + A_{ij}^{2y}) h_c^y & j=3 \end{cases} \tag{18}$$

where h_c^y is the CO_2 emission cost corresponding to each mile driven (measured by dollar per mile) in year y. Note that we assume the alternative mileage by BEV consumes gasoline.

3.3. Optimization model

Let us now define the main components of the system costs used in the optimization model. The total purchase subsidy required is

$$M = \sum_{y=1} \sum_{i} \sum_{j} V_{ij}^{y} s_{j}^{y}. \tag{19}$$

The total fuel cost is

$$G = \sum_{y=1}^{Y} \sum_{i} \sum_{j} Q_{ij}^{y} g_{ij}^{y}. \tag{20}$$

The total investment on charging stations is

$$I = \sum_{y} \sum_{l=1}^{2} u_{l}^{y} h_{pl}^{y}, \tag{21}$$

where h_{pl}^{y} is the unit cost of building one charging station corresponding to location l in year y.

The total consumer time spent on charging can be represented as

$$T = \sum_{y} \sum_{i} \sum_{j} Q_{ij}^{y} t_{ij}^{y}, \tag{22}$$

and finally the total cost of carbon emission is

$$C = \sum_{y} \sum_{i} \sum_{j} Q_{ij}^{y} c_{ij}^{y}. \tag{23}$$

The optimization model can be formulated as follows:

$$\min z = \gamma_g G + \gamma_t T + \gamma_c C = \sum_{y} \sum_{i} \sum_{j} \left(Q_{ij}^{y} \times (\gamma_g g_{ij}^{y} + \gamma_t t_{ij}^{y} + \gamma_c c_{ij}^{y}) \right) \tag{24a}$$

$$\text{subject to:} \quad x_l^y = x_l^{y-1} + u_l^y, \quad \forall l = 1, 2; \quad y = 1, 2, \ldots \tag{24b}$$

$$Q_{ij}^{y} = \sum_{y'=y-l_j+1}^{y} V_{ij}^{y'}, \quad \forall i, j, \quad y = 1, 2, \ldots \tag{24c}$$

$$V_{ij}^{y} = \left(q_i^y + \sum_{j} V_{ij}^{y-l_j+1} \right) P_{ij}^{y}, \quad \forall i, j, \quad y = 1, 2, \ldots \tag{24d}$$

$$x_l^Y \le \kappa_l, l = 1, 2, \ldots \tag{24e}$$

$$M + I = \sum_{y} \sum_{i} \sum_{j} V_{ij}^{y} s_j^{y} + \sum_{y} \sum_{l} u_l^y h_{pl}^y \le B \tag{24f}$$

$$s_j^y \ge 0, \forall j, y; \quad x_l^y \ge 0 \quad \forall l, y \tag{24g}$$

The objective function is a weighted cost inclusive of fuel, charging time and CO_2 costs.[10] The decision maker has the flexibility of adjusting the coefficients γ_g, γ_t and γ_c to reflect how it values each of the cost components. For instance, setting $\gamma_c = 0, \gamma_t = 0, \gamma_c = 1$ leads to an aggressive emission reduction policy, whereas setting $\gamma_c = 1, \gamma_t = 1, \gamma_c = 1$ represents a more balanced approach. It is not the purpose of this paper to promote any particular objective function, however.

Constraints (24b) and (24c)–(24d) describe the dynamic evolution of charging stations and vehicles, respectively. Constraints (24e) and (24f) provide the limits on the number of charging stations built and total budget available for incentives, respectively. We note that Constraint (24e) only applies the limit on the last year of the analysis period Y, because meeting the limit in year Y implies the limit must also be met in all $y < Y$.

In the above model, s_j^y and u_l^y are decision variables, and x_l^j, V_{ij}^y and Q_{ij}^y are state variables. Note that x_l^0, $V_{ij}^y, y \le 0$ and $q_i^y, y \le 1$ are given inputs.

4. Solution method

Due to the nature of the market evolution dynamics and logit-based choice model, the optimization problem (24) is not only highly nonlinear, but also very likely non-convex. Consequently, it is difficult to establish the uniqueness of a global optimal solution. However, to support policy-making, a satisfactory improvement over existing decisions is probably

[10] The vehicle price is not included here because we assume that AFV-related polices target “social costs”, such as emission, gasoline consumption and user time. Yet, the cost of acquiring vehicles can be easily incorporated into the formulation.

sufficient in most cases. Therefore, a steepest descent direction algorithm is proposed here for finding a local optimum. We note that, to avoid being trapped in a miserable local optimum, a modeler can always try different starting points until a satisfactory solution is obtained.

4.1. Derivatives

We first derive the derivative of the objective function z with respect to $u_l^e, l = 1, 2, e = 1, \dots, Y,$ and $s_k^t, k = 1, \dots J, e = 1, \dots, Y$. Note that

$$\frac{\partial z}{\partial u_l^e} = \sum_y \sum_i \sum_j \left(\frac{\partial Q_{ij}^y}{\partial u_l^e} (\gamma_g g_{ij}^y + \gamma_t t_{ij}^y + \gamma_c c_{ij}^y) + Q_{ij}^y \left(\frac{\partial (\gamma_g g_{ij}^y + \gamma_t t_{ij}^y + \gamma_c c_{ij}^y)}{\partial u_l^e} \right) \right) \tag{25}$$

$$\frac{\partial Q_{ij}^y}{\partial u_l^e} = \frac{\sum_{y'=y-l_j+1}^{y} \partial V_{ij}^{y'}}{\partial u_l^e} \tag{26}$$

We first examine the derivative of V_{ij}^y with respect to u_l^e. Note that the derivative is zero whenever $y < e$. From Constraint (24d) we know that

$$V_{ij}^y = \left(q_i^y + \sum_j V_{ij}^{y-l_j+1} \right) P_{ij}^y$$

Thus,

$$\frac{\partial V_{ij}^y}{\partial u_l^e} = \frac{\partial \sum_j V_{ij}^{y-l_j+1}}{\partial u_l^e} P_{ij}^y + \left(q_i^y + \sum_j V_{ij}^{y-l_j+1} \right) \frac{\partial P_{ij}^y}{\partial u_l^e}$$

The reader is referred to Appendix B for details about how to compute $\frac{\partial V_{ij}^y}{\partial u_l^e}, \frac{\partial g_{ij}^y}{\partial u_l^e}, \frac{\partial c_{ij}^y}{\partial u_l^e}$, and $\frac{\partial t_{ij}^y}{\partial u_l^e}$ in closed forms. To compute the derivative with respect to s_k^e, note

$$\frac{\partial z}{\partial s_k^e} = \sum_y \sum_i \sum_j \left(\frac{\partial Q_{ij}^y}{\partial s_k^e} (\gamma_g g_{ij}^y + \gamma_t t_{ij}^y + \gamma_c c_{ij}^y) + Q_{ij}^y \left(\frac{\partial (\gamma_g g_{ij}^y + \gamma_t t_{ij}^y + \gamma_c c_{ij}^y)}{\partial s_k^e} \right) \right), \tag{27}$$

$$\frac{\partial Q_{ij}^y}{\partial s_k^e} = \frac{\sum_{y'=y-l_j+1}^{y} \partial V_{ij}^{y'}}{\partial s_k^e}; \quad \frac{\partial (\gamma_g g_{ij}^y + \gamma_t t_{ij}^y + \gamma_c c_{ij}^y)}{\partial s_k^e} = 0, \tag{28}$$

where

$$\frac{\partial V_{ij}^y}{\partial s_k^e} = \frac{\partial \sum_j V_{ij}^{y-l_j+1}}{\partial s_k^e} P_{ij}^y + \left(q_i^y + \sum_j V_{ij}^{y-l_j+1} \right) \frac{\partial P_{ij}^y}{\partial s_k^e}.$$

First, note that $\frac{\partial V_{ij}^y}{\partial s_{j'}^e} = 0$ whenever $e \neq y$; that is, the subsidy provided to a vehicle type j in year y can only affect the vehicle choice in that year. Second,

$$\frac{\partial P_{ij}^y}{\partial s_k^e} = \sum_{j'=1}^{J} \frac{\partial P_{ij}^y}{\partial U_{ij'}^y} \frac{\partial U_{ij'}^y}{\partial s_k^e},$$

where $\frac{\partial P_{ij}^y}{\partial U_{ij'}^y}$ is given in Eq. (41) and

$$\frac{\partial U_{ij'}^y}{\partial s_k^e} = -\frac{\beta_i^p}{w_i^y}.$$

Finally the derivative of M and I with respect to the decision variables can be evaluated as follows:

$$\frac{\partial M}{\partial u_l^e} = \sum_{y'=e}^{y} \sum_i \sum_j \frac{\partial V_{ij}^{y'}}{\partial u_l^e} s_k^{y'}; \quad \frac{\partial M}{\partial s_k^e} = \sum_y \sum_i \sum_j \frac{\partial V_{ij}^y s_j^y}{\partial s_k^e} = \sum_i \left(V_{ik}^e + \sum_j \frac{\partial V_{ij}^e}{\partial s_k^e} s_j^e \right) \tag{29a}$$

$$\frac{\partial I}{\partial s_k^e} = 0; \quad \frac{\partial I}{\partial u_l^e} = h_{pl}^e \tag{29b}$$

We close this section by noting that the derivatives defined in (25) and (27) can be evaluated in a single ascending pass of time, along with all the state variables.

4.2. A steepest descent direction algorithm

To solve (24) we first construct its Lagrangian by dualizing the two capacity constraints (24e) and (24f). Associating Constraint (24f) with multiplier γ and (24e) with multiplier ρ_l, the Lagrangian is written as (note that the dynamic evolution conditions are incorporated implicitly in the objective function):

$$\mathcal{L} = z + \gamma(M + I - B) + \sum_l \rho_l \left(x_l^Y - \kappa_l\right). \tag{30}$$

The Karush–Kuhn–Tucker (KKT) conditions are

$$\frac{\partial \mathcal{L}}{\partial s_k^e} \equiv \frac{\partial z}{\partial s_k^e} + \gamma \frac{\partial M}{\partial s_k^e} \geq 0 \qquad s_k^e \frac{\partial \mathcal{L}}{\partial s_k^e} = 0; \quad \forall k, e \tag{31a}$$

$$\frac{\partial \mathcal{L}}{\partial u_l^e} \equiv \frac{\partial z}{\partial u_l^e} + \rho_l + \gamma \left(h_{pl}^e + \frac{\partial M}{\partial u_l^e}\right) \geq 0 \qquad u_l^e \frac{\partial \mathcal{L}}{\partial u_l^e} = 0; \quad \forall l, e \tag{31b}$$

$$\gamma(M + I - B) = 0; \quad \gamma \geq 0; \quad M + I - B \leq 0 \tag{31c}$$

$$\rho_l\left(x_l^Y - \kappa_l\right) = 0; \quad \rho_l \geq 0; \quad x_l^Y - \kappa_l \leq 0; \quad \forall l \tag{31d}$$

The following result gives regularity conditions under which KKT conditions guarantee at least a local minimum.

Proposition 1 (Necessity)**.** *If* $[s^*, u^*]$ *is a local minimum to Problem (24) that satisfies one of the following conditions: (i) Constraint (24f) is not binding; (ii) there exists at least one* $s_j^{y*} > 0$*; or (iii) Constraint (24e) is not binding for* $l = 1$ *and* $l = 2$*; then* $[s^*, u^*]$ *must also satisfy the KKT conditions (31).*

Proof. See Appendix A. □

We now propose a specialized algorithm that makes use of the gradient of $\mathcal{L}$ and the KKT conditions to guide the descending course from an initial solution to a local optimum. The basic algorithmic idea can be described as follows.

In each iteration, we first find the decision variable that, when deviated from its current value but kept within the feasible set, promises the maximum possible reduction in the objective function (24a). Let Γ_s^+ and Γ_s^- be defined as

$$\Gamma_s^+ = \left\{(j, y), \text{s.t.} \frac{\partial \mathcal{L}}{\partial s_j^y} > 0\right\}; \quad \Gamma_s^- = \left\{(j, y), \text{s.t.} \frac{\partial \mathcal{L}}{\partial s_j^y} < 0\right\}, \tag{32}$$

Similarly, we can define

$$\Gamma_u^+ = \left\{(l, y), \text{s.t.} \frac{\partial \mathcal{L}}{\partial u_l^y} > 0\right\}; \quad \Gamma_u^- = \left\{(l, y), \text{s.t.} \frac{\partial \mathcal{L}}{\partial u_l^y} < 0\right\} \tag{33}$$

The KKT conditions dictate that $\Gamma_s^- = \emptyset$, $\Gamma_u^- = \emptyset$, $s_j^y = 0$ for all $(j, y) \in \Gamma_s^+$ and $u_l^y = 0$ for all $(l, y) \in \Gamma_u^+$. For each of the above four sets, the deviation and the maximum possible reduction in the objective can be defined respectively as follows:

$$\Delta s_j^y = -\min\left(s_j^y, \tau \frac{\partial \mathcal{L}}{\partial s_j^y}\right); dz_s^+ = \left\{\min\left(-\frac{\partial z}{\partial s_j^y} \times \Delta s_j^y\right), \forall (j, y) \in \Gamma_s^+\right\} \tag{34}$$

where τ is a predetermined step size, and $\min(s_j^y, \tau \frac{\partial \mathcal{L}}{\partial s_j^y})$ defines the maximum possible reduction in s_j^y at the given step size and the gradient.

$$\Delta s_j^y = -\tau \frac{\partial \mathcal{L}}{\partial s_j^y}; \quad dz_s^- = \left\{\min\left(\frac{\partial z}{\partial s_j^y} \times \Delta s_j^y\right), \forall (j, y) \in \Gamma_s^-\right\} \tag{35}$$

$$\Delta u_l^y = -\min\left(u_l^y, \tau \frac{\partial \mathcal{L}}{\partial u_l^y}\right); \quad dz_u^+ = \left\{\min\left(-\frac{\partial z}{\partial u_l^y} \times \Delta u_l^y\right), \forall (l, y) \in \Gamma_u^+\right\} \tag{36}$$

$$\Delta u_l^y = \min\left(\kappa_l - x_l^Y, -\tau \frac{\partial \mathcal{L}}{\partial u_l^y}\right); \quad dz_u^- = \left\{\min\left(\frac{\partial z}{\partial u_l^y} \times \Delta u_l^y\right), \forall (l, y) \in \Gamma_u^-\right\} \tag{37}$$

where again τ is the step size and $\kappa_l - x_l^Y$ represents the maximum additional number of chargers allowed at location l. In the next, we find $\min(dz_s^-, dz_u^-, dz_s^+, dz_u^+)$ and adjust the corresponding decision variables to get a new solution. The algorithm is terminated when the solution is sufficiently close to a KKT point or the maximum number of iterations is reached. The details of the solution algorithm, in the form of pseudo code, are given in Algorithm 1. The step size τ is pre-determined in this study based on the iteration index n as follows:

$$\tau = \frac{1}{n^\eta} \tag{38}$$

where $\eta \in (0, 1]$ is a parameter.

Algorithm 1 Steepest decent direction algorithm.

Input: Convergence criterion ϵ, maximum number of inner iterations M and main iterations N.
Output: Optimal subsidy s_l^{y*} and optimal number of new charging stations u_l^{y*}.
initialize:
Set main iteration index $n = 0$, the convergence measure $g_r = \infty$, $\gamma = 0$, and $\rho_l = 0, l = 1, 2$. Initialize $s_l^y(0)$, $u_l^y(0)$.
while $g_r > \epsilon$ and $n \le N$ **do**
  Compute V_{ij}^y, Q_{ij}^y, x_l^y, as well as all derivatives $\partial z/\partial u_l^y$, $\partial z/\partial s_j^y$, $\partial (M+I)/\partial u_l^y$ and $\partial (M+I)/\partial s_l^y$.
  Compute the relative gap g_r as follows:

$$g_r = \sum_l \sum_y \left| \frac{\partial \mathcal{L}}{\partial u_l^y} u_l^y \right| / \kappa_l + \sum_j \sum_y \left| \frac{\partial \mathcal{L}}{\partial s_j^y} s_j^y \right| / (M+I) + \gamma |M+I-B|/B + \sum_l \rho_l |x_l^Y - \kappa_l| / \kappa_l \tag{48}$$

  Compute $dz_s^-, dz_u^-, dz_s^+, dz_u^+$ and the corresponding deviations Δu_l^y and Δs_l^y.
  if dz_s^- or $dz_s^+ = \min(dz_s^-, dz_u^-, dz_s^+, dz_u^+)$ **then**
    Set the corresponding $s_j^y(n+1) = s_j^y(n) + \Delta s_j^y$, where Δs_j^y is defined in (34) or (35).
    Estimate the change in the budget by $\Delta(M+I) = \frac{\partial M+I}{\partial s_j^y} \Delta s_j^y$
    if $M + I + \Delta(M+I) \le B$ **then** set $\gamma = 0$;
    else
      set (as per KKT conditions (31)) $\gamma = -\frac{\partial z}{\partial s_j^y} / \frac{\partial M}{\partial s_j^y}$, $\rho_l = -\frac{\partial z}{\partial u_l^e} - \gamma \left(h_{pl}^e + \frac{\partial M}{\partial u_l^e} \right), l = 1, 2$
    end if
  else
    Set the corresponding $u_l^y(n+1) = u_l^y(n) + \Delta u_l^y$, where Δu_l^y is defined in (36) or (37).
    Estimate the change in the budget by $\Delta(M+I) = \frac{\partial M+I}{\partial u_l^y} \Delta u_l^y$
    if $M + I + \Delta(M+I) \le B$ **then** set $\rho = 0$
    else
      set $\gamma = -\frac{\partial z}{\partial s_j^y} / \frac{\partial M}{\partial s_j^y}$
    end if
    if $x_l^Y + \Delta u_l^y < \kappa_l$ **then** set $\rho_l = 0$
    else
      set $\rho_l = -\frac{\partial z}{\partial u_l^e} - \gamma \left(h_{pl}^e + \frac{\partial M}{\partial u_l^e} \right)$
    end if
  end if
  Set $n = n + 1$, update step size according to Eq. (38).
end while
Set $s_j^{y*} = s_j^y(n-1)$, $u_l^{y*} = u_l^y(n-1)$.

5. Numerical experiments

In this section a hypothetical case study is constructed to test the proposed optimization model. Our focus here is not to produce a realistic model ready for decision making support, but rather to prove the concept and demonstrate its potential. In the following, we first introduce the setting of the case study and how the necessary input parameters are obtained. Then, the optimization model is tested under different scenarios. For each scenario, the (locally) optimal incentive policies produced by the optimization model are compared against the state-of-the-practice policies. Finally, a sensitivity analysis is conducted to examine the sensitivity of the optimal solution to energy price and environmental cost.

5.1. Case study

The case study models the choice of three types of vehicles, namely CGV ($j = 1$), PHEV ($j = 2$) and EV ($j = 3$), in a period of $Y = 30$ years. The main characteristics of the vehicles in the base year are reported in Table 1.

Consumers are differentiated exclusively by their travel pattern. Specifically, we divide consumers into three classes: modest drivers ($i = 1$), average drivers ($i = 2$), and frequent drivers ($i = 3$). Fig. 2 shows the probability density functions (PDF) of each consumer class' daily trip distance, directly adopted from the MA3T model (Lin and Greene, 2010). Based on the MA3T model, we estimate the average ratio between the populations of the three classes of consumers is 0.35:0.33:0.32. Without loss of generality, the total population of all drivers at the base year is set at 1,000,000. Table 2 summarizes

Table 1
Vehicle characteristics in base year.

Vehicle type	CGV $j=1$	PHEV $j=2$	BEV $j=3$
Price (p_j^0) in $[a]	21,000	35,000	31,000
Terminal value (ξ_j^0) in $	2000	3000	2500
Electric range (r_j^0) in mile	0	20	75
Gas efficiency in gallon/mile[a]	0.03	0.03	0
Electricity efficiency in kWh/mile[a]	0	0.23	0.23
CO_2 emission in kg/mile[b]	0.5	0.5	0
Life (l_j^0)	10	10	10

[a] MA3T model inputs, see http://cta.ornl.gov/ma3t/, last visited: 11-21-2014.
[b] US Environmental Protection Agency, http://www.epa.gov/cleanenergy/energy-resources/refs.html, last visited: 07-31-2014. We note that these are tailpipe emission rate (tank-to-wheels), not full fuel cycle (well to wheels) rate. However, upstream emission may be taken into account by changing these parameters.

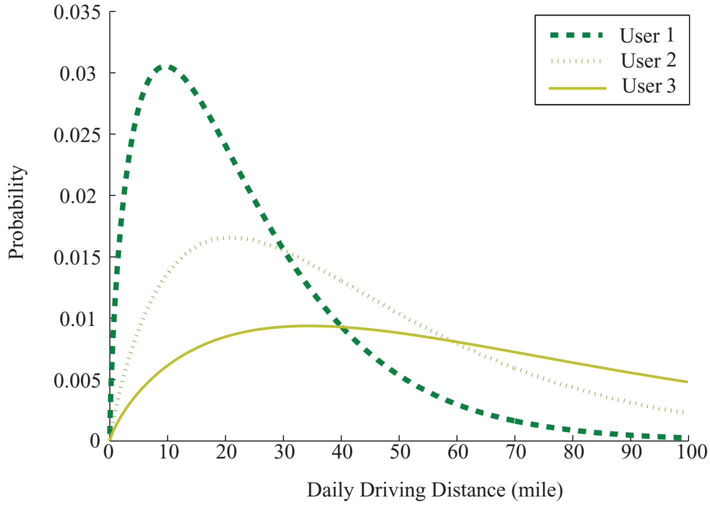


Fig. 2. Probability density functions of daily trip length for different consumer classes. All distributions are fit to a gamma distribution with the following parameters. User 1: Modest traveler (mean = 23.47 miles, variance = 334.5 mile^2), User 2: Average traveler (mean = 40 miles, variance = 900 mile^2, User 3: Frequent traveler (mean = 75 miles, variance = 3200 mile^2).

the values of all other input parameters adopted in the case study, along with sources used to estimate them, whenever applicable.

The coefficients in the vehicle choice model have critical influences on the outcomes of the model. Usually, they should be calibrated from empirical behavioral data. In this study, these coefficients are synthesized from the literature as follows. We start by setting the coefficient for vehicle price $\beta_i^p = -1$. Then, the other coefficients are determined based on their relative magnitude against β_i^p, as revealed in the literature. Tables 3 and 4 report the consumer- and vehicles-specific coefficients used in this case study, respectively. Details regarding how these values are determined can be found in Appendix C.

Once all the parameters are set, the constants β_j^0 are calibrated so as to replicate the current aggregated market share of the three types of vehicles in the base year. In this study, the initial market share is assumed to be $Q_{i1}^0 : Q_{i2}^0 : Q_{i3}^0 = 0.92 : 0.07 : 0.01, \forall i$. In reality, the market share for different classes of consumers may vary. However, no data is readily available that differentiates the aggregate market share for the consumers types defined in this paper. The least squares method (see Section 5.5, Ben-Akiva and Lerman, 1985) is used to perform the calibration and the results are reported in the last row in Table 4.

Finally, unless otherwise specified, $\gamma_g = \gamma_t = \gamma_c = 1.0$. In this case, the objective function may be considered a surrogate of "social cost" associated with vehicle use.

5.2. Optimal policy vs. alternative policies

In this section we compare the optimal policies provided by the proposed model with the following incentive policies.

Table 2
Other input parameters.

Definition	Value	Unit
Energy price		
Gasoline[a]	3.2	(\$/gallon)
Electricity[b]	0.08	(\$/kWh)
CO_2[c]	200	(\$/ton)
Wage rate ($\bar{w}_i^0$)[d]	15	(\$/h)
Cost of acquiring alternative transport[a]	30	(\$/time)
Charging-related		
Charger installation cost[e]	500	(\$/kW)
Charge power[e]	50	kW
Fixed charge station construction cost[e]	150,000	(\$)
Number of chargers per station	4	–
Miscellaneous		
Diameter of the imaginary city (L)	50	(miles)
Full-accessibility home-station distance for intracity (d_m)	2	(mile)
Full-accessibility station space for intercity(s_m)	10	(mile)
Average population density(ζ)	500	(person/mile^2)
Per-capita highway millage[f]	0.0005	(mile/person)

[a] Administration (2014).
[b] MA3T model inputs, see http://cta.ornl.gov/ma3t/, last visited: 11-21-2014.
[c] See Appendix B for the estimation of CO_2 price.
[d] The base year wage rate is estimated from per capita income in US, which is close \$30,000 in 2013, see e.g. http://www.deptofnumbers.com/income/us/, last visited: 12-15-2014.
[e] See Nie and Ghamami (2013). A charging station with a power of 50 kW belongs to Type III (or fast) charging facility.
[f] Estimated based on the total US population and the total length of the national highway system (approximately 160,000 mile, see http://www.fhwa.dot.gov/ohim/hs01/hm41.htm, last visited: 12-15-2014).

Table 3
Consumer-specific coefficients in the vehicle choice model.

	β_i^p (capital)	β_i^g (fuel)	β_i^c (CO_2)	β_i^t (time)
Default	−1	−0.7	−1	−0.5
Modest ($i = 1$)	−1	−0.5	−1	−0.7
Average ($i = 2$)	−1	−0.7	−1	−0.5
Frequent ($i = 3$)	−1	−0.9	−1	−0.3

Table 4
Vehicle-specific coefficients in the vehicle choice model.

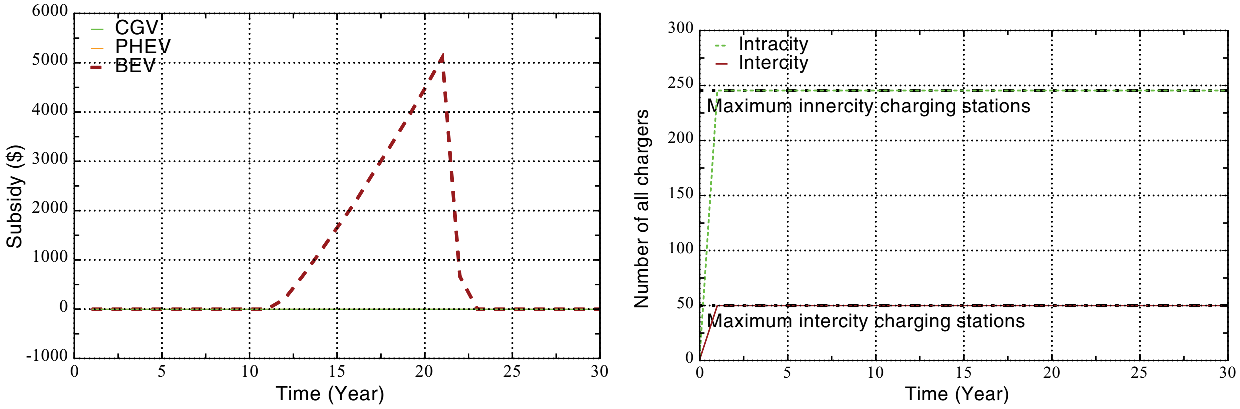
	CGV ($j = 1$)	PHEV ($j = 1$)	BEV ($j = 1$)
β_{j1}^d (charging density (intracity)	0	0.3	0.6
β_{j2}^d (charging density (intercity)	0	0.2	0.4
β_j^0	2.34	−0.37	−1.97

- Current policy (CURRENT): we set \$4000 purchase subsidy for BEVs (s_3^y) and \$2500 subsidy for PHEVs (s_2^y) in $y = 1, \ldots, 10$.[11] The scaled number of charging stations at the base year is set to 4 for intracity (x_1^0, full accessorily require $\kappa_1 \sim 250$) and 1 for intercity (x_1^0, full accessibility requires $\kappa_2 \sim 50$). The additional number of charging stations built in year $y = 1, \ldots, 30$ is 2.6 for intracity (u_1^1) and 0.5 for intercity (u_2^1), with an annual growth rate of 0.1%.[12]
- Zero investment policy (ZERO): this policy is a do-nothing policy, introduced to benchmark the efficacy of investment.
- Higher subsidy policy (HISUB): this policy deviates from CURRENT in that the subsidies to BEV are increased to \$10,000.[13]

[11] Existing subsides for new EV buyers range between \$2500 and \$7500, see http://www.irs.gov/Businesses/Plug-In-Electric-Vehicle-Credit-28IRC-30-and-IRC-30D29, last visited: 12-03-2014. On average the states are providing \$2500 to \$4000 subsidy to PEV owners, see http://www.afdc.energy.gov/laws/state, last visited: 12-03-2014.

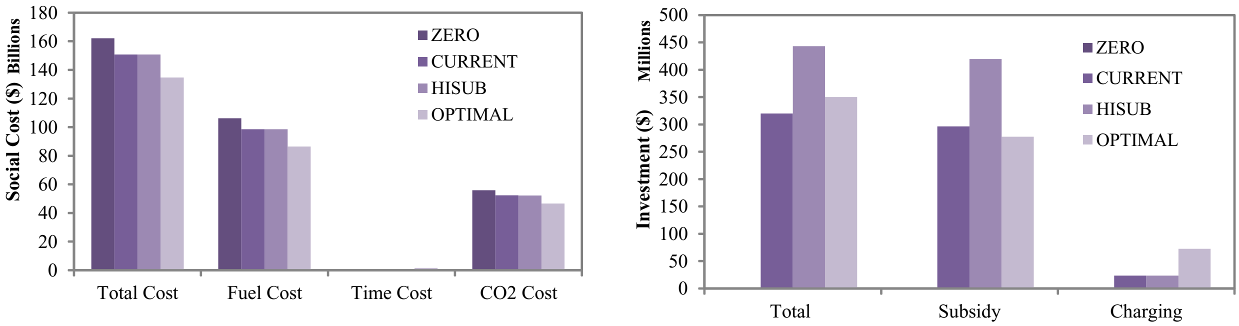
[12] The charging station data provided by US Department of Energy was used to derive the initial number of charging stations, annual increase and growth rate, see http://www.afdc.energy.gov/data/10332, last visited: 12-03-2014. Since the available data is for the entire US, they were scaled based on the ratio between the study area (estimated from the total population and population density) and the land area of the US. Then, the charging stations are distributed among intercity and intracity based on κ_1/κ_2.

[13] It has been proposed that the maximum subsidy to plug-in EVs be increased from \$7500 to \$10,000, see http://content.usatoday.com/communities/driveon/post/2012/02/president/-obama-budget-/electric-car-/subsidies-chevrolet-volt/1#.VICyEb64lcw, last visited: 12-03-2014.



(a) Purchase subsidy (b) Cumulative charge stations

Fig. 3. Optimal policy in the BASE scenario (OPTIMAL, or minimizing total cost).



(a) Social costs (b) Investment

Fig. 4. Comparison of four incentive policies.

In all cases, the following time-varying changes to the input parameters are introduced (note that inflation effect has been excluded):

- Population (N): an annual growth rate of 0.85% is assumed (as per the population growth data published by The World Bank, 2013).
- Hourly wage rate ($\bar{w}_i^y$): an annual growth rate of 1.2% is assumed (as per National Average Wage Index published by Social Security Administration, 2013).
- Energy prices: an annual increase rate of 3.8% is assumed for the gasoline price, according to MA3T (http://cta.ornl.gov/ma3t/).
- Vehicle price: based on MA3T, the prices of PHEV and BEV are assumed to decrease by an annual rate of 0.9% and 0.6% respectively from the base year price. The price of CGV is assumed to increase by 0.1% each year.

In what follows, the above input setting will be referred to as BASE scenario. In the scenario, all the other parameters are assumed to remain constant throughout the analysis period.

To ensure the comparison is meaningful, the market evolution is first simulated based on the CURRENT policy for the BASE scenario to obtain the cumulative investment. The simulation results suggest the CURRENT policy spent in total $320 per capita, with $296.5 on subsidy, and $23.5 on charging stations. Accordingly, we set the maximum budget for the optimization model as $350 per capita, just slightly higher than the CURRENT policy.

Fig. 3 depicts the optimal policy found by the proposed model. Clearly, building charging stations as many as possible and as early as possible is crucial. In fact, the policy suggests increasing the number of charging stations to the maximum value in the first year. This makes sense intuitively, because a charging station, once built, is there to stay. Therefore, it benefits more if it is built earlier. Fig. 3 also suggests spending the rest of the budget exclusively on subsidizing BEV sales. The optimal solution suggests that the subsidy ought to start around $y = 11$, and linearly increase to slightly more than $5000 (interestingly, this is comparable to the current level of subsidy) before it is discontinued after $y = 22$.

Fig. 4 compares various cost components and investment for the four different policies, with more details presented in Table 5. Compared to the OPTIMAL policy, the “no-nothing” policy (ZERO) increases the total cost by more than 20%, whereas

Table 5
Relative changes in social cost, investment and market shares of alternative policies compared to the OPTIMAL policy.

Policies	ZERO (%)	CURRENT (%)	HISUB (%)
Social cost			
Total	20.42	12.01	11.99
Fuel	22.91	13.94	13.97
Time	−99.72	−95.41	−95.32
CO_2	19.94	12.13	12.03
Investment			
Total	−100	−8.57	26.60
Subsidy	−100	6.85	51.21
Charging station	−100	−67.59	−67.59
Final market share			
CGV	16.22	14.86	14.86
PHEV	−25.00	−25.00	−25.00
BEV	−80.00	−70.00	−70.00

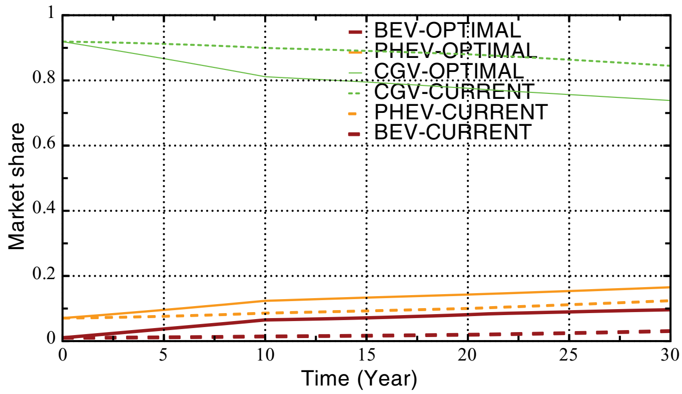


Fig. 5. Total market share evolution under OPTIMAL and CURRENT policies.

for a comparable budget, the CURRENT policy still results in a total cost that is 12% higher. Importantly, with a much larger budget (25% more), the HISUB policy virtually makes no difference in the total cost. Similarly, the three alternative policies also lead to higher market share for CGV and lower market share for PHEV and BEV at the end of the analysis period (see Table 5). For the market share of BEV, in particular, the ZERO and CURRENT policies would lead to a 80% and 70% loss compared to the OPTIMAL policy, respectively.

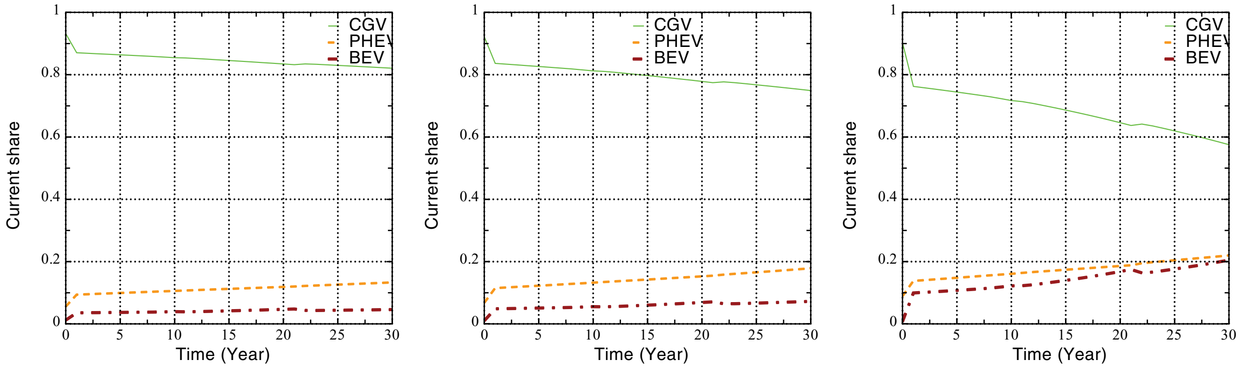
The above observation highlights *the promise of an optimization approach and the potential pitfalls of a well-intended but poorly informed policy such as arbitrarily increasing the purchase subsidy*. Note that the cost reduction brought by the incentive programs mostly come from fuel and CO_2 savings, thanks to the market shift to PHEVs and BEVs. As expected, the time cost actually increases since the adoption of PHEVs and BEVs introduces charging time.

Also worth noting here is the fact that the OPTIMAL policy spends more money on charging stations and less on the subsidy, compared to the CURRENT and HISUB policies. *Giving priority to building more charging stations seems to play an essential role in helping the adoption of the alternative fuel vehicles while reducing the total cost.*

Fig. 5 visualizes the evolution of the overall market share under the CURRENT and OPTIMAL policies. The comparison not only confirms that the OPTIMAL policy outperforms the CURRENT policy in shifting the market share to PHEV and BEV, but also shows that it does so mostly in the first ten years. Notably, the distinct slopes of the OPTIMAL curves before and after $y = 10$ can be attributed to the dramatic increase of charging stations in year $y = 1$ whose effect went away after ten years, when the vehicles bought in $y \geq 1$ began to retire from the fleet. Fig. 6 plots the evolution of the choice probabilities of the three consumers under the OPTIMAL policy. The plot shows that consumers are increasingly in favor of PEVs, as they drive more on average and make long trips with higher frequency. For frequent drivers, the probability of choosing PEVs rises to almost 50% at the end of analysis period. This is expected because the frequent drivers benefit more from the savings in fuel and CO_2 costs brought by electric vehicles.

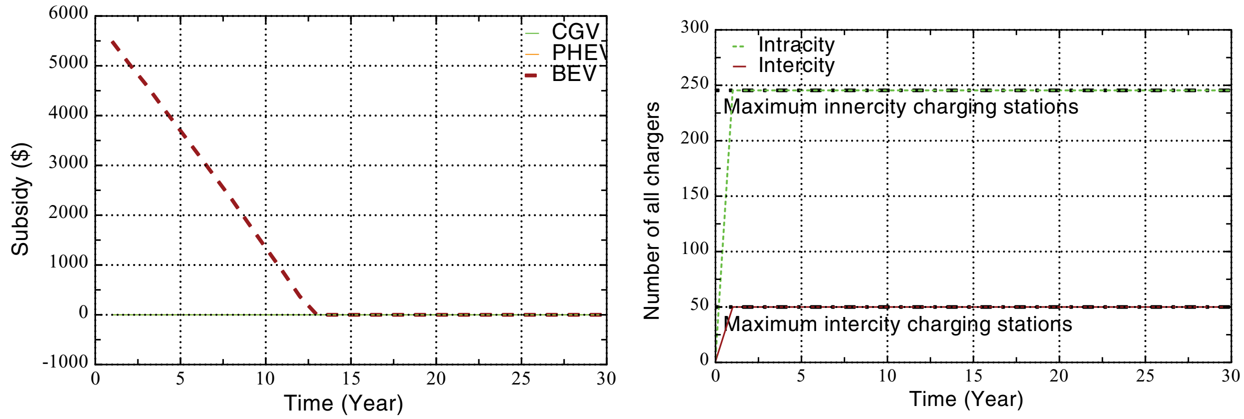
5.3. Impact of "social cost"

The default setting of our experiments is to minimize a weighted social cost that values fuel, CO_2 and time equally ($\gamma_g = \gamma_t = \gamma_c = 1.0$). In this section, we will compare the optimal policy from the default model to one that more aggressively



(a) Modest Driver (b) Average Driver (c) Frequent Driver

Fig. 6. Evolution of consumer-specific vehicle choice probability under the OPTIMAL policy.



(a) Purchase subsidy (b) Cumulative charge stations

Fig. 7. Optimal policy in the BASE scenario (CO_2OPT, or minimizing CO_2 cost).

target environment protection, namely minimizing CO_2 cost only (called CO_2OPT, i.e. $\gamma_g = \gamma_t = 0, \gamma_c = 1$). We emphasize that the purpose here is not to discuss whether a particular objective is appropriate, but rather to demonstrate the model is capable of producing policies based on specified objectives, which gives the decision makers an opportunity to understand the consequences of pursuing certain objectives.

Fig. 7 visualizes the CO_2OPT policy. A comparison with Fig. 3 indicates that the optimal investment policy for charging stations remains unchanged. Yet, the subsidies to BEV have a quite different temporal pattern. Instead of linearly increasing between $11 < y < 22$, the subsidy is now given in the first 12 years and is linearly decreasing from a maximum value of about $5500. As expected, the new policy does lower the CO_2 cost by 0.05% compared to the OPTIMAL policy. However, it also increases the total cost by about 0.05%. In any case, the differences in the costs associated with two objectives are rather small, possibly because fuel and CO_2 are strongly related to each other.

5.4. Computational performance

In this section we examine the computational performance of the proposed algorithm for finding OPTIMAL and CO_2OPT policies. The algorithm was coded in C++ and tested on a Workstation with an Intel Xeon CPU @2.8 GHz and 24 GB RAM. In both cases, the ZERO policy is used as the initial solution. However, it is worth noting that other initial solutions were also tested but differences found in the final solution were practically negligible.

Fig. 8(a) and (b) plots, respectively, objective function values and relative gap in each iteration when Algorithm 1 is used to find OPTIMAL and CO_2OPT polices for the BASE scenario. In both cases, the algorithm was terminated when the relative gap reaches 10^{-6} (the target convergence criterion), which took about 850 iterations for CO_2OPT2 and about 1050 iterations for OPTIMAL. The computation time was less than 30 s in both tests. Whereas the objective function decreases monotonically as the algorithm proceeds, the relative gaps are subject to much greater fluctuations. This is expected since by design the algorithm aims at reducing the objective function value in each iteration following the steepest descent direction. Fig. 8(c) shows how the step size decreases with the number of iterations, which corresponds to $\eta = 0.3$ in Eq. (38). Fig. 8(d) reveals

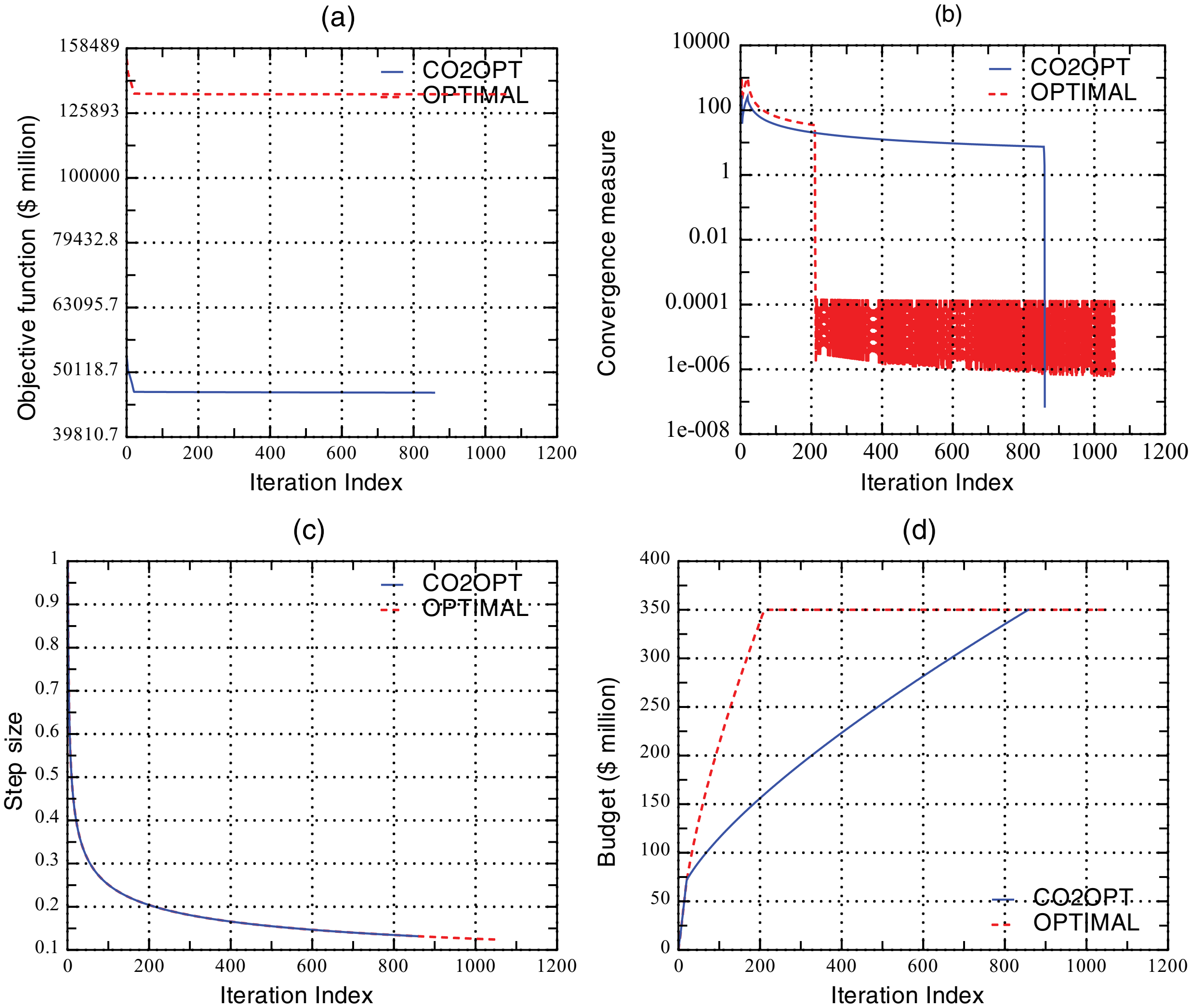


Fig. 8. Computational performance of the proposed algorithm for BASE scenario.

Table 6
Scenarios tested for the sensitivity analysis.

Scenarios	Description
1	BASE scenario
2	Same as BASE except the gasoline prices increases by 7.6% annually (DOUBLEGAS)
3	Same as BASE except the CO_2 price increases by 3.8% annually (CO_2GROWTH)
4	Same as BASE except PHEV sale price decreases by −1.5% annually (PHEVFAST)
5	Same as BASE except BEV sale price decreases by −1% (BEVFAST)

how the total used budget increases from zero to the maximum value ($350 per capita) as the algorithm drives the incentive policy from ZERO to the respective optimums.

5.5. Sensitivity analysis

In this section, we test how sensitive the optimal solution is to the growth rate of the gasoline price, CO_2 price, and the price of electric vehicles. Table 6 shows the details of the five tested scenarios: Scenario 2 doubles the growth rate of the gasoline price in the BASE scenario; Scenario 3 assumes the CO_2 price increases at a rate comparable to that of the gasoline in the BASE scenario; in Scenarios 3 and 4, the prices of PHEV and BEV are assumed to decline at a higher rate than that in the BASE scenario.

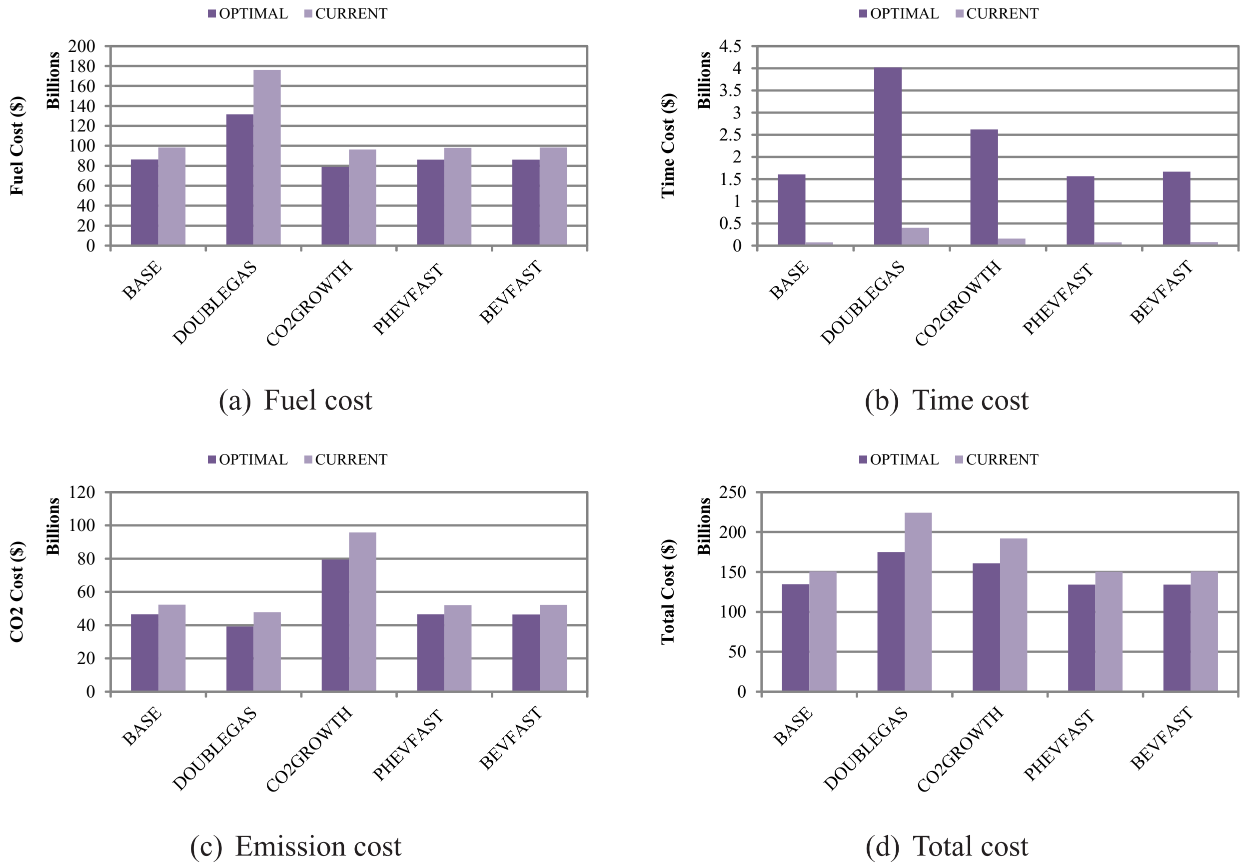


Fig. 9. Social costs of the five scenarios under CURRENT and OPTIMAL policies.

Table 7
Costs and final market shares in the five scenarios under OPTIMAL policy.

	Costs (million dollar)				Final market share		
	Fuel	Time	Emission	Total	CGV	PHEV	EV
BASE	86,417.1	1607.65	46,642.3	134,667	0.74	0.16	0.1
DOUBLEGAS	131,600	4015.83	39,220	174,836	0.46	0.23	0.31
CO_2GROWTH	78,862.4	2619.04	79,482.8	160,964	0.6	0.22	0.18
PHEVFAST	86,158.5	1565.59	46,572.6	134,297	0.73	0.18	0.09
BEVFAST	86,104.4	1666.2	46,488.6	134,259	0.74	0.16	0.1

Fig. 9(a)–(d) compares various cost components in the five scenarios under CURRENT and OPTIMAL policies. First, note that the costs from PHEVFAST and BEVFAST are almost identical to those from BASE, which seems to indicate that a modestly accelerated decline (compared to the current predictions) in electric vehicle prices is unlikely to make any meaningful difference. On the contrary, when the gasoline growth rate is doubled, the total cost increases almost 30% under the OPTIMAL policy, and about 50% under the CURRENT policy. The rise of CO_2 price also has a significant impact, leading to about 20% and 27% increases in the total cost for the OPTIMAL and CURRENT policies, respectively. Fig. 9 also reveals that the OPTIMAL policy seems to create more savings for higher total costs, such as in DOUBLEGAS and CO_2GROWTH scenarios.

Table 7 reports the costs and final market shares in the five scenarios under OPTIMAL policy. For PHEVFAST, PHEV gains about 2% in the market share at the expense of BEV and CGV (each loses 1%). For BEVFAST, the gain by BEV is so small that it is hardly noticeable. On the other hand, DOUBLEGAS creates the largest market shift in favor of BEV: its market share would triple from 10% to 31%, and exceed the share of PHEV. The adoption of BEV also benefits more from the rising price of CO_2. In CO_2GROWTH, BEV gains 8% in the market share while PHEV only gains 6%.

6. Conclusions

In this paper we propose a modeling framework for optimizing publicly funded incentive policies that aim to accelerate the adoption of plug-in electric vehicles (PEVs). The impact of the incentives on the evolution of consumer vehicle market is captured by integrating a simplified vehicle choice model and a macroscopic travel and charging model. The resulting optimization model is formulated as a nonlinear and non-convex program. While finding the global optimum is not guaranteed, we prove that the KKT conditions are necessary for a local optimum under mild regularity conditions. A specialized algorithm is then developed to search for a local optimum. Main findings from a case study that involves three vehicle types and three consumer classes are summarized as follows.

- The proposed algorithm provides satisfactory convergence performance and is reasonably efficient in all tested scenarios. In theory, the algorithm could converge to a different local optimum when starting at different initial solutions. However, we have not observed such a case in the experiments.
- The optimal solution from the proposed model consistently outperforms the alternative policies that mimic the state-of-the-practice by a large margin, in terms of both the total savings in social costs and the market share of electric vehicles. Thus, implementing optimal incentive policies could make meaningful differences in practice.
- In all tested scenarios, the optimal policy always sets the investment priority to building charging stations. In fact, the charging stations should be built up to the level that allows full accessibility as soon as possible. In contrast, providing purchase subsidy, which is a widely used policy, does not seem to be cost efficient. A striking example from the case study shows that increasing subsidy to BEV from $4000 to $10,000 achieves virtually nothing at a cost of about $123,000,000 (or more than 35% increase in the total budget).
- The adoption rate of EV, especially that of BEV, is more sensitive to the fuel price than to the vehicle sale prices. The cost of EV charging time is rather small and does not seem to play an important role in our model.
- The consumers who drive more and make long trips with higher frequency tend to prefer EVs more than others.

This research can be extended in many directions. First of all, the proposed model may have multiple local optimums in theory. It is unclear to what extent the existence of such local optimums would impact the application of the model, e.g. how likely an algorithm such as proposed in this paper would be trapped at a "bad" solution? A future investigation can attempt to better understand the analytical properties of the model, or seek a different modeling approach. As a reviewer of an earlier draft pointed out, dynamic programming may be a good fit since our problems involve "sequential decision making along the temporal dimension". Second, more vehicle types and consumer classes, as well as more sophisticated modeling structures, can be considered in the vehicle choice model. For example, a nested multinomial logit model may be used to capture the commonality between vehicles of similar features. While increasing the number of consumer classes and vehicle options is relatively straightforward, it can bring about significant computational challenges (note that MA3T has over 1000 consumer classes and 39 vehicle types) that may require more efficient and robust solution algorithms. Last but not least, a future study can directly calibrate all coefficients in the vehicle choice model with the same set of empirical data so that it better captures real consumer preferences.

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Appendix A. Proof of Proposition 1

We will show that when one of the conditions (i–iii) is met, Problem (24) satisfies the so-called linear independence constraint quality (LICP), which states that the gradients of the active inequality constraints and the gradients of the equality constraints are linearly independent at the KKT point (Peterson, 1973). First note constraints (24b)–(24d) can be merged into the objective function. Thus, we will focus on the inequality constraints (24e)–(24g). Let us denote

- $a_i = [a_i^1, a_i^2, a_i^3]^T, i = 1, \ldots, \Omega_1$ as the gradient corresponding to the ith active nonnegative constraint for vector u_1; where $a_i^1 = [0, \ldots, 1, \ldots 0]$ and $a_i^2 = [, \ldots, 0, \ldots]$ are both vectors of length Y, and $a_i^3 = [, \ldots, 0, \ldots]$ is a vector of length $J \times Y$.
- $b_i = [b_i^1, b_i^2, b_i^3]^T, i = 1, \ldots, \Omega_2$ as the gradient corresponding to the ith active nonnegative constraint for vector u_2; where $b_i^1 = [, \ldots, 0, \ldots]$ and $b_i^2 = [, \ldots, 1, \ldots]$ are both vectors of length Y, and $a_i^3 = [, \ldots, 0, \ldots]$ is a vector of length $J \times Y$.
- $c_i = [c_i^1, c_i^2, c_i^3]^T, i = 1, \ldots, \Omega_3$ as the gradient corresponding to the ith active nonnegative constraint for vector s; where $c_i^1 = [, \ldots, 0, \ldots]$ and $c_i^2 = [, \ldots, 0, \ldots]$ are both vectors of length Y, and $c_i^3 = [, \ldots, 1, \ldots]$ is a vector of length $J \times Y$.
- $d = [d^1, d^2, d^3]^T$ as the gradient corresponding to Constraint (24e) of location 1; where $d^1 = [1 \ldots, 1, \ldots 1]$ and $d^2 = [, \ldots, 0, \ldots]$ are both vectors of length Y, and $d^3 = [, \ldots, 0, \ldots]$ is a vector of length $J \times Y$.

- $e = [e^1, e^2, e^3]^T$ as the gradient corresponding to Constraint (24e) of location 2; where $e^1 = [0\ldots,0,\ldots 0]$ and $e^2 = [1,\ldots,1,\ldots 1]$ are both vectors of length Y, and $e^3 = [,\ldots,0,\ldots]$ is a vector of length $J \times Y$.
- $f = [f^1, f^2, f^3]^T$ as the gradient corresponding to Constraint (24f); where $f^1 = [\frac{\partial M+I}{\partial u_1^1},\ldots,\frac{\partial M+I}{\partial u_1^Y}]$ and $f^2 = [\frac{\partial M+I}{\partial u_2^1},\ldots,\frac{\partial M+I}{\partial u_2^Y}]$ are both vectors of length Y, and $f^3 = [,\ldots,\frac{\partial M+I}{\partial s_j^y},\ldots]$ is a vector of length $J \times Y$.

  Clearly, vectors $a_1,\ldots a_{|\Omega_1|}, b_1,\ldots b_{|\Omega_2|}, c_1,\ldots c_{|\Omega_3|}$ are linearly independent per the above definition.

- For condition (i), i.e. Constraint (24f) is not binding, vector f is not considered. Suppose Constraint (24e) of location 1 is binding, we need to show that d cannot be expressed as linear combination of $a_1,\ldots a_{|\Omega_1|}, b_1,\ldots b_{|\Omega_2|}, c_1,\ldots c_{|\Omega_3|}$. Note that d are only related to $a_1,\ldots a_{|\Omega_1|}$. However since $\Omega_1 < Y$ (otherwise it contradicts with the assumption that Constraint (24e) is active), d cannot be expressed by $a_1,\ldots a_{|\Omega_1|}$. The case when (24e) of location 2 is binding can be proven similarly.
- For condition (ii), i.e., there exist at least one $s_j^{y*} > 0$. Without loss of generality, suppose Constraint (24f) is binding, as well as Constraints (24e) for both $l = 1$ and 2. This means that we need to check

  $$a_1,\ldots a_{|\Omega_1|}, b_1,\ldots b_{|\Omega_2|}, c_1,\ldots c_{|\Omega_3|}, d, e, f$$

  First we note that all components of vector f are positive, as per (29). Since there is one $s_j^{y*} > 0$, $|\Omega_3| < Y \times J$. It follows that f cannot be expressed as linear combination of other vectors.
- For condition (iii), i.e., Constraint (24e) is not binding for $l = 1$ and $l = 2$. Suppose Constraint (24f) is binding. Thus we need to check

  $$a_1,\ldots a_{|\Omega_1|}, b_1,\ldots b_{|\Omega_2|}, c_1,\ldots c_{|\Omega_3|}, f$$

  If any $u_1^y > 0$ (or $u_1^y > 0$), it would imply Ω_1 (or Ω_2) $< Y$, which in turn suggests linear independence of the above vectors. If all $u_l^y = 0$, then the binding budget constraint implies that at least one $s_j^y > 0$. Invoking condition (ii) above leads to linear independence.

Therefore, we have proven that if any of the three conditions (i–iii) is satisfied, LICQ is met and the KKT conditions are necessary for local optimum. □

Appendix B. Derivation of derivatives

The first part in the above can be evaluated recursively, if the derivatives are computed following the increasing order of time. To compute the second part, note that

$$\frac{\partial P_{ij}^y}{\partial u_l^e} = \sum_{j'} \frac{\partial P_{ij}^y}{\partial U_{ij'}^y}\frac{\partial U_{ij'}^y}{\partial \lambda_l^y}\frac{\partial \lambda_l^y}{\partial x_l^y}\frac{\partial x_l^y}{\partial u_l^e} \tag{40}$$

$$\frac{\partial P_{ij}^y}{\partial U_{ij'}^y} = \begin{cases} P_{ij}(1-P_{ij'}) & j = j' \\ -P_{ij}P_{ij'} & j \neq j' \end{cases} \tag{41}$$

$$\frac{\partial U_{ij}^y}{\partial \lambda_l^y} = \frac{\beta_i^g}{w_i^y}\frac{\partial \hat{g}_{ij}^y}{\partial \lambda_l^y} + \frac{\beta_i^t}{w_i^t}\frac{\partial \hat{t}_{ij}^y}{\partial \lambda_l^y} + \frac{\beta_i^c}{w_i^y}\frac{\partial \hat{c}_{ij}^y}{\partial \lambda_l^y} + \beta_j^d \tag{42}$$

$$\frac{\partial \lambda_l^y}{\partial x_l^y} = \frac{1}{\kappa_l} \tag{43}$$

$$\frac{\partial x_l^y}{\partial u_l^e} = \begin{cases} 1 & e \le y \\ 0 & e > y \end{cases} \tag{44}$$

Moreover,

$$\frac{\partial \hat{g}_{ij}^y}{\partial \lambda_1^y} = \frac{\partial \sum_{y'=y}^{y+l_j} \hat{g}_{ij}^{yy'}}{\partial \lambda_1^y} = \sum_{y'=y}^{y+l_j} \begin{cases} 0 & j=1 \\ -S_{ij}^1 h_o^{y'} + S_{ij}^1 h_e^{y'} & j=2 \\ -h_b\mu_{ij}^1 + S_{ij}^1 h_e^{y'} & j=3 \end{cases}, \quad \frac{\partial \hat{g}_{ij}^y}{\partial \lambda_2^y} = \frac{\partial \sum_{y'=y}^{y+l_j} \hat{g}_{ij}^{yy'}}{\partial \lambda_2^y} = \sum_{y'=y}^{y+l_j} \begin{cases} 0 & j=1 \\ 0 & j=2 \\ -h_b\mu_{ij}^2 + S_{ij}^2 h_e^{y'} & j=3 \end{cases}$$

$$\frac{\partial \hat{t}_{ij}^y}{\partial \lambda_1^y} = \frac{\partial \sum_{y'=y}^{y+l_j} \hat{t}_{ij}^{yy'}}{\partial \lambda_1^y} = 0, \quad \frac{\partial \hat{t}_{ij}^y}{\partial \lambda_2^y} = \frac{\partial \sum_{y'=y}^{y+l_j} \hat{t}_{ij}^{yy'}}{\partial \lambda_2^y} = \sum_{y'=y}^{y+l_j} \begin{cases} 0 & j=1,2 \\ S_{ij}^2 \alpha_i^{y'} \bar{w}_i^{y'} & j=3 \end{cases}$$

$$\frac{\partial \hat{c}_{ij}^y}{\partial \lambda_1^y} = \frac{\partial \sum_{y'=y}^{y+l_j} \hat{c}_{ij}^{yy'}}{\partial \lambda_1^y} = \sum_{y'=y}^{y+l_j} \begin{cases} 0 & j=1 \\ -S_{ij}^1 h_c^{y'} & j=2,3 \end{cases}, \quad \frac{\partial \hat{c}_{ij}^y}{\partial \lambda_2^y} = \frac{\partial \sum_{y'=y}^{y+l_j} \hat{c}_{ij}^{yy'}}{\partial \lambda_2^y} = \sum_{y'=y}^{y+l_j} \begin{cases} 0 & j=1,2 \\ -S_{ij}^2 h_c^{y'} & j=3 \end{cases}$$

Similarly, the derivative in the second term in (25) can be computed as follows:

$$\frac{\partial g_{ij}^y}{\partial \lambda_1^y} = \begin{cases} 0 & j=1 \\ -S_{ij}^1 h_o^y + S_{ij}^1 h_e^y & j=2 \\ -h_b \mu_{ij}^1 + S_{ij}^1 h_e^y & j=3 \end{cases}, \quad \frac{\partial g_{ij}^y}{\partial \lambda_2^y} = \begin{cases} 0 & j=1 \\ 0 & j=2 \\ -h_b \mu_{ij}^2 + S_{ij}^2 h_e^y & j=3 \end{cases}$$

$$\frac{\partial t_{ij}^y}{\partial \lambda_1^y} = 0, \quad \frac{\partial t_{ij}^y}{\partial \lambda_2^y} = \begin{cases} 0 & j=1,2 \\ S_{ij}^2 \alpha_i^y \bar{w}_i^y & j=3 \end{cases}$$

$$\frac{\partial c_{ij}^y}{\partial \lambda_1^y} = \begin{cases} 0 & j=1 \\ -S_{ij}^1 h_c^y & j=2,3 \end{cases}, \quad \frac{\partial c_{ij}^y}{\partial \lambda_2^y} = \begin{cases} 0 & j=1,2 \\ -S_{ij}^2 h_c^y & j=3 \end{cases}$$

Appendix C. Coefficients for the vehicle choice model

First note that by construction all $\beta_j^p = -1$.

- β_i^g: Ziegler (2012) shows that the ratio of the coefficient for the purchase price to that for the fuel cost is 0.77 (see Table 3b, flexible multinomial probit model). Converted to the scale of the corresponding costs in our model (which uses life-time costs), the ratio is estimated at about 0.7, which is used for average drivers. The magnitude of β_i^g is slightly increased for frequent drivers and reduced for modest drivers (each by 0.2).
- β_{jl}^d: Table 3b in Ziegler (2012) shows that a charging network with a density similar to that of gas stations would be valued at roughly € 28,500; or \$34,200 (at an exchange rate of 1.2), which is comparable to an annual income for a wage rate of \$15/h. We thus conclude that $\beta_{31}^d + \beta_{32}^d$ should be about 1.0 for EVs in our model (recall the cost terms are scaled by the average annual income). Due to the lack of data, we split the total effect between intracity and inter-city locations somewhat arbitrarily, at 0.6 and 0.4, respectively. The coefficients for PHEV is assumed to be half of those for EV, based on the conjecture that the charging density is less important to PHEV drivers than to EV drivers.
- β_i^c: Table 3b in Ziegler (2012) also shows that consumers would value 1 kg of CO_2 roughly at € 0.57, or \$ 0.69, of fuel cost. This gives the price of CO_2 at \$690 $\times$ 0.7 = \$480/ton (where 0.7 converts the fuel cost to the purchase cost). This value is rather high compared to those from other studies, such as Interagency Working Group on Social Cost of Carbon (2013), which prices the social cost of CO_2 at roughly \$50/ton. In light of this, a middle value, \$200/ton is selected for CO_2 price, and accordingly β_i^c is set to β_i^p.
- β_i^t: the literature on the impact of charging time on vehicle choice is quite limited. In this study, we estimate it based on the value of travel time. Small and Verhoef (2007) suggest that the value of travel time may be represented as a fraction of the wage rate, usually between 20% and 90% and averaging around 50%. Since the travel cost is defined based on the wage rate, the default value for β_i^t is set to 0.5. The magnitude of β_i^t is slightly decreased for frequent drivers and increased for modest drivers (each by 0.2).

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