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Estimating the influence of common disruptions on urban rail transit networks

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ABSTRACT

With the continuous expansion of urban rapid transit networks, disruptive incidents—such as station closures, train delays, and mechanical problems—have become more common, causing such problems as threats to passenger safety, delays in service, and so on. More importantly, these disruptions often have ripple effects that spread to other stations and lines. In order to provide better management and plan for emergencies, it has become important to identify such disruptions and evaluate their influence on travel times and delays. This paper proposes a novel approach to achieve these goals. It employs the tap-in and tap-out data on the distribution of passengers from smart cards collected by automated fare collection (AFC) facilities as well as past disruptions within networks. Three characteristic types of abnormal passenger flow are divided and analyzed, comprising (1) "missed" passengers who have left the system, (2) passengers who took detours, and (3) passengers who were delayed but continued their journeys. In addition, the suggested computing method, serving to estimate total delay times, was used to manage these disruptions. Finally, a real-world case study based on the Beijing metro network with the real tap-in and tap-out passenger data is presented.

1. Introduction

As the most reliable and energy-efficient transportation system, urban rapid transit has come to be regarded as the best solution for alleviating road congestion; thus it plays an increasingly important role in many large cities around the world (Kang et al., 2015). With the continuous expansion and overload operation of urban rail transit networks, disruptions often arise, leading to difficulties for both passengers and managers. For example, in the first quarter of 2015, disruptive incidents were up 146% over the previous year in Beijing, China. Therefore it is an important part of transportation management to be able to identify such problems (e.g., abnormal passenger flows, train delays, etc.) quickly and effectively and to estimate their impact.

Once an abnormal incident has occurred, train service will be affected, including network disruption, increased travel time, and changes in travel behavior. From the point of view of the network's structure, an abnormal incident will lead to the disruption of lines or stations. Researchers have attempted to simulate different scenarios of network disruption and to measure their impact on network resilience. However, most of them focus on large-scale emergencies or catastrophes, where all passengers have to evacuate. For example, Derrible and Kennedy (2010a,b) analyzed the robustness of subway

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systems, which is useful in studying the choice of travel path under disruption. Rodriguez-Nunez and Garcia-Palomares (2014) emphasized the importance of circular lines in the network structure for the management and resolution of disruptions. Jenelius and Cats (2015) presented a methodology for assessing the value of new links to support the robustness of public transport networks. They considered disruptions of all lines and links, including the new links themselves. However, the results obtained by these researchers do not reflect the ways in which disruption affects passenger behavior. Therefore other investigators have tried to develop new simulation methods (Galea and Galparsoro, 1994; Almodóvar and García-Ródenas, 2013; Lo et al., 2014; Wales and Marinov, 2015).

Although these studies can help us to understand passenger behavior when an emergency occurs, they do not yield enough detail with regard to the flow of passengers. Therefore there may be different responses to disruptive incidents, and these may have different implications. Nikos and Nicolas (2004) searched for effective emergency rescue plans by the simulation method and completed a case study involving a fire in a metro station. Jiang et al. (2009) developed a numerical simulation of an emergency evacuation in an urban metro station, taking the Beijing transit network as an example. Also Li et al. (2016) modeled and analyzed the fire emergency response in urban rail transit. Guo et al. (2012) studied passenger flow assignment in an urban rail network under emergency response conditions. In addition, the behavior of passengers boarding trains will change during emergencies and must also be considered. Pnevmatikou and Karlaftis (2011) presented an analysis of changes in passenger demand that arise when a metro network is disrupted. Duan et al. (2012) established the effect of passenger density on each type of station: originating stations, intermediate stations, and transfer stations. In addition, dell'Olio et al. (2013) studied the possible behavior of both passengers and crew while facing emergencies. Cadarso et al. (2013) studied the disruption recovery problem of urban rail transit networks. Jin et al. (2014) studied a practical problem involving the integration of a local bus service with a metro network in an effort to enhance the network's resilience and avoid disruptions in service. However, the previous work fails to offer a clear estimate the effects of disruption on a network because it does not include an analysis of passenger choice. In addition, not all common disruption situations will trigger a response which would depend on the basis of degree of seriousness as shown in Table 1. In this paper, we give the categorization amongst the common disruptions on the basis of degree of incident delay time. Generally, the common disruption is defined as the delay time less than 30 min caused by the incident. Moreover, the response of passenger behavior to the disruption will be triggered when the delay time between 10 and 30 min in the real application.

Besides, without the support of real passenger travel data, many passengers behavior in the face of disruption are not discovered. Therefore the proposed methods are difficult to apply directly in actual management. Recently a new issue related to passenger tap-in and tap-out data is receiving much attention because it offers a more accurate description of passenger behavior. The analysis of smart-card data in transportation systems has pointed to the importance of traffic data in management and the prediction of transit passenger flow (Pelletier et al., 2011). In the urban rail transit network, Park et al. (2008) studied the demand characteristics of various public transport modes in Seoul, South Korea. In particular, they considered the rapid transit system based on smart-card data. A more detailed review of smart-card data in public transit can be found in Pelletier et al. (2011). Also Sun et al. (2012) presented a methodology to analyze the dynamic demand distribution based on smart-card data collected in Singapore. The result is significant for responding to failure in a timely and effective manner. Supported by the AFC data, we can model passenger volume according to changes in passenger behavior in the station where a disruption has occurred.

This paper comprises a quantitative method for estimating the effects of common disruptions (not catastrophes or large-scale emergencies) on passengers and stations from the spatial and temporal points of view, including a review of the behavior of 3 types of passengers: (1) missed passengers who leave the system, (2) passengers who take detours, and (3) passengers who are delayed but remain on course to their destinations. The behaviors of these passengers are modeled and analyzed. In the model, Automated Fare Collection (AFC) data are used to identify the disruption and calibrate its parameters using the Bayesian method. In addition, the paths affected by the disruption are determined; thus the number of affected passengers can be calculated. The proposed measurements can be applied to the evaluation of the influence of common disruptions on an urban transit network.

2. Using the Bayesian model to identify an incident

Generally, the operator knows the whole operating situation by the historical data analysis. However, in the day train scheduling, the operator need to know the real time situation with the on-line tap-in and tap-out data. Therefore, an incident identification model should be developed to monitor the operation of metro system by the trends of passenger flow at stations. In many cities, smart-card data can be collected by AFC machines at entrances and exits which contains precise information as to the time and place of both boarding and alighting (Ma et al., 2013). For example, Beijing's smart card data include card ID, date, boarding station and time, and alighting station and time.

Fig. 1 shows the number of passengers at a particular station over 4 working days in March 2014, indicating that the demand had a stable trend; this helps us to identify the disruption and estimate its effects. We can see 2 peaks: the morning peak and the evening peak. In fact, during a typical working day, most of trips represent commuters traveling to and from work. This makes for a stable pattern of flow. However, when a disruption occurs, this pattern is disturbed, and we can identify the incident by this change in flow. In Fig. 1, the tap-in passenger volume between 7:45 a.m. and 9:00 p.m. drops, indicating that the abnormal incident occurred on that day, March 15, 2014.

Table 1
The effects of delay time and seriousness degree on the passenger behavior.

		Delay time		
		5 min	10–15 min	20–30 min
Seriousness degree		Small	Medium	Large
Passenger behavior	Left	Almost all	Majority	Most
	Detoured	Almost no	Minority	Part
	Missed	Almost no	Minority	Part

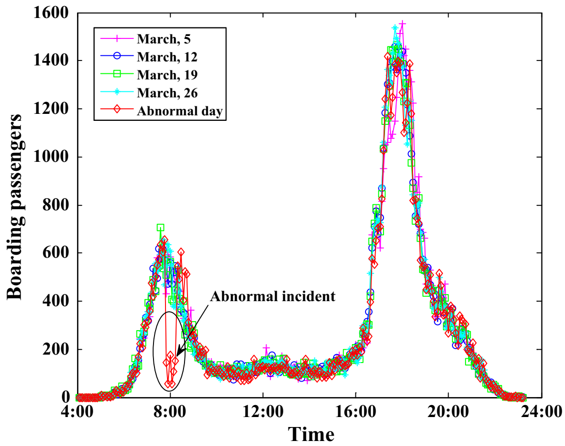

Fig. 1. Temporal boarding patterns of passengers on different days at a station of the Beijing metro network.

2.1. Description of the model

In assembling our statistical results, the arrival rate during the same time period, such as 15 min or one schedule day follows a normal distribution. Let the arrival passengers y be a random variable with $y \sim N(\mu, \sigma^2)$ on a certain day, where μ and σ^2 represent the mean value and variance. However, the passenger distributions will be different over days in one week. Therefore, the parameter μ would also be a random variable with a mean value of θ for a given several weeks data.

2.2. Estimation of parameters

Let $p(\theta)$ be the prior probability of the parameter θ. Then the posterior probability distribution of parameter θ is obtained with $p(\theta|D) \propto p(\theta)L(D|\theta)$ according to the sample data. Let T be the data sample size; then the estimated parameter θ is subject to a normal distribution with the following formulation (Ma, 2013):

$$\mathrm{var}(D_{t+1}|D_t) = E(\mathrm{var}(D_{t+1}|\theta, D_t)|D_t)) + \mathrm{var}(E(D_{t+1}|\theta)|D_t) = \sigma^2 + \tau_t^2 \quad (1)$$

$$E(D_{t+1}|D_t) = E(E(D_{t+1}|\theta, D_t|D_t)) = E(\theta|D_t) = \mu_t \quad (2)$$

Next, we estimate passenger volume y at time $t+1$ with $N(\mu_t, \sigma^2 + \tau_t^2)$. Besides, the experienced prior probability distribution $\theta \sim N(\mu_0, \tau_0^2)$ can be determined using the following formula:

$$p(\theta) \propto \exp\left[-\frac{1}{2\tau_0^2}(\theta - \mu_0)^2\right] \quad (3)$$

For the passenger volume x_ω over ω days, we collected the sample set $D = \{x_1, \ldots, x_\omega\}$. Based on the Bayesian model, the posterior probability can be calculated by

$$p(\theta|D) = \frac{p(D|\theta)p(\theta)}{p(D)} \quad (4)$$

where $p(D) = \int p(D|\theta)p(\theta)d\theta$.

Let

$$\alpha = \frac{1}{p(D)} \quad (5)$$

In addition, $D = \{x_1, \ldots, x_\omega\}$ is an independent and identically distributed sample; therefore the formula is written as:

$$\begin{aligned} p(\theta|D) &= \alpha p(D|\theta)p(\theta) = \alpha \prod_{i=1}^{\omega} \frac{1}{\sqrt{2\pi}\sigma} \exp\left[-\frac{1}{2\sigma^2}(x_i-\theta)^2\right] \times \frac{1}{\sqrt{2\pi}\tau_0} \exp\left[-\frac{1}{2\tau_0^2}(\theta-\mu_0)^2\right] \\ &= \alpha' \exp\left[-\frac{1}{2}\left(\frac{1}{\sigma^2}\sum_{i=1}^{\omega} x_i^2 - \frac{2}{\sigma^2}\theta\sum_{i=1}^{\omega} x_i + \frac{\omega}{\sigma^2}\theta^2 + \frac{1}{\tau_0^2}\theta^2 - \frac{2}{\tau_0^2}\mu_0\theta + \frac{1}{\tau_0^2}\mu_0^2\right)\right] \\ &= \alpha'' \exp\left\{-\frac{1}{2}\left[\left(\frac{\omega}{\sigma^2}+\frac{1}{\tau_0^2}\right)\theta^2 - 2\left(\frac{1}{\sigma^2}\sum_{i=1}^{\omega} x_i + \frac{\mu_0}{\tau_0^2}\right)\theta\right]\right\} \end{aligned} \tag{6}$$

Clearly $p(\theta|D)$ follows the normal distribution and is represented by

$$p(\theta|D) \sim N(\mu_\omega, \tau_\omega^2) \tag{7}$$

Therefore the formula is derived as follows:

$$p(\theta|D) = \frac{1}{\sqrt{2\pi}\tau_\omega} \exp\left[-\frac{1}{2}\left(\frac{\theta-\mu_\omega}{\tau_\omega}\right)^2\right] \tag{8}$$

Comparing formulas (6) and (8), we can get

$$\tau_\omega^2 = \frac{\sigma^2\tau_0^2}{\omega\tau_0^2+\sigma^2} \tag{9}$$

$$\mu_\omega = \frac{\tau_0^2}{\omega\tau_0^2+\sigma^2}\sum_{i=1}^{\omega} x_i + \frac{\sigma^2\mu_0}{\omega\tau_0^2+\sigma^2} \tag{10}$$

2.3. Detection of anomalies

Generally the normal data will be in the range of $(\mu - 3 \times \sqrt{\sigma^2+\tau_\omega^2}, \mu + 3 \times \sqrt{\sigma^2+\tau_\omega^2})$, which is called the "thrice-standard-error principle." For a random variable y following the normal distribution, the 99.47% probability of y will be in this range. Once a disruption occurs, the passenger volume will be affected even if it differs from the normal shape. That is to say, an abnormal passenger flow can be identified according to this principle. In the urban rail transit network, the passenger flows follow a normal distribution at each given time of day. If the real number of passengers is outside of the thrice-standard-error principle, we consider the data to be abnormal.

3. Model estimating the effects of disruption

The effects of a disruption are modeled from the viewpoint of the affected passengers and the length of delay in the network. This section summarizes the results of this model.

3.1. The response of passengers for the delay and the trip distance

In the urban rail transit network, the choice behavior of the affected passengers will have close relationship with the delay situation caused by the disruption, especially for the different O-D pairs. We have performed a survey for different delay time at different stations (urban area and suburbs) and different time (peak hours and non-peak hours) as shown in Fig. 2(a). Obviously, we can see that passengers will have different behavior for the additional waiting and travel time. In addition, the choice behavior of 200 commuters in different O-D is analyzed according to the historical data for two incident days as shown in Fig. 2(b). In the figure, the distance is represented by the numbers of stations in the route approximately. It is found that the O-D distance will have a great influence on the passenger behavior, e.g., waiting in the station, detouring in another station and giving up the urban rail transit network.

The behavior will be diverse with the disruption degree, the line (downtown line or suburbs line) and the station characteristics (ordinary stations or transfer stations), etc. Therefore, it is difficult to get a uniform result for this. However, by the history data, we can obtain that 75% passengers will give up the metro system when the number of stations in their trip is less than 7.

3.2. Different types of affected passengers

Depending on the nature and extent of the disruptive incident, passengers will behave differently. In general, passengers will manifest 1 of 5 behaviors. They may (1) continue to take urban rail transit to reach their destination by the original route, (2) continue to take unban rail transit but change their destination; (3) continue to take urban rail transit but make

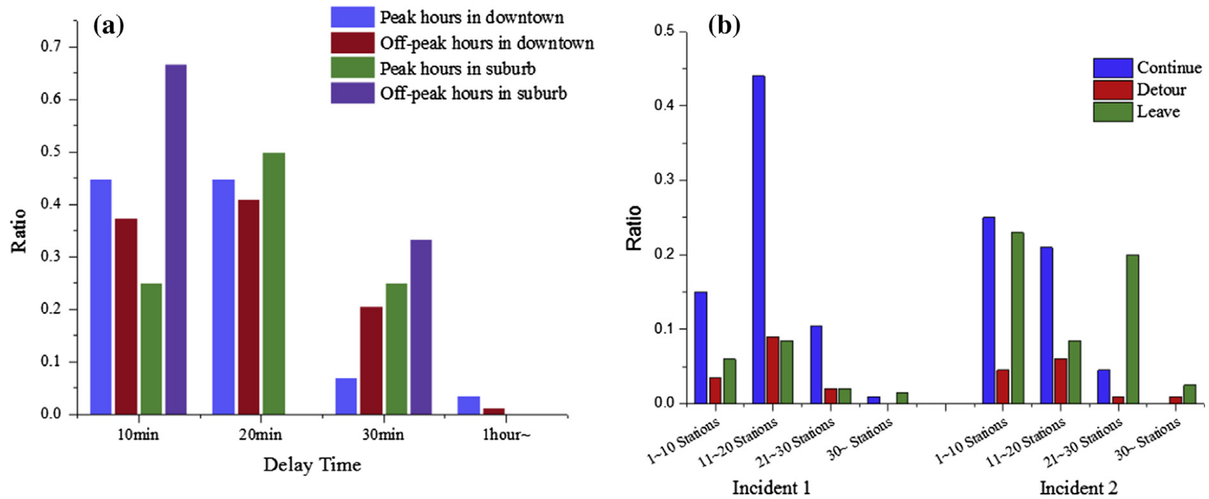

Fig. 2. The response of passengers. (a) The tolerance of passengers for the delay time and (b) the choice behavior of the affected passengers for different distances.

a detour to reach their destination; (4) select an adjacent normally functioning station from which to continue their trip, or (5) leave the urban rail transit system.

Therefore the 3 types of passengers are classified as (a) passengers who have left the system, (b) passengers who have taken detours, and (c) passengers who were delayed. Delayed passengers who had already been in the urban rail transit network are likely to fit into cases (1) to (3); passengers who took detours are likely to fit into cases (2) and (3). However, passengers who were not originally in the urban rail transit network are likely to be delayed passengers for cases (1) to (4); cases (2) to (4) are likely to be passengers who took detours; and passengers who left the system would fit into case (5).

Let $M(t,h)$ be the number of passengers on path h at time t. The starting and ending times of an incident are represented by t_s and t_e, respectively. Because AFC data contain only information on passengers who have been in the urban rail transit system, passengers' behavior regarding choice of route is difficult to identify or predict. For simplicity we use the shortest path.

In order to estimate the number of affected passengers, it is necessary to determine how many passengers were in urban rail transit network when the disruption occurred. Let $D_b(t,h)$ be the demand matrix on path h at time t before the disruption happened. For any passenger who has not completed his or her trip at time t, we let $D_b(t,h) = M(t,h)$. Otherwise, $D_b(t,h) = 0$. We give the detailed descriptions of the 3 types of passengers below.

3.2.1. Passengers who have left the system

Once an incident occurs, some passengers, on having to wait a longer time, will give up using the urban rail transit system. These passengers can be defined as

$$Q_{Miss} = \sum_{s\in s_d}\sum_{t\in[t_s,t_e]} In_t^{sn} - \sum_{s\in s_d}\sum_{t\in[t_s,t_e]} In_t^{se} \tag{11}$$

where In_t^{sn} and In_t^{se} represent the arrival passengers at time t in station s under normal and abnormal conditions respectively. s_d is a set of affected stations which can be determined by the Bayesian identification model mentioned previously. Then the proportion of missed passengers is defined by:

$$\alpha = \frac{Q_{Miss}}{\sum_{s\in s_d}\sum_{t\in[t_s,t_e]} In_t^{sn}} \tag{12}$$

3.2.2. Detoured passengers

There are 3 cases in the analysis of detoured passengers, as follows:

Case 1: Disruption Occurs at the Start Station of One's Travel Path.

If the starting station of one's travel path is affected by an incident, passengers may go to another starting station, reenter the system from there, and then finish their trips. Let β be the proportion of detoured passengers and $\mathbf{Z}_e^o$ be a set of affected starting stations. If the starting station s in the passenger path h is affected, $Z_e^o(h) = 1$. Otherwise, $Z_e^o(h) = 0$. Therefore the following equation can be derived:

$$Q_O(t) = \sum_h \beta \times M(t,h) \times Z_e^o(h), \quad t \in [t_s, t_e] \tag{13}$$

Case 2: Disruption Occurs at the End Station of One's Travel Path.

Let $\mathbf{Z}_e^d$ be the matrix for the ending stations. If the terminal station in the passenger path is affected, $Z_e^d(h) = 1$. Otherwise, $Z_e^d(h) = 0$.

The estimation of affected passengers can be calculated by the following equation:

$$Q_D(t) = \begin{cases} \sum_h (1-\alpha-\beta) \times M(t,h) \times Z_e^d(h), & t \in [t_s, t_e] \\ \sum_h D_b(t,h) \times Z_e^d(h), & t < t_s \end{cases} \tag{14}$$

Case 3: Disruption Occurs at the Crossing Station of One's Travel Path

Let $\mathbf{Z}_e^m$ be the matrix for the crossing stations. If the affected station is the crossing station in the passengers' path, $Z_e^m(h) = 1$. Otherwise, $Z_e^m(h) = 0$.

The estimation of affected passengers can be calculated by

$$Q_M(t) = \begin{cases} \sum_h (1-\alpha-\beta) \times D(t) \times Z_e^m(h), & t \in [t_s, t_e] \\ \sum_h D_b(t) \times Z_e^m(h), & t < t_s \end{cases} \tag{15}$$

3.2.3. Delayed passengers

An incident will affect the departure and the arrival of trains and lead to the delay. The number of delayed passenger can be calculated by

$$Q_{Delay} = \sum_{i=1}^{N} n_i \tag{16}$$

where Q_{Delay} represents the total quantity of delayed passenger flow, as travel time increases after the emergency occurs and $n_i = 1$ when the travel time of passenger i increases compared with the normal case. Otherwise, $n_i = 0$. In addition, N is the total number of passengers. In fact, it need a lot of time to determine n_i for each passenger i due to the great volume of passengers and O-D pair. For simplicity, in this paper, we first calibrate the distribution of travel time between arbitrary two affected stations according to the historical AFC data. Then, for the passengers who have a trip in the incident day, we can obtain their travel time with boarding time and alighting time in the recorded abnormal day data. Compared with the travel time between normal day and abnormal day, we can calculate the total number of delay passengers.

3.3. Estimation of delay

Here, 3 indicators are proposed to estimate the effects of a disruption on time delay in an urban rail transit system: the average delay time, the maximum delay time, and the rate of punctual arrival. For passenger i, the delay time can be represented by the difference between the real travel time t_i^{travel}, on the abnormal day, and the normal travel time, t_h, of path h. That is:

$$t_i^{delay} = t_i^{travel} - t_h \tag{17}$$

3.3.1. Average delay time

The average delay time of passengers Q_{Delay} caused by the disruption, defined as $\overline{t_{delay}}$ is

$$\overline{t_{delay}} = \frac{\sum_{i=1}^{Q_{Delay}} t_i^{delay}}{Q_{Delay}} \tag{18}$$

3.3.2. Maximum delay time

The maximum delay time of the passenger $t_{\max}^{delay}$ caused by the disruption is defined by

$$t_{\max}^{delay} = \max\{t_i^{delay}\} \tag{19}$$

3.3.3. Punctuality rate

Let P be the punctuality rate of passenger in urban rail transit system in an abnormal situation, it is formulated as follows

$$P = \frac{N - Q_{Delay}}{N} \tag{20}$$

3.4. Estimation of financial losses

For the operator, once the incident occurs, it will have the financial losses due to the passengers who have left the urban rail transit system. Let C_{Losses} be the total financial losses and $P_{Average}$ be the average price for the left passengers. We can formulate the estimated financial losses as follows:

$$C_{Losses} = Q_{Miss} \times P_{Average} \tag{21}$$

4. Case study: the Beijing rail transit network

4.1. Data description and incident identification

The following case study considers the Beijing rail transit network, which included 17 two-directional lines and 334 stations in 2014, as shown in Fig. 3. Based on the analysis of the AFC data on a working day, as shown in Fig. 1, the distribution of passenger flow in the Beijing rail transit system shows a clear periodicity and stable characteristics. In this paper a case study of an incident that occurred at Xi'erqi Station on line 13 of the Beijing metro at 7:55 p.m. is presented.

4.1.1. Data preparation

Datasets that represent the tap-in passengers at Xi'erqi Station on a working day between 7:45 and 8:00 p.m., within 8 weeks, were collected, as shown in Table 2. Groups 1–6 and sample group 7 represent the number of passengers on a normal day; the test group represents the number of passengers on an abnormal day. The a priori information by using groups 1–6 is first obtained. Then the sample data are used to obtain the a posteriori information using the Bayesian model.

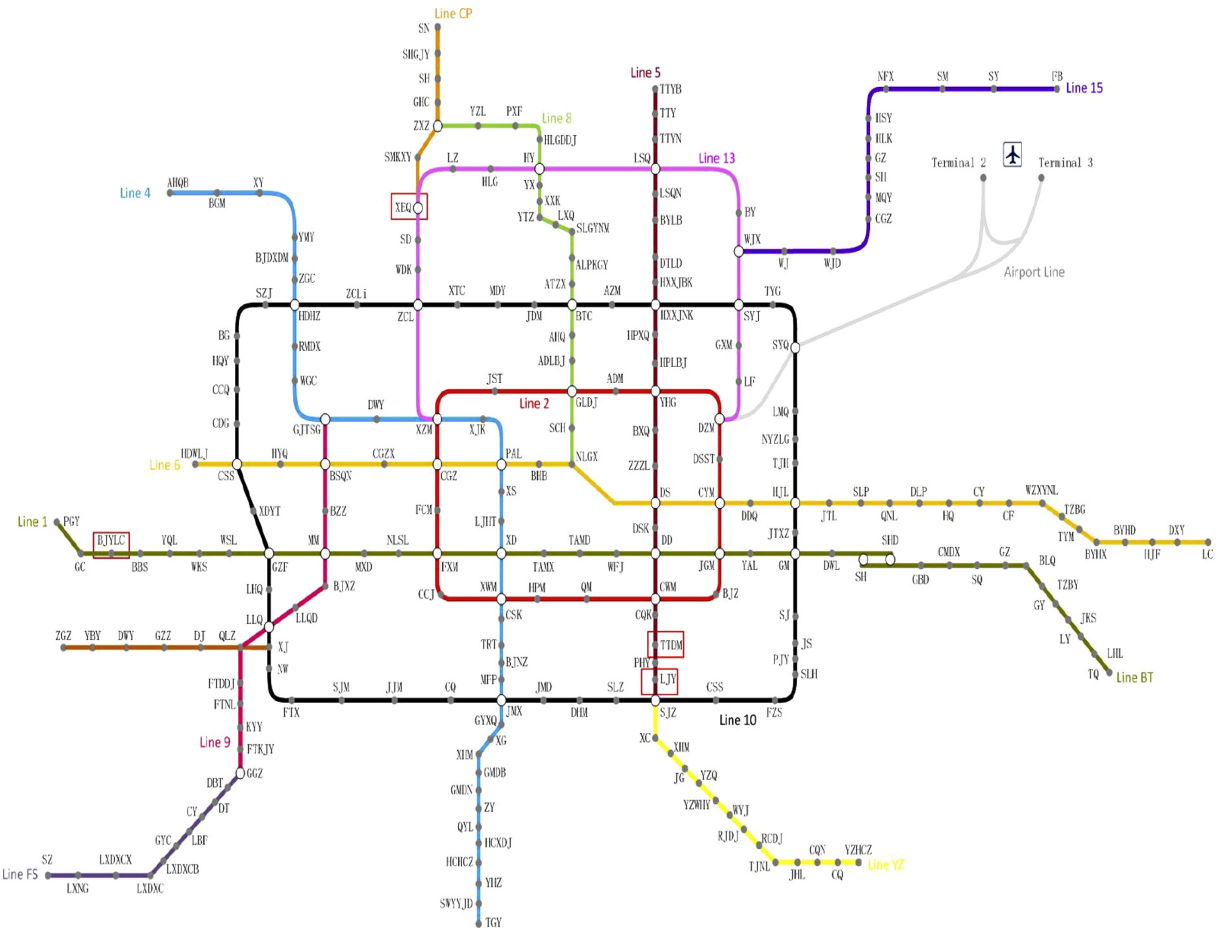

Fig. 3. A map of the Beijing rail transit network. The stations marked with red boxes are Xi'erqi (XEQ), BaJiaoYouLeYuan (BJYLY), LiuJiaYao (LJY), and TianTanDongMen (TTDM). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)

Table 2
The Number of Tap-in Passengers from 7:45 a.m. to 8:00 p.m. at Xi'erqi Station.

Group	Monday	Tuesday	Wednesday	Thursday	Friday
1	1630	1593	1580	1691	1619
2	1678	1685	1752	1675	1737
3	1783	1786	1671	1646	1655
4	1709	1709	1667	1737	1553
5	1697	1716	1647	1716	1602
6	1573	1673	1694	1730	1738
Sample	1591	1697	1724	1534	1732
Test	1716	1601	1648	1405	1707

4.1.2. Identification of the incident

The a priori probability for the first 6 weeks from the historical data is first calculated. To identify the 2 arrival data sets with a common distribution, Fig. 4 shows the quantile-quantile graph of tap-in passengers on working days during the 4 weeks from 1, March 1 to March 31, when the passenger flow followed a normal distribution. By using the Bayesian forecasting method, the general model is calibrated from the sample data (the first cycle of the 5 days) and the estimated value is obtained using the "thrice-standard-error principle," as shown in Fig. 5. Through the recursion of the sample data, the safety threshold changes over time, which indicates that the model can reflect the data characteristics of historical and real time.

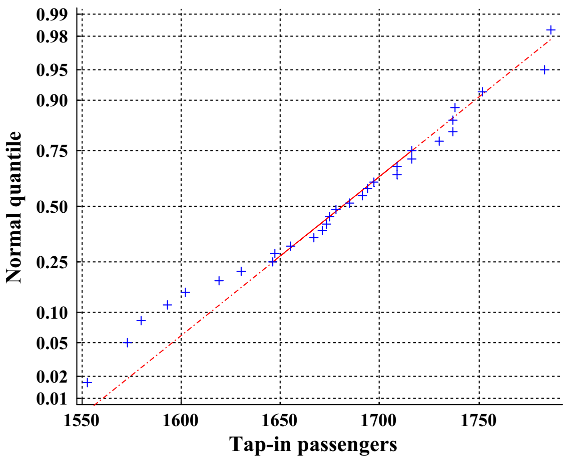

Fig. 4. Quantile–quantile graph of tap-in passengers.

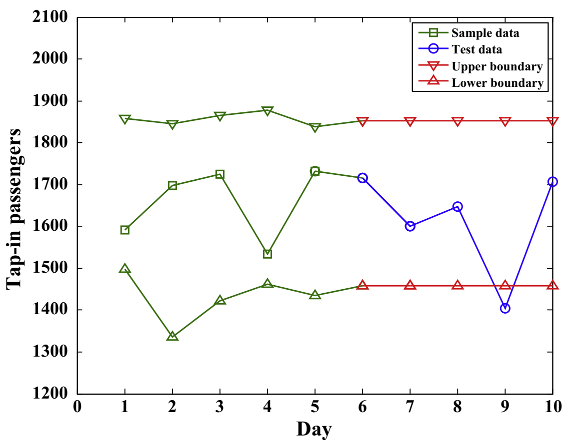

Fig. 5. Outlier detection for tap-in passenger flow.

Therefore it can be used to describe the normal threshold range of tap-in passenger flow accurately. The dataset on the ninth day clearly indicates the abnormal point, because it is beyond the safety threshold.

4.2. Effects of a disruption

In this paper, the mean of tap-in passengers for 4 consecutive weeks as the normal passenger flow and origin-destination matrix (OD) are studied. The local network of the Beijing rail transit was adopted, as shown in Fig. 6. First, the total travel time between each origin and destination OD should be calculated, including the walking time from the AFC machine to the platform, the waiting time on the platform, the in-train time, and the transfer time from one line to another.

4.2.1. Model of total travel time

Passengers will choose the travel path between each OD pair according to the total travel time T_TOTAL, which includes the walking time at the station, T_WALKING; the waiting time, T_WAITING; the in-train time, T_INTRAIN; and the transfer time, T_TRANSFER. Generally, the in-train time can be determined by checking the published train schedule. The waiting time can be estimated as half the headway, and the walking time is the time that one walks from the ticket machine at the entrance to the platform and vice versa at the exit. The transfer time is determined by the walking speed in the transfer passageway between 2 lines. Therefore the total travel time could be calculated as follows:

$$\mathrm{T\_TOTAL} = T\mathrm{\_WALKING} + \mathrm{T\_WAITING} + \mathrm{T\_INTRAIN} + \mathrm{T\_TRANSFER} \tag{22}$$

According to the total travel time model, the reasonable path between any arbitrary OD pair could be calculated using the k-shortest path algorithm. Table 3 gives the average travel time in different path of the local network.

4.2.2. The detour probability β

Let β be the probability that passengers will choose to go to the neighboring station. This is generally difficult to determine in a real operation. However, it could be calibrated by using the historical AFC data for abnormal days. Here, two incidents that occurred at the BaJiaoYouLeYuan Station on February 19, 2014, are selected, and at LiuJiaYao Station caused by an incident at TianTanDongMen Station on February 24, 2014. The response of passengers to these 2 incidents is analyzed according to the recorded passengers ID. The detour probability for passengers who chose the neighboring station was about 13.5% and 11.5% respectively. For simplicity the average value of $\beta = 12.5\%$ was selected. Therefore, for this case study, the estimated passenger who detour to the neighbor of disruption station and finish their trip will be about 212.

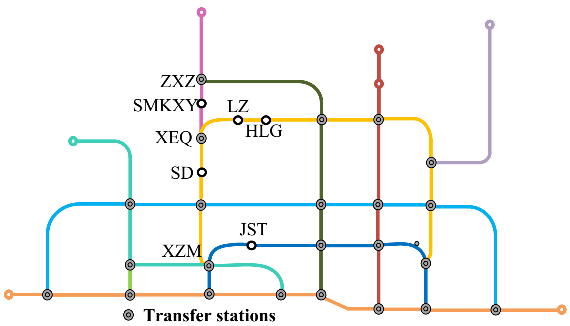

Fig. 6. A local network of the Beijing rail transit (the various paths are marked in different colors).

Table 3
Average travel time in different path.

Path number	Path	Travel time(s)
1	ZXZ-SMKXY	290
2	ZXZ-SMKXY-XEQ	696
3	ZXZ-SMKXY-XEQ-XEQ-SD	1018
4	ZXZ-SMKXY-XEQ-XEQ-LZ	993
5	ZXZ-SMKXY-XEQ-XEQ-HLG	1128
6	ZXZ-SMKXY-XEQ-XEQ-SD-WDK-ZCL-DZS-XZM-XZM-JST	5308
...	...	...

Note: [dashed box] means the transfer station.

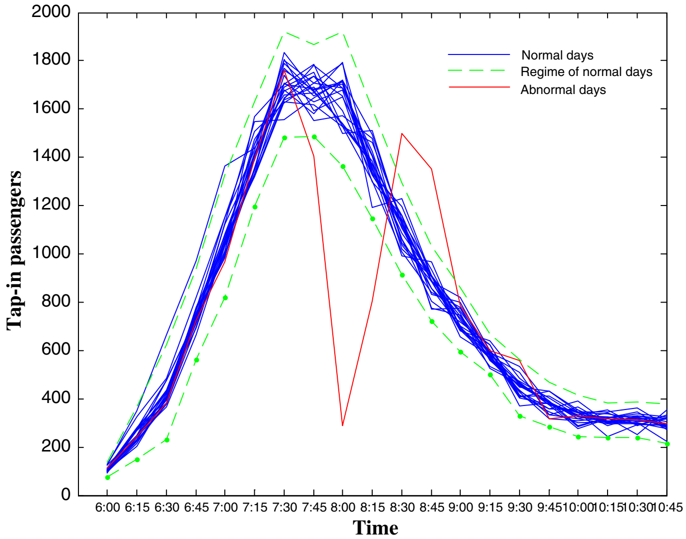

Fig. 7. Temporal boarding patterns of passengers at Xi'erqi Station between 6:00 a.m. and 10:45 p.m.

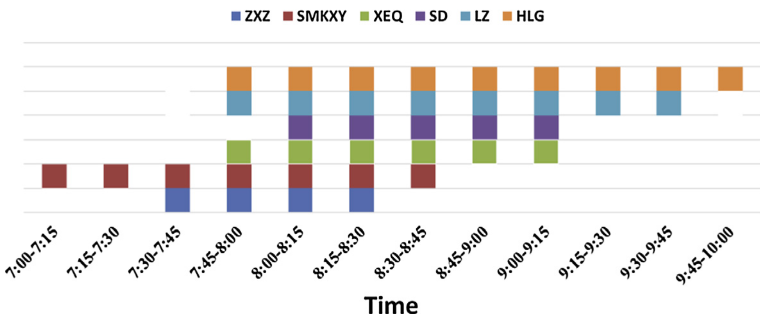

Fig. 8. The stations affected by the disruption and the lengths of the disruptions.

4.2.3. Estimation of the impact of a disruption

4.2.3.1. Influence scope of the network. By using the Bayesian detection method, the affected arriving passengers at Xi'erqi Station are as shown in Fig. 7, which gives the temporal boarding patterns of passengers at the Xi'erqi station. The red[1] line in Fig. 7 represents the number of arriving passengers at the Xi'erqi Station on the day of the disruption, and the blue line represents the number of arriving passengers at the station on a typical working day within a month. The green dotted lines are the upper and lower boundaries within the "thrice-standard-error principle" of arriving passengers on normal days. The passenger distribution from 7:45 a.m. to 9:00 p.m. is comparable to the normal situation.

Once the incident occurred, it quickly spread throughout the network. We can identify the affected stations by using Bayesian detection method. The affected period and scope are shown in Figs. 8 and 9.

4.2.3.2. Estimation of affected passenger flow. Table 4 lists a part of the demand matrix $M(t,h)$ on path h at time t, which represents the number of passengers on different paths over time. By applying the model proposed in Section 3, the matrices $\mathbf{Z}_e^o$, $\mathbf{Z}_e^m$, and $\mathbf{Z}_e^d$ could be calculated, as shown in Table 5.

The missed passengers in different stations and in the rail network generally over time are shown in Figs. 10 and 11. It could be observed that the number of arriving passengers is in accordance with the trend, in which decreased at first and then increased within the duration of the disruption compared with the normal level. The minimum value of cumulative arriving passenger flow appeared at 9:00 p.m., which means that the disruption was coming to a close. The total number of missed passengers in the network was about $Q_{Miss} = 8920$. The ratio of missed passengers is about $\alpha = 15.7\%$, as calculated by Eq. (12).

[1] For interpretation of color in Figs. 7 and 8, the reader is referred to the web version of this article.

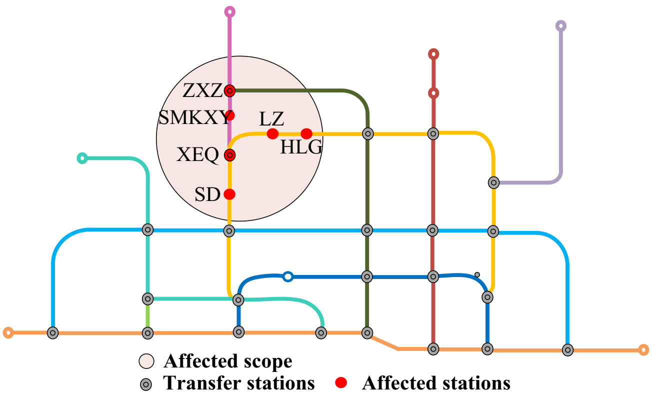

Fig. 9. Other stations affected by an incident at XEQ Station.

Table 4
Part of the matrix $M(t,h)$.

Time	Path index							
	1	2	3	4	5	6	7	...
6:45–7:00	6	25	10	0	1	2	0	...
7:00–7:15	8	69	7	0	0	3	1	...
7:15–7:30	5	118	16	0	0	6	1	...
7:30–7:45	6	106	16	1	0	4	1	...
7:45–8:00	12	211	23	0	0	6	2	...
8:00–8:15	11	223	33	2	1	2	2	...
8:15–8:30	10	169	14	1	1	1	1	...
8:30–8:45	2	99	12	0	1	0	1	...
8:45–9:00	8	95	10	3	1	1	1	...

Table 5
Some of the affected paths.

	Path index								
	1	2	3	4	5	6	7	8	...
$\mathbf{Z}_e^o$	0	0	1	1	1	0	0	0	...
$\mathbf{Z}_e^m$	0	1	1	1	1	1	1	0	...
$\mathbf{Z}_e^d$	0	0	0	0	0	0	0	0	...

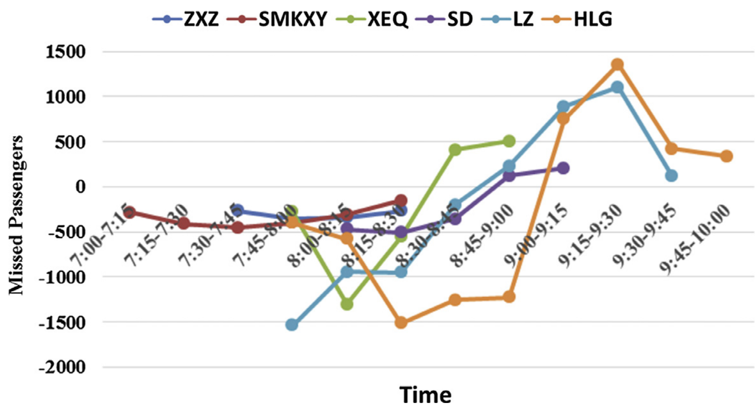

Fig. 10. Missed passengers at affected stations.

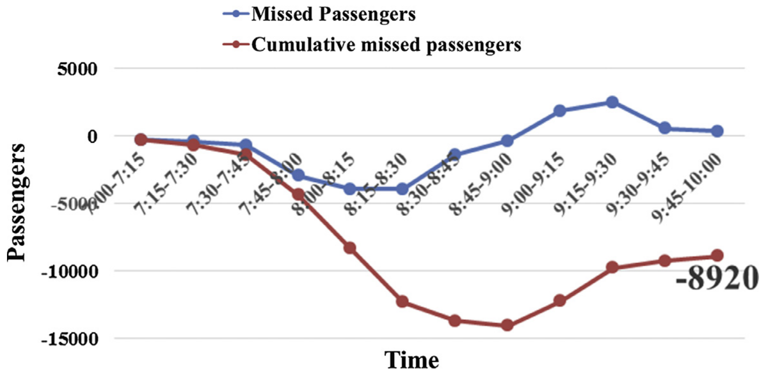

Fig. 11. Total number of missed passengers.

Table 6 presents the number of affected passengers in the network. It was found that the total number of passengers affected by the incident was about 45,823. The number of passengers whose starting station was affected was 3801; the number whose terminal station was affected was 20,526, and the number whose middle station was affected was 21,496.

4.2.3.3. Estimation of the delay time. Because the travel time is affected mainly during the period of the incident, the delay time is estimated between 7:00 and 8:30 p.m. The delay time for each affected passenger is plotted in Fig. 12. Table 7 gives the total average delay time, maximum delay time, and punctuality rate. As we can see, about 25,533 passengers were delayed during the period of the incident. The average delay time was 630 s. The maximum delay time was 254 s, and the punctuality rate was only 26.25%.

Table 6
Affected passengers during different time periods.

	5:45–6:00	6:00–6:15	6:15–6:30	6:30–6:45	6:45–7:00	7:00–7:15
Q_O	0	0	0	0	0	314
Q_D	0	3	39	514	1603	1897
Q_M	9	41	168	853	2026	2226
Total	9	44	207	1367	3629	4437
	7:15–7:30	7:30–7:45	7:45–8:00	8:00–8:15	8:15–8:30	Total
Q_O	485	663	825	808	706	3801
Q_D	2447	3163	3813	3822	3225	20,526
Q_M	2714	3121	3586	3632	3120	21,496
Total	5646	6947	8224	8262	7051	45,823

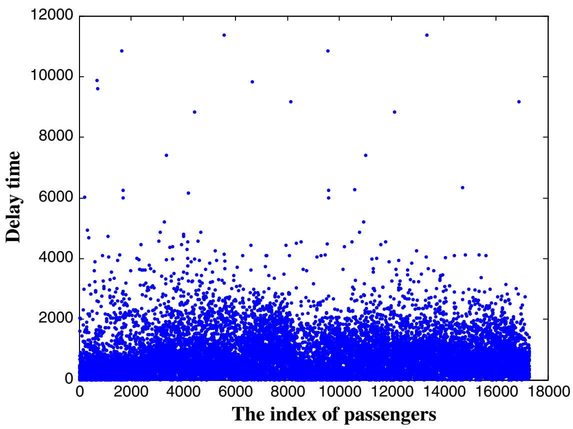

Fig. 12. Distribution of delay time per passenger.

Table 7
The delay time caused by disruption.

Q_{delay} (people)	$\overline{t_{delay}}(s)$	$t_{\max}^{delay}(s)$	P
25,533	630	2548	26.25%

4.2.4. Estimation of the financial losses

Before 2015, the Beijing adopted a flat-rate subway fare with unlimited transfers. A single-ride ticket costs only two RMB. Therefore, we can estimate the financial losses easily for single-ride ticket. However, fares were adjusted based on distance in 2015. In this situation, we should estimate the average price according to the affected O-D pair and the total number of passengers. Because the incident used in this paper occurred in 2014, the estimated financial losses of operator will be about 17,840 RMB with Eq. (21).

4.3. Response to the disruption

In general, once an incident occurs, operators will adjust the timetable to meet the change of passenger volume and network structure. At the same time, the limitation measure of passengers will be implemented at the affected stations as shown in Fig. 8 (stations with red color).

5. Conclusions

This paper describes the development of a model that estimates the effects of a disruption in an urban rail transit network taking into account the spatiotemporal factors involved. Based on the AFC data, an efficient Bayesian method was introduced to identify the disruption according to the number of tap-in passengers collected by AFC system. To estimate the effects of a disruption, passenger behaviors were divided into 3 groups: missed passengers, detoured passengers, and delayed passengers. Then a model was developed to analyze the delay caused by the disruption, including average delay time, maximum delay time, and rate of punctual arrivals. By using that model, the affected stations could be determined. Finally, the validity of the model and method was verified by a case study involving the Beijing rail transit network. The results show that the model can be used to estimate the effects of a disruption on the urban rail transit network mathematically and quantitatively.

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