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# Measuring vulnerability of urban metro network from line operation perspective

Daniel (Jian) Sun [a,b], Shituo Guan [b,*]

[a] State Key Laboratory of Ocean Engineering, School of Naval Architecture, Ocean and Civil Engineering, Shanghai Jiao Tong University, Shanghai 200240, China
[b] Transportation Research Center, Shanghai Jiao Tong University, Shanghai 200240, China

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ABSTRACT

Urban metro systems are subject to recurring service disruption for various reasons, such as mechanical or electrical failure, adverse weather, or other accidents. In recent years, studies on metro networks have attracted increasing attention because the consequence of operational accidents is barely affordable. This study proposes to measure the metro network vulnerability from the perspective of line operation by taking the Shanghai metro network as a case study. As opposed to previous studies that focused largely on disruption of important nodes or links, this study investigates the disruption from the line operation perspective. Betweenness centrality (BC) and passenger betweenness centrality (PBC), number of missed trips, weighted average path length, and weighted global efficiency were analyzed considering relative disruption probability of each line. Passenger flow distribution and re-distribution were simulated for different disruption scenarios based on all-or-nothing assignment rule. The results indicate that the metro lines carrying a large number of passengers generally have a significant impact on the network vulnerability. The lines with circular topological form also have a significant influence on passenger flow re-distribution in case of a disruption. The results of this study provide suggestions on metro system administration for potential improvement of the performance of operation, and passengers may meanwhile have an improved alternate plan for their commute trip when a disruption occurs.



## 1. Introduction

Public transport networks are generally indispensable for mobility in urban areas, and metro networks are vital components of transit systems in major cities, acting as a key solution in supporting commuter traffic demand within metropolis area. In addition to huge capacity, metro systems also provide improved service experience such as punctuality and fast speed. The dependence on metro systems keeps growing in several cities over the world. According to the *Shanghai comprehensive traffic operation annual report, 2014*, 6.585 billion trips were made during the year in Shanghai, i.e., around 18.04 million trips per day, in which the Shanghai metro system accounts for approximately 43%, overtaking regular road transit for the first time. It also indicates that if the metro system fails, the consequence is serious and barely affordable. To guarantee efficient operation of a metro system, it is important to assess the vulnerability of the metro network to potential disruptions and identify lines whose incidents may have a crucial impact on both metro networks and travelers.

* Corresponding author at: A304, Ruth Mulan Chu Chao Building, No. 800 Dongchuan Road, Shanghai 200240, China.
*E-mail addresses:* danielsun@sjtu.edu.cn (D. (Jian) Sun), guantt@sjtu.edu.cn (S. Guan).

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Disruptions in a metro network may be caused not only by operational degradations of physical infrastructure, such as electrical failures and malfunctioning vehicles, but also from service degradations, including crew strikes, terrorist attacks, adverse weather, or other accidents (Cats and Jenelius, 2015). Taking Shanghai Metro as an example, on March 10, 2015, a train in Line 2 traveling in the direction towards Guanglan Rd. was forced to stop because of the sudden failure of a pantograph, causing loss of electrical power. The service disruption lasted for more than 5 h, affecting a large number of passengers.

Vulnerability and resilience are two widely used indices to measure network performance (Mattsson and Jenelius, 2015). Reggiani et al. (2015) reviewed recently emerging concepts of resilience and vulnerability in transportation. Network vulnerability in transportation system is defined as the susceptibility to incidents that may result in considerable reduction in network serviceability. In the context of metro network, resilience is correspondingly related to the ability to withstand unexpected incidents with acceptable reduction in operating performance, which is generally measured by the decrease of capacity and the efforts for disruption recovery (Berdica, 2002). Consequently, vulnerability is more about the susceptibility of a system and resilience concerns more with the response of a system. This study mainly focuses on the perspective of vulnerability for metro networks.

Extensive studies on network vulnerabilities have been carried out in many disciplines, while research on metro networks are mainly based on graph theory (Derrible and Kennedy, 2009, 2010a; Gattuso and Miriello, 2005). Gattuso and Miriello (2005) investigated the metro networks of 13 metropolitan areas using graphs and geographical indicators. Comparatively, Derrible and Kennedy (2009) studied the relationship between ridership and network design using updated graph theory concepts, concluding that the network topology plays a key role in attracting travelers to public transit. Various concepts of graph theory were used to describe characteristics of state, form, and structure using new or existing network indicators by studying 33 metro systems in the world (Derrible and Kennedy, 2010a).

Topology vulnerability of metro networks has drawn increasing attention in recent years (Deng et al., 2013; Yang et al., 2015). The topology characteristics and functional properties of the Nanjing metro network in the Jiangsu Province, China, were studied with the Space L model concluding that the network is robust against random attacks, but vulnerable to malicious attacks, similar to power networks (Chen et al., 2010) and air networks (Lordan et al., 2014; Janić, 2015). By investigating the 33 metro network systems throughout the world, Derrible and Kennedy (2010b) found that most metro networks were indeed scale-free (with scaling factors ranging from 2.10 to 5.52) and small-world networks. Two parameters, namely the functionality loss and connectivity of subway lines were used to measure transport functionality and connectivity by taking Lines 4 and 7 of the Shanghai metro, as examples (Zhang et al., 2011), in which the highest betweenness node-based attack was found, to cause the most serious damage to metro networks among the different attack protocols. These studies took important stations or links as disruption subjects to assess function loss of the network. However, no link or station is independent of others. Moreover, disruption of important links or nodes may result in operational failure of the entire line.

While the aforementioned studies only considered the network topology issues, researchers recently began incorporating passenger flow and travel cost to measure the metro vulnerability. Jenelius and Mattsson (2015) proposed to analyze the road network vulnerability, and the impact of disruption scenarios were evaluated from an economic point of view. Sun et al. (2015) introduced origin destination (OD) flows into vulnerability investigation focusing on station vulnerability and proposed a method for identifying important stations within the metro network. Rodríguez-Núñez and García-Palomares (2014) focused on link vulnerability by simulating targeted attack. Riding time and missed trips of disruption scenario were analyzed and results indicated that links carrying many trips and circular line played an important role in network vulnerability. Cats and Jenelius (2014) developed a dynamic and stochastic notion of public transport network vulnerability and found that the importance of links varied depending on the real-time-information provision schemes. Moreover, being devoid of standards in assessing disruption handling efficiency, effectiveness evaluation of a strategic capacity increase on alternative public-transport-network links was proposed to mitigate the impact of unexpected network disruptions (Cats and Jenelius, 2015). De-Los-Santos et al. (2012) evaluated passenger robustness in a rail transit network using time ratio as an evaluation index for two different cases: with and without bridging interruptions. Unfortunately, these studies generally ignored the overall performance of the network, although considerable attention was paid to the details of the reliability or vulnerability assessment.

Another branch of transport network analyses in recent years was on the recovery of disruption. Cadarso et al. (2013) proposed a two-step approach that combined an integrated optimization model (for the timetable and rolling stock) with a passenger behavior model for studying the disruption management problem of rail rapid transit networks. For this consideration, Kepaptsoglou and Karlaftis (2009) discussed the algorithms and models in bus bridging route designing to obtain good results. However, few studies deal with the riding duration and the number of missed trips. Thus, there is still a limitation in considering stations or links as a separate research subject.

Any type of service interruptions occurring on metro networks would affect daily normal functionality (Lou and Zhang, 2011). Previous vulnerability-related researches focus largely on passenger flow and travel time/distance. Unfortunately, little attention was paid from the perspective of line operation, which considers an entire metro line, rather than certain stations or links as an investigating subject during disruptions. In this study, a metro line with one non-operational line during a disruption is studied. The study intends to fill the gap from line operation perspective by taking actual passenger flow of a network into consideration, using the Shanghai metro network as a case study. The overall objective is to measure the vulnerability of metro networks from the line operation perspective. The consequences of disruption in each line were analyzed to determine the impact on the network vulnerability. Moreover, three sub-objectives of the study are as follows:

- To study a metro system considering passenger traffic flow based on a complex network and graph theory.
- To propose an effective method for estimating the vulnerability of metro networks from the perspective of line operation.
- To analyze passenger flow distribution/re-distribution under different disruption scenarios.

The remainder of the paper is organized as follows. Section 2 proposes a methodology for measuring the vulnerability of the metro system as a theoretical network. Section 3 provides the details of the Shanghai metro network and demonstrates the results of vulnerability analysis from the perspective of line operation. Finally, discussion and conclusions are included in Section 4.

## 2. Methodology

In general, metro network can be represented as an undirected graph, $G = \{V, L\}$, where the node set $V = \{v_i | i = 1, 2, \ldots, n\}$ ($n$ denotes the number of nodes) represents metro stations or stops, and the link set, $L = \{l_{ij} | \text{there is a link between } v_i,\ v_j \in V\}$, represents direct connections between nodes, $v_i$ and $v_j$.

The importance of a link or node is determined not only by topology location in the network and availability of appropriate alternatives, but also from the amount of passengers it handles (Jenelius, 2009). To this end, passenger flow is used as an essential element in metro network vulnerability studies. In this study, an all-or-nothing passenger flow assignment principle was used to calculate the passenger flow of each link from OD data during which all passengers would choose the shortest distance route, indicating the shortest travel time and the minimum economic cost from origin to destination. If there are more than one shortest paths between two stations, the passengers were assumed to be distributed on these paths equally. In addition, if two lines are intersecting at a station, an extra link was assumed in between to represent a penalty cost of transferring. Thereafter, the Dijkstra algorithm was used to calculate the shortest path based on a network adjacency matrix.

### 2.1. Basic topological property

Based on a graph theory, the degree of a node is the number of links directly connected to the node. Apparently, nodes with larger degree are generally of higher importance for the connectivity of a network. For a network $G$, containing $n$ nodes, definitions of some performance indicators are provided below.

#### 2.1.1. Betweenness Centrality (BC) and Passenger Betweenness Centrality (PBC)

BC is an indicator reflecting the centrality of a node or link in a network from the network topological perspective. For a node, it represents the share of shortest paths from any node to all others that pass through the node. In general, a node with high BC has a large influence on transferring passengers through a network under the assumption that the passenger transfer follows the shortest paths (Chen et al., 2010). However, as passenger flow is an indispensable element in measuring metro network performance, an improved indicator, namely passenger betweenness centrality (PBC), which considers passenger flow, was introduced. Comparing with BC, PBC is a rather effective indicator of measuring importance of the nodes in a metro system, while BC represents the topological importance of the nodes. Nodes with high PBC serve many passenger trips. If one node suffers disruption, all trips through the node have to take a detour, indicating a longer travel distance (or time) or even transfer to other modes. Here, $PBC_i$ denotes the PBC of node $i$, and the average PBC of each line is denoted by $PBC^l_{ave}$ with $l$ denoting the line index. $BC_i$ and $BC^l_{ave}$ denote betweenness centrality of node $i$ and average betweenness centrality of line $l$, respectively. A line with higher $PBC^l_{ave}$ means the stations and links of this line are used more frequently, indicating that the line is relatively more important. For comparison, both BC and PBC are calculated. The mathematical expressions of $BC_i$, $PBC_i$ and $BC^l_{ave}, PBC^l_{ave}$ are obtained as follows.

$$BC_i = \frac{\sum_{j=1}^{n}\sum_{k=1}^{n} n^i_{jk}}{\sum_{j=1}^{n}\sum_{k=1}^{n} n_{jk}}, \quad (j \neq k;\ i, j, k \in V) \tag{1}$$

$$BC^l_{ave} = \frac{1}{m^l}\sum_{i \in V^l} BC_i, \quad (V^l \subseteq V) \tag{2}$$

$$PBC_i = \frac{\sum_{j=1}^{n}\sum_{k=1}^{n} f^i_{jk}}{\sum_{j=1}^{n}\sum_{k=1}^{n} f_{jk}}, \quad (j \neq k;\ i, j, k \in V) \tag{3}$$

$$PBC^l_{ave} = \frac{1}{m^l}\sum_{i\in V^l} PBC_i, \quad (V^l \subseteq V) \tag{4}$$

where

$n_{jk}$ denotes the total number of shortest paths from node $j$ to node $k$,
$n^i_{jk}$ denotes the number of shortest paths through node $i$ of all shortest paths $n_{jk}$,
$V^l$ denotes the set of stations on the $l^{\text{th}}$ line,
$m^l$ denotes the number of stations on the $l^{th}$ line,
$f_{jk}$ denotes the total number of passengers from node $j$ to node $k$,
$f^i_{jk}$ denotes the number of passengers through node $i$ of all passengers $f_{jk}$.

### 2.1.2. Average path length

The shortest path, $d_{ij}$, between the nodes $i$ and $j$ is the minimum length from nodes $i$ to $j$. In this study, the path length is the summation of the link distance (km) to be traveled; consequently, $d_{ij}$ denotes the length of the shortest path connecting the nodes $i$ and $j$. If there is a change between different lines in one path, a fixed penalty distance is added to the path, which is denoted as $d_{ij}$. By defining the maximum shortest path between any two nodes as network diameter, denoted by $D$, the global connectivity of the network could be quantified. Similarly, the average distance between any two nodes was defined as the average path length denoted by $APL$. In case of a disruption, if no feasible path(s) exist between two nodes that initially had one, the penalty path length between the two nodes is assumed as $D$ in the study.

$$D = \max_{i,j} d_{ij}, \quad (i \neq j;\ i,j \in V) \tag{5}$$

$$APL = \frac{1}{n(n-1)}\sum_i^n\sum_j^n d_{ij}, \quad (i \neq j;\ i,j \in V) \tag{6}$$

In actual metro networks, passenger flow is an important attribute that influences the performance of service. Passengers who cannot complete their trips by metro during disruption were assumed with a travel path, $D$. Considering passenger flow, the weighted average path length, $APL_f$, can be obtained using Eq. (7).

$$APL_f = \frac{1}{\sum_i^n\sum_j^n f_{ij}}\sum_i^n\sum_j^n d_{ij}f_{ij}, \quad (i \neq j;\ i,j \in V) \tag{7}$$

### 2.1.3. Global efficiency

Global efficiency is a comprehensive index for evaluating network performance, denoted by $E$. The calculation is similar to the average path length, but involves taking the reciprocal of $d_{ij}$. When there is no feasible path(s) between two stations that initially had one, the distance, $d_{ij}$, is similarly set as D. Thereafter, passenger flow is introduced to obtain the weighted global efficiency, $E_f$.

$$E = \frac{1}{n(n-1)}\sum_i^n\sum_j^n \frac{1}{d_{ij}}, \quad (i \neq j;\ i,j \in V) \tag{8}$$

$$E_f = \frac{1}{\sum_i^n\sum_j^n f_{ij}}\sum_i^n\sum_j^n \frac{1}{d_{ij}}f_{ij}, \quad (i \neq j;\ i,j \in V) \tag{9}$$

## 2.2. Method for evaluating vulnerability

In this section, vulnerability is measured at a line operation level in the sense that when an important station or link disrupts, the entire line would be seriously affected; it may even go completely out of service. Rodríguez-Núñez and García-Palomares (2014) have shed light on the influence of entire line disruption on a network. $PBC^l_{ave}$, was used to evaluate the importance of a line in a network from passengers traveling perspective with $BC^l_{ave}$ as a cross reference from topological perspective considering passenger flow. As metro systems are commuter transit systems, passenger flow is a key factor for measuring network vulnerability. From this perspective, variation of passengers involved and the corresponding travel distance should be incorporated into an impact evaluation based on each line disruption scenario. When a line is assumed to be disrupted, all trips that originally used the stations belonging to the disruption line are assumed to be cancelled and the rest have to be reassigned to other lines based on the all-or-nothing assignment rule.

Once a metro line is out of service, passengers have to take a detour or transfer to other travel modes. Passengers who can take a detour during the disruption are assumed to be well aware of metro operation status and are willing to perform the detour. Considering that the metro lines often suffer disruption abruptly and the duration cannot be estimated accurately, passengers who cannot take a detour during the disruption are assumed to transfer to other modes. As interchange between different travel modes is not the focus of this study, a fixed penalty distance was assumed for the passengers who prefer a transfer to other modes during disruption.

Previous studies (Chen et al., 2010; Zhang et al., 2011) used the node-based attack strategy to investigate the topological vulnerability of a metro network. This study focuses on the vulnerability of a metro network from line operation perspective, and the disruption probability based on the number of stations was introduced. The relative disruption probability is assumed to be proportional to the summation of stations along the line, denoted as *prob*.

$$prob = \frac{m^l}{\sum_k m^k} \tag{10}$$

Let $P(\sigma_0)$ denote the number of passengers served in baseline scenario without any disruptions. Correspondingly, $P(\sigma)$ denotes the passengers during an interruption scenario, $\sigma$. Based on this consideration, variation of passengers can be normalized as $\frac{P(\sigma)-P(\sigma_0)}{P(\sigma_0)}$. Generally, the passengers on the disruption line transferring to alternative lines have longer travel distance and travel time. The variation of the weighted average path length, $APL_f$, and the weighted global efficiency, $E_f$, were used to denote the travel time and the comprehensive performance for the network. Generally, as disruption duration cannot be accurately predicted and publicized, passengers who cannot take a detour during disruptions were assumed to transfer to other travel modes. The normalized variation of the weighted average path length and the weighted global efficiency were obtained similar to the variation of passengers.

To quantitatively assess the consequence of a disruption, the impact of each disruption scenario, $\sigma$, from passengers' perspective was measured as the total difference between the disruption scenario, $\sigma$, and the baseline scenario, $\sigma_0$, denoted as $\Delta W(\sigma)$.

$$\Delta W(\sigma) = prob \cdot \left( \mu_P \frac{P(\sigma) - P(\sigma_0)}{P(\sigma_0)} + \mu_{APL} \frac{APL_f(\sigma_0) - APL_f(\sigma)}{APL_f(\sigma_0)} + \mu_E \frac{E_f(\sigma) - E_f(\sigma_0)}{E_f(\sigma_0)} \right) \tag{11}$$

$$\mu_P + \mu_{APL} + \mu_E = 1 \tag{12}$$

where

$P(\sigma_0)$: denotes the service passengers in baseline scenario, $\sigma_0$,
$P(\sigma)$: denotes the service passengers in disruption scenario, $\sigma$,
$APL_f(\sigma_0)$: denotes the weighted average path length in baseline scenario, $\sigma_0$,
$APL_f(\sigma)$: denotes the weighted average path length in disruption scenario, $\sigma$,
$E_f(\sigma_0)$: denotes the weighted global efficiency in baseline scenario, $\sigma_0$,
$E_f(\sigma)$: denotes the weighted global efficiency in disruption scenario, $\sigma$,
$\mu_P$, $\mu_{APL}$, $\mu_E$: denote the weight coefficients.

$W(\sigma)$ is a negative number indicating vulnerability from the line operation perspective. As the value of $\Delta W(\sigma)$ tends to zero, smaller is the influence of the disruption. A negative value of $\Delta W(\sigma)$ indicates the loss of the network function. Eq. (12) is the constraint of Eq. (11) for ensuring normalization of $\Delta W(\sigma)$. The values of $\mu_P$, $\mu_{APL}$, $\mu_E$ may be obtained based on different investigating objectives.

## 3. Vulnerability analysis of Shanghai metro network

### 3.1. Shanghai metro network

By the end of 2013, the Shanghai metro consisted of 11 lines, 259 stations, and 295 links with a total length of 467.24 km. Of all the lines within the network, seven are transversal (Lines 1, 3, 5, 6, 7, 8, and 11), three are radial (Lines 2, 9, and 10), and one is circular (Line 4), as shown in Fig. 1. Line 5 has the minimum number of stations (11 stations) and the minimum line length (17.2 km), while Line 11 has the maximum number of stations (36 stations) and the maximum length (72 km). The average link distance ranges from 1.2 km (Line 10) to 2.21 km (Line 2). The network is one of the most concentrated metro systems in the world with a complex structure and a large size, 31 stations intersected by two lines, four stations with three lines and one station with four lines. OD (origin-destination) trips from AM peak (7:30–8:30), September 16, 2013, were obtained from the Shanghai Shentong Metro Group Co., Ltd., which contain more than 530,000 trips, as well as the flow distribution. The network diameter is 86.26 km, and the average path length is 22.51 km, while the weighted average path length is 14.37 km. The penalty distance for the transfers is assumed as the average link distance, which is 1.58 km.

$BC^l_{ave}$, defined in Eq. (2) and $PBC^l_{ave}$, defined in Eq. (4), were calculated to evaluate the importance of each line as shown in Table 1. Line 4 was found with the largest $BC^l_{ave}$, 0.0063. Comparison with the rest of the lines ranging from 0.0035 to 0.0059, indicates that Line 4 is more important from the topological perspective. When it comes to the passengers traveling perspective, Line 1 has the largest $PBC^l_{ave}$ value, with Line 2 and Line 4 ranking behind. The transversal Line 1 and radial Line 2 are the

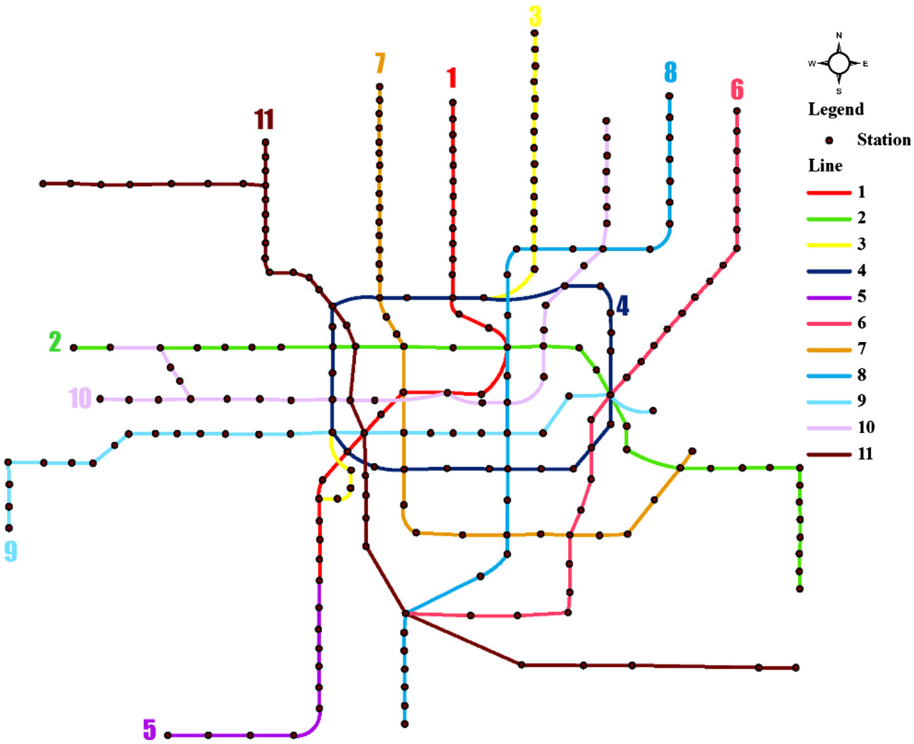


**Fig. 1.** Topological structure of the Shanghai metro network (schematic diagram).

**Table 1**
BC and PBC values of each Line.

| Line | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 | 11 |
|---|---|---|---|---|---|---|---|---|---|---|---|
| $BC^l_{ave}$ | 0.0054 | 0.0054 | 0.0053 | **0.0063** | 0.0026 | 0.0036 | 0.0045 | 0.0043 | 0.0053 | 0.0035 | 0.0059 |
| $PBC^l_{ave}$ | **0.0082** | **0.0067** | 0.0033 | **0.0061** | 0.0018 | 0.0027 | 0.004 | 0.0045 | 0.0057 | 0.004 | 0.0031 |

earliest lines put into operation, and they are regarded as the most important commuter lines in the Shanghai metro. Previous research (Rodríguez-Núñez and García-Palomares, 2014) had demonstrated the importance of a circular line for the network robustness considering the Madrid metro network. In order to analyze the vulnerability importance of each line, disruption scenarios of each line in the Shanghai metro were studied in the remainder of this section.

### *3.2. Vulnerability analysis from perspective of line*

The network vulnerability was investigated under disruption of each metro line. $\Delta W(\sigma)$ was calculated using Eq. (11) by assuming all the values of $\mu_P$, $\mu_{APL}$, $\mu_E$ as 1/3 (Deng et al., 2013), with the corresponding numerical results presented in Table 2.

The column of passenger trips ($P$) in Table 2 shows that disruption of Lines 1 and 2 has the largest passenger loss, indicating both lines are the most important commuter lines within the network. Disruption scenarios of Lines 1 and 2 also have the largest changes in the weighted average path length $APL_f$ and the weighted global efficiency $E_f$; the disruption scenario of Line 4 ranks behind that of Lines 1 and 2. The differences in disruption probability may reflect the topological attribute for the network. The disruption probability of Line 11 is 12.38%, ranking the first among all 11 lines, as it has the largest number of stations. Combing these three indicators, it was found that the disruption of Line 2 has the lowest value of $\Delta W(\sigma)$, with disruption of Line 1 ranking second. Table 3 provides three scenarios by considering different weight coefficients. The results show that disruption of Line 2 still has the greatest influence on the overall network vulnerability under each scenario.

The results of this section indicate that radial Line 2 is the most influential line among all the lines, as it handles the largest number of passengers compared to the rest of the lines. The results of $PBC^l_{ave}$ and $BC^l_{ave}$ values are somewhat different, which reflect the importance of a line from the passengers' traveling perspective and the topological perspective, respectively.

**Table 2**
Vulnerability analyses from line perspective.

| Scenario | $P$ | $APL_f$ | $E_f$ | $Prob$ | $\Delta W(\sigma)$ | $\Delta \boldsymbol{W}(\boldsymbol{\sigma})^*$ $prob = 1$ |
|---|---|---|---|---|---|---|
| Baseline scenario | 525,153 | 14.373 | 0.111 | – | – | – |
| Disruption of Line 1 | 436,525 | 20.16 | 0.094 | 9.12% | −2.22% | −24.33% |
| Disruption of Line 2 | **404,039** | **20.934** | **0.089** | 9.77% | −**2.9%** | −**29.67%** |
| Disruption of Line 3 | 483,820 | 16.499 | 0.105 | 9.45% | −0.89% | −9.38% |
| Disruption of Line 4 | 459,230 | 18.362 | 0.097 | 8.47% | −1.51% | −17.81% |
| Disruption of Line 5 | 504,589 | 15.276 | 0.109 | 3.58% | −0.15% | −4.22% |
| Disruption of Line 6 | 485,301 | 16.492 | 0.105 | 9.12% | −0.85% | −9.35% |
| Disruption of Line 7 | 463,055 | 17.75 | 0.101 | 10.75% | −1.6% | −14.91% |
| Disruption of Line 8 | 462,296 | 17.79 | 0.1 | 9.77% | −1.49% | −15.25% |
| Disruption of Line 9 | 447,432 | 18.104 | 0.1 | 7.49% | −1.27% | −16.99% |
| Disruption of Line 10 | 483,633 | 16.88 | 0.102 | 10.1% | −1.13% | −11.2% |
| Disruption of Line 11 | 483,884 | 16.207 | 0.106 | 12.38% | −1.04% | −8.43% |

Note: $\Delta \boldsymbol{W}(\boldsymbol{\sigma})^*$ represents the impact of disruption scenario with *prob* of each line being equal to unity.

**Table 3**
Vulnerability analysis from perspective of lines considering different weight coefficients.

| Scenario | $\Delta \boldsymbol{W}(\boldsymbol{\sigma})$ | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|
| | $\mu_P$ **0** | $\mu_{APL}$ **½** | $\mu_E$ **½** | $\mu_P$ **½** | $\mu_{APL}$ **0** | $\mu_E$ **½** | $\mu_P$ **½** | $\mu_{APL}$ **½** | $\mu_E$ **0** |
| Disruption of Line 1 | −2.56% | | | −1.49% | | | −2.61% | | |
| Disruption of Line 2 | −**3.22%** | | | −**2.12%** | | | −**3.36%** | | |
| Disruption of Line 3 | −0.96% | | | −0.63% | | | −1.07% | | |
| Disruption of Line 4 | −1.73% | | | −1.09% | | | −1.71% | | |
| Disruption of Line 5 | −0.16% | | | −0.11% | | | −0.18% | | |
| Disruption of Line 6 | −0.93% | | | −0.61% | | | −1.02% | | |
| Disruption of Line 7 | −1.77% | | | −1.14% | | | −1.9% | | |
| Disruption of Line 8 | −1.65% | | | −1.07% | | | −1.75% | | |
| Disruption of Line 9 | −1.35% | | | −0.94% | | | −1.53% | | |
| Disruption of Line 10 | −1.3% | | | −0.82% | | | −1.28% | | |
| Disruption of Line 11 | −1.08% | | | −0.78% | | | −1.28% | | |

Considering all the parameters, Line 2 is recognized as the most important line in the Shanghai metro system, while Line 1 ranks second. Hence, Line 2 was selected as a case for further investigation on passengers' distribution and redistribution across the network. The Circular Line 4 with numerous transfer stations generally provides alternative route choices with similar or even shorter travel distances. Considering the special topological form of Line 4 and decent performance in $PBC^l_{ave}$ and $BC^l_{ave}$, Line 4 is selected as another case study for further analysis.

### *3.3. Passenger flow distribution of Lines 2 and 4*

In addition to the largest passenger flow, Line 2 has the longest average station distance, which is 2.21 km. Taking into account that the passenger flow represents dependence on metro, and link distance represents travel time, it is concluded that Line 2 is the most important line in operation. The circular Line 4 is a collinear circular line, including 9 links from Baoshan Rd. to Yishan Rd. overlapped with Line 3. For simplicity, all the overlapped stations were regarded as belonging to Line 4. Of the 26 stations on Line 4, 17 stations (accounting for probably 65% of all the stations on Line 4) are referred to as transfer stations, which infer that disruption of Line 4 would have a significant impact on both passenger flow distribution and network topological properties. The passenger flow distribution was studied under the disruption scenarios of Line 2 and Line 4.

This study assumes that the passenger flow redistribution follows an all-or-nothing assignment rule. When a line is disrupted, it is assumed that the travel time remains constant for the non-disrupted lines, similar to the study by Rodríguez-Núñez and García-Palomares (2014). The all-or-nothing assignment rule for a metro network indicates that all the passengers choose the shortest path without considering the congestion situation, which seems somewhat inconsistent practically. However, commuting passengers in Shanghai generally pay more attention to travel time instead of congestion during AM peak hours, and consequently, the all-or-nothing assignment rule becomes reasonable. Once disruption of any line occurs, the passenger flow will be redistributed. In this case, the Shanghai metro system was supposed to have had sufficient experience in dealing with disruption scenarios and releasing relevant information. Therefore, all passengers are assumed to be well informed regarding the disruption scenarios and alternative routes. Fig. 2 presents (a) passenger trip distributions for baseline scenario, (b) re-distributions for Line 2 disruption scenario, and (c) Line 4 disruption scenario.

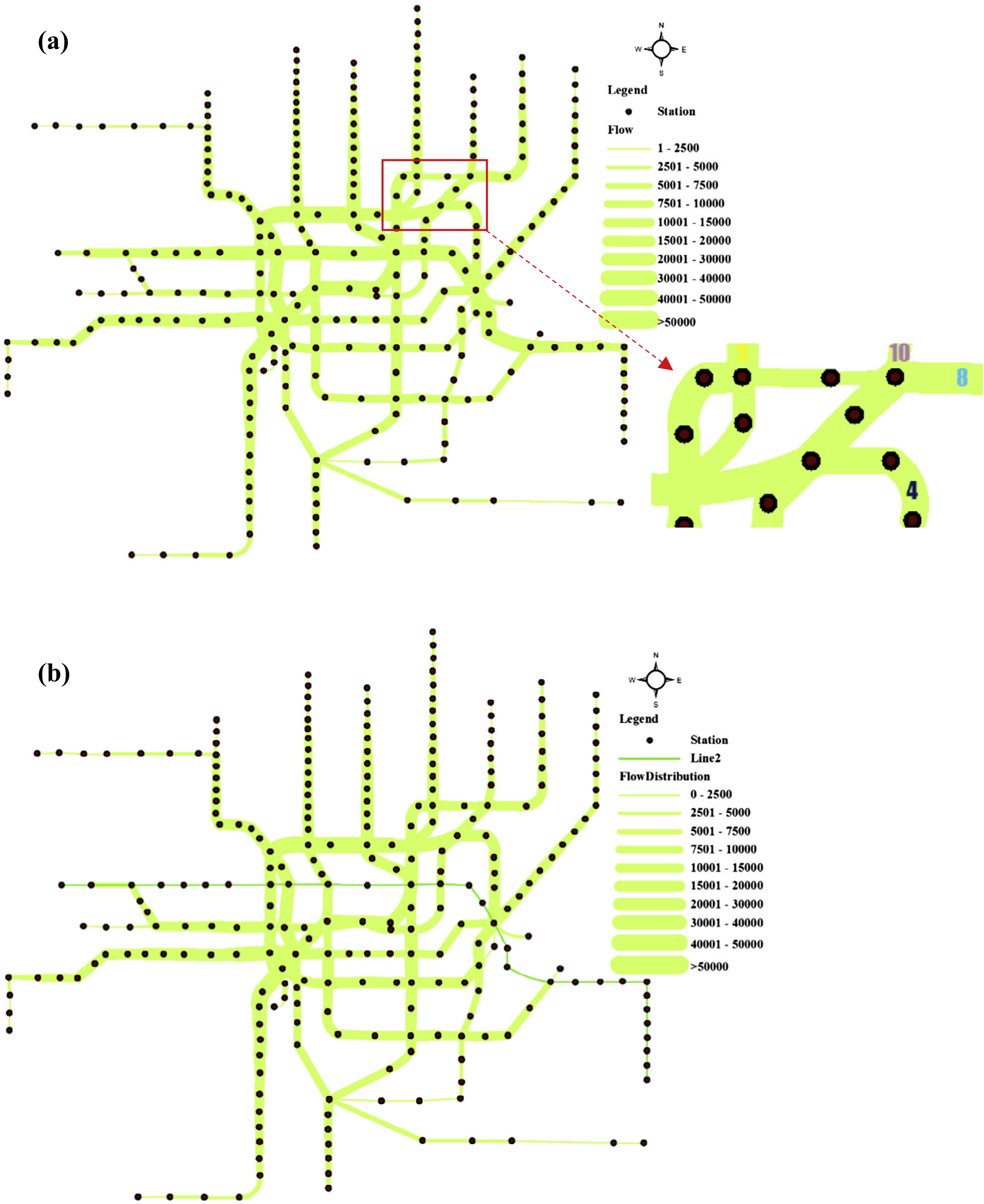


**Fig. 2.** Passenger flow distribution (a) Baseline scenario, (b) Line 2 disruption scenario, and (c) Line 4 disruption scenario.

As presented in Fig. 2(a), the passenger flow gradually gathered in the direction from peripheral to city center zones. Lines 4, 2, and 1 have the largest average number of trips per link, which is more than 20,000 trips per hour. Fig. 2(b) provides flow distribution of the Line 2 disruption scenario. Enlarged portion of the red[1] rectangle on the right side is provided to display the variation of passenger flow distribution more clearly, as presented in Fig. 2(a) and (c). Line 4 carries a large number of

[1] For interpretation of color in Fig. 2, the reader is referred to the web version of this article.

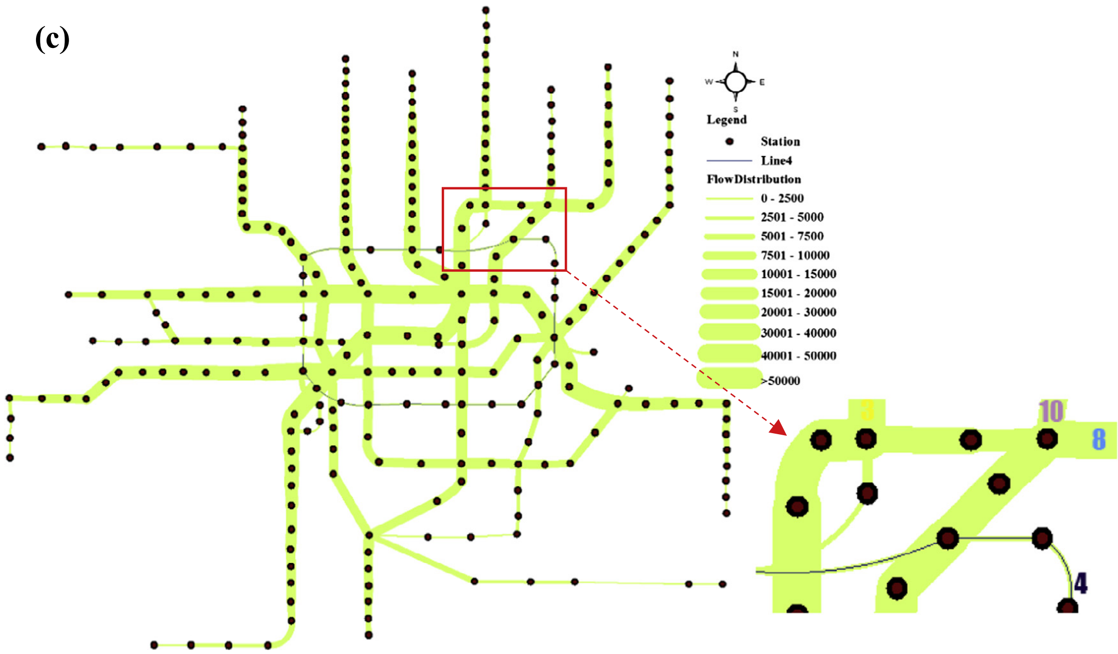


**Fig. 2** (*continued*)

passengers in the baseline scenario and the lines that intersect with Line 4 carry a smaller number of passengers. When disruption occurred on Line 4, the passenger distribution undergoes large changes—passenger flow suffers a sharp decline, even to zero on the links of Line 3 connecting with Line 4; the opposite situation occurs on Lines 8 and 10 leading to the city center.

The flow variation schematic under disruption condition is presented in Fig. 3. It is established that the trips increase almost on each link of Line 4, while it decreases on most of the other links in the network. The reason for this phenomenon is mainly that Line 4 has a rather large number of transfer stations offering several alternative routes.

The major difference between the circular and normal lines (radial or transversal) is that several stations on the circular line are transfer stations with rather good connectivity performance. A large number of passengers could be channeled through the circular line in order to reach their destinations through the shortest path. As the circular Line 4 has the largest topological importance, passengers traveling towards the city center in Line 4 may have to travel a much longer distance via other routes, when it is out of service.

As the result of this redistribution, passenger flow on the circular line periphery reduces, as seen in Fig. 3(b), while the inner periphery increases. The largest difference between the two figures in Fig. 3 is that disruption of Line 2 results in an increase of trips on Line 4, while disruption of Line 4 results in trip increases on the adjacent links. The reason is that the two lines have different topology roles within the network. Line 2 represents a common line mainly transporting commuters, while circular Line 4 represents a specific line not only for transporting commuters, but also for connecting with other lines that offer alternative transfers.

## 4. Discussion and conclusion

This study proposed a methodology for measuring the vulnerability of metro network from line operation perspective. Field passenger traffic data were adopted to measuring vulnerability of metro network. Previous works (Deng et al., 2013; Sun et al., 2015; Zhang et al., 2011) have paid attention on simulation of important stations and links for measuring the vulnerability of metro networks. However, this study expands the previous research (Rodríguez-Núñez and García-Palomares, 2014) on public transport network vulnerability and provides an innovative perspective to understand network vulnerability from line operation with consideration of disruption probability. Line operation is a concept corresponding to stations/links operation, which assumes the entire line would be out of work during disruption.

A methodology is proposed for identifying the most influential lines and measuring vulnerability of the metro network. Taking Shanghai Metro Network as an example, network topology considering $BC^l_{ave}$ (the average betweenness centrality of each line) was analyzed to determine the most topologically important line. The result indicates that circular Line 4 has the highest value of $BC^l_{ave}$ for its specific morphology. An improved indicator, $PBC^l_{ave}$, which took passenger flow into consideration, shows that Line 1 has the highest value, followed by Line 2 and Line 4. A reasonable explanation for this situation is that passenger flow in Line 1 is the second highest, and the passengers on Line 5 became unconnected with the network

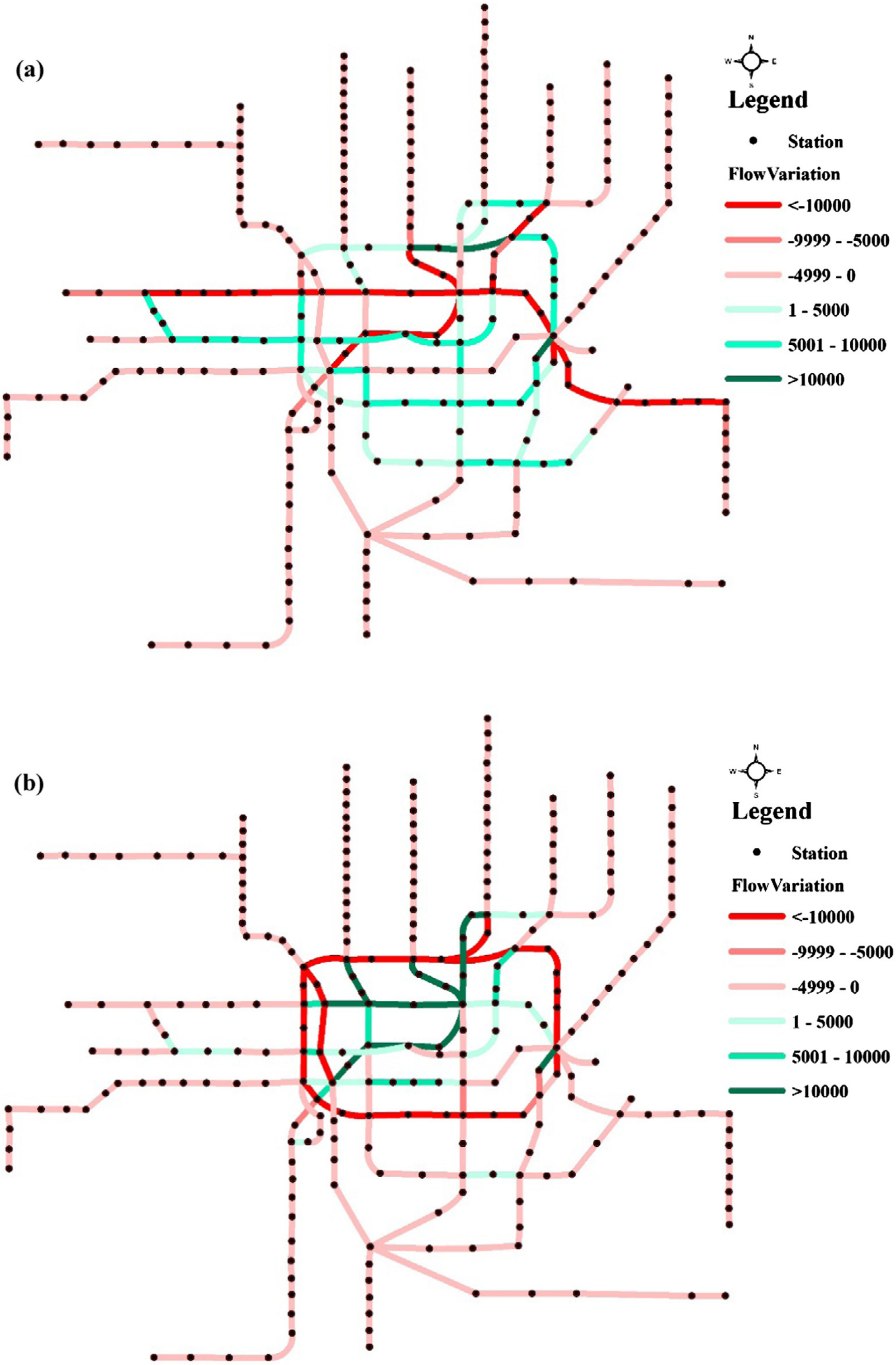


**Fig. 3.** Variation of number of trips under disruption condition (a) Line 2 disruption scenario and (b) Line 4 disruption scenario.

under the disruption scenario. Thereafter, the vulnerability indicators were calculated under scenarios of each line with disruption with others remaining in operation. The disruption of Line 2 has the lowest value of $\Delta W(\sigma)$ indicating the largest functional influence during disruption; Line 1 follows in the second place. Finally, the passenger flow redistribution with disruption of Lines 2 and 4 were visualized in ArcGIS software and the topological properties were analyzed under the baseline scenario and the disruption scenario of Line 4. The conclusions drawn are as follows:

- Passenger flow is the key factor for network vulnerability. Metro network is a passenger-based transport system, in which disruption of lines having a large number of trips would have a comparably larger influence.
- Large differences in passenger flow redistribution exist between disruption of the ordinary metro (such as Line 2) and circular lines (such as circular Line 4), in which the circular line plays a key role because of the capability to provide multiple alternative routes to redistribute flows.

This study fills the research gap in the metro network vulnerability analysis from a line operation perspective. The methodology proposed in this study may provide good guidelines for metro network vulnerability analysis and offer useful information for the potential incidents. Identifying important lines within the network facilitates management and protection tasks to ensure operations safely. Moreover, redistribution of the passenger flow on the network is helpful for managing potential increase of other lines or the trips transferring to other travel modes. In addition, metro operation agencies may pay attention to vulnerable lines and make contingency plans for unexpected disruption scenarios, and passengers can have a backup plan for transferring to other lines or travel modes when encountering disruptions. To deal with line disruptions, strategies, such as providing shuttle buses may be incorporated, which can be a future research topic.

Although the results are promising, this study can be improved from several aspects. First, the disruption of the line operation without considering part of the line operation may not be reliable in practice and requires further improvement. Moreover, disruption is seldom limited to one line because of the shared stations and links; other lines can be inevitably influenced to some extent. The generalized travel cost in this study is simplified to travel distance by ignoring monetary travel cost; the disruption probability is assumed related to the number of stations without considering link distance, which may bring some adverse effects. This is an issue which future research needs to address.

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