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Review

Fracture properties of asphalt mixtures using semi-circular bending test: A state-of-the-art review and future research

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HIGHLIGHTS

- Presented current state-of-the-art regarding SCB test pertinent to asphalt mixtures.
- Discussed fundamental fracture assessment based on load-deformation characteristics.
- Documented analytical procedures to deduce fracture parameters for asphalt mixes.
- SCB test is a promising crack propagation assessment candidate for asphalt material.

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ABSTRACT

Although many fracture test procedures are available, the semi-circular bending (SCB) test has drawn a growing interest in the pavement community due to its simplicity, reproducibility, and flexibility in testing and evaluation. In this direction, fracture properties of asphalt mixtures are currently being evaluated using SCB test with the application of fracture mechanics to characterize low-temperature and fatigue fracture using the standard semi-circular bending protocols. However, several research studies have employed various sets of specifications suiting practical convenience that calls for a critical review of the procedures that have been followed to date. This review article presents the current state-of-the-art regarding the utilization of SCB test to evaluate fracture properties of different asphalt mixtures. The fundamental assessment of fracture through the SCB test, which was based on load-deformation characteristics of asphalt mixes, was discussed in detail. The analytical procedures employed to deduce fracture parameters for asphalt mixes to understand the fracture performance was also documented. Overall, the SCB test procedure was found to be a promising crack propagation assessment candidate to evaluate asphalt mix fracture properties. It was recommended that future studies must concentrate on developing cyclic SCB test to investigate the dynamic fatigue behaviour along with viscoelastic properties. Certainly, there exists scope for advancing the current state-of-the-art pertaining to the SCB test procedure that actually simulates the field performance characteristics in conjunction with mechanistic based flexible pavement designs.

Contents

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1. Introduction

Fracture forms an integral part of fatigue and low-temperature cracking mechanisms in flexible asphalt pavements. A robust and rational pavement design necessitates incorporation of an important fundamental property such as fracture to ensure its successful implementation in predicting long-term pavement performance. In this context, lower fracture performance of an asphalt material will entail diminished service life of the pavements, and eventually results in a premature failure.

The current flexible pavement design practices [1–7] fundamentally assess fracture of asphalt mixtures based on the limiting strain (for fatigue) and stress (for low-temperature cracking) criteria within the linear elastic regime. It has been a practice to use fracture mechanics principles to derive and analyze fracture properties such as fracture toughness and fracture energy parameters in the cracking processes. However, these successful design methodologies emphasize crack initiation and total failure stages through basic stress–strain analyses. Essentially, the processes do not directly account for the time-dependent viscoelastic properties and the associated fracture behaviour by targeting crack propagation phase.

Fracture phenomenon has been historically investigated through laboratory experimentations, numerical simulations, and field evaluation studies. In the last few decades, laboratory investigations along with analytical and numerical simulations have taken a major share in asphalt mix fracture characterization research. Currently, fracture characteristics of asphalt mixtures are being evaluated in the laboratory using: Single Edge Notch Beam (SENB); Disc-shaped Compact Tension (DCT); and Semi-Circular Bending (SCB) tests. Owing to the several merits such as repeatability, reproducibility, consistency, and simplicity in terms of specimen preparation and testing, the SCB test has received a growing interest by the research community to characterize fracture properties of asphalt mixtures. Furthermore, the success in generating the requisite parameters for fracture assessment has ensued in the development of an SCB standard protocol for monotonic loading conditions [8,9].

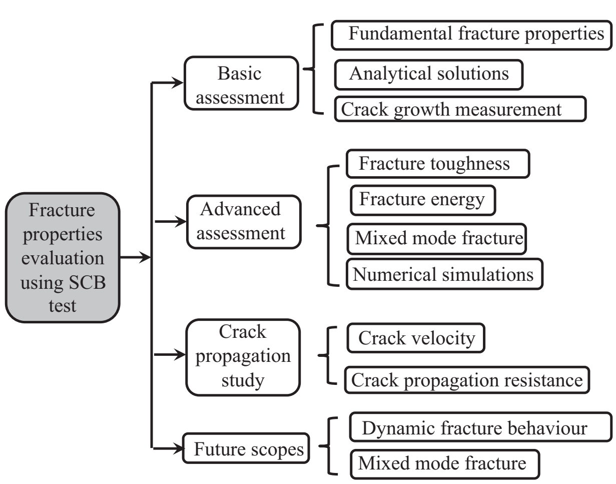

Fig. 1. Research review outline.

Due to its increasing popularity, monotonic SCB tests have been conducted extensively on different types of asphalt mixtures, and the obtained fracture properties have been analyzed using fracture mechanics principles to characterize fracture behaviour. Concurrently, studies have also used various other specifications as inputs for testing purposes and digressing from the standard protocols, which simply call for a critical review of the deviated procedures to assess if those findings can actually correlate with field performance. Thus, a consolidated discussion summary on the various test specifications used in conducting SCB test and its applications (findings) is needed. This collated review will help researchers and practitioners to comprehensively understand the tangibility of the test technique to assess fracture-cracking behaviour of asphalt mixtures. Moreover, it is also envisaged that this compilation will offer necessary inputs to further advance the state-of-the-art pertinent to fracture in asphalt mixtures so that new design practices can be developed incorporating SCB test parameters and associated mechanistic principles.

The main purpose of this research review paper was to assemble and present the current knowledge about the utilization of SCB test to evaluate fracture properties of asphalt mixtures, which is also aimed at taking forward this research area for implementation in flexible pavement designs. Although limited research is available regarding the SCB test and the associated findings on asphalt mixtures' fracture properties, it is envisaged that this methodology turns out to be a promising candidate test to assess fracture performance. Note that this review discussion focuses only on SCB test technique and other test procedures will not be discussed. Fig. 1 presents the review scope outline. The paper is divided into four major heads: (i) brief description of the SCB test background and standard protocol; (ii) fracture failure experimental and numerical investigation findings from the associated studies; (iii) advanced fracture resistance characterization using SCB test fracture parameters obtained during various investigations; and (iv) fracture crack propagation properties evaluation. A discussion summary regarding the current state-of-the-art is provided at the end of the review along with future prospects of the SCB test methodology that illustrate its worthiness of being a promising fracture properties assessor for asphalt mixtures.

2. Background to static SCB test

SCB test was first employed by material scientists as a means to determine fracture resistance of rock materials and reported in [10]. Later, it was adopted by pavement engineers to understand fracture characteristics of different asphalt mixtures, which led to the development of standard protocols for monotonic loading conditions. EN12697-44: 2010 [8] and AASHTO TP105-2013 [9]

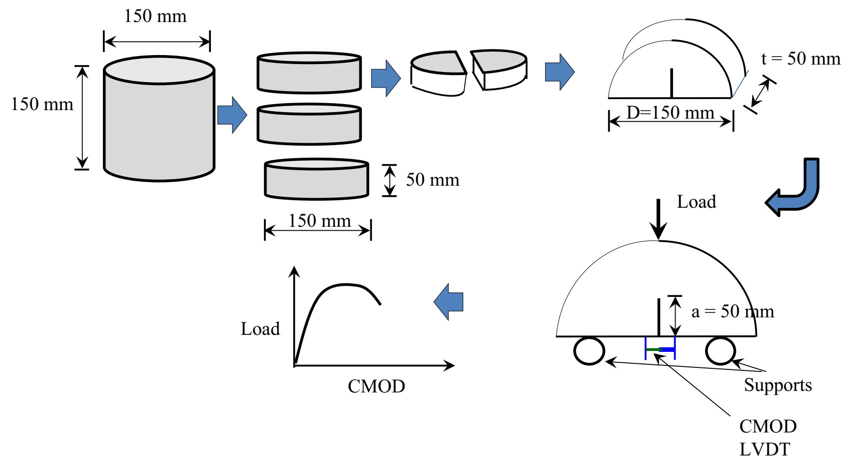

Fig. 2. Schematic of SCB specimen preparation and test procedure. *Note: D = diameter, t = thickness, a = notch depth, CMOD = crack mouth opening displacement.*

provide specifications pertinent to static (monotonic) SCB test for asphalt mixtures. The test parameters such as loading rate, specimen geometry, support conditions, and fracture toughness and energy estimations are undertaken during the experimentation procedure using [8,9].

The premise of the test technique lies in the basic theory of fracture mechanics. In this test, a semi-circular specimen geometry having a central notch is mounted on the roller supports, and is loaded from the top of the specimen. Due to loading, the tensile stress develops and a crack initiates at the tip of the notch. A sequential schematic of SCB specimen procedure and test conduction are illustrated in Fig. 2. For linear elastic materials, stress distribution near the crack tip can be explained using the stress intensity factor, K. K is a function of the applied stress and geometric factor of the specimen. K increases with increasing applied stress and reaches a critical value (K_{IC}) when failure occurs. K_{IC}, also termed as fracture toughness is an intrinsic property of the material, which helps explain the material's resistance against fracture. The maximum stress at failure (σ_{max}) and K_{IC} are calculated using the following Equations based on [8] with conditions of the ratio of the support span to the specimen diameter being 0.8.

$$\sigma_{max} = \frac{4.263\ P_{max}}{D \times t} \tag{1}$$

$$K_{IC} = \sigma_{max} \times f\left(\frac{a}{w}\right) \tag{2}$$

$$f\left(\frac{a}{w}\right) = -4.9965 + 155.58\left(\frac{a}{w}\right) - 799.94\left(\frac{a}{w}\right)^2 + 2141.9\left(\frac{a}{w}\right)^3 - 2709.1\left(\frac{a}{w}\right)^4 + 1398.6\left(\frac{a}{w}\right)^5 \tag{3}$$

where:

σ_{max} = Maximum stress at failure, N/mm^2
P_{max} = Maximum load at failure, N
D = Diameter of specimen, mm
t = Thickness of specimen, mm
K_{IC} = Fracture toughness, N/mm$^{3/2}$
a = Notch length of specimen, mm
w = Width of specimen, mm

The constant term (*4.263*) in Eq. (1) includes square root of (πa). Since the stress $\sigma_{\max}$ incorporates the square root of a by the means of the constant term, the product of the $\sigma_{\max}$ with units N/mm^2 and square root of a with units mm$^{1/2}$ produces the unit of K, which is given by (N/mm$^{3/2}$). Note that the concept is based on [8]. It is noteworthy that several research studies investigated the process of fracture following various research-specific specifications and not conforming completely to the standard protocols. The studies conducted to evaluate the fracture fatigue performance of asphalt mixtures using SCB test technique is summarized in a chronological manner as shown in Table 1.

3. Fracture properties: basic assessment

During the first phase of utilization of SCB test on asphalt mixtures, fracture properties were obtained by basic load-deformation characteristics. However, at a later stage, fracture properties obtained from the test were simulated using analytical solutions based on mechanics. Attempts were also made to understand the fracture mechanism through digital image correlations that led to the development of crack propagation concept.

3.1. Fundamental fracture failure properties

During the latter half of 1990s, the SCB test was fundamentally employed to investigate and determine fracture resistance of asphalt mixtures using load-deformation characteristics. Essentially, the pertaining studies focused on crack initiation and propagation pattern, which provided the basic understanding of fracture failure trends of asphalt materials. Krans et al. [11] used SCB test on asphalt mixes in static and dynamic loading conditions. All the tests were conducted on dense-graded semi-circular asphalt specimens with 25 mm thickness and at 20 °C. Although the specimens were not notched, the cracks were reported as initiated at the mid-span of the support. It was noted that the length of the stable crack was considered to be two-thirds of the specimen height. The static SCB test produced an average failure load of 4.7 ± 0.4 kN with a force per unit width of 190 N/mm, which is typical of a dense-graded asphalt mix. In another study [12], it was observed that the SCB test practice had the potential to assess the realistic mode of fracture experienced in the field. Further, the failure load of the SCB specimen decreased with increasing notch depth.

Elsewhere, fracture crack initiation and propagation resistance of asphalt materials were studied based on failure load criteria [13]. In the first phase, the load increment that resulted in a visible form of micro-crack and void coalescence without any load reduction was identified as the crack initiation load (P_{int}). Afterwards, it was found that the specimen continued to sustain increasing loads till it attained the ultimate or peak load (P_{ult}). Thus, P_{int} was employed to measure the crack initiation strength whereas P_{ult} was implemented to characterize the overall tensile strength of

Table 1
Summary of the SCB research studies.

Study findings	Test type	Authors	Year
Introduction of SCB in rock materials	Static	Chong and Kuruppu [10]	1984
Analytical solution of fracture parameters	Static	Lim et al. [40]	1993
Fracture failure load and cycle investigation	Static & Dynamic	Krans et al. [11]	1996
J integral determination	Static	Mull et al. [31]	2002
Fracture toughness and crack propagation	Static & Dynamic	Hofman et al. [20]	2003
Loading rate dependency of fracture toughness	Static	Molenaar et al. [26]	2003
J integral determination and its sensitivity	Static	Mohammad et al. [12]	2004
Analytical solutions and tensile strength evaluation	Static	Huang et al. [14]	2005
J integral evaluation and its sensitivity	Static	Wu et al. [36]	2005
J_c and fracture resistance characterization	Static	Othman [32]	2006
Crack pattern study and digital image correlation	Static	Birgisson et al. [16]	2008
Fracture toughness and its potential to assess fatigue performance	Static	Arabani and Ferdowsi [18]	2008
Crack velocity and effect of mix variables on fracture properties	Static	Tarefdar et al. [13]	2009
Analytical solutions and stiffness modulus	Static	Huang et al. [15]	2009
Fracture properties and its effect on mix composition	Static	Behbahani et al. [21]	2009
Fracture toughness and fracture energy evaluation	Static	Khalid and Monney [28]	2009
Low temperature fracture resistance	Static	Li and Marasteanu [30]	2010
Fracture toughness determination	Static	Othman [33]	2010
Frequency effect on fatigue life	Dynamic	Hassan and Khalid [47]	2010
J_C determination and fracture characterization	Static	Liu [34]	2011
Fracture toughness and fracture energy on field core asphalt mixes	Static	Biligiri et al. [22]	2012
Mix mode fracture determination	Static	Aliha et al. [39]	2012
J_c and its potential in predicting field cracking	Static	Mohammad et al. [35]	2012
Mixed mode fracture characterization	Static	Sallam and Abd-Elhady [43]	2012
Fatigue life using Paris' law parameter	Dynamic	Huang et al. [24]	2013
Cohesive zone modeling and fracture energy characterization	Static	Im et al. [41]	2013
FEM analysis on crack propagation	Static	Im et al. [41]	2013
XEFM modeling and creep compliance	Static	Lancaster et al. [44]	2013
XEFM analysis and J integral	Static	Wang et al. [45]	2013
Cyclic J integral and Paris' law parameter	Dynamic	Hassan [48]	2013
Mix mode fracture properties determination	Static	Im et al. [23]	2014
Fracture properties and its sensitivity	Static	Minhajuddin et al. [29]	2015
Crack propagation based on fracture energy	Static	Saha and Biligiri [46]	2015

the asphalt mixtures. Additionally, the difference between P_{ult} and P_{int} represented the resistance of the specimen to failure as shown in Fig. 3. This concept was employed to analyze the fracture properties of three asphalt mixtures under dry and wet conditions. The difference in the two loads for the mixes was consistently higher for wet than dry specimens indicative that wet specimens offered better fracture resistance that dry specimens.

3.2. Analytical solutions

Analytical solutions to obtain the basic fracture parameters were developed by [14,15], which were used to validate laboratory experimental results. Huang et al. [14] provided an analytical solution to estimate horizontal stresses and strains for asphalt mixtures with semi-circular geometry. During the process, the free body diagram of the SCB geometry was first drawn and later simplified by replacing the support reaction with distributed shear stresses along the inclined boundary. Eqs. (4) and (5) present the gist of the research reported in [14]. The relationship between β and l is used as a boundary condition to solve few parameters to obtain the original stress function. In addition, the loading system was further simplified assuming uniformly loaded distribution on the top of the geometry instead of a concentrated load. Since the normal stress at the ends is zero in a semi-circular geometry, a sinusoidal form of normal stress was assumed by the investigators to calculate the maximum stress (σ) at the middle portion of the lower surface as given by [14]:

$$\sigma_{max} = \frac{-2q\beta h(\frac{1}{2}\beta l)\sinh(\beta h)\cosh(\beta h)}{\beta^2 h^2 - cosh^2(\beta h) sinh^2(\beta h)} \tag{4}$$

$$\varepsilon_x = 0.807\ \frac{\delta_v}{D} \tag{5}$$

where:
σ_{max} = maximum tensile stress, kPa
ε_x = horizontal tensile strain, mm/mm
$q = \pi P/2l$, N/m
P = load, N
$\beta = \pi/l$, m^{-1}
δ_v = vertical deformation, mm
l = length of the specimen, mm
h = height of specimen, mm

Further, researchers [15] also performed an analytical investigation to determine σ_{max} by verifying the accuracy of Eq. (6) with the basic assumption of basic mechanics that the plane sections remained plane even after bending has taken place in the specimen. The prediction of σ_{max} based on Eq. (4) at a support condition of 0.7 D (D being the diameter of the specimen) was consistent with the results obtained from finite element method where the error increased with the deviation of the support length from 0.7 D to D. On the other hand, estimation of σ_{max} as per Eq. (6) was very close (maximum error of 2%) to the results obtained from finite element analysis, and was found to be effectively implementable to assess σ_{max} using the specimen diameter and thickness as inputs [15].

$$\sigma_{max} = \frac{6Pl}{tD^2} \tag{6}$$

where:
P = load, N
l = spacing between supports, mm
t = thickness of specimen, mm
D = diameter of specimen, mm

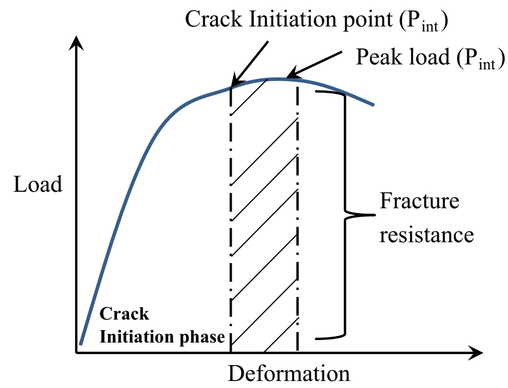

Fig. 3. SCB load-deformation schematic relationship (recreated based on [13]).

3.3. Crack growth measurement

Based on the fundamental fracture parametric assessment, limited research is available that provides a methodology to estimate the crack growth and helps understand the crack propagation component in fracture process. In this direction, Birgisson et al. [16] applied digital image analysis (DIC) technique to investigate the crack growth pattern in an SCB test. A comparison between strain gauges and DIC method showed a good agreement in measuring the horizontal strains with an accuracy of 0.034%. It is important to note that the horizontal strain measured as a ratio of horizontal deformation to the gauge length by a strain gauge was fixed at the bottom of the SCB specimen in the direction of mode-I crack opening. On the other hand, DIC employs the comparison of the deformed sample with the initial/un-deformed specimen using photogrammetry-based method. It was also reported that DIC could be effectively applied to measure the crack initiation and crack propagation in an SCB test. Thus, it can be understood that although the analytical solutions discussed earlier can provide the requisite fracture parameters (σ_{max}, ε_x, and K_{IC}) with the help of load-deformation relationship; crack growth would serve as a better input for crack propagation analyses.

4. Fracture properties: advanced assessment

In principle, fracture properties of asphalt mixtures using a performance test such as the SCB method must involve the assessment through the association of linear elastic fracture mechanics (LEFM) and elastic plastic fracture mechanics (EPFM). Generally, for brittle materials, the small scale yielding (SSY) governs the fracture process where LEFM assumptions remain valid. But, for quasi-brittle materials, high scale yielding (HSY) at the crack tip requires EPFM that considers larger fracture process zone (FPZ) for fracture assessment. The parameters such as stress intensity factor K, fracture toughness K_{IC}, and fracture energy G_{IC} are commonly used in LEFM analyses; while the critical energy rate J_c and J integral are used for analyses in the EPFM domain. With this background, several researchers [17,18,20–24,26–33] have utilized one or both methods to evaluate fracture properties of asphalt mixtures based on the SCB test outcomes.

4.1. Fracture toughness (K_{IC})

K_{IC} of asphalt mixes was the main fracture assessment parameter used by several researchers under various test conditions and input specifications. Most of the studies were found to have digressed from the standard protocols suiting experimental and practical convenience. For example, specimen geometry was a crucial factor in evaluating fracture toughness [17,18]. Although [8,9], respectively standardize 25 and 50 mm as the specimen thickness to run SCB test, K_{IC} was found to be not dependent on specimen thickness in the range of 25–75 mm as reported in [17]. Also, the dependency of K_{IC} was found to be relatively higher pertaining to specimen diameter than its thickness. However, [18] recommended validating the true value of K_{IC} determined using SCB test through the ASTM protocol prescribed in [19]. K_{IC} obtained from the test (termed as apparent fracture toughness, K_{IQ}) was supposed to meet a set of conservative requirements before being adjudged as the true fracture toughness (K_{IC}). The requirements for validation purposes were as follows [19]:

$$\frac{P_{max}}{P_o} \leqslant 1.10 \tag{7}$$

$$a \geqslant 2.5\left(\frac{K_{IQ}}{\sigma_{ys}}\right)^2 \tag{8}$$

$$w = 5\left(\frac{K_{IQ}}{\sigma_{ys}}\right)^2 \tag{9}$$

where:
P_{max} = maximum load, N
P_o = peak load, N
K_{IQ} = apparent fracture toughness, N/mm$^{3/2}$
σ_{ys} = yield stress, N/mm^2
w = width of specimen, mm

Again, based on the experimental findings, it was concluded that K_{IC} was not dependent on specimen thickness in the range of 25–75 mm at lower than 15 °C [18]. This non-dependency on the type of thickness ensured the plane strain fracture condition of the asphalt materials and served as intrinsic material properties.

In other studies, K_{IC} was used as an assessor to measure the effect of mix variables on the fracture resistance of asphalt mixtures [20–22]. Hofman et al. [20] reported that K_{IC} determination using LEFM approach was not applicable for high asphalt contents in the mix due to their increased viscoelastic properties [20]. Also, it was indicated that an increase in air voids of the asphalt mixtures reduced K_{IC}. Limestone and siliceous types of aggregates with different sizes along with asphalt binders were considered to understand the sensitivity of K_{IC} obtained using static SCB test at −15 °C [21]. The reduction of K_{IC} with respect to air voids was more pronounced for siliceous aggregates than limestone. Additionally, K_{IC} increased when stiffer asphalt binders were used since these binders increased the resistance of asphalt mixtures against cracking. Similarly, larger aggregates resulted in higher K_{IC} due to an overall increase in the amount of aggregates in the mix matrix that was attributed to higher stiffness. Biligiri et al. [22] showed that an increase in asphalt content in the range of 4.4–5.4% decreased the magnitude of K_{IC} at −10, 0, and 10 °C. However, these research studies did not clearly demarcate the applicability of LEFM with reference to the mix properties, specifically, asphalt content, which has a pronounced effect due to temperature.

The effects of SCB test configurations on K_{IC} such as loading rate, specimen geometry, and support conditions were also determined by [17,23,24]. K_{IC} was found not to be dependent on diameter when the loading rates were 0.3 and 3.0 mm/min with a combination of sample diameter exceeding 150 and 220 mm, respectively. In addition, sensitivity analysis of K_{IC} based on support conditions revealed that spacing between two support rollers from 0.67 to 0.9 times of the specimen diameter produced similar stress distributions [24]. Thus, there was an insignificant effect due to support conditions and complied with standard protocols [8,9].

The repeatability and reproducibility factors of K_{IC} had an effect due to a change in laboratory experimentation but did not have influence based on mixture compositions [20]. In simple words, the magnitudes of K_{IC} had no influence due to mix compositions within or between laboratories, but were affected if laboratory methodologies were changed to obtain K_{IC}. The tensile strength obtained using standard indirect diametral tensile (IDT) test conforming to [25] and K_{IC} obtained from SCB test were compared for reproducibility property by [26]. The exercise found that K_{IC} obtained from SCB test was more reproducible than the IDT tensile strength. In another research [18], K_{IC} of asphalt mixes were correlated with fatigue performance, which indicated that K_{IC} exhibited good correlation with the stiffness modulus as obtained from Nottingham Asphalt Tester (NAT).

4.2. Fracture energy

Fracture properties of asphalt materials are also being characterized by fracture energy approach through LEFM and EPFM. Critical energy release rate, denoted by G_I, represents the external energy required for a crack to grow which is calculated by considering the area under the load-deformation curve as in the LEFM approach. On the other hand, *J* integral, which is defined as the critical fracture energy is calculated accounting for the area under the load-deformation curve until the peak load, and then plotted against the notch depths (in *x*-axis) if one uses the SCB specimen geometry. A linearly decreasing trend of strain energy can be obtained, and thereof the critical fracture resistance is calculated as the slope of the fracture energy curve using [27]:

$$J_c = -\left(\frac{1}{b}\right)\frac{dU}{da} \tag{10}$$

where:
J_c = critical fracture energy, kJ/mm^2
b = specimen thickness, mm
U = total strain energy to failure, kJ/mm^2
a = notch depth, mm

A few studies [22,28,29] employed G_I to understand the fracture resistance of asphalt mixtures using static (monotonic) SCB test. In general, G_I increased with increasing temperature from −20 to 0 °C, and then decreased. The change of the trend was ascribed to the frozen moisture entrapment during conditioning at temperatures less than 0 °C [28]. Additionally, G_I increased with increasing asphalt content at temperatures greater than 0 °C; and a reverse trend was observed at temperatures less than 0 °C [22].

In another study [29], crack propagation property for conventional and modified asphalt mixtures was studied extensively by calculating total fracture energy. It was found that modified asphalt mixtures produced approximately 1.5–2 times higher total fracture energy at intermediate temperatures than conventional dense graded mixtures. Furthermore, the ratio (designed fracture energy ratio) of critical fracture energy (until failure) to total fracture energy was calculated indicative of fracture resistance during the crack propagation phase. Modified asphalt mixes showed lower fracture energy ratio than dense graded ones illustrating that the modified mixes would offer higher resistance against crack propagation. Researchers [30] have also accounted for the variations in the notch depth that may distinguish fracture energy properties of asphalt mixtures.

J integral, which is part of the EPFM approach, was also extensively used to characterize fracture resistance of modified asphalt mixtures [12,31–34]. Theoretically, *J* integral is a path independent line integral around the crack which is used to measure the strain energy release rate where LEFM assumption does not valid. Although the theoretical concept demands a path independent line integral, it can be also measured as per Eq. (10) for laboratory evaluation purposes. One of the major differences between the LEFM and EPFM approaches to calculate *J* integral is the notch depth that is variable in the EPFM practice during the estimation process of *J* integral. Although various notch depths can be found in literature; 5, 15, 20, 25.4, 31.8, and 38.2 mm notch depths have been commonly used. At lower temperatures, rubber-modified mixtures prepared using the wet process resulted in a higher J_c than that of the dry process and control mixtures [34].

Research studies [28,35] also found the *J* integral to be a reproducible parameter using the static SCB test. J_c showed higher degree of sensitivity as compared to peak load, strain energy, and deformation with respect to asphalt binder, nominal maximum aggregate size (NMAS), and compaction effect (N_{design}) [34,36]. Furthermore, J_c obtained from SCB tests exhibited good correlations (R^2 = 0.58) with field fatigue cracking indicating that J_c is a promising index to explain the field fatigue cracking performance [30,37].

4.3. Mixed mode fracture parameters

Owing to the successful simulation of Mode-I cracking as reported in previous studies, SCB test was also examined to produce other modes of fracture, i.e., mode-II and mixed mode fracture. Several researchers [23,38,39] attempted to evaluate the mix mode fracture of asphalt mixes using static SCB test. Research [38] reported that the mixed mode fracture, which is a combination of modes I and II cracking can be obtained by unequal support distance and asymmetric notch. Aliha et al. [39] conducted a numerical study to investigate the mixed mode cracking of asphalt mixtures using asymmetric SCB test. The authors replaced the notch position from the conventional symmetric specimen (mode-I and [8]) setup with an unequal support distance from the notch as shown in Fig. 4. Two stress intensity functions, K_I and K_{II} were evaluated to account for modes-I and II fracture properties, respectively, using:

$$K_I = Y_I \frac{P}{Dt}\sqrt{\pi a} \tag{11}$$

$$K_{II} = Y_{II} \frac{P}{Dt}\sqrt{\pi a} \tag{12}$$

where:
K_I = stress intensity factor for mode-I, N/mm$^{3/2}$
K_{II} = stress intensity factor for mode-II, N/mm$^{3/2}$

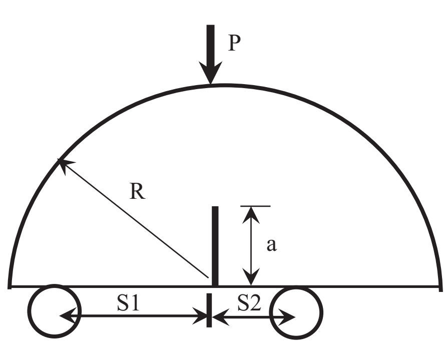

Fig. 4. Asymmetric SCB test setup reported in [39]. *Note: s_1, s_2 = support distances from notch.*

Table 2
Various SCB test configurations in mixed mode fracture evaluation [23].

Test configuration	Mode	s/r ratio	a/r ratio	Inclination angle, α (degree)
A	I	0.8	0.33	0
B	II	0.4	0.33	45
C	II	0.4	0.33	50

Y_I = geometric factor for mode-I
Y_{II} = geometric factor for mode-II
P = load, N
D = diameter, mm
t = thickness, mm
a = notch depth, mm

Mode of mixing between modes I and II is measured by the mode mixity parameter, M as designated in [38]. For pure mode-I, the value of M becomes zero whereas mode-II being 1. The researchers showed that higher mode of mixity can be obtained using SCB sample than an SENB specimen, and is a function of notch angle and 2 D/S ratio (D = specimen diameter and S = support distance).

The geometric factors for both modes as a function of a/r, s_1/r, and s_2/r (where s_1 and s_2 are distances between the supports and notch; r being radius = $D/2$) were estimated using finite element analysis [39]. It was noted that when $s_1 = s_2$, it resulted in a pure mode-I fracture but when they were unequal, it caused a mixed mode fracture failure. Further, the analyses revealed that Y_I (or K_I) becomes zero at a particular value of s_2 indicating a pure mode-II characteristic. Thus, it was concluded that asymmetric SCB test had the potential to assess the realistic mixed mode fracture experienced in a flexible pavement system.

Im et al. [23] examined the potential of SCB test in evaluating the mode-II fracture of fine asphalt matrix (with maximum aggregate size of 1.19 mm). Three test configurations were employed to investigate modes-I and II fracture characteristics as summarized in Table 2. Static SCB tests were carried out at a constant deformation rate of 10 mm/min and 21 °C, and experimental results indicated that K_{IIC} was almost three times higher than K_{IC}. As indicated in the Table 2, the test configuration A resulted in vertical crack propagation in accordance with mode-I while test configurations B and C produced crack initiations at the tip of the notch, and propagated to the loading point that followed mode-II fracture.

4.4. Numerical simulations

Numerical modeling also played an integral part of asphalt mixtures' fracture properties characterization and evaluation through finite element method (FEM) for SCB geometry. The fracture parameters Y_I, K, K_{IC}, and J_c in pure and mixed mode fracture were determined for various semi-circular geometric dimensions, which was only possible with the help of numerical analyses. For example, Lim et al. [40] provided solutions for SCB test geometric factors to calculate stress intensity factors at various a/r and s/r ratios. A numerical solution developed to estimate Y_I based on geometric ratios was given by:

$$Y_{1\{S_o/r\}} = C_1 + C_2\left(\frac{a}{r}\right) + C_3 e^{\left(C_4\left(\frac{a}{r}\right)\right)} \tag{13}$$

where:
a = notch length, mm
r = radius of specimen, mm
s = support distance, mm
C_1, C_2, C_3, and C_4 = geometry parameters

Other researchers [23,41,42] also considered cohesive zone modeling or CZM (crack evaluation at the localized crack tip region) based EPFM approach using FEM to simulate the critical strain energy of asphalt mixtures based on material properties: modulus of elasticity and Poisson's ratio. Further, it was found that the choice of displacement measurement, specimen geometry and boundary conditions would result in erroneous estimations of the critical energy release rate [23,41]. To circumvent the difficulty, it was recommended to calculate the critical energy release rate along the FPZ instead of the conventional load-deformation curve. Furthermore, finite element based CZM showed that strain energy decreased with faster loading rates in the range of 0–20 °C. But, a reverse trend was observed in case of faster loading rates at the ambient temperature range (21–30 °C). At higher temperature, the FPZ size near the crack tip increases which in turn reduces the critical energy of mix. But, the faster rate of loading possibly results in prompting reduced size of FPZ near the rack tip, thus increasing the strain energy with faster.

Huang et al. [24] correlated the effective crack length of SCB geometry with the vertical load line displacement (LLD) using FEM. The modeling technique offered an alternative solution to estimating the effective crack length which is otherwise cumbersome to measure by experimental means. Furthermore, the authors also showed that the observed stress intensity contour in SCB geometry using FEM analysis confirmed the mode-I cracking mechanism.

Contemporaneously, extended finite element method (XFEM) was also employed to study the discontinuous behaviour of crack propagation in the SCB test on asphalt mixtures [41,43–45]. In general, XFEM models the crack propagation without having to remesh the crack growth, and hence, the crack propagation takes place in a mode-dependent cracking path. Research findings revealed that XEFM in conjunction with viscoelastic materials' inputs was capable of simulating the load-deformation relationship, and the stress intensity variation of various asphalt mixes [44]. In addition, K_{IC} of asphalt mixtures was also numerically simulated using FEM for different asphalt mixes in which the laboratory findings exhibited a good correlation with the numerically simulated K_{IC} [46].

5. Crack propagation study

As mentioned previously, fracture assessment involves crack initiation and propagation to finally arrive at failure stage in an asphalt mix. However, very little knowledge about crack propagation is available that sheds light on the comprehensive picture of the innate mechanisms of fracture. Thus, the investigations which exclusively focused on crack propagation process for asphalt mixtures are summarized as follows.

5.1. Crack velocity

Crack velocity was used as a crack resistance assessor by [13], who defined it as the distance that the crack travelled with respect to time. The research also examined the crack initiation and crack propagation of asphalt mixtures with the help of multiple Linear Variable Differential Transducers (LVDT) fixed parallel to the specimen base. The experiments encompassed static SCB tests and the crack velocity was given by:

$$v = \frac{\Delta l}{\Delta t} \tag{14}$$

where:
v = velocity of the crack, mm/s
Δl = length of the crack, mm
Δt = time elapsed, s

The asphalt mixtures under dry and wet conditions produced an average velocity of 12.0 and 7.7 mm/s, respectively. Although the crack velocity was found to be increasing with increasing air voids content, the correlation was weak. In addition, the crack initiation was observed to be occurring along the interfacial aggregate boundary, and the crack velocity was not found significantly sensitive to the aggregate gradation. However, the study presented a methodology to measure the crack propagation velocity that is possibly helpful to get an understanding of the crack propagation resistance of asphalt mixes.

In a recent study, crack propagation resistance of asphalt mixtures in post-crack initiation stage was investigated using static SCB methodology at six varying temperatures [46]. It was reported that HSY would best be described by EPFM concept and can also explained the fracture phenomenon of asphalt mixtures using FPZ concept. In the post-crack initiation regime, the cumulative energy dissipation was modeled as a function of percentage fracture damage using power relationship as follows:

$$\text{Dissipated energy} = \alpha \times (\%\ \text{fracture damage})^{\beta} \quad (15)$$

where:
α and β = materials' fracture parameters
α and β provided the necessary knowledge about the crack propagation resistance at different stages of fracture crack propagation. It was also found that the fracture parameters could be determined in terms of their fundamental materials inputs such as asphalt content, aggregate gradation, binder viscosity, temperature, and modification (rubber/polymer).

5.2. Dynamic SCB test

In order to simulate the actual field fracture fatigue performance and characterize the fracture properties of asphalt mixtures, cyclic loading conditions are essential. In this connection, dynamic SCB tests were also conducted by [11,20,24,47,48]. Nonetheless, it is noteworthy that the dynamic SCB tests were conducted at varying load magnitudes and frequencies, as there is no prescribed standard protocol to perform the test.

Initially, dynamic SCB test was attempted using 9.8 and 29.3 Hz with varying loading levels; and the number of cycles to failure was found inversely related to the load magnitude [11]. Another research study [20] investigated the crack growth rate by dynamic SCB test where a haversine load of 3 kN (maximum) and 0.3 kN (minimum) at 29.3 Hz were applied and crack growth rate was recorded optically by a digital camera. An irregular cracking pattern was reported as the major difficulty towards measuring the crack growth. It was found that higher asphalt contents reduced the crack growth rate by providing higher crack propagation resistance than the one with lower asphalt contents in the asphalt mixes.

In order to advance fracture analyses of asphalt materials, repeated load fracture properties necessitates the use of Paris' law parameters using LEFM and EPFM approaches (Eq. (16)) [49]. A study conducted by [24] estimated Paris' law parameters of four dense graded asphalt mixes by applying a sinusoidal load with maximum and minimum loads of 1.38 and 0.0045 kN, respectively, at 5 Hz and 25 °C. Research findings showed that Paris' law parameters obtained from SCB tests were sensitive to asphalt content of the mix, which also were in good agreement with the previous magnitudes obtained from other fracture tests on asphalt mixtures.

$$\frac{dc}{dN} = a\ \Delta K^{n} \quad (16)$$

where:
c = crack length, mm
a, n = material constants
ΔK = stress intensity factor = $K_{max} - K_{min}$, N/mm$^{3/2}$

Dynamic SCB study conducted by [47,48] evaluated the viscoelastic fatigue performance using Paris' law parameters. The viscoelastic behaviour of asphalt mixes was studied using modified Paris' law with the association of J integral as represented in Eq. (17). One of the major outcomes of the study was that the modified Paris' law constant n_j was found to be related to the creep compliance exponent of time for a mix.

$$\frac{dc}{dN} = a_j\ \Delta J^{n_j} \quad (17)$$

where:
a_j, n_j = material constants
J = J integral
n_j = 6.2 − 1/m
m = creep compliance exponent

Wang et al. [50] also studied the effect of frequency on crumb-rubber modified asphalt mixtures using dynamic SCB test. Three frequency levels: 5, 10, and 15 Hz were used in the study. Although a change in the frequency had no effect on the failure load, higher frequency resulted in an extended fatigue life. Further, an increase in the temperature increased the crack growth rate, thereby reducing the fatigue life of asphalt mixes.

6. Future prospects of SCB test – fracture evaluation of asphalt mixes

Static and dynamic SCB test techniques have been employed as versatile methodologies to characterize fracture properties in the context of low temperature and fracture fatigue cracking mechanisms. In this direction, two standard protocols [8,9] provide a strong foundation towards practical experimentation of the static SCB test. However, it is worth noting that several research studies performed the experiment suiting regional and/or similar research convenience. Owing to rationality and simplicity of the SCB test procedure in evaluating fracture properties of asphalt mixtures, there is scope for further research to advance the current state-of-the-art test procedure that simulates actual field performance characteristics. In a nutshell, the future scope of research regarding SCB tests to evaluate fracture properties of asphalt mixtures is summarized below.

- Future additional studies are required to characterize and validate dynamic fracture properties through the use of cyclic SCB test.
- The suitability of LEFM to understand fracture behaviour should be investigated further taking into account the viscoelastic characterization associated with asphalt binder as the deciding material property using the well-established concept [49].
- An extensive study is required to further understand the dependency of fracture properties on sample geometry such as thickness, notch depth, support distance, loading rate, frequency, and temperature pertinent to dynamic SCB tests as all of the influencing factors have been accounted in the development of static SCB tests.
- Additional research is required to understand the correlations between dynamic SCB fracture parameters and surface energy to investigate moisture damage, fatigue, and durability of asphalt mixtures using the methodologies proposed in [51,52], and several others.
- Different materials including modifiers (and proportions) must be used in future to investigate the applicability of the SCB test procedure to corroborate the actual field performance of modified mixes that outperform the conventional ones.
- Test configurations, namely, unequal support distance, inclined notch in conjunction with numerical simulations are required to estimate mixed mode fracture properties of asphalt mixtures.

7. Conclusions

There has been an increasing interest to utilize the SCB test to assess fracture performance of asphalt mixtures in the pavement community due to its simplicity and rational approach. This review article presents the current state-of-the-art regarding the utilization of SCB test to evaluate fracture properties of different asphalt mixtures. Although several research studies are available in the ambit of fracture characterization of asphalt mixtures based on the monotonic SCB test technique, the consolidated discussion provides a comprehensive understanding of the state-of-the-art for completeness purposes.

The first part of the review focused on the fundamental assessment of fracture through the static SCB test, which was based on load-deformation characteristics of asphalt mixes, usually used in LEFM approach. Next, analytical solutions and application of fracture mechanics in evaluating fracture properties of asphalt mixes that led to the development of a standard monotonic SCB test protocol was discussed. The advanced analytical procedures usually employed to deduce fracture parameters for asphalt mixes to understand the fracture performance were also documented based on LEFM and EPFM approaches.

It was noted that majority of the studies utilized static (or monotonic) SCB test technique to determine the fracture resistance of asphalt materials at various test specifications. Although static SCB test evaluation provides a preliminary knowledge about the ultimate failure load and fracture resistance; fatigue performance characterization is not completely understood using this technique. In this direction, it is recommended that future studies continue on developing and using cyclic SCB tests to investigate the dynamic fatigue behaviour in association with viscoelastic properties.

Overall, the dynamic SCB test procedure is a potential crack propagation assessment candidate in the areas of asphalt mix fracture characterization. In essence, there is scope for advancing the current state-of-the-art pertaining to the dynamic SCB test procedure that actually simulates the field performance characteristics. One of the major accomplishments of developing a dynamic SCB test will help incorporate the relevant deduced fracture parameters into the mechanistic-based flexible pavement design approach.

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