Transportation Research Part E 93 (2016) 212–231

Contents lists available at ScienceDirect

Transportation Research Part E

journal homepage: www.elsevier.com/locate/tre

# Pricing strategies for a taxi-hailing platform

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ARTICLE INFO

*Article history:*
Received 29 February 2016
Received in revised form 20 April 2016
Accepted 22 May 2016
Available online 20 June 2016

*Keywords:*
Taxi
Market equilibrium
E-hailing
Pricing strategy

ABSTRACT

Taxi hailing apps that facilitate taxi-customer matching quickly become popular in recent years. By combining the theories of two-sided market and taxi market, this paper models the taxi market in the presence of a single taxi hailing app through an aggregate and static approach. Based on the equilibrium model, the existence and stability of equilibria are examined, and a partial-derivative-based sensitivity analysis is conducted to quantitatively evaluate the impacts of the platform's pricing strategies to the taxi market performance. The features of desirable price perturbations that improve social welfare and/or the platform's profitability are also characterized.



## 1. Introduction

With the rapid development of mobile and wireless communication technologies, various taxi hailing applications (apps), such as Didi KuaiDi-taxi, Uber-taxi, and Ola Cabs, have emerged in recent years to global popularity. Now, unlike in previous days when people had to stand on the street to hail a taxi, customers can now publish their travel demand to nearby taxis through a taxi hailing app, and the taxis who are logged in the app can instantaneously receive the nearby demand and determine whether they would like to take the e-hailing orders or keep cruising and look for roadside hailing customers. The convenient and instantaneous information exchange facilitated by taxi hailing apps greatly mitigates the previous information barriers caused by spatial deviation between customers and taxi drivers, therefore is widely believed to be a powerful instrument for improving the taxi market efficiency.[1,2]

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[1] To some extent, taxi hailing apps play a similar role as a taxi call center in reducing information asymmetry between customers and taxi drivers. However, as radio-call orders are usually manually handled, the information exchange between customers and taxi drivers is not so convenient and speedy as through a taxi hailing app. In most cases, the taxi call center works in a dispatching mode, i.e., the center directly assigns vacant taxis to customers. While some of the existing taxi hailing apps work in the dispatching mode as well, this study focus on apps that works as an information platform (i.e., distributing e-hailing orders to nearby taxis, and allowing drivers to determine whether or not they take the order). Another difference between taxi call center and taxi hailing app is their operators. While the taxi hailing apps today are operated by platform companies, the taxi call centers are usually operated by taxi companies who have no clear incentive to increase radio-call orders. So the popularity of radio-call mode in the old days is not comparable with the popularity of e-hailing mode today.

[2] This advancement in information technology also facilitates matching between customers and part-time drivers, which allow many e-hailing apps, such as Uber, Didi Kuaidi and Lyft, to use private car drivers to provide taxi-like services. The participation of part-time drivers greatly increases the supply of taxi-like services and therefore improves customers' experiences by making taxi services more readily available. However, concerns over safety, road congestion, and the significant impact on the taxi industry are common. Whether or not private car drivers should be allowed to provide taxi-like service remains a controversial issue. Indeed, many countries, such as Germany, France and India, have banned low-cost Uber services. To be clear, in this study, we focus only on the pricing strategies of a taxi hailing app dedicated to the taxi industry.

http://dx.doi.org/10.1016/j.tre.2016.05.011
1366-5545/

Existing taxi hailing apps can be mainly categorized into two types. The first type works as an information platform that distributes e-hailing orders to nearby taxis, and allows drivers to determine whether or not they take the order. The most popular e-hailing apps tend to adopt this model because it provides drivers with more freedom and is therefore more acceptable to drivers. The second type of e-hailing app works as a dispatching center and assigns orders to drivers based on a matching algorithm. Drivers must take the orders they are assigned. This model sacrifices drivers' freedom, but guarantees a much higher rate of successful matching, and is, therefore, more favorable to customers. In this study, we focus our study on taxi hailing apps that work as an information platform.

The popularity of taxi hailing apps relies not only on advanced technology, but also on pricing strategy. In order to increase the size of user pools, many taxi hailing apps provide subsidies to both customers and taxi drivers. For example, from January 10, 2014 to March 27, 2014, Didi and Kuaidi, the two most popular taxi hailing apps in China, spent over 2 billion RMB in subsidizing customers and taxi drivers. For each e-hailing order, both the customer and taxi driver received between 5 and 20 RMB (Chen, 2014). Such generous subsidies allowed some customers to receive free taxi rides and ultimately increased the total number of e-hailing accounts in China to over 49 million by the 2nd quarter of 2014 (CNNIC, 2014). Didi and Kuaidi merged into one company, Didi KuaiDi, in 2015, effectively capturing over 99% of the taxi hailing market share in China. However, even with over 3 million daily taxi orders transacted through their platform today, the company does not impose any positive charge on either taxi drivers or customers for fear of losing business, which clearly demonstrates the significance of pricing strategies in e-hailing apps.

As an information platform that connects customers and taxi drivers, a taxi hailing app can charge different rates[3] for customers seeking rides and taxis seeking passengers. Charging schemes can be based on membership and/or usage. If a platform charges based on membership, then users pay a set of membership fee up front and use the e-hailing service without incurring any further fees during the defined membership period. If a platform charges based on usage, then users pay the platform each time they use the app service. As almost all existing e-hailing apps only charge users for successful transactions, we restrict our attention to usage fees only, so that for each customer-taxi meeting facilitated by an e-hailing app, the app charges respective rates to both customers and taxi drivers.

Determining an appropriate pricing strategy can be difficult. Even after receiving generous subsidies from taxi hailing apps, customers will not continue to use the app if it is hard to find a taxi through the app, because customers consider not only the e-hailing charges and subsidies, but also waiting times. For example, if a taxi hailing app provides a 2 dollar subsidy to customers and then charges taxi drivers 2 dollars for each order, few taxi drivers may be willing to take e-hailing orders, causing customer waiting times from the e-hailing mode to be very long. Consequently, even though the taxi hailing app has provided subsidies to customers and completely sacrificed its profit with zero net income for each order, the transaction volume through the app would not be high due to the long customer waiting time. What if the platform increases its subsidies on the customer side to 5 dollars and meanwhile charges taxi drivers 5 dollars per order? Apparently, customers will be more stoic about long wait times in light of the increased subsidy, even while increased charges make taxi drivers ever more unwilling to take e-hailing orders. Increased subsidies and customer willingness to endure long wait times may make it more difficult for taxi drivers to find roadside-hailing customers. In this case, many taxi drivers may be forced to take e-hailing orders, and the transaction volume of the taxi hailing app could be increased even though the aggregate price level per order is the same as the previous example. Therefore, designing appropriate pricing strategies for an e-hailing platform is much more complex than that for a traditional commodity because it is necessary to determine not only an aggregate price for each order, but also the price share between customers and taxi drivers. According to Rochet and Tirole (2004), "*if holding constant the sum of the prices faced by the two sides, any change in the price share between the two groups [will] affect the volume of transactions*", then the market is called a two-sided market. In view of the above discussion, the markets in which customers and taxi drivers interact through an e-hailing platform is thus a two-sided market. A crucial factor in a two-sided market is cross-group (or inter-group) externality: "*the attractiveness of the platform to users in each group is governed by the number of users on the other side of the platform*" (Caillaud and Jullien, 2003; Parker and Van Alstyne, 2005; Armstrong, 2006). So in many cases in reality, a platform may even subsidize users on one side to attract users on the other side. And to expand their user pool at the beginning stage, more and more platforms choose to subsidize both sides of users.

Numerous two-sided markets, such as computer operating systems (e.g. Windows, iOS) which connect computer users and software developers, credit card services (e.g. Visa, MasterCard) which connect cardholders and affiliated merchants, and e-commerce platforms (e.g., Alibaba, e-bay) which connect retailers and consumers, have emerged in recent decades. However, the theory of two-sided markets (or, equivalently, platform economics) was not developed until the early 2000s. In 2004, Rochet and Tirole first pointed out the commonalities between seemingly different markets and provided a clear characterization of what constitutes a two-sided market. Rochet and Tirole (2004) then built a canonical model of two-sided markets in which one platform encompassed both usage and membership externalities. Caillaud and Jullien (2003), Rochet and Tirole (2003), and Armstrong (2006) explored platform competition under different shapes of utility and cost functions, investigated how the platforms' pricing strategy was affected by platform governance (for-profit vs. not-for-profit), the end users' cost of multi-homing, platform differentiation, the platforms' ability to use volume-based pricing, the presence of same-side externalities, and platform compatibility.

[3] In this study, charge rates could be positive or negative. Negative charge rates indicate subsidies.

After this initial and crucial research, more recent research has focused on diverse issues related to two-sided markets, such as variant assumptions about timing, price instruments and externalities (Roson, 2005), and different service modes in various fields, e.g. media industries (Kaiser and Wright, 2004), electronic intermediaries (Jullien, 2005), operating systems, and health and education sectors (Bardey and Rochet, 2010). Due to the fairly recent emergence of taxi hailing apps, the taxi market under hybrid modes of e-hailing and roadside hailing has never been examined from a two-sided market perspective.

Previous studies of two-sided markets cannot be easily applied to the development of a taxi hailing platform due to the more complex nature of taxi service. For instance, most previous studies of two-sided markets assume that the attractiveness of a platform to one user group increases with the size of the user group on the other side. However, in the taxi service platform, the customers' (taxi drivers') utility depends on their waiting time (vacant taxi hours), which is determined not only by the number of searching taxis (waiting customers), but also by the number of waiting customers competing for available taxis (the number of available taxis competing for e-hailing orders). Therefore, both cross-group externality and intra-group externality need to be considered. Additionally, in other previous studies of two-sided markets, the contribution of each user to the attractiveness of a platform is relatively long-term.[4] However, with a taxi hailing platform, the influence of each customer or taxi driver on the attractiveness of the platform is short-term. All taxi demand disappears once being served and all available taxis become unavailable after meeting a customer. Therefore, each e-hailing customer and taxi driver contributes to the matching efficiency of the taxi hailing app only during her/his waiting and searching time. Furthermore, the taxi industry tends to be regulated by local city governments which restrict taxi fleet sizes and set fixed taxi fare rate. Ultimately, a large number of occupied taxis results in a lessening of available taxis on the platform. Thus, the platform's matching rate has a negative effect on the number of available taxis at any given moment. To the authors' knowledge, no previous research of two-sided markets has studied a market that exhibits properties similar to those of the taxi service.

On the other hand, the traditional taxi market with roadside hailing or ranked waiting mode has been extensively studied since early 1970s. It is commonly acknowledged that the taxi market does not necessarily fit into the conventional analysis of a regular market because taxi prices do not play a typical role in market clearing. Due to information asymmetry and spatial deviation among customers and taxi drivers, there are always vacant taxis searching for customers, which results in the total service hours supplied always being larger than the hours demanded. The excessive taxi service hours determine the average taxi availability at each instant, which further affect customer waiting times. And the customer waiting times and taxi fare rates in turn influence taxi demand. The complex correlations among taxi availability, customer waiting times, and taxi demand under any given taxi fleet sizes and taxi fare rates are shown in Fig. 1. Based on a common acknowledgment of the above correlations, continuous efforts by researchers, economists in particular, have been made to better understand and characterize the mechanism of how taxi demand and supply are equilibrated, and the impact of different market configurations and regulatory regimes on such equilibria (e.g. Douglas, 1972; De vany, 1975; Beesley, 1973; Beesley and Glaister, 1983; Schroeter, 1983; Manski and Wright, 1967). And in the 1990s, Yang, Wong, and their collaborators further incorporated the spatial structure of taxi services into existing models in order to better understand the equilibrium of the taxi market. Interested readers are referred to their substantial stream of research that characterizes taxi movements in a road network for a given origin–destination demand pattern (e.g., Yang and Wong, 1998; Yang et al., 2010; Wong and Yang, 1998; Wong et al., 2002, 2008).

In 2010, Yang and his colleagues firstly explicitly introduced meeting functions to spell out the searching and meeting frictions between customers and taxi drivers (Yang et al., 2010, 2014; Yang and Yang, 2011). A meeting function ascertains the customer-taxi meeting rate (i.e., the number of meetings during each time period) as an increasing function of the number of waiting customers and available taxis at any given moment. The meeting rate as well as the number of waiting customers at any given moment jointly determines the customer waiting times. So as shown in Fig. 2, the customer waiting time is now dependent on not only taxi availability, but also on the number of waiting customers. This is a significant improvement as it takes both cross-group and intra-group externalities into consideration. As we can see from Fig. 2, while an increase in available taxis always contributes to lower customer waiting times, the impact of waiting customers on the overall customer waiting time can be either positive or negative depending on the elasticity of meeting rates with respect to the number of waiting customers.

This study focuses on taxi systems in the context of taxi hailing apps. Most cities still employ a traditional roadside-hailing taxi industry, meaning that the e-hailing mode competes with the traditional roadside hailing mode for both customers and taxi drivers. The customer/taxi searching times of the two modes and the app's pricing strategy govern the mode choices of customers/taxi drivers. And for each mode, the number of customers and taxi drivers determine the meeting rate, which further affect the customer/taxi searching times. Such complex interactions at taxi market equilibrium under hybrid modes of e-hailing mode and roadside hailing is modeled in He and Shen (2015). They adopted a network model considering the spatial distribution of demand, and examined the mode choices and taxi movements at network equilibrium under any given pricing strategy of the e-hailing platform. However, the pricing strategy of a taxi hailing app is not explicitly considered in their analysis, and, consequently, the impact of the platform's pricing strategies in relation to the taxi system remains unexamined.

[4] By 'long-term,' we mean the contribution of a user to a platform's attractiveness will not disappear after a short period of time. For example, the attractiveness of visa services to a customer depends on the number of merchants affiliated with visa. A merchant is assumed contributive to the attractiveness of visa service to customers once it becomes affiliation to visa.

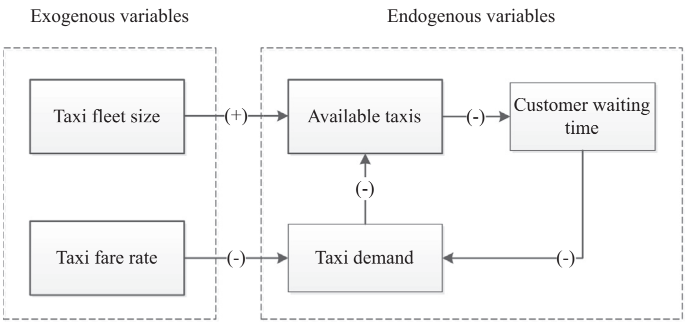


**Fig. 1.** Assumed correlations of variables in early studies.

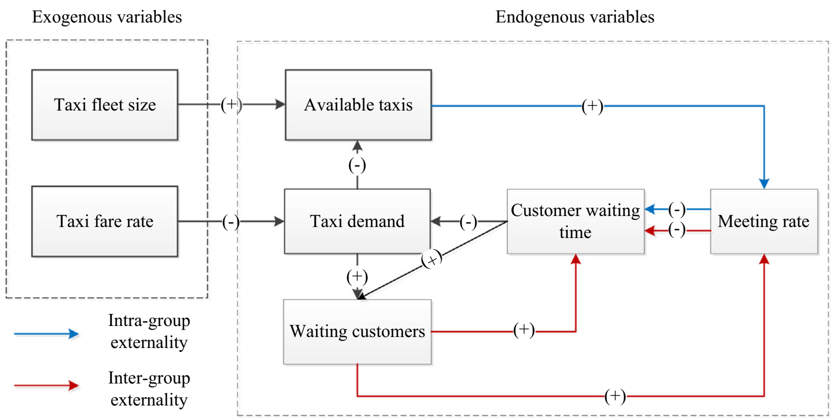


**Fig. 2.** Correlations in a taxi market using searching and meeting frictions.

This paper makes the first attempt to quantitatively investigate the pricing strategies of a taxi hailing platform in the context of the overall taxi market equilibrium (consisting of both e-hailing and roadside hailing modes). Specifically, this paper sequentially addresses the following three questions: 1. Does a stable taxi market equilibrium exist under any pricing strategy of a taxi hailing platform?; 2. How does the taxi system vary in response to a price perturbation in the platform?; and 3. How should a platform adjust its pricing strategies in order to improve the platform's profitability and/or social welfare?

Section 2 of this paper models the taxi market equilibrium in the presence of a single taxi hailing app through an aggregate and static approach. Under any given pricing strategy of the taxi hailing platform, the complex interactions among the demand, supply, and service quality of each mode at taxi market equilibrium are formulated into a system of nonlinear equations. In Section 3, the existence of equilibrium solutions are established and sufficient conditions for the stability of equilibrium are examined. Based on the system of nonlinear equations, a partial-derivative-based sensitivity analysis is conducted in Section 4 in order to reveal the variance of the taxi system in response to any price perturbation in the taxi hailing platform. In Section 5, the sensitivity analysis results are used to investigate desirable price perturbations that improve the platform's profitability or the public's social welfare. Section 6 concludes the paper with summaries of key findings and discussions of future research.

## 2. The basic model of taxi market equilibrium

Consider a one-hour modeling period for an aggregate taxi market with a fixed taxi fleet size of $N$ and a fixed taxi fare rate of $\tau$. The total taxi demand rate, indicated by $D$, is assumed constant.[5] As a preliminary study, we assume only one e-hailing platform exists in the taxi market, and all customers and taxi drivers have easy access to the taxi hailing app. The app works as an information platform: by distributing each e-hailing order to all taxis within a certain distance radius of the customer. The taxi driver who responds (to the order) in the earliest time gets the order. The roadside hailing mode is still allowed, so vacant and unreserved[6] taxis can take either roadside hailing or e-hailing customers, and customers can hail taxis by either hailing

[5] This is not a restrictive assumption for most developing cities with poor public transit systems. Little cross-elasticity of demand exists between the taxicab and mass transit industries (Dempsey, 1996) in these cities, as most taxi demand are very time sensitive and/or subject to no alternative choice.

[6] By 'vacant,' we mean no customer is on board. By 'unreserved,' we mean the taxi is not on the way to pick up an e-hailing customer. In this study, we assume only vacant and unreserved taxis are allowed to take e-hailing orders.

from the roadside or sending e-hailing orders. During the period of study, all taxis (whether vacant or occupied) cruise on the streets[7] while also monitoring their taxi hailing apps. For simplicity, we assume that all customers and taxi drivers keep their appointments once the customers' e-hailing orders are accepted by a taxi driver through the platform.[8] Let $I = \{i = 0, 1\}$ be the set of available modes, where '0' stands for roadside hailing mode, and '1' for e-hailing mode. In the following study, all the parameters and variables with subscripts '$i$' are correlated with mode $i$. Regardless of hailing modes, the average taxi ride time is assumed to be a constant $l$, and the average taxi riding fare for each trip is constantly $F$.[9] When the e-hailing mode is utilized, both customers and taxi drivers must pay the taxi hailing app for information services, rendered as $a_1^c$ and $a_1^t$ respectively. Throughout the paper, $\mathbf{a} = (a_1^c, a_1^t) \in \Re^2$ is referred to as the pricing strategy of the e-hailing platform.

Following the settings and assumptions outlined above, the rest of this section discusses the taxi market equilibrium under any given pricing strategy of an e-hailing platform. To ease reading, a schematic description of the complex interactions among the taxi system variables at equilibrium is provided in Fig. 3, and lists of endogenous and exogenous variables are provided in Appendix A.

### 2.1. Matching efficiency of e-hailing and roadside hailing

We begin by discussing the matching efficiency of both e-hailing and roadside hailing modes. The measure of matching efficiency is the number of successful matches/meetings between customers and taxi drivers in one time period (i.e., one hour). For the traditional taxi industry, which relies solely on the roadside hailing mode, spatial deviation and limited information between customers and taxi drivers necessitates an extensive amount of time and effort being lost to searching and waiting. Such bilateral searching and matching between two groups of players is also a popular topic in economics (e.g. trading and labor markets). To explicitly account for the nature of matching frictions, the matching function has been widely used to characterize the relationship between the number of users in each group and the aggregate meeting rate (Mortensen and Pissarides, 1994; Petrongolo and Pissarides, 2001; Burdett et al., 2001). As mentioned in Section 1, Yang et al. (2010) were the first to apply an explicit meeting function to the taxi industry in order to model the meeting rate with search frictions between unserved customers and vacant, available taxis at a macroscopic level:

$$m_0 = M_0(N_0^c, N_0^t), \quad \partial M_0/\partial N_0^c > 0, \ \partial M_0/\partial N_0^t > 0 \tag{1}$$

where $m_0$, $M_0(N_0^c, N_0^t)$, $N_0^c$, and $N_0^t$ respectively indicate the meeting rate, meeting function, the number of roadside hailing customers, and the number of vacant taxis searching for roadside hailing customers. The meeting rate between customers and taxi drivers during one time period is positively proportional to the number of waiting customers and searching taxis at each instant.

Although taxi hailing apps are useful in revealing the location of customers and taxi drivers, the spatial deviation between them remains unaltered, meaning matching friction still exists. When a customer submits an e-hailing demand she/he is still required to wait for response as there may be no vacant and unreserved taxis in close proximity, or currently available taxis (i.e., vacant and unreserved taxis) may not be willing to take the order. Generally speaking, the more waiting customers and searching taxis on the platform, the higher matching efficiency can be expected. Let $N_1^c$ and $N_1^t$ respectively denote the number of e-hailing customers and the number of vacant and unreserved taxis that are waiting for e-hailing orders at any given moment,[10] and let $m_1$ be the matching efficiency (or meeting rate) of e-hailing. Then the meeting rate $m_1$ of the e-hailing mode can be given as a function of $N_1^c$ and $N_1^t$:

$$m_1 = M_1(N_1^c, N_1^t), \quad \partial M_1/\partial N_1^c > 0, \quad \partial M_1/\partial N_1^t > 0 \tag{2}$$

where $M_1(N_1^c, N_1^t)$ is the meeting function of the e-hailing mode, which increases with $N_1^c$ and $N_1^t$. The increasing relationship between the matching rate and the number of users in each group holds steady for most platforms, so that Eq. (2) is widely acknowledged in platform economics (or two-sided market economics) (e.g., Rochet and Tirole, 2003, 2004).

[7] Although an e-hailing mode would seemingly reduce the necessity of cruising, almost all taxis in Chinese cities have kept cruising on the streets since the emergence of taxi hailing apps. The reasons are threefold: First, e-hailing orders are received by the nearest taxis in a shorter amount of time. Taxi drivers are therefore motivated to get closer to areas with higher demand densities, which is also typical of traditional roadside hailing taxis. Second, there is generally a lack of parking near areas with high demand densities. Thus, taxi drivers are not able to park nearby and wait for e-hailing orders. Third, cruising on streets provides taxi drivers with additional opportunities to meet customers in close proximity, which could be more attractive than waiting for e-hailing orders at a fixed location.

[8] It is worthwhile to point out that the reliability of customers and taxi drivers keeping their appointments is a very influential factor in the attractiveness of the e-hailing mode. The previous taxi calling center in Shenzhen (China) also worked as a platform: telephone operators distributed customer demand information and let taxi drivers respond to and take the orders. However, it was very difficult to get a taxi through this calling center because taxi drivers stopped responding to calls due to being frequently stood up by customers. As time passed, fewer and fewer taxi drivers became willing to take taxi calling orders. In order to enhance the ratio of successful meetings, most existing e-hailing platforms impose restrictions on taxi drivers' and customers' behavior, e.g., customers' e-hailing accounts can be banned for a set period if she/he abandon confirmed orders several times.

[9] For clarification, the taxi fare is paid by customers to taxi drivers for riding services, and is irrelevant in terms of the hailing mode. Payment can be conducted either through the taxi hailing app, or not. Either method ensures that money goes to the taxi drivers.

[10] It is worthwhile to point out that although we assumed all taxi drivers were on the platform at any given moment, only a fraction of vacant and unreserved vacant taxis were willing to take e-hailing orders at any given time because for some drivers it is less costly to take roadside hailing customers.

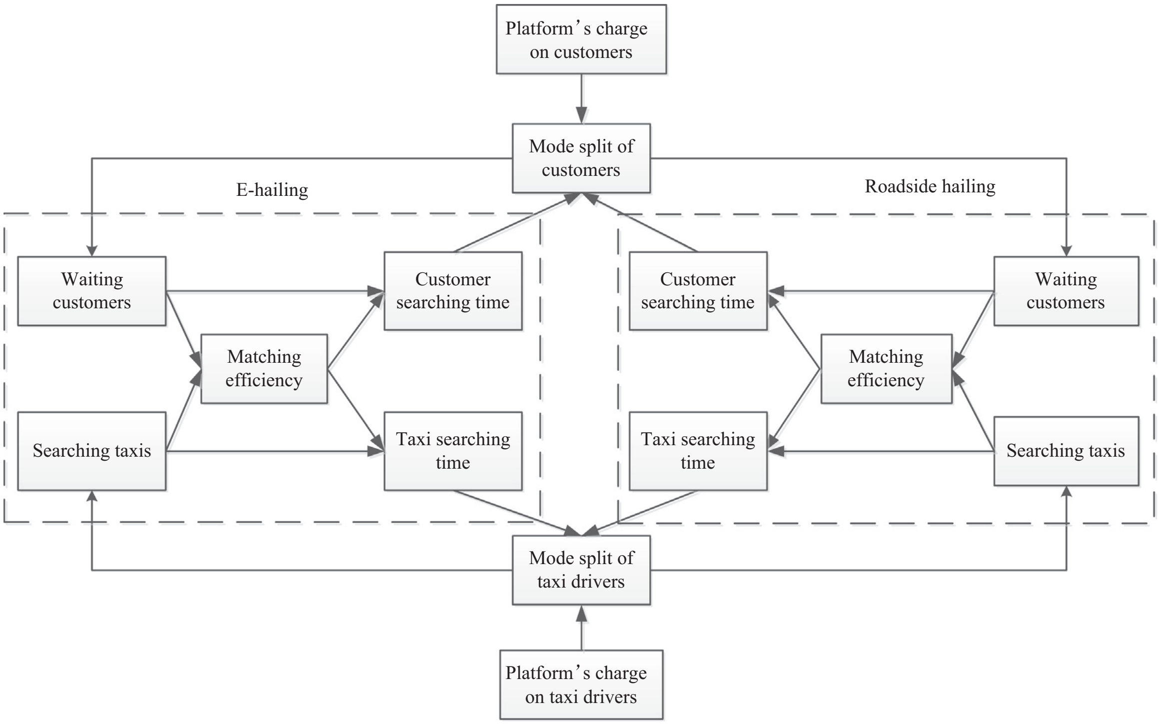


**Fig. 3.** Complex interactions among taxi system variables at equilibrium.

Furthermore, in order to enable a more concrete analysis, we assumed the matching functions of both modes followed a Cobb–Douglas type production function[11] (Varian, 1992), which exhibits constant elasticity:

$$M_i(N_i^c, N_i^t) = A_i(N_i^c)^{\alpha_i^c}(N_i^t)^{\alpha_i^t}, \ i = 0, 1 \tag{3}$$

where $A_i$ is a positive model parameter that depends on the spatial characteristics of the market and the hailing mode,[12] and $\alpha_i^c$ and $\alpha_i^t$ respectively represent the constant elasticity of meeting rates with respect to the number of waiting customers and available taxis for the hailing mode, $i = 0, 1$:

$$\alpha_i^c = \frac{\partial M_i}{\partial N_i^c}\frac{N_i^c}{M_i} \tag{4}$$

$$\alpha_i^t = \frac{\partial M_i}{\partial N_i^t}\frac{N_i^t}{M_i} \tag{5}$$

Furthermore, it is generally the case that $0 < \alpha_i^c, \alpha_i^t \leqslant 1, \ i = 0, 1$. A meeting function which is homogeneous of degree $(\alpha_i^c + \alpha_i^t)$ can exhibit increasing, constant, or decreasing returns to scale for cases of $\alpha_i^c + \alpha_i^t > 1, \ \alpha_i^c + \alpha_i^t = 1$ and $\alpha_i^c + \alpha_i^t < 1$, respectively.

As the e-hailing mode is generally more efficient than the roadside hailing mode, the meeting rate of the e-hailing mode is always higher than that of the roadside hailing mode, assuming that each mode has the same number of waiting customers and searching taxis:

$$M_1(N^c, N^t) = A_1(N^c)^{\alpha_1^c}(N^t)^{\alpha_1^t} \geqslant M_0(N^c, N^t) = A_0(N^c)^{\alpha_0^c}(N^t)^{\alpha_0^t}, \ \forall N^c \geqslant 0, \ N^t \geqslant 0 \tag{6}$$

In order to reflect the advantageous matching efficiency of the e-hailing mode, we assume $A_1 \geqslant A_0$, $\alpha_1^t \geqslant \alpha_0^t$ and $\alpha_1^c \geqslant \alpha_0^c$, so that Eq. (6) always holds.

[11] In previous studies of the traditional taxi market, a Cobb–Douglas type function has been explicitly utilized (e.g., Yang and Yang, 2011). In previous studies of two-sided market, a special form of the Cobb–Douglas function with $M_i(N_i^c, N_i^t) = N_i^c N_i^t$ is frequently adopted (e.g., Rochet and Tirole, 2003, 2004).

[12] This parameter can be negatively associated with the size of the searching and meeting area (Yang and Yang, 2011).

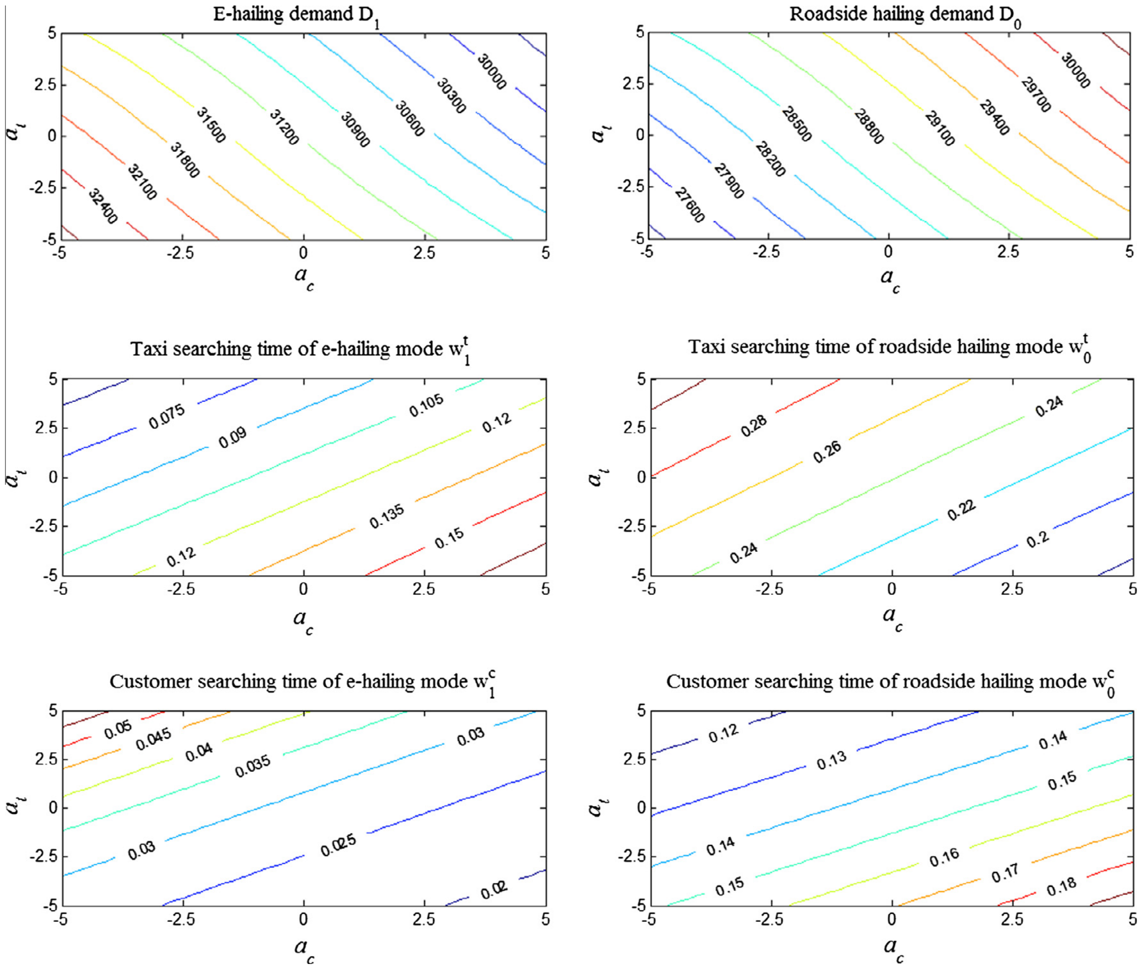


**Fig. 4.** Taxi market equilibria for Example 1.

### 2.2. Customer/taxi searching time

With a number of $N_i^c$ customers waiting for taxis via hailing mode $i$ at any moment and meeting taxis at a rate of $m_i$, the customer searching time of mode $i$, $i = 0, 1$, indicated by $w_i^c$, can be approximated by

$$w_i^c = \frac{N_i^c}{m_i}, \quad i = 0, 1 \tag{7}$$

Similarly, with a number of $N_i^t$ vacant taxis searching for customers through hailing mode $i$ at any moment, and meeting customers at a rate of $m_i$, the average taxi vacant hour, denoted by $w_i^t$, can be given as

$$w_i^t = \frac{N_i^t}{m_i}, \ i = 0, 1 \tag{8}$$

To be clear, customer searching time, indicated by $w_i^c, \ i = 0, 1$, is defined as the time it takes for a customer to 'find' a taxi, and taxi searching time is defined as the time a taxi driver takes to 'find' a customer. For the roadside hailing mode, customers and taxis meet precisely when they find each other, so customer searching time is identical to the customer waiting time, i.e., the time a customer takes to meet the taxi, and the taxi searching time is identical to vacant taxi hours, i.e., the time a taxi takes to 'meet' the next customer after dropping off the previous customer. However, in the e-hailing mode customers and taxis find each other through the app first and meet on the road at a later time, meaning an additional waiting time is required for the customers to be picked up by the reserved taxis. Let $r$ indicate the average driving time from the taxi's location to the customer's location after an e-hailing order is confirmed. The average vacant taxi hour and customer waiting time of the e-hailing mode are thus $w_1^t + r$ and $w_1^c + r$, respectively. For simplicity, we assume $r$ is constant in this study.

### 2.3. Mode split of customers and taxi drivers

In the presence of both e-hailing and roadside hailing, both taxis and customers will split between the two modes. After dropping off customers, taxi drivers can always choose to serve roadside hailing customers or take e-hailing orders. The decision making process can be very complex, but eventually all vacant taxis will take e-hailing orders or meet roadside hailing customers. As the searching and meeting cost is a major concern for taxi drivers, it is reasonable to assume that the final divergence of taxi drivers between the two modes is governed by the difference between searching and meeting costs of the two modes. Let $c_i^t$ indicate the searching cost (or disutility) of mode $i = 0, 1$ for taxi drivers, then from the previous discussion about vacant taxi hour and recalling the additional e-hailing fee $a_1^t$, we have

$$\begin{aligned} c_0^t &= \beta^t w_0^t + \varepsilon_0^t \\ c_1^t &= \beta^t \left(w_1^t + r\right) + a_1^t + \varepsilon_1^t \end{aligned}$$

where $\beta^t$ is the unit-time operation cost of taxis, which is assumed to be identical for both vacant and occupied taxis, and $\varepsilon_i^t$ is the random term of the searching cost of mode $i = 0, 1$. Let $T^{vt}$ be the total number of vacant and unreserved taxi trips, and $T_1^{vt}$ be the number of vacant taxi trips that end with meeting e-hailing customers. Assuming that $\varepsilon_0^t$ and $\varepsilon_1^t$ follow independent and identical distributions, the taxi drivers' mode split between the two modes can be approximated by the following binary Logit model (Ben-Akiva and Lerman, 1985):

$$T_1^{vt} = \frac{T^{vt}}{1 + \exp\left[\theta^t\left(\beta^t\left(w_1^t - w_0^t + r\right) + a_1^t\right)\right]} \tag{9}$$

where $\theta^t$ is a positive constant reflecting the degree of uncertainty in the searching costs of the e-hailing and roadside hailing modes. Accordingly, the total number of vacant taxi trips that ultimately take roadside hailing customers, denoted by $T_0^{vt}$, can be given as

$$T_0^{vt} = T^{vt} - T_1^{vt} \tag{10}$$

Furthermore, since the total number of taxis is fixed, the service time per unit modeling period is limited to $N$. Therefore, an additional service time conservation constraint must hold:

$$T_0^{vt}\left(w_0^t + l\right) + T_1^{vt}\left(w_1^t + r + l\right) = N \tag{11}$$

where the first term $T_0^{vt}\left(w_0^t + l\right)$ indicates the total number of hours taxis spend searching for and serving roadside hailing customers, and where the second term $T_1^{vt}\left(w_1^t + r + l\right)$ indicates the total number of hours taxis spend searching for, picking up, and serving e-hailing customers.

On the demand side, the customers' waiting cost, denoted by $c_i^c,\ i = 0, 1$, can be given as

$$\begin{aligned} c_0^c &= \beta^c w_0^c + \varepsilon_0^c \\ c_1^c &= \beta^c (w_1^c + r) + a_1^c + \varepsilon_1^c \end{aligned}$$

where $\beta^c$ is customers' value of time, and $\varepsilon_i^c$ is the random term of the waiting cost of mode $i = 0, 1$. Let $D_i$ be the number of customers using hailing mode $i = 0, 1$. Assuming that $\varepsilon_0^c$ and $\varepsilon_1^c$ follow identical and independent distribution of extreme values, and that each customer chooses the mode with a lower waiting cost, then the mode split of customers can be approximated by the following binary Logit model:

$$D_0 = \frac{D}{1 + \exp\left[\theta^c\left(\beta^c w_0^c - \beta^c\left(w_1^c + r\right) - a_1^c\right)\right]}, \quad D_1 = D - D_0 \tag{12}$$

where $\theta^c$ is a positive constant reflecting the degree of uncertainty in the waiting costs of the two modes.

### 2.4. Taxi market equilibrium

The mode split between roadside hailing and e-hailing among customers and taxi drivers reaches market equilibrium by matching the two sides of the market at the prevailing vacant taxi hours and customer waiting times of the two modes. Therefore, in addition to the above mentioned relationships, we have

$$T_i^{vt} = D_i = m_i,\ i = 0, 1 \tag{13}$$

Inserting Eqs. (7), (8), and (13) into Eqs. (3) and (9)–(12), the equilibrium conditions can be reduced to the following system of nonlinear equations after some simple transformations:

$$w_1^c = (A_1)^{-\frac{1}{\alpha_1^c}}(D_1)^{\frac{1-\alpha_1^t-\alpha_1^c}{\alpha_1^c}}(w_1^t)^{-\frac{\alpha_1^t}{\alpha_1^c}} \tag{14}$$

$$w_0^c = (A_0)^{-\frac{1}{\alpha_0^c}}(D_0)^{\frac{1-\alpha_0^t-\alpha_0^c}{\alpha_0^c}}(w_0^t)^{-\frac{\alpha_0^t}{\alpha_0^c}} \tag{15}$$

$$w_0^t - w_1^t = r - \frac{1}{\theta^t\beta^t}\ln\left(\frac{D}{D_1}-1\right) + \frac{a_1^t}{\beta^t} \tag{16}$$

$$w_0^t = \frac{N - Dl - D_1(w_1^t - w_0^t + r)}{D} \tag{17}$$

$$D_1 = \frac{D}{1+\exp\left[\theta^c(-\beta^c(w_0^c - w_1^c - r) + a_1^c)\right]} \tag{18}$$

$$D_0 = D - D_1 \tag{19}$$

where all endogenous variables are nonnegative.

## 3. Existence and stability of taxi market equilibrium

Based on the above system of nonlinear equations that must hold at the taxi market equilibrium, this section discusses the existence and stability of taxi market equilibrium.

**Proposition 1.** *If the customer waiting time functions in Eqs. (14) and (15) are continuous with respect to $D_i \in R^+$ and $w_i^t \in R$, $i = 0, 1$, then Eqs. (14)–(19) provide at least one solution.*

**Proof.** Validating the existence of solutions in Eqs. (14)–(19) is equivalent to establishing the existence of solutions to the following fixed point problem of $\Omega_{D_1} = \{D_1 | 0 \leqslant D_1 \leqslant D\}$:

$$D_1 = f(D_1) = \frac{D}{1+\exp\left[\theta^c\left(-\beta^c\left(w_0^c(D_1) - w_1^c(D_1) - r\right) + a_1^c\right)\right]} \tag{20}$$

where $w_0^c(D_1)$ and $w_0^c(D_1)$ satisfy Eqs. (14)–(17). According to Brouwer's fixed point theorem (Fuente, 2000), a fixed point problem, $x = f(x)$, admits at least one solution on $\Omega$ if, and only if, $\Omega$ is compact and non-empty, $f(x)$ is continuous on $\Omega$, and $f(x) \in \Omega$ for any $x \in \Omega$. In this problem, $\Omega_{D_1}$ is compact and non-empty, and $f(x)$ is non-expansive on $\Omega_{D_1}$. So in order to validate the existence of solutions to the above fixed point problem, we need only to show that $f(D_1)$ is continuous with $D_1$.

From Eq. (16), we can see that $g(D_1) = w_0^t - w_1^t$ is continuous with respect to $D_1$. Inserting $g(D_1)$ and Eq. (19) into Eq. (17), we have $w_0^t = [N - Dl - D_1(g(D_1) + r)]/D$, so $w_0^t$ can be written into a continuous function of $D_1$, i.e., $w_0^t = w_0^t(D_1)$, and the continuity of $w_1^t(D_1) = w_0^t(D_1) - g(D_1)$ can thus be immediately verified. Substituting $w_0^t(D_1)$ and $w_1^t(D_1)$ into Eqs. (14) and (15) respectively, we can conclude the continuity of $w_0^c(D_1)$ and $w_1^c(D_1)$, which further implies the continuity of $f(D_1)$ in Eq. (20). This completes the proof. □

Proposition 1 establishes the existence of solutions to the system of nonlinear equations (14)–(19) under any pricing strategy. It is worthwhile to point out that the solutions to Eqs. (14)–(19) are not necessarily taxi market equilibria. A mathematically suitable solution to Eqs. (14)–(19) may imply a negative taxi/customer waiting time (which is impossible in reality) for some taxi fleet size, $N$ (for example, if $N < Dl$, then Eq. (17) implies $w_0^t < 0$). While determining an appropriate taxi fleet size that admits positive equilibrium solutions is an interesting topic, it is beyond the focus of this paper. So in the following analysis, we only focus on the cases where the existence of taxi market equilibrium is guaranteed under the given taxi fleet size $N$.

Furthermore, not all equilibrium solutions are necessarily stable. If an equilibrium solution is unstable, then a minor perturbation may break the equilibrium state, and all discussions about taxi systems in an unstable equilibrium state is meaningless. In the following proposition, an explicit condition on the stability of taxi market equilibrium is provided.

**Proposition 2.** *The taxi market equilibrium is locally asymptotically stable if*

$$S = \left|\frac{\theta^c\beta^c Z D_1 D_0}{D}\right| < 1 \tag{21}$$

*where*

$$Z = -\frac{w_0^c}{\alpha_0^c}\left[\frac{1-\alpha_0^c-\alpha_0^t}{D_0} + \frac{\alpha_0^t}{w_0^t}\left(\frac{1}{\beta^t\theta^t D_0} - \frac{w_1^t - w_0^t + r}{D}\right)\right] - \frac{w_1^c}{\alpha_1^c}\left[\frac{1-\alpha_1^t-\alpha_1^c}{D_1} + \frac{\alpha_1^t}{w_1^t}\left(\frac{w_1^t - w_0^t + r}{D} + \frac{1}{\beta^t\theta^t D_1}\right)\right] \tag{22}$$

*and the taxi market equilibrium is unstable if*

$$S = \left|\frac{\theta^c \beta^c Z D_1 D_0}{D}\right| > 1.$$

**Proof.** See Appendix B.

As the system of nonlinear equations (14)–(19) is highly nonlinear and non-convex, no existing algorithms are capable of calculating the equilibrium solution with guaranteed convergence. Nevertheless, when a stable equilibrium exists, the following simple fixed point iterations show a satisfying convergence rate in our numerical tests:

**Step 1**. Set $n = 0$ and $D_1^{(0)}$;

**Step 2**. Sequentially update the value of $(w_0^t - w_1^t)^{(n)}$, $w_0^{t(n)}$, $w_1^{t(n)}$, $w_1^{c(n)}$, and $w_0^{c(n)}$ using the following:

$$(w_0^t - w_1^t)^{(n)} = r - \tfrac{1}{\theta^t \beta^t} \ln\left(\tfrac{D}{D_1^{(n)}} - 1\right) + \tfrac{a_1^t}{\beta^t}$$

$$w_0^{t(n)} = \tfrac{N - Dl - D_1\left[(w_1^t - w_0^t)^{(n)} + r\right]}{D}$$

$$w_1^{t(n)} = (w_0^t - w_1^t)^{(n)} - w_0^{t(n)}$$

$$w_1^{c(n)} = (A_1)^{-\frac{1}{\alpha_1^c}} (D_1^{(n)})^{\frac{1-\alpha_1^t-\alpha_1^c}{\alpha_1^c}} \left(w_1^{t(n)}\right)^{-\frac{\alpha_1^t}{\alpha_1^c}}$$

$$w_0^{c(n)} = (A_0)^{-\frac{1}{\alpha_0^c}} (D_0^{(n)})^{\frac{1-\alpha_0^t-\alpha_0^c}{\alpha_0^c}} \left(w_0^{t(n)}\right)^{-\frac{\alpha_0^t}{\alpha_0^c}}$$

**Step 3**. Calculate $D_1^{(n+1)}$ by inserting $w_1^{c(n)}$ and $w_0^{c(n)}$ into Eq. (18). If $|D_1^{(n+1)} - D_1^{(n)}| < \varepsilon$, do not continue; otherwise, set $n = n + 1$ and move on to Step 2. □

At the end of this section, we deliver the following numerical example to demonstrate the impacts of platform's pricing strategy to the taxi market performance at equilibrium.

**Example 1.** Consider a taxi market with a total taxi demand of $D$ = 60,000 trips/h and a total taxi fleet size of $N$ = 30,000. The average trip time is $l$ = 0.3 h. As taxi customers are usually time-sensitive, their value of time is set relatively high with $\beta^c$ = 60 RMB/h. For taxis, we assume the hourly taxi operating cost is $\beta^t$ = 50 RMB/h. Both e-hailing and roadside hailing modes exhibit increasing returns to scale with $\alpha_i^c = 1$ and $\alpha_i^t = 1,\ i = 0, 1$. However, since the e-hailing mode is generally more efficient than the roadside hailing mode, we assume $A_1 = 10A_0 = 10^{-2}$ in the meeting functions of the two modes. The taxi reserved time interval is set at $r$ = 0.05 h, and the rest of the parameters related to the mode choice of customers and taxi drivers are set as $\theta^c$ = 0.02 and $\theta^t$ = 0.02.

When the platform's charging rates $a_1^c$ and $a_1^t$ vary between −5 and 5, the taxi market equilibrium at any $(a_1^c, a_1^t)$ is calculated based on the solution algorithm provided above, and the resulting taxi market performance is depicted in Fig. 4. In this example, when the platform's charge on customers, $a_1^c$, increases, the number of e-hailing customers, $D_1$, always decreases. The reduced e-hailing demand further results in a reduced intra-group competition among e-hailing customers and increased intra-group competition among e-hailing taxis, so the customer waiting time $w_1^c$ is decreased and the taxi searching time $w_1^t$ is increased. Accordingly, the demand and taxi/customer searching time of roadside hailing mode change in an opposite direction. And for any increase of platform's charge on taxis, $a_1^t$, the same trend of variance can be observed in this case.

In this example, the minimal customer waiting time (longest taxi searching time) of e-hailing mode is reached when the taxi drivers are subsidized to the utmost and the e-hailing customers are charged to the utmost, i.e., $(a_1^c, a_1^t) = (5, -5)$. In this case, there are lowest demand and highest supply of taxis serving e-hailing mode. However, at this pricing strategy of the platform, the roadside hailing customers will suffer most, as most taxis will be induced to wait and serve e-hailing customers, whereas few customers are affordable for e-hailing service. On the contrary, if the taxi drivers are charged to the utmost and the e-hailing customers are subsidized to the utmost, i.e., $(a_1^c, a_1^t) = (-5, 5)$, then the situation get reversed. At this new price level, the longest customer waiting time and taxi searching time for e-hailing mode would be observed, as the high subsidy induces most customers to e-hailing mode while the positive charge on the taxi side prevent most taxi drivers to take e-hailing orders. Meanwhile, for the roadside hailing mode, there are lowest demand and highest supply of taxis, so the corresponding customer waiting time is shortest in this case.

Obviously as we can see from this example, the impacts of the platform's pricing strategy on the taxi market performance are significant. However, are these phenomena observed in this example universal for all taxi markets under hybrid modes of e-hailing and roadside hailing? What're the determinants to the variance of the taxi market in response to price perturbations? The next section is devoted to answer these questions.

## 4. Sensitivity analysis

To quantify the variance of the taxi system in accordance to a platform's price perturbation, a partial-derivative based sensitivity analysis is conducted in this section. Let $da = (da_1^c, da_1^t)$ be a minor perturbation of $a$, and let $dD_1$, $dw_0^c$, $dw_1^c$, $dw_0^t$, and $dw_1^t$ be the resulting variance of $D_1$, $w_0^c$, $w_1^c$, $w_0^t$, and $w_1^t$ respectively. Taking derivative on both sides of Eqs. (14)–(19) with respect to $a_1^c$ and $a_1^t$, we obtain the following sensitivity results (the detailed calculation process is provided in Appendix C):

$$dD_1 = -\frac{da_1^c}{B\beta^c} - \left[D_1\frac{\alpha_0^t}{w_0^t}\frac{w_0^c}{\alpha_0^c} + D_0\frac{\alpha_1^t}{w_1^t}\frac{w_1^c}{\alpha_1^c}\right]\frac{da_1^t}{DB\beta^t} \tag{23}$$

$$dw_0^t = -\left(\frac{w_1^t - w_0^t + r}{D} - \frac{1}{\beta^t\theta^t D_0}\right)dD_1 + \frac{D_1}{D\beta^t}da_1^t \tag{24}$$

$$dw_1^t = -\left(\frac{w_1^t - w_0^t + r}{D} + \frac{1}{\beta^t\theta^t D_1}\right)dD_1 - \frac{D_0}{D\beta^t}da_1^t \tag{25}$$

$$dw_0^c = -\frac{w_0^c}{\alpha_0^c}\left(\frac{1-\alpha_0^c-\alpha_0^t}{D_0}dD_1 + \frac{\alpha_0^t}{w_0^t}dw_0^t\right) \tag{26}$$

$$dw_1^c = \frac{w_1^c}{\alpha_1^c}\left(\frac{1-\alpha_1^c-\alpha_1^t}{D_1}dD_1 - \frac{\alpha_1^t}{w_1^t}dw_1^t\right) \tag{27}$$

where

$$\begin{aligned} B = {} & \frac{D}{D_1 D_0 \theta^c\beta^c} + \left[\frac{1-\alpha_0^c-\alpha_0^t}{D_0} + \left(\frac{1}{\beta^t\theta^t D_0} - \frac{(w_1^t + r - w_0^t)}{D}\right)\frac{\alpha_0^t}{w_0^t}\right]\frac{w_0^c}{\alpha_0^c} \\ & + \left[\frac{1-\alpha_1^c-\alpha_1^t}{D_1} + \left(\frac{1}{\beta^t\theta^t D_1} + \frac{(w_1^t + r - w_0^t)}{D}\right)\frac{\alpha_1^t}{w_1^t}\right]\frac{w_1^c}{\alpha_1^c} \end{aligned} \tag{28}$$

Based on the above sensitivity results, it is easy to predict variances in the taxi market in response to any price perturbation. Additionally, ascertaining desirable price perturbations that lead a taxi system to a better state is also easily accomplished. However, before making any predictions or determinations, it is necessary to examine the continuity of the taxi market equilibrium with respect to the platform's pricing strategy. According to the definition of continuous vector function of multiple variables (Sydsater et al., 2005), the taxi market equilibrium is continuous with the platform's pricing strategy, $a$, if at any pricing strategy $a_0$, we can find a $\delta > 0$ for any $\varepsilon > 0$, such that $|dD_i| < \varepsilon$, $|dw_i^c| < \varepsilon$ and $|dw_i^t| < \varepsilon$, $i = 0, 1$ for any $\|a - a_0\| < \delta$. From Eqs. (23)–(27), the continuity condition is obviously violated if $B = 0$ and/or $D_i = 0$, $i = 0, 1$. Nevertheless, as established in the following lemma, $D_i = 0$, $i = 0, 1$ would never appear at taxi market equilibrium.

**Lemma 1.** *Under hybrid modes, both e-hailing and roadside hailing modes admit positive utilization at taxi market equilibrium, i.e., $0 < D_1 < D$.*

**Proof.** Please refer to Appendix D. □

Based on Lemma 1, the following proposition establishes the necessary and sufficient conditions of the continuity of taxi market equilibrium with respect to the platform's pricing strategy.

**Proposition 3.** *Under hybrid modes, the taxi market equilibrium varies continuously with platform's pricing strategy if and only if $B \neq 0$, where $B$ is defined by Eq. (28).*

**Proof.** From Lemma 1, under any given pricing strategy, $a_0$, of the platform, we always have $0 < D_1 < D$ and $0 < D_0 < D$ at taxi market equilibrium. By inserting $0 < D_1 < D$ and $0 < D_0 < D$ into Eqs. (14)–(19), it is easy to verify that $w_i^c < +\infty$ and $w_i^t < +\infty$, $i = 0, 1$. In view of Eqs. (23)–(27), we know that if $B \neq 0$ then the boundedness of $w_i^c$, $w_i^t$, and $D_i$, $i = 0, 1$ implies that for any $\varepsilon > 0$, we can always find a $\delta > 0$, so that $|dD_i| < \varepsilon$, $|dw_i^c| < \varepsilon$ and $|dw_i^t| < \varepsilon$ stand for any $da \in B(\delta, a_0)$. However, if $B = 0$, then from Eq. (23), we have $dD_1 \to \infty$ for any $da \neq 0$. Therefore, the taxi market equilibrium varies continuously with $a$ if and only if $B \neq 0$. This completes the proof. □

As $B = 0$ rarely occurs in practice, we can generally assume that the taxi market equilibrium is continuous with the platform's pricing strategy. However, despite the continuous variation of the taxi market equilibrium, *a minor perturbation in the platform's pricing strategy may change the taxi system from a stable equilibrium into an unstable equilibrium* because the stability condition in Proposition 2 may no longer be held at the new equilibrium point. In such a case, it becomes very difficult (if not impossible) to achieve an equilibrium solution, even if the change in the platform's pricing strategy is minor. The following numerical example is provided to first validate the sensitivity results, and then emphasize the risk of having an unstable equilibrium after a minor price perturbation.

**Example 2.** Still consider the same taxi market as in Example 1. When $a_1^t$ is fixed at 2 RMB and $a_1^c$ varies between $-10$ and 15 RMB, we calculate the taxi market equilibrium based on two methods: the first method is to directly solve the system of nonlinear equations in Eqs. (14)–(19) based on the solution algorithm in Section 3; the second method is to estimate the value of $D_1$, $w_i^c$, and $w_i^t$, $i = 0, 1$ after a price perturbation by adopting the sensitivity results in Eqs. (23)–(27).[13] As shown in Fig. 5(a)–(c), the curves of $D_1$, $w_i^c$, and $w_i^t$ are obtained by using the first method, and the curves of EST-$D_1$, EST-$w_i^c$, and EST-$w_i^t$ are obtained by using the second method. The two curves are almost identical, which verifies the derivative results of the sensitivity analysis.

When $a_1^c$ varies between $-10$ and 15 RMB, all the taxi system variables will vary continuously with $a_1^c$, as shown in Fig. 5 (a)–(c). Furthermore, the equilibrium states will all be locally asymptotically stable, which is consistent with Fig. 5(d) where the value of $S$ (defined by Eq. (21)) is always less than 1. However, as we can see from Fig. 5(d), $S$ is too close to 1 and will decrease with $a_1^c$ at $a_1^c = -10$. Therefore, the stability of the taxi market equilibrium will likely no longer hold if $a_1^c$ is further reduced. Indeed, as shown in Fig. 6, a minor price perturbation, $da = (-1.5, 0)$, will prevent the taxi market from converging and forming a stable equilibrium.

If the taxi market equilibrium is unstable, then any discussions of the properties of the taxi market equilibrium are meaningless. As such, we only focus on stable equilibria. Using the stability conditions established in Proposition 2, we can derive the following lemma.

**Lemma 2.** *In any locally asymptotically stable equilibrium, we always have $B > 0$ with $B$ defined by Eq. (28).*

**Proof.** Using Proposition 2, if the taxi market equilibrium is locally asymptotically stable, then

$$Z < \frac{D}{\theta^c \beta^c D_1 D_0} \tag{29}$$

Meanwhile, using the definition of $B$ and $Z$ in Eqs. (28) and (22), respectively, we have

$$B = \frac{D}{D_1 D_0 \theta^c \beta^c} - Z$$

Inserting Eq. (29) into the above equation yields $B > 0$. This completes the proof. □

In view of Proposition 3 and Lemma 2, we can determine that in any locally asymptotically stable equilibrium, the taxi system variables must vary continuously with both $a_1^c$ and $a_1^t$. Additionally, interesting properties concerning the variance of the taxi market in response to any price perturbation can be derived.

**Proposition 4.** *In any locally asymptotically stable equilibrium, the taxi system variables must vary continuously with both $a_1^c$ and $a_1^t$. Furthermore, in any locally asymptotically stable equilibrium,*

(1) *If the platform increases the price level on one side without reducing its charge on the other side, then the e-hailing demand must be decreased, i.e. $\partial D_1/\partial a_1^c < 0$ and $\partial D_1/\partial a_1^t < 0$;*
(2) *If the platform keeps the aggregate price level at $a^{agg} = a_1^c + a_1^t$ unchanged while manipulating the price share between customers and taxi drivers, i.e. $da_1^c = -da_1^t$, then reducing the platform's charge rate on the customer side will increase e-hailing demand if, and only if,*

$$Y = \left( D_1 \frac{\alpha_0^t}{w_0^t} \frac{w_0^c}{\alpha_0^c} + D_0 \frac{\alpha_1^t}{w_1^t} \frac{w_1^c}{\alpha_1^c} \right) \frac{1}{D\beta^t} - \frac{1}{\beta^c} < 0 \tag{30}$$

**Proof.** As established in Lemma 2, $B > 0$ in any locally asymptotically stable equilibrium. So using Proposition 3, the continuity of taxi market equilibrium can be validated. Inserting $B > 0$ into Eq. (23), the first part of the proposition can also be readily established. For the second part of the proposition, inserting $da_1^c = -da_1^t$ into Eq. (23), we have

$$dD_1 = \left[ \left( D_1 \frac{\alpha_0^t}{w_0^t} \frac{w_0^c}{\alpha_0^c} + D_0 \frac{\alpha_1^t}{w_1^t} \frac{w_1^c}{\alpha_1^c} \right) \frac{1}{D\beta^t} - \frac{1}{\beta^c} \right] \frac{da_1^c}{B}$$

Since $B > 0$, we must have $dD_1 > 0$ for any $da_1^c < 0$ if Eq. (30) holds. This completes the proof. □

[13] To be clear, let $X^n$ be the equilibrium state at $a^n$. The second method then derives the equilibrium solution $X^{n+1}$ by calculating $X^{n+1} = X^n + (dX/da_1^c)|_{a^n} da_1^c + (dX/da_1^t)\big|_{a^n} da_1^t$.

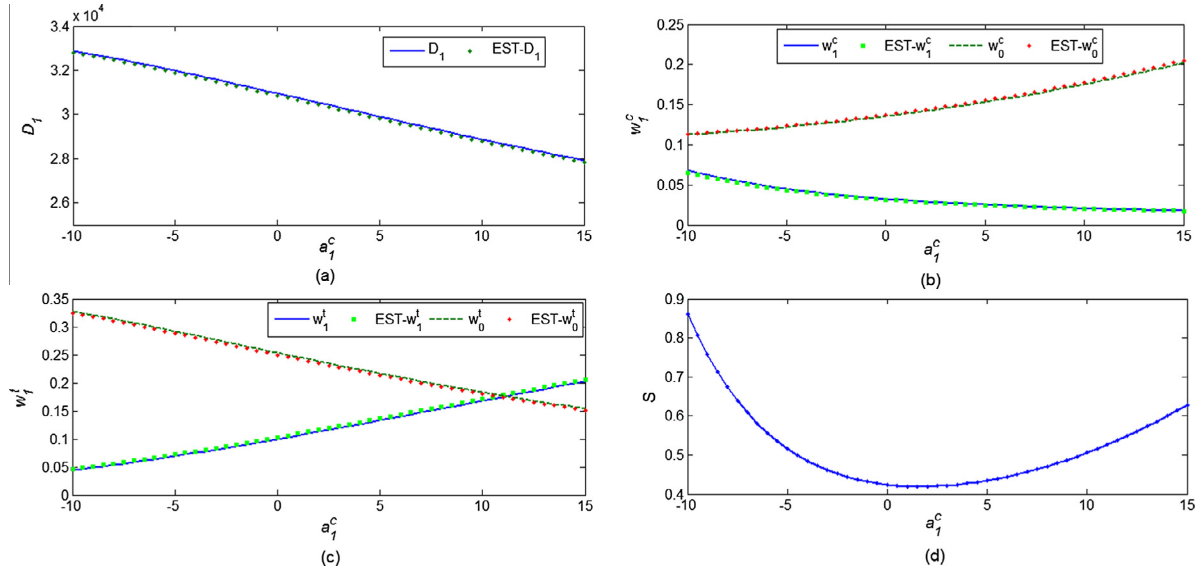


**Fig. 5.** Variance of the taxi system at $a_1^c \in [-10, 15]$ and $a_1^t = 2$.

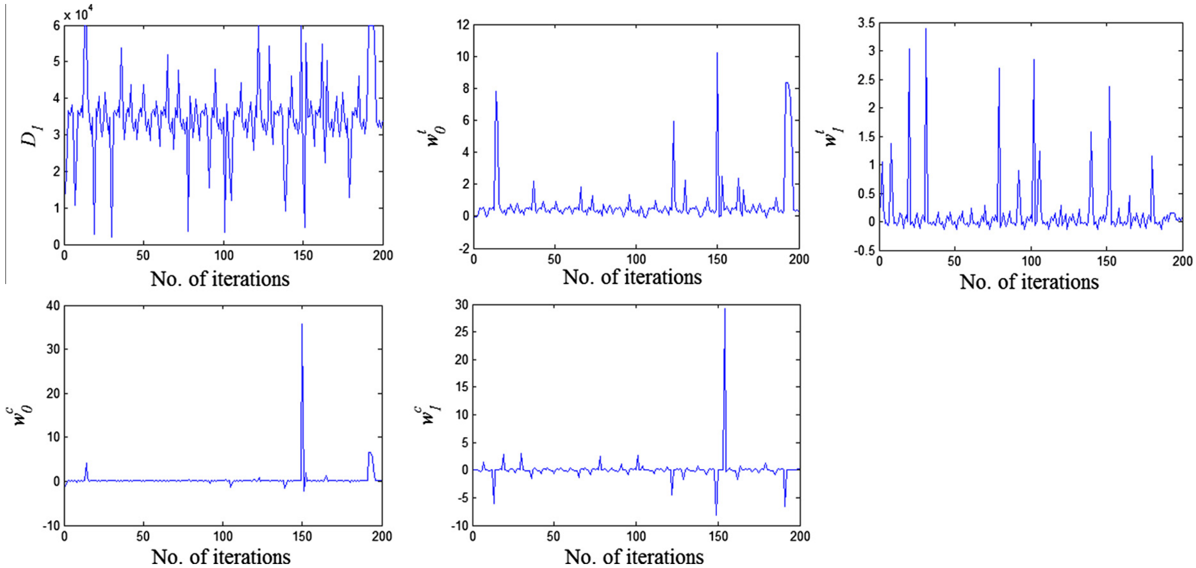


**Fig. 6.** Instability of taxi market equilibrium at $a_1^c = -11.5$ and $a_1^t = 2$.

As Proposition 4 clearly illustrates, in any stable equilibrium, if the platform increases the charge rate on one side while keeping the charge rate on the other side unchanged, or increases the charge rates on both sides, the e-hailing demand will be reduced. This is consistent with our observation from Fig. 4 of Example 1. However, if the platform increases the charge rate on one side and reduces the charge rate on the other side by the same amount, then whether the e-hailing demand will increase or decrease depends on the relative impacts of $da_1^c$ and $da_1^t = -da_1^c$ on $D_1$. In Eq. (30), the first term $\left(D_1 \frac{\alpha_0^t}{w_0^t}\frac{w_0^c}{\alpha_0^c} + D_0 \frac{\alpha_1^t}{w_1^t}\frac{w_1^c}{\alpha_1^c}\right)(D\beta^t)^{-1}$ and second term $-(\beta^c)^{-1}$ are respectively correlated with the impacts of $da_1^c < 0$ and $da_1^t = -da_1^c > 0$ on $D_1$, so $dD_1 > 0$ if and only if the combinatory effect is positive. For the taxi market described in Example 1, Fig. 7 depicts the value of $Y$ for $a_1^c \in [-5, 5]$ and $a_1^t \in [-5, 5]$. In this example, $Y < 0$ within the entire price range, so any price perturbation with $da_1^c < 0$ and $da_1^t = -da_1^c > 0$ leads an increase of $D_1$, being consistent with Fig. 4. □

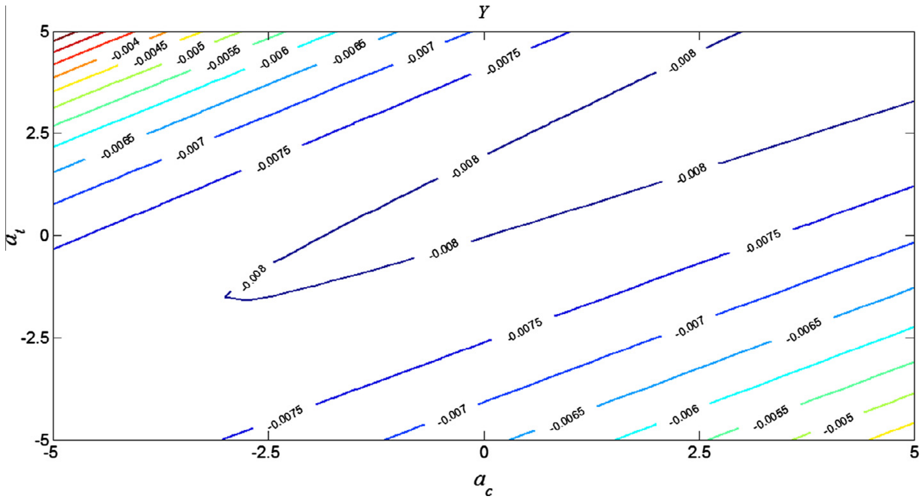


**Fig. 7.** The value of $Y$ in Eq. (30) for Example 1.

## 5. Enhancing social welfare vs. increasing a platform's profit

Based on the sensitivity results in Section 4, we are now ready to examine desirable price perturbations from both the public's and platform's perspectives. While the public wishes the taxi system to be improved, the e-hailing company usually focuses on its own profit. This creates conflict between the public's and platform's interests. The objective of this section is to derive desirable price perturbations in respect to both the public's and platform's perspectives, and to highlight the necessity of regulation over a platform's pricing strategies.

We begin by discussing pricing strategies that improve the platform's profit. Let $K$ indicate the total operation cost of the taxi hailing platform, then together with the total revenue of the platform, $(a_1^c + a_1^t)D_1$, derived from charging both sides of users (drivers and riders), the platform's profit can be given by

$$\Pi(a) = (a_1^c + a_1^t)D_1 - K \tag{31}$$

Taking the derivatives of $\Pi$ with respect to $a_1^c$ and $a_1^t$ respectively yields

$$\frac{\partial \Pi}{\partial a_1^c} = (a_1^c + a_1^t)\frac{\partial D_1}{\partial a_1^c} + D_1 = -\frac{1}{B\beta^c}(a_1^c + a_1^t) + D_1 \tag{32}$$

$$\frac{\partial \Pi}{\partial a_1^t} = (a_1^c + a_1^t)\frac{\partial D_1}{\partial a_1^t} + D_1 = -\left(D_1 \frac{w_0^c}{\alpha_0^c}\frac{\alpha_0^t}{w_0^t} + D_0 \frac{w_1^c}{\alpha_1^c}\frac{\alpha_1^t}{w_1^t}\right)\left(\frac{a_1^c + a_1^t}{DB\beta^t}\right) + D_1 \tag{33}$$

Therefore, in any price perturbation, $da$, the resulting variance of the platform's profit is

$$d\Pi = \left[D_1 - \frac{1}{B\beta^c}(a_1^c + a_1^t)\right]da_1^c + \left[D_1 - \left(D_1 \frac{w_0^c}{\alpha_0^c}\frac{\alpha_0^t}{w_0^t} + D_0 \frac{w_1^c}{\alpha_1^c}\frac{\alpha_1^t}{w_1^t}\right)\left(\frac{a_1^c + a_1^t}{DB\beta^t}\right)\right]da_1^t \tag{34}$$

which directly leads to the following proposition on the properties of desirable price perturbations that improve the platform's profit:

**Proposition 5.** *A price perturbation, $da = (da_1^c, da_1^t)$, increases the platform's profit if and only if*

$$\left(-\frac{a_1^c + a_1^t}{B\beta^c} + D_1\right)da_1^c > \left[\left(D_1 \frac{w_0^c}{\alpha_0^c}\frac{\alpha_0^t}{w_0^t} + D_0 \frac{w_1^c}{\alpha_1^c}\frac{\alpha_1^t}{w_1^t}\right)\left(\frac{a_1^c + a_1^t}{DB\beta^t}\right) - D_1\right]da_1^t \tag{35}$$

*where $B$ is defined by Eq. (28).*

However, from the public's perspective, an optimal pricing strategy should maximize social welfare. In a fixed demand case, the gross benefit of taxi customers is constant, and, as the taxi operating cost for both occupied and vacant taxis is assumed to be the same, the total taxi operating cost is also a constant under the fixed taxi fleet size. Therefore, within the framework of this study, maximizing social welfare is equivalent to reducing average customer waiting times:

$$\bar{w}(a) = \frac{w_0^c(a)D_0(a) + (w_1^c(a) + r)D_1(a)}{D} \tag{36}$$

Taking the derivative of $\bar{w}(a)$ with respect to $a_1^c$ and $a_1^t$ respectively yields

$$\frac{\partial \bar{w}}{\partial a_1^c} = -\frac{C}{D\beta^c B} \tag{37}$$

$$\frac{\partial \bar{w}}{\partial a_1^t} = \frac{1}{D^2\beta^t}\left[\frac{w_1^c}{\alpha_1^c}\frac{\alpha_1^t}{w_1^t}\left(D_0D_1 - \frac{D_0}{B}C\right) - \frac{w_0^c}{\alpha_0^c}\frac{\alpha_0^t}{w_0^t}\left(D_0D_1 + \frac{D_1}{B}C\right)\right] \tag{38}$$

where

$$C = r + (1-\alpha_1^t)\frac{w_1^c}{\alpha_1^c} - (1-\alpha_0^t)\frac{w_0^c}{\alpha_0^c} + \left(\frac{w_0^c}{\alpha_0^c}\frac{\alpha_0^t}{w_0^t}D_0 + \frac{w_1^c}{\alpha_1^c}\frac{\alpha_1^t}{w_1^t}D_1\right)\frac{(w_1^t + r - w_0^t)}{D} + \left(\frac{w_1^c}{\alpha_1^c}\frac{\alpha_1^t}{w_1^t} - \frac{w_0^c}{\alpha_0^c}\frac{\alpha_0^t}{w_0^t}\right)\frac{1}{\beta^t\theta^t} \tag{39}$$

Therefore, any price perturbation, $da$, causes a variance in average customer waiting times, which can be predicted by

$$d\bar{w} = -\frac{C}{D\beta^c B}da_1^c + \frac{1}{D^2\beta^t}\left[\frac{w_1^c}{\alpha_1^c}\frac{\alpha_1^t}{w_1^t}(D_0D_1 - \frac{D_0}{B}C) - \frac{w_0^c}{\alpha_0^c}\frac{\alpha_0^t}{w_0^t}\left(D_0D_1 + \frac{D_1}{B}C\right)\right]da_1^t \tag{40}$$

By employing Eq. (40), it becomes possible to easily determine what price perturbations would improve social welfare in a non-optimal taxi market equilibrium.

**Proposition 6.** *A price perturbation, $da = (da_1^c, da_1^t)$, reduces the average customer waiting time if and only if*

$$\left[\frac{w_1^c}{\alpha_1^c}\frac{\alpha_1^t}{w_1^t}\left(D_0D_1 - \frac{D_0}{B}C\right) - \frac{w_0^c}{\alpha_0^c}\frac{\alpha_0^t}{w_0^t}\left(D_0D_1 + \frac{D_1}{B}C\right)\right]da_1^t < \frac{\beta^t DC}{\beta^c B}da_1^c$$

*where B and C are defined in Eqs. (28) and (39) respectively.*

While Propositions 5 and 6 provide directional guidelines for desirable price perturbations that would improve the platform's profit and/or the public's social welfare, the step size of perturbation is not addressed. As all taxi system variables vary nonlinearly in relation to the platform's pricing strategy, one should better adopt a small step size when adjusting pricing strategies based on the above results. Furthermore, as demonstrated in Example 1, the stability condition in Proposition 2 may be violated after even a minor perturbation, so one should also be careful about the stability issue. Using Proposition 2, let $dS$ be the variance of $S$ defined by Eq. (21) in response to any price perturbation, $da$. After a price perturbation, the taxi market equilibrium will remain stable if, and only if, $|S + dS| < 1$. Using Eqs. (23)–(27), we have

$$dS = \frac{\theta^c\beta^c ZD_1D_0}{DS}\left[\frac{\theta^c\beta^c Z(D_0 - D_1)}{D}dD_1 + \frac{\theta^c\beta^c D_1D_0}{D}dZ\right] \tag{41}$$

where $dD_1$ is defined by Eq. (23), and $dZ$ is defined in Appendix E. In view of Eq. (41), we readily reach the proposition provided below.

**Proposition 7.** *In a locally asymptotically stable equilibrium, a price perturbation, $da = (da_1^c, da_1^t)$, will lead to another locally asymptotically stable equilibrium if, and only if,*

$$-1 - S < dS < 1 - S$$

*where S is defined by Eq. (21), and dS is defined by Eq. (41).*

Based on Eqs. (34) and (40), we are close to determining the optimal pricing strategies that respectively maximize social welfare or the platform's profit. However, as observed in our numerical tests, such pricing strategies will likely not permit a stable equilibrium. In the case of an unstable equilibrium, it is meaningless to discuss social welfare or the platform's profit. Thus, when determining the optimal pricing strategies that maximize platform's profit or social welfare, one should focus on a feasible price region of $\Omega_a$ within which the existence of stable equilibrium is guaranteed (i.e. Eq. (21)). Determining the optimal pricing strategies within $\Omega_a$ involves significant additional work, which we leave for future study.

In concluding this section, we recall Example 1 in order to highlight the different optimal pricing strategies found between the public's and platform's points of view, and demonstrate how we can determine desirable price perturbations based on the results derived in this section.

**Example 1 (continued).** Consider the same taxi market as described in Example 1. Assuming that the total operation cost of the taxi hailing platform is $K = 10^5$ RMB, when $a_1^c$ and $a_1^t$ vary between −5 and 5, the average customer waiting time and the platform's profit are depicted in Fig. 8. In this example, both customers and taxi drivers are not sensitive enough to the cost difference between the two modes (i.e., the values of $\theta^c$ and $\theta^t$ are relatively small), so although e-hailing demand decreases as $a_1^c$ and $a_1^t$ increase (see Fig. 4), the platform's profit calculated by Eq. (31) is increasing. Therefore, if the platform is operated by a private company, it will charge both customers and taxi drivers as much as possible within the given price range, i.e., $(a_1^c, a_1^t) = (5, 5)$ (point B), so that the profit of the platform is maximized to $\Pi = 1.96 \times 10^5$. At this price level, the equilibrium state is

$$D_1 = 29578,\ D_0 = 30422,\ w_1^c = 0.0298,\ w_0^c = 0.1396,\ w_1^t = 0.1136,\ w_0^t = 0.2354$$

and the average customer waiting time at this price level is 0.1101 h. However, if the platform is operated by a public company to minimize average customer waiting time, then it will take care of the waiting time of both types of customers. Given

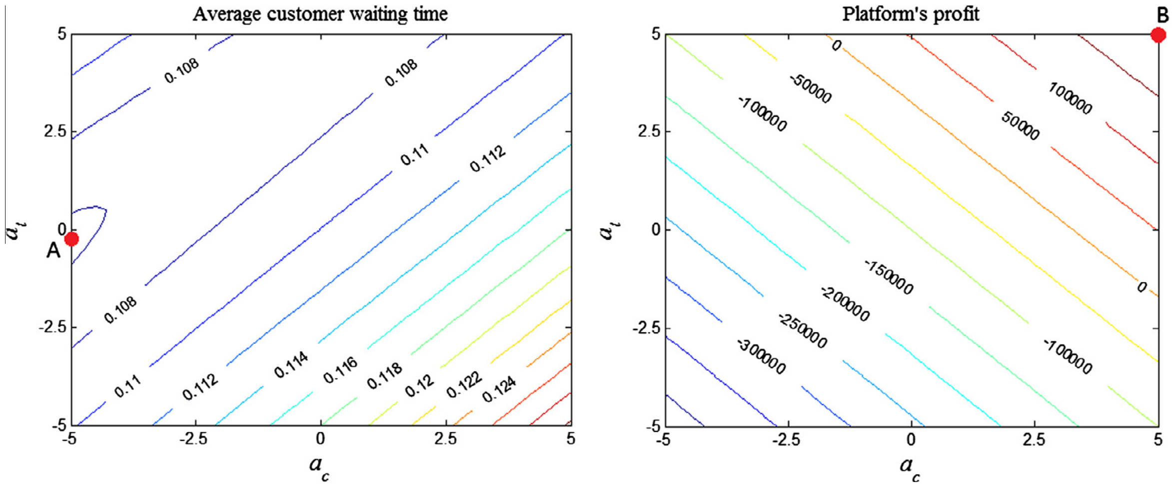


**Fig. 8.** Variance of social welfare and the platform's profit.

the different variation trend of $w_1^c$ and $w_0^c$ with respect to $a_1^c$ and $a_1^t$ (see Fig. 4), the optimal pricing strategy that minimizes average customer waiting time within the given price range is $(a_1^c, a_1^t) = (-5, -0.25)$ (point A). At this price level, the equilibrium state is

$$D_1 = 32253,\ D_0 = 27747,\ w_1^c = 0.0375,\ w_0^c = 0.1295,\ w_1^t = 0.0827,\ w_0^t = 0.2782$$

and the minimal average customer waiting time is 0.1069 h. Unfortunately, the shorter average customer waiting time results in a negative profit for the platform ($\Pi = -2.69 \times 10^5$). Such conflicts characterize the relationship between social welfare and a platform's profitability.

Now suppose the private platform is willing to sacrifice 5% of its profit to reduce the average customer waiting time, what's the highest reduction of $\bar{w}$ can be achieved within the given price range? Inserting the equilibrium state at $(a_1^c, a_1^t) = (5, 5)$ into Eqs. (40) and (34), we have

$$d\bar{w} = 0.0011da_1^c - 0.0011da_1^t$$
$$d\Pi = 2.746 \times 10^4 da_1^c + 2.850 \times 10^4 da_1^t$$

Then by solving the following linear optimization problem:

$$\min_{da<0} d\bar{w} = 0.0011da_1^c - 0.0011da_1^t,\ \text{s.t.},\ 2.746 \times 10^4 da_1^c + 2.850 \times 10^4 da_1^t > -5\%\Pi$$

it is easy to obtain the best price adjustment $(da_1^c, da_1^t) = (-0.36, 0)$. At this new pricing strategy $(a_1^c, a_1^t) = (4.64, 5)$, the average customer waiting time is reduced as much as possible to 0.1097, while the platform's profit is reduced to $1.86 \times 10^5$.

## 6. Conclusion

With the rapid development and broad distribution of smartphones and wireless communication technologies, various taxi hailing apps have emerged within the taxi service. These apps, together with their underlying social networking infrastructures, constitute powerful e-hailing platforms that alleviate the information asymmetry inherent to taxi services, and facilitate taxi-customer matching. The result is a taxi market served by hybrid modes of roadside hailing and e-hailing. The pricing strategies of the e-hailing platform play a significant role in driving the overall taxi market equilibrium and determining taxi market performance.

To reveal the impacts of the pricing strategy of an e-hailing platform in taxi service, this paper incorporated theories of two-sided market into studies of the taxi industry, introduced matching functions to model the cross-group externalities between customers and taxi drivers on an e-hailing platform, and proposed a system of nonlinear equations to describe the taxi market equilibrium under hybrid modes of e-hailing and roadside hailing. Based on the proposed model, it is found that the existence of equilibrium solutions is guaranteed under any given pricing strategy, but the locally asymptotic stability of equilibrium solution only holds conditionally. So a price perturbation may turn the taxi system from a stable equilibrium into an unstable equilibrium. For any small perturbation of the platform's pricing strategy, a partial-derivative-based sensitivity analysis is conducted to quantify the variance of the taxi market equilibrium, the changes in the profitability of the e-hailing platform and the implications for social welfare. At any stable equilibrium, our results show that if the platform manager increases the platform's charge on one side while keeping that on the other side unchanged, then the e-hailing demand must be decreased; and if the platform increases its charge rate on one side while reducing the charge rate on the other side by the same amount, then e-hailing demand can either increase or decrease depending on the satisfaction of the provided conditions. Based on the sensitivity results, the features of desirable price perturbations that improve social welfare and/or platform's profitability are readily characterized.

Based on the models and results derived in this paper, there are many possible extensions. For example, as almost all taxi hailing platforms running with profit, whether they can eventually arrive at positive profits has been an important issue in reality. The sensitivity of e-hailing demand with respect to the platform's pricing strategy revealed in this paper is essential to the determination of platform's maximally achievable profit. The methodology adopted in this paper also provides a basis to examine the taxi market under multiple taxi hailing platforms. Given the fierce competition among taxi hailing apps in reality, revealing the taxi market performance in the presence of multiple e-hailing platforms is not only interesting in theory but also meaningful in practice. Furthermore, nowadays many taxi hailing apps, such as Uber, Lyft, and Didi Kuaidi, allow private cars to take e-hailing orders, which significantly impact the taxi industry. As this study focused on a taxi hailing app that worked as an information platform for taxi services only, our model and results cannot be directly applied to such e-hailing apps, but they still serve as a good basis for further investigations into the transportation market and the pricing strategies of e-hailing platforms that allow private cars to take e-hailing orders.

## Acknowledgements

The work described in this paper is supported by grants from National Natural Science Foundation of China under Projects #71401102, and Hong Kong's Research Grants Council under Project #T12140616.

## Appendix A

List of endogenous variables of the equilibrium model in Section 2:

| Endogenous variables | Notations |
|---|---|
| $D_i$ | Taxi demand of mode $i = 0, 1$ |
| $T_i^{vt}$ | Vacant taxi trips of mode $i = 0, 1$ |
| $T^{vt}$ | Total vacant taxi trips |
| $w_i^c$ | Customer waiting time of mode $i = 0, 1$ |
| $w_i^t$ | Taxi searching time of mode $i = 0, 1$ |
| $N_i^c$ | The number of waiting customers of mode $i = 0, 1$ at any time moment |
| $N_i^t$ | The number of searching taxis of mode $i = 0, 1$ at any time moment |
| $m_i$ | Meeting rate of mode $i = 0, 1$ |

List of exogenous variables of the equilibrium model in Section 2:

| Exogenous variables | Notations |
|---|---|
| $D$ | Total taxi demand |
| $N$ | Taxi fleet size |
| $a_1^c$ | The platform's charge on customers for each e-hailing order |
| $a_1^t$ | The platform's charge on taxis for each e-hailing order |
| $l$ | Average taxi ride time per trip |
| $r$ | The average reserved taxi hour, that is, the average taxi travel time from a responding taxi to its e-hailing customer |
| $\beta^c$ | Taxi customers' value of time |
| $\beta^t$ | Taxis' unit time operation cost |
| $\theta^c$ | A positive constant reflecting the degree of uncertainty in the customer waiting costs of the two modes |
| $\theta^t$ | A positive constant reflecting the degree of uncertainty in the taxi searching costs of the two modes |
| $\alpha_i^c$ | Constant elasticity of meeting rate with respect to the number of waiting customers of mode $i = 0, 1$ |
| $\alpha_i^t$ | Constant elasticity of meeting rate with respect to the number of available taxis of mode $i = 0, 1$ |
| $A_i$ | A positive model parameter in the meeting function of mode $i = 0, 1$ |

## Appendix B. Proof of Proposition 2

**Proof.** Let $D_1^*$ be a solution to the following fixed point problem (20). According to the theories of stability of difference equations, the equilibrium solution $D_1^*$ is locally asymptotically stable if $|f'(D_1^*)| < 1$, and unstable if $|f'(D_1^*)| > 1$ (Sydsater et al., 2005). In order to establish the proposition, we must first determine $f(D_1)$. Using Eq. (16), we have

$$d(w_0^t - w_1^t) = \frac{D}{\theta^t \beta^t D_0 D_1} dD_1 \tag{42}$$

Inserting Eq. (42) into Eq. (17) yields

$$dw_0^t = \left(-\frac{w_1^t - w_0^t + r}{D} + \frac{1}{\theta^t \beta^t D_0}\right) dD_1 \tag{43}$$

which further gives rise to

$$dw_1^t = -\left(\frac{w_1^t - w_0^t + r}{D} + \frac{1}{\beta^t \theta^t D_1}\right) dD_1 \tag{44}$$

Furthermore, by inserting Eqs. (43) and (44) into Eqs. (15) and (14) respectively, we have

$$dw_1^c = \frac{w_1^c}{\alpha_1^c}\left[\frac{1 - \alpha_1^t - \alpha_1^c}{D_1} + \frac{\alpha_1^t}{w_1^t}\left(\frac{w_1^t - w_0^t + r}{D} + \frac{1}{\beta^t \theta^t D_1}\right)\right] dD_1 \tag{45}$$

and

$$dw_0^c = -\frac{w_0^c}{\alpha_0^c}\left[\frac{1 - \alpha_0^c - \alpha_0^t}{D_0} + \frac{\alpha_0^t}{w_0^t}\left(\frac{1}{\beta^t \theta^t D_0} - \frac{w_1^t - w_0^t + r}{D}\right)\right] dD_1 \tag{46}$$

Additionally, Eq. (18) implies that

$$f'(D_1) = \frac{D}{1 + \exp\left[\theta^c\left(-\beta^c(w_0^c - w_1^c - r) + a_1^c\right)\right]} = \frac{\theta^c \beta^c D_1 D_0}{D} \frac{d(w_0^c - w_1^c)}{dD_1} \tag{47}$$

Therefore, by inserting Eqs. (45) and (46) into Eq. (47), we have

$$f'(D_1) = -\frac{\theta^c \beta^c Z D_1 D_0}{D}$$

where $Z = d(w_0^c - w_1^c)/dD_1$ is defined by Eq. (22). Apparently, $|f'(D_1)| < 1$ if Eq. (21) holds. This completes the proof. □

## Appendix C. Partial derivative-based sensitivity analysis

This appendix introduces how we derive the sensitivity results in Eqs. (23)–(27). Firstly, by taking derivative with respect to $a_1^c$ on both sides of Eqs. (14)–(18), we have

$$\frac{\partial D_1}{\partial a_1^c} = \frac{\alpha_1^c}{w_1^c}\left(D_1 \frac{\partial w_1^c}{\partial a_1^c} + w_1^c \frac{\partial D_1}{\partial a_1^c}\right) + \frac{\alpha_1^t}{w_1^t}\left(D_1 \frac{\partial w_1^t}{\partial a_1^c} + w_1^t \frac{\partial D_1}{\partial a_1^c}\right) \tag{48}$$

$$\frac{\partial D_0}{\partial a_1^c} = \frac{\alpha_0^c}{w_0^c}\left(D_0 \frac{\partial w_0^c}{\partial a_1^c} + w_0^c \frac{\partial D_0}{\partial a_1^c}\right) + \frac{\alpha_0^t}{w_0^t}\left(D_0 \frac{\partial w_0^t}{\partial a_1^c} + w_0^t \frac{\partial D_0}{\partial a_1^c}\right) \tag{49}$$

$$\frac{\partial(w_0^t - w_1^t)}{\partial a_1^c} = \frac{D}{\beta^t \theta^t D_1 D_0} \frac{\partial D_1}{\partial a_1^c} \quad \frac{\partial(w_0^t - w_1^t)}{\partial a_1^t} = \frac{D}{\beta^t \theta^t D_1 D_0} \frac{\partial D_1}{\partial a_1^t} + \frac{1}{\beta^t} \tag{50}$$

$$\frac{\partial D_0}{\partial a_1^c} w_0^t + \frac{\partial w_0^t}{\partial a_1^c} D_0 + \frac{\partial D_1}{\partial a_1^c}(w_1^t + r) + \frac{\partial w_1^t}{\partial a_1^c} D_1 = 0 \tag{51}$$

$$\frac{\partial(w_0^c - w_1^c)}{\partial a_1^c} = \frac{D}{D_1 D_0 \theta^c \beta^c} \frac{\partial D_1}{\partial a_1^c} + \frac{1}{\beta^c} \tag{52}$$

where $\alpha_i^c$ and $\alpha_i^t$ are defined in Eqs. (4) and (5). Substituting Eq. (50) into Eq. (51) yields

$$\frac{\partial w_0^t}{\partial a_1^c} = \left(\frac{1}{\beta^t \theta^t D_0} - \frac{w_1^t - w_0^t + r}{D}\right) \frac{\partial D_1}{\partial a_1^c} \tag{53}$$

and

$$\frac{\partial w_1^t}{\partial a_1^c} = -\left(\frac{1}{\beta^t \theta^t D_1} + \frac{w_1^t - w_0^t + r}{D}\right) \frac{\partial D_1}{\partial a_1^c} \tag{54}$$

Putting the above two equations back into Eqs. (48) and (49), we have

$$\frac{\partial w_1^c}{\partial a_1^c} = \frac{w_1^c}{D_1 \alpha_1^c}\left[(1-\alpha_1^c-\alpha_1^t) + \frac{\alpha_1^t}{w_1^t}\left(\frac{1}{\beta^t\theta^t} + \frac{D_1(w_1^t - w_0^t + r)}{D}\right)\right]\frac{\partial D_1}{\partial a_1^c} \tag{55}$$

$$\frac{\partial w_0^c}{\partial a_1^c} = -\frac{w_0^c}{\alpha_0^c D_0}\left[(1-\alpha_0^c-\alpha_0^t) + \frac{\alpha_0^t}{w_0^t}\left(\frac{1}{\beta^t\theta^t} - \frac{D_0(w_1^t - w_0^t + r)}{D}\right)\right]\frac{\partial D_1}{\partial a_1^c} \tag{56}$$

Substituting Eqs. (55) and (56) into Eq. (52) yields

$$\frac{\partial D_1}{\partial a_1^c} = -\frac{1}{B\beta^c} \tag{57}$$

where

$$\begin{aligned} B &= \frac{D}{D_1 D_0 \theta^c \beta^c} + \left[\frac{1-\alpha_0^c-\alpha_0^t}{D_0} + \left(\frac{1}{\beta^t\theta^t D_0} - \frac{(w_1^t + r - w_0^t)}{D}\right)\frac{\alpha_0^t}{w_0^t}\right]\frac{w_0^c}{\alpha_0^c} + \left[\frac{1-\alpha_1^c-\alpha_1^t}{D_1} + \left(\frac{1}{\beta^t\theta^t D_1} + \frac{(w_1^t + r - w_0^t)}{D}\right)\frac{\alpha_1^t}{w_1^t}\right]\frac{w_1^c}{\alpha_1^c} \\ &= -Z + \frac{D}{D_1 D_0 \theta^c \beta^c} \end{aligned}$$

By substituting Eq. (57) back into Eqs. (53)–(56), we readily obtain the partial derivatives of $w_1^c,\ w_i^t,\ i = 1, 2$ with respect to $a_1^c$.

Following exactly the same procedure as we did above, one can easily obtain the following partial derivatives of taxi market equilibrium with respect to $a_1^t$:

$$\frac{\partial w_0^t}{\partial a_1^t} = \frac{D_1}{D\beta^t} - \left(\frac{w_1^t - w_0^t + r}{D} - \frac{1}{\theta^t\beta^t D_0}\right)\frac{\partial D_1}{\partial a_1^t} \tag{58}$$

$$\frac{\partial w_1^t}{\partial a_1^t} = -\left(\frac{1}{\theta^t\beta^t D_1} + \frac{w_1^t - w_0^t + r}{D}\right)\frac{\partial D_1}{\partial a_1^t} - \frac{D_0}{D\beta^t} \tag{59}$$

$$\frac{\partial w_1^c}{\partial a_1^t} = \frac{w_1^c}{\alpha_1^c D_1}\left[(1-\alpha_1^c-\alpha_1^t) + \frac{\alpha_1^t}{w_1^t}\left(\frac{1}{\theta^t\beta^t} - \frac{D_1(w_0^t - w_1^t - r)}{D}\right)\right]\frac{\partial D_1}{\partial a_1^t} + \frac{w_1^c}{\alpha_1^c}\frac{\alpha_1^t}{w_1^t}\frac{D_0}{D\beta^t} \tag{60}$$

$$\frac{\partial w_0^c}{\partial a_1^t} = -\frac{w_0^c}{\alpha_0^c D_0}\left[(1-\alpha_0^c-\alpha_0^t) + \frac{\alpha_0^t}{w_0^t}\left(\frac{1}{\theta^t\beta^t} - \frac{D_0(w_1^t - w_0^t + r)}{D}\right)\right]\frac{\partial D_1}{\partial a_1^t} - \frac{w_0^c}{\alpha_0^c}\frac{\alpha_0^t}{w_0^t}\frac{D_1}{D\beta^t} \tag{61}$$

$$\frac{\partial D_1}{\partial a_1^t} = -\frac{D_1 x_0 + D_0 x_1}{DB\beta^t} \tag{62}$$

And combining the above partial derivatives, we can derive the following full derivative of $dD_1$, $dw_i^t$ and $dw_i^c,\ i = 0, 1$:

$$\begin{aligned} dD_1 &= -\tfrac{1}{B\beta^c}da_1^c - \tfrac{D_1x_0+D_0x_1}{DB\beta^t}da_1^t \\ dw_0^t &= \left(\tfrac{w_1^t-w_0^t+r}{D} - \tfrac{1}{\beta^t\theta^t D_0}\right)\left[\tfrac{1}{B\beta^c}da_1^c + \tfrac{D_1x_0+D_0x_1}{DB\beta^t}da_1^t\right] + \tfrac{D_1}{D\beta^t}da_1^t \\ dw_1^t &= \left(\tfrac{1}{\beta^t\theta^t D_1} + \tfrac{w_1^t-w_0^t+r}{D}\right)\left[\tfrac{1}{B\beta^c}da_1^c + \tfrac{D_1x_0+D_0x_1}{DB\beta^t}da_1^t\right] - \tfrac{D_0}{D\beta^t}da_1^t \\ dw_0^c &= \tfrac{w_0^c}{\alpha_0^c D_0}\left[1-\alpha_0^c-\alpha_0^t + \tfrac{\alpha_0^t}{w_0^t}\left(\tfrac{1}{\beta^t\theta^t} - \tfrac{D_0(w_1^t-w_0^t+r)}{D}\right)\right]\left(\tfrac{da_1^c}{B\beta^c} + \tfrac{D_1x_0+D_0x_1}{DB\beta^t}da_1^t\right) - \tfrac{w_0^c\alpha_0^t}{\alpha_0^c w_0^t}\tfrac{D_1}{D\beta^t}da_1^t \\ dw_1^c &= -\tfrac{w_1^c}{D_1\alpha_1^c}\left[1-\alpha_1^c-\alpha_1^t + \tfrac{\alpha_1^t}{w_1^t}\left(\tfrac{1}{\beta^t\theta^t} + \tfrac{D_1(w_1^t-w_0^t+r)}{D}\right)\right]\left(\tfrac{da_1^c}{B\beta^c} + \tfrac{D_1x_0+D_0x_1}{DB\beta^t}da_1^t\right) + \tfrac{w_1^c\alpha_1^t}{\alpha_1^c w_1^t}\tfrac{D_0}{D\beta^t}da_1^t \end{aligned}$$

## Appendix D. Proof of Lemma 1

**Proof.** Suppose $D_1 = 0$ at taxi market equilibrium. Then from Eq. (16), we have $w_1^t = +\infty$ and $(w_1^t - w_0^t)D_1 = 0$. Inserting $D_1 = 0$ and $(w_1^t - w_0^t)D_1 = 0$ into Eq. (17), (15) and (19) yields $D_0 = D$, $w_0^t = (N - Dl)D^{-1}$ and $w_0^c = (A_0)^{-\frac{1}{\alpha_0^c}}(D)^{\frac{1-\alpha_0^t-\alpha_0^c}{\alpha_0^c}}(w_0^t)^{-\frac{\alpha_0^t}{\alpha_0^c}}$. From Eq. (14), if $1-\alpha_1^c-\alpha_1^t \geqslant 0$, then $w_1^t = +\infty$ and $D_1 = 0$ imply that $w_1^c = 0$; if $1-\alpha_1^c-\alpha_1^t > 0$, $w_1^c(D_1)$ in Eq. (14) takes the form of $\infty/\infty$ at $D_1 = 0$. By L'Hospital rule, we have

$$\begin{aligned} \lim_{D_1\to 0} w_1^c(D_1) &= \lim_{D_1\to 0}(A_1)^{-\frac{1}{\alpha_1^c}}\frac{-\frac{\alpha_1^t}{\alpha_1^c}(w_1^t(D_1))^{-\frac{\alpha_1^t}{\alpha_1^c}-1}}{\frac{\alpha_1^t+\alpha_1^c-1}{\alpha_1^c}(D_1)^{\frac{\alpha_1^t-1}{\alpha_1^c}}}\frac{dw_1^t}{dD_1} \\ &= \lim_{D_1\to 0}\frac{\alpha_1^t w_1^c}{(\alpha_1^t+\alpha_1^c-1)w_1^t}\left(\frac{D_1(w_1^t-w_0^t)+D_1 r}{D} + \frac{1}{\beta^t\theta^t}\right) \\ &= \lim_{D_1\to 0} w_1^c(D_1)\cdot 0 \end{aligned}$$

where the second inequality can be derived based on Eq. (25). The above equation implies that $w_1^c(D_1) \to 0$ as $D_1 \to 0$ in the case of $1 - \alpha_1^c - \alpha_1^t > 0$. So in summary, for any type of returns to scale, Eq. (14) implies that $w_1^c \to 0$ if $D_1 \to 0$. However, from Eq. (18), if $D_1 = 0$ at equilibrium, then we must have $w_1^c = +\infty$. Such contradiction excludes the possibility of having $D_1 = 0$ at taxi market equilibrium.

Following a similar reasoning, it can be easily proven that $D_1 = D$ is not an equilibrium state as well. This completes the proof. □

## Appendix E

From Eq. (22), we have

$$\begin{aligned} dZ = &-\frac{dw_0^c}{\alpha_0^c}\left[\frac{1-\alpha_0^c-\alpha_0^t}{D_0}+\frac{\alpha_0^t}{w_0^t}\left(\frac{1}{\beta^t\theta^t D_0}-\frac{w_1^t-w_0^t+r}{D}\right)\right]-\frac{dw_1^c}{\alpha_1^c}\left[\frac{1-\alpha_1^t-\alpha_1^c}{D_1}+\frac{\alpha_1^t}{w_1^t}\left(\frac{w_1^t-w_0^t+r}{D}+\frac{1}{\beta^t\theta^t D_1}\right)\right] \\ &-\frac{w_0^c}{\alpha_0^c}\left[-\frac{1-\alpha_0^c-\alpha_0^t}{(D_0)^2}dD_0-\frac{\alpha_0^t}{(w_0^t)^2}\left(\frac{1}{\beta^t\theta^t D_0}-\frac{w_1^t-w_0^t+r}{D}\right)dw_0^t+\frac{\alpha_0^t}{w_0^t}\left(-\frac{dD_0}{\beta^t\theta^t(D_0)^2}-\frac{dw_1^t-dw_0^t+r}{D}\right)\right] \\ &-\frac{w_1^c}{\alpha_1^c}\left[-\frac{1-\alpha_1^t-\alpha_1^c}{(D_1)^2}dD_1-\frac{\alpha_1^t}{(w_1^t)^2}\left(\frac{w_1^t-w_0^t+r}{D}+\frac{1}{\beta^t\theta^t D_1}\right)dw_1^t+\frac{\alpha_1^t}{w_1^t}\left(\frac{dw_1^t-dw_0^t+r}{D}-\frac{dD_1}{\beta^t\theta^t(D_1)^2}\right)\right] \end{aligned}$$

where $dD_1$, $dw_i^c$ and $dw_i^t$, $i = 0, 1$ satisfy Eqs. (23)–(28).

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