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Improved approaches to the network design problem in regional hazardous waste management systems



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ABSTRACT

This paper investigates the network design problem arising from the regional hazardous waste management system. The problem is to identify the locations of various waste facilities, and determine the transportation routes of hazardous wastes and waste residues between those waste facilities. Aiming at minimizing jointly the total cost and total risk, the problem is formulated as a multi-objective mixed integer linear programming model. By exploiting the advantages of the model, three multi-objective optimization approaches are customized to find highly qualified non-dominated solutions. The effectiveness and efficiency of the approaches are examined both on a hypothetical case and a realistic case. © 2016 Elsevier Ltd. All rights reserved.

1. Introduction

In China, hazardous wastes are categorized as the solid and liquid wastes exhibiting at least one of the following five characteristics: corrosivity, toxicity, ignitability, reactivity and infectivity. The other solid and liquid wastes which possess unclear hazardous characteristics but present harmful effects on the environment and human health have to be managed as per hazardous wastes. Along with the industrial advancement in the past ten years, a large number of hazardous wastes diverging in types have been generated in the industrial production and manufacturing processes in China. According to environmental statistics annual report 2013 released by the Ministry of Environmental Protection of China, Chinese industries produced 31.57 million tons of hazardous wastes belonging to 49 types in 2013. Of this number of hazardous wastes, 53.9% is recycled and reused, 22.2% is treated and disposed, and 25.7% is stored. The top three hazardous waste types in terms of quantity are the asbestos waste mainly generated from the nonmetal mining industry, the acid waste mainly from the chemical manufacturing industry, and the caustic waste mainly from the paper manufacturing industry.

The biggest difference distinguishing the hazardous waste from the ordinary waste is its hazardous characteristic. If the hazardous waste is not properly managed, it will endanger the ecological environment and human health, and even hinder the sustainable development of the economy. Nowadays many developing countries (e.g. China, India, Turkey) in the world still take the industry as the main force to drive their economic development. As the country becomes more industrialized, the more hazardous wastes will be generated in the industrial processes. Hence, how to manage the hazardous waste in a safe and cost-effective manner is a significant problem in an industrialized country. The hazardous waste management mainly involves the collection, transportation, recycling, treatment and disposal processes. Each process involves complex

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http://dx.doi.org/10.1016/j.tre.2016.02.002 1366-5545/© 2016 Elsevier Ltd. All rights reserved. decisions and imposes immediate or long-term risk to the surrounding environment and population. Meanwhile, the decisions of all processes relate with each other and determine the quality and safety of the management of hazardous wastes simultaneously.

In this paper, we investigate the network design problem arising from the regional hazardous waste management system. The framework of the system is a multi-stage multi-facility logistics network as illustrated in Fig. 1. Our problem starts with the waste origins that produce a number of and multiple types of hazardous wastes at every certain time (e.g. a month, half a year, or 1 year). Each type of hazardous waste at each origin is further classified as the recyclable, treatable and disposable portions. Recyclable wastes are transported directly to recycling centers to be recycled and reused. Treatable wastes have to be transported firstly to treatment centers where their potential risk is reduced. Disposable wastes are transported directly to disposal centers with security landfills. Then, each treatment center can be equipped with multiple types of technology, such as solidification and incineration, see LaGrega et al. (2010). However, each type of hazardous waste can only be treated with certain technologies. For example, highly reactive wastes cannot be incinerated. Therefore, treatment centers utilize comparable technologies to treat hazardous wastes as waste residues. Waste residues are also classified as the recyclable and disposable portions which are transported directly to recycling centers are transported to disposal centers such that the management processes of hazardous wastes are completed ultimately.

We define the regional hazardous waste network design problem to identify the locations of recycling centers, treatment centers and disposal centers from their respective candidate sites, and determine the transportation routes of hazardous wastes and waste residues from waste origins to those centers and between those centers, while all corresponding operational and safety constraints are respected. The problem is inherently a multi-objective optimization problem as the solution to the problem should optimize multiple objectives to reflect different perspectives of the stakeholder. In our setting, the environmental protection department of the regional government takes full responsibility for the management of hazardous wastes in the region. The government firstly designs the regional hazardous waste management system, and then attracts private companies to (co-)construct and (co-)operate hazardous waste facilities. Accordingly, on one hand, given the compelling obligation of the government to build a harmonious and livable society, the best solution should impose the least risk to the environment while the economic cost is controlled as low as possible. On the other hand, to encourage private companies to join the hazardous waste market so as to alleviate the government financial burden in the management of hazardous wastes, the best solution should require the least economic cost while the environmental protection regulations imposed by the national standards are satisfied strictly. Hence, it is necessary to consider a multi-objective network design problem to obtain an implementable regional hazardous waste management system.

We would like to call our problem the network design problem (Nema and Gupta, 1999) rather than the location–routing problem as by many related works, i.e. Zografros and Samara (1989), Alumur and Kara (2007), etc., since our problem differs actually from the classical logistics location–routing problem (Nagy and Salhi, 2007) in that the vehicle routing decision is not considered explicitly in our setting. Our problem has received great attentions during the last twenty-five years. A lot of early works on the hazardous waste management considered simplified network design problems. Some researches focused only on the locations of undesirable hazardous waste facilities (e.g. incinerators in treatment centers, landfills in disposal centers), in which the transportation routes between facilities given that the locations of facilities emphasized only the transportation sof hazardous waste facilities significantly affect the transportation cost and risk between facilities. Therefore, the location and transportation decisions of the network design problem in the hazardous waste management should be optimized simultaneously. In what follows, we restrict our attention to the early works focusing on complete hazardous waste network design problems. The related works can be characterized by the framework of the management system,



Fig. 1. Framework of the regional hazardous waste management system.

the compositions of the objective, the components of the constraint, and the multi-objective optimization algorithms used to solve the problem, leading to different hazardous waste network design approaches.

Zografros and Samara (1989) was the first work to address the network design problem with single type of hazardous waste. They proposed a goal programming model with three objectives including the travel time, transportation risk and location risk to determine the locations of disposal facilities and the routes between waste origins and disposal facilities. Later, many researchers investigated similar problems as in Zografros and Samara (1989), differing mainly in the objective composition and solution algorithm. ReVelle et al. (1991) studied the problem of locating storage facilities and routing spent fuel rods from nuclear reactors. They formulated a zero-one programming model to minimize a convex combination of the transportation cost and transportation risk. Stowers and Palekar (1993) developed a continuous location model to an obnoxious facilities (e.g. landfills) cannot be located arbitrarily from an environmental perspective. Wyman and Kuby (1995) presented a location–allocation model with an additional decision of treatment technologies. Their model minimized the total cost, total risk and location equity, which was solved by a weighted-sum algorithm and a constraint algorithm, respectively. Boyer et al. (2013) provided a model to minimize the total cost and transportation risk over a complex management system incorporating recycling, treatment and disposal facilities. In their paper, efficient solutions were obtained by a weighted-sum algorithm.

Jacobs and Warmerdam (1994), Current and Ratick (1995), Giannikos (1998), and Cappanera et al. (2004) considered almost the same class of hazardous waste network design problem with single type of waste facility. They suggested a continuous network flow model to optimize the routing of hazardous wastes from generation nodes to located waste facilities. Jacobs and Warmerdam (1994) formulated a multi-objective model to minimize the total cost and total risk simultaneously, and they solved their model using a constraint approach. Current and Ratick (1995) further incorporated the transportation equity and location equity into their model which was solved by a weighted-sum approach. Giannikos (1998) exhibited a goal programming model to minimize four objectives including the total cost, transportation risk, transportation equity and location equity. Cappanera et al. (2004) utilized the Lagrangian relaxation and branch and bound approaches to obtain an optimal solution with minimal total cost. Zhao and Verter (2015) established a model with vehicle routing considerations to configure the used-oil management system including storage and integrated facilities. In this research, the model minimized jointly the total economic cost and total environmental risk. A goal programming approach was developed to explore efficient solutions.

List and Mirchandani (1991) firstly investigated the hazardous waste network design problem with multiple types of hazardous waste and multiple types of treatment technology. They exploited a path-based model to identify the locations and technologies of treatment facilities and determine the routes of hazardous materials and wastes to these facilities. Their model considered jointly the minimal transportation cost, total risk and total equity. The efficient solutions were found by utilizing a weighted-sum approach. Later, many efforts contributed to the topic initialized in List and Mirchandani (1991) by either extending the management system framework or imposing more realistic constraints. Alidi (1992) presented a general-purpose goal programming model with one objective to minimize the total cost for a hazardous waste management system with recycling, treatment and disposal facilities. Alidi (1996) specialized the goal programming model developed in Alidi (1992) to the hazardous waste network design problem in the petrochemical industry. Emek and Kara (2007) studied the problem of locating incinerators in a hazardous waste management system with recycling and treatment facilities. They explained an additional constraint of the air pollution standards imposed by the government regulations. They further formulated an integer programming model to minimize the transportation cost.

Nema and Gupta (1999, 2003), and Alumur and Kara (2007) addressed the problem of efficiently designing a hazardous waste management system with treatment and disposal facilities. Nema and Gupta (1999) firstly introduced the waste-waste and waste-technology comparability requirements. They formulated a continuous network flow model to minimize a composite utility function with respect to the total cost and total risk. Nema and Gupta (2003) reformulated the model in Nema and Gupta (1999) as a goal programming model to overcome the difficulty in determining weights in the utility function. Alumur and Kara (2007) firstly modelled linear constraints for the waste-technology comparability requirements introduced in Nema and Gupta (1999). They provided a multi-objective model with the total cost and transportation risk as objectives. Samanlioglu (2013) and Ghezavati and Morakabatchian (2015) both described the hazardous waste network design problem in a complex management system with recycling, treatment and disposal facilities. Samanlioglu (2013) formulated a location–allocation model to minimize simultaneously the total cost, transportation risk and location risk. A lexicographic weighted Tchebycheff approach was developed to search efficient solutions. Ghezavati and Morakabatchian (2015) further separated the generation nodes into true waste origins and actual collection centers. They extended the model in Samanlioglu (2013) by adding vehicle routing decisions with fuzzy service level requirements from waste origins to collection centers.

Previous works on the hazardous waste network design problem are compared in Table 1 in terms of the management system framework, objective composition, constraint component, and solution algorithm. Firstly, as can be seen from Table 1, most of the previous works describe a simplified hazardous waste management system compared with that in Fig. 1, in which either some types of critical facility are not located or some bunches of waste and residue flows are not routed. Secondly, most of the early efforts consider only the capacity of the hazardous waste facilities (node capacity), but ignore the capacity of the transportation links between those facilities (link capacity). It can be visualized that some bottleneck links without capacity limitations may suffer such great risk that their surrounding people are not willing to accept. We remark

Table 1

Literature review on the hazardous waste network design problem.

Author (year)	System framework ^a	Objective composition ^b	Constraint component ^c	Solution algorithm ^d
Zografros and Samara (1989)	G+D, SW+ST	Min: TC, TR, LR	LV+PR	GP
ReVelle et al. (1991)	G+D, SW+ST	Min: TC+TR	PR	IP
Stowers and Palekar (1993)	G+D, SW+ST	Min: TR+LR	_	AA
Jacobs and Warmerdam (1994)	G+D, SW+ST, RO	Min: TC+LC, TR+LR	NV	CA
Wyman and Kuby (1995)	G+D, SW+MT	Min: TC+LC, TR+LR, LE	NV	WS+CA
Current and Ratick (1995)	G+D, SW+ST, RO	Min: TC+LC, TR, LR, TE, LE	NV	WS
Giannikos (1998)	G+D, SW+ST, RO	Min: TC+LC, TR, TE, LE	NV	GP
Cappanera et al. (2004)	G+D, SW+ST, RO	Min: TC+LC	NV+NR	IP
Boyer et al. (2013)	G+R+T+D, SW+ST	Min: TC+LC, TR	NV	WS
Zhao and Verter (2015)	G+T+D, SW+ST, RO	Min: TC+LC, TR+LR	NV	GP
List and Mirchandani (1991)	G+D, MW+MT, RO	Min: TC, TR+LR, TE+LE	PR	WS
Alidi (1992)	G+R+T+D, MW+ST	Min: TC+LC	NV+LV+PR	GP
Alidi (1996)	G+R+T+D, MW+ST	Min: TC+LC	NV+LV	GP
Nema and Gupta (1999)	G+T+D, MW+MT, RO	Min: TC+LC, TR+LR	NV+CR	WS
Nema and Gupta (2003)	G+T+D, MW+MT, RO	Min: TC+LC, TR+LR	NV+CR	GP
Alumur and Kara (2007)	G+T+D, MW+MT, RO	Min: TC+LC, TR	NV+CR	WS
Emek and Kara (2007)	G+R+D, MW+ST	Min: TC	NV+NR+PR	IP
Samanlioglu (2013)	G+R+T+D, MW+MT	Min: TC+LC, TR, LR	NV+CR	ТА
Ghezavati and Morakabatchian (2015)	G+R+T+D, MW+MT	Min: TC+LC, TR, LR	NV+CR	WS

^a G(R, T, D): Generation (Recycling, Treatment, Disposal) facility, S(M)W: Single (Multiple) waste type(s), S(M)T: Single (Multiple) technology type(s), RO: Route optimization.

^o TC(R, E): Transportation cost (risk, equity), LC(R, E): Location cost (risk, equity).

^c NV(R): Node volume (risk) capacity, LV: Link volume capacity, PR: P-value restriction, CR: Comparability requirement.

^d AA: Analytical approach, IP: Integer programming, GP: Goal programming, WS: Weighted-sum approach, TA: Tchebycheff approach, CA: Constraint approach.

that the link capacity is a valid constraint from a social equity perspective. Thirdly, many former studies formulate the hazardous waste network design problem as a location–allocation model by directly transporting wastes and residues through the shortest routes. The model can simplify the problem without losing the optimality if the problem is uncapacitated or only node capacitated. However, it cannot guarantee an optimal or even a feasible solution in case the problem is link capacitated. Moreover, the multi-objective hazardous waste network design problem should be solved by finding highly qualified efficient solutions. Many initial attempts simply utilize customized multi-objective optimization algorithms (e.g. weighted-sum approach) and just illustrate their algorithms can identify representative efficient solutions. No initial attempts do evaluate or compare the quality of the efficient solutions of their algorithms.

In view of the shortages of previous works, we consider a new and complicated multi-objective network design problem arising from a comprehensive regional hazardous waste management system as shown in Fig. 1 with multiple types of hazardous waste, waste facility and treatment technology. Meanwhile, we reflect simultaneously many real-life constraints in the hazardous waste management including the node capacity, link capacity and comparability requirement. As far as we know, compared with the previous works in Table 1, we may be addressing the most challenge hazardous waste network design problem. The objectives of this paper are to develop more general models and customize more efficient algorithms to the hazardous waste network design problem using advanced multi-objective optimization approaches. Specifically, we try to formulate the problem as a general and compact multi-objective optimization model incorporating the facility location, waste allocation and waste routing decisions simultaneously. Meanwhile, we expect to theoretically customize and numerically compare several existing general-purpose multi-objective optimization approaches to find representative efficient solutions with high quality.

The contributions of our paper to the literature are mainly twofold. Firstly, we formulate the problem as a large-scale multi-objective mixed integer linear programming model to minimize jointly the total cost and total risk in the transportation and location processes. The model is quite general in the sense that it can capture all decisions and constraints, and regard many other models proposed in the literature as special cases. The model is also linear and compact so that its single objective version can efficiently be solved for large-scale problem instances within reasonable computation times by invoking state-of-the-art commercial optimization software. Secondly, we customize three existing multi-objective integer programming approaches including a weighted-sum approach, an augmented weighted Tchebycheff approach and an augmented ε -constraint approach to explore highly qualified efficient solutions. To the best of our knowledge, the latter two approaches, even widely recognized as good multi-objective integer programming algorithms, are firstly applied to the hazardous waste network design problem. We further reveal that the weighted-sum approach adopted by many previous works in the literature is actually not a good alternative algorithm to our problem.

The rest of the paper is organized as follows. Section 2 presents a detailed problem description, and specifies the main problem assumptions. In Section 3, we formulate our problem as a large-scale multi-objective mixed integer programming model with all decisions, objectives and constraints. After a brief review on the multi-objective optimization theory, Section 4

customizes a weighted-sum approach, an augmented weighted Tchebycheff approach and an augmented ε -constraint approach to identify competitive efficient solutions, respectively. In Section 5, a small-scale hypothetical case collected from the literature and a large-scale realistic case constructed based on the Sichuan province in the Southwest region of China are used to test the proposed multi-objective approaches, and also to evaluate the effect of critical parameters on our problem. Finally, Section 6 concludes the main research works in this paper and provides future research directions.

2. Problem description

The regional hazardous waste network design problem is defined on a transportation network consisting of nodes and links. The nodes of the network are classified into five parts representing the generation nodes, candidate recycling centers, candidate treatment centers, candidate disposal centers and other intermediate nodes, respectively. A generation node, the place where hazardous wastes are produced or collected, is generally a factory, a hospital or a collection center. A recycling (treatment or disposal) center is the place to recycle (treat or dispose) hazardous wastes and waste residues. The recycling centers, treatment centers and disposal centers can only be discretely located at prescribed candidate nodes. Without loss of generality, the generation node, recycling center, treatment center and disposal center can be available at the same node. A link connecting two adjacent nodes in the network represents the physical road which can be used to transport hazardous wastes and waste residues between these two nodes. A lot of population areas (e.g. city, town and village) are distributed randomly in the transportation network, which simultaneously suffer risk from the processes of transporting hazardous wastes and waste residues on links and locating recycling centers, treatment centers and disposal centers at nodes.

The regional hazardous waste network design problem to be investigated in this paper is formally described as follows. Given the transportation network (nodes, links and link attributes), hazardous wastes (origins, types, amounts and compositions) and candidate facilities (locations, capacities and attributes), the regional hazardous waste network design problem mainly answers the following questions: where to locate hazardous waste recycling centers, where to locate hazardous waste treatment centers with which types of treatment technology, where to locate hazardous waste disposal centers, how to route recyclable (treatable and disposable) hazardous wastes from generation nodes to recycling (treatment and disposal) centers, how to route recyclable (disposable) waste residues from treatment centers to recycling (disposal) centers, and how to route disposable waste residues from recycling centers. The problem aims at satisfying all corresponding operational and safety constraints such that the total cost and total risk in the transportation and location processes can be minimized simultaneously.

We optimize a multi-objective optimization problem to achieve implementable solutions. The problem has two objectives to minimize the total cost and total risk. The total cost is composed of the total transportation cost and total location cost. The total transportation cost includes the transportation cost of three bunches of hazardous waste flows and three bunches of waste residue flows. The total location cost contains the fixed cost of locating and the variable cost of operating recycling, treatment and disposal centers. The total risk is also incurred simultaneously by the transportation and location processes. The total transportation risk is calculated by the same way to count the total transportation cost, namely, it computes the transportation risk of the same six waste and residue flows. The total location risk differs slightly from the total location cost in that only the variable risk of operating recycling, treatment and disposal centers are measured.

In order to design a cost-effective and environmental-friendly regional hazardous waste management system, we mainly consider four kinds of constraints containing the mass balance, node capacity, link capacity and comparability requirement constraint. Firstly, the mass balance constraint is introduced to guarantee that all hazardous wastes and waste residues are recycled, treated and disposed at proper centers. Secondly, the node capacity constraint ensures that each candidate center can only be located and operated if its minimal waste (residue) amount requirement and maximum waste (residue) amount capacity are met simultaneously. Thirdly, the link capacity constraint is modelled from an environmental perspective requiring that the transportation risk involved on each link should not exceed a prescribed maximal risk tolerance capacity. Moreover, the waste-technology comparability requirement constraint specifies that each hazardous waste can only be treated at proper treatment centers with comparable technologies.

Designing a regional hazardous waste management system is not a trivial task, since several aspects such as the economic, environmental and social aspects should be incorporated together, and each aspect could be estimated by many measurement methods. In this paper, we try to provide an optimization-based decision-making framework from a strategic decision perspective so that many tactical and operational issues are not addressed thoroughly. To facilitate the model formulation, the following assumptions are introduced.

- (1) The region to be designed a hazardous waste management system can be described by a transportation network consisting of generation nodes, candidate recycling (treatment and disposal) centers, intermediate nodes and transportation links.
- (2) The cost of transporting one unit of hazardous waste or waste residue is linearly correlated to the network distance traversed. The fixed cost of locating and variable cost of operating a recycling (treatment or disposal) center are independent of and linearly dependent on the workload at the center, respectively.

- (3) The link transportation risk and node location risk both satisfy the linearity and additivity properties. They are linearly dependent on the link traffic volume and node workload, respectively.
- (4) The detailed transportation processes on links and the detailed recycling, treatment and disposal processes at nodes of hazardous wastes and waste residues are not modelled explicitly.

3. Mathematical model

We propose a multi-objective continuous network flow model with all decisions, objectives and constraints to the regional hazardous waste network design problem. The sets, indices, parameters and decision variables to be used in the mathematical model are explained below. Furthermore, the decision variables in the mathematical model are graphically depicted in Fig. 2. Note that our problem has a large number of parameters and variables. To ease the understanding of notations, each parameter and variable are defined as a single letter with two subscripts separated by comma. Besides, similar parameters or variables are introduced by the same single letter but with different subscripts. Specifically, for each notation, the first subscript before the comma indicates the waste facility (facilities) to which it corresponds, where capital letters H, R, T and D represent the generation, recycling, treatment and disposal node, respectively. Meanwhile, the second subscript after the comma expresses the index (indices) of the notation.

Sets:

 $\mathcal{N}(\mathcal{V}, \mathcal{A})$: regional hazardous waste transportation network, where \mathcal{V} is the set of nodes, and \mathcal{A} the set of links.

 $\mathcal{R} \subseteq \mathcal{V}$: set of candidate recycling centers indexed by *r*.

 $\mathcal{T} \subseteq \mathcal{V}$: set of candidate treatment centers indexed by *t*.

 $\mathcal{D} \subseteq \mathcal{V}$: set of candidate disposal centers indexed by *d*.

 $\mathcal{O} \subseteq \mathcal{V}$: set of intermediate nodes without indices.

 \mathcal{L} : set of treatment technologies indexed by l.

W: set of hazardous waste types indexed by w. It is classified by certain property, such as hazardous characteristic, treatment technology, and industrial distribution.

 \mathcal{H} : set of hazardous wastes indexed by h. Each generation node produces multiple types of hazardous waste. However, any $h \in \mathcal{H}$ represents only one type of hazardous waste produced by one generation node.

Parameters:

 $c_{\text{HR,wij}}$: cost of transporting one unit of recyclable hazardous waste type $w \in W$ from generation nodes on link $(i, j) \in A$. $c_{\text{HT,wij}}$: cost of transporting one unit of treatable hazardous waste type $w \in W$ from generation nodes on link $(i, j) \in A$. $c_{\text{HD,wij}}$: cost of transporting one unit of disposable hazardous waste type $w \in W$ from generation nodes on link $(i, j) \in A$. $c_{\text{TR,ij}}$: cost of transporting one unit of recyclable waste residue from treatment centers on link $(i, j) \in A$.

 $c_{\text{TD},ij}$: cost of transporting one unit of disposable waste residue from treatment centers on link $(i,j) \in A$.

 $c_{RD,ij}$: cost of transporting one unit of disposable waste residue from recycling centers on link $(i,j) \in A$.

 $f_{R,r}$: fixed cost of locating a recycling center at candidate node $r \in \mathcal{R}$.

 $f_{T,tl}$: fixed cost of locating a treatment center at candidate node $t \in \mathcal{T}$ with technology $l \in \mathcal{L}$.

 $f_{D,d}$: fixed cost of locating a disposal center at candidate node $d \in D$.

 $v_{R,r}$: variable cost of operating a recycling center at candidate node $r \in \mathcal{R}$.

 $v_{T,t}$: variable cost of operating a treatment center at candidate node $t \in \mathcal{T}$ with technology $l \in \mathcal{L}$.

 $v_{\text{D},d}$: variable cost of operating a disposal center at candidate node $d \in \mathcal{D}$.



Fig. 2. Illustration of decision variables in the mathematical model.

 $p_{\text{HR,wij}}$: risk of transporting one unit of recyclable hazardous waste type $w \in W$ from generation nodes on link $(i,j) \in A$. $p_{\text{HT,wij}}$: risk of transporting one unit of treatable hazardous waste type $w \in W$ from generation nodes on link $(i,j) \in A$. $p_{\text{HD wii}}$: risk of transporting one unit of disposable hazardous waste type $w \in W$ from generation nodes on link $(i, j) \in A$. $p_{\text{TR},ij}$ risk of transporting one unit of recyclable waste residue from treatment centers on link $(i,j) \in A$. $p_{\text{TD},ii}$: risk of transporting one unit of disposable waste residue from treatment centers on link $(i,j) \in A$. $p_{\text{RD},ii}$: risk of transporting one unit of disposable waste residue from recycling centers on link $(i,j) \in A$. s_{Rr} : variable risk of operating a recycling center at candidate node $r \in \mathcal{R}$. $s_{T,tl}$: variable risk of operating a treatment center at candidate node $t \in \mathcal{T}$ with technology $l \in \mathcal{L}$. $s_{\text{D},d}$: variable risk of operating a disposal center at candidate node $d \in \mathcal{D}$. $o_{H,h}$: origin of hazardous waste $h \in \mathcal{H}$. $\tau_{\text{H},h}$: type of hazardous waste $h \in \mathcal{H}$. $q_{\text{H},h}$: amount of hazardous waste $h \in \mathcal{H}$. $\alpha_{\mathrm{H},h}$: recyclable percentage of hazardous waste $h \in \mathcal{H}$. $\beta_{\text{H},h}$: treatable percentage of hazardous waste $h \in \mathcal{H}$. $\gamma_{\mathrm{H},h}$: disposable percentage of hazardous waste $h \in \mathcal{H}$. $\eta_{T,by}$: mass variation percentage of hazardous waste type $w \in W$ treated with technology $l \in \mathcal{L}$. $\kappa_{T,lw}$: recyclable percentage of hazardous waste type $w \in W$ treated with technology $l \in \mathcal{L}$. $\sigma_{T,lw}$: disposable percentage of hazardous waste type $w \in W$ treated with technology $l \in \mathcal{L}$. $\delta_{\text{T},lw}$: 1 if hazardous waste type $w \in \mathcal{W}$ can be treated with technology $l \in \mathcal{L}$; 0 otherwise. $\lambda_{\mathbf{R},\mathbf{r}}$: average recycling percentage at candidate node $\mathbf{r} \in \mathcal{R}$. $a_{R,r}$: minimum waste amount requirement of locating a recycling center at candidate node $r \in \mathcal{R}$. $a_{r,t}$: minimum waste amount requirement of locating a treatment center at candidate node $t \in T$ with technology $l \in \mathcal{L}$. $a_{\text{D},d}$: minimum waste amount requirement of locating a disposal center at candidate node $d \in \mathcal{D}$. $b_{\text{B}r}$: maximum waste amount capacity of locating a recycling center at candidate node $r \in \mathcal{R}$. $b_{T,t}$: maximum waste amount capacity of locating a treatment center at candidate node $t \in \mathcal{T}$ with technology $l \in \mathcal{L}$. $b_{D,d}$: maximum waste amount capacity of locating a disposal center at candidate node $d \in D$. $b_{A,ii}$: maximum allowable risk tolerance capacity of link $(i, i) \in A$. $\theta_{i\nu}$: 1 if node *i* belongs to set \mathcal{V} (i.e. $i \in \mathcal{V}$); 0 otherwise. ϑ_{ii} : 1 if node *i* is equal to node *j*; 0 otherwise. Decision variables: $x_{\text{HR},hii}$: amount of recyclable hazardous wastes $h \in \mathcal{H}$ transported through link $(i, j) \in \mathcal{A}$. $x_{\text{HT}hii}$: amount of treatable hazardous wastes $h \in \mathcal{H}$ transported through link $(i, j) \in \mathcal{A}$. $x_{\text{HD,hij}}$: amount of disposable hazardous wastes $h \in \mathcal{H}$ transported through link $(i, j) \in \mathcal{A}$. $x_{\text{TR,tij}}$: amount of recyclable waste residues in candidate treatment center $t \in \mathcal{T}$ transported through link $(i, j) \in \mathcal{A}$. $x_{\text{TD,tii}}$: amount of disposable waste residues in candidate treatment center $t \in \mathcal{T}$ transported through link $(i, j) \in \mathcal{A}$. $x_{\text{RD,rij}}$: amount of disposable waste residues in candidate recycling center $r \in \mathcal{R}$ transported through link $(i,j) \in \mathcal{A}$. $y_{\text{HR} hi}$: amount of recyclable hazardous wastes $h \in \mathcal{H}$ recycled at node $i \in \mathcal{V}$.

 $y_{\text{HT,hil}}$: amount of treatable hazardous wastes $h \in \mathcal{H}$ treated at node $i \in \mathcal{V}$ with technology $l \in \mathcal{L}$.

 $y_{\text{HD},hi}$: amount of disposable hazardous wastes $h \in \mathcal{H}$ disposed at node $i \in \mathcal{V}$.

 $y_{\text{TR,}i}$: amount of recyclable waste residues in candidate treatment center $t \in \mathcal{T}$ recycled at node $i \in \mathcal{V}$.

 $y_{\text{TD},ii}$: amount of disposable waste residues in candidate treatment center $t \in \mathcal{T}$ disposed at node $i \in \mathcal{V}$.

 $y_{RD,ri}$: amount of disposable waste residues in candidate recycling center $r \in \mathcal{R}$ disposed at node $i \in \mathcal{V}$.

 $y_{R,r}$: amount of hazardous wastes (waste residues) recycled at candidate node $r \in \mathcal{R}$.

 $y_{T,thw}$: amount of hazardous waste type $w \in W$ treated at candidate node $t \in T$ with technology $l \in \mathcal{L}$.

 $y_{T,tl}$: amount of hazardous wastes treated at candidate node $t \in T$ with technology $l \in \mathcal{L}$.

 $y_{\text{D},d}$: amount of hazardous wastes (waste residues) disposed at candidate node $d \in \mathcal{D}$.

 $z_{R,r}$: 1 if a recycling center is located at candidate node $r \in \mathcal{R}$; 0 otherwise.

 $z_{T,tl}$: 1 if a treatment center is located at candidate node $t \in T$ with technology $l \in \mathcal{L}$; 0 otherwise.

 $z_{D,d}$: 1 if a disposal center is located at candidate node $d \in D$; 0 otherwise.

With the above notations, the regional hazardous waste network design problem (O) can be formulated as the following multi-objective mathematical optimization model:

$$\begin{split} \min : & \sum_{V' \in \{R,T,D\}} \sum_{h \in \mathcal{H}(i,j) \in \mathcal{A}} c_{HV,\tau_{H,h}jj} X_{HV',hij} + \sum_{V' \in \{R,D\}} \sum_{t \in \mathcal{T}} \sum_{(i,j) \in \mathcal{A}} c_{TV',ij} x_{TV',ij} + \sum_{r \in \mathcal{R}(i,j) \in \mathcal{A}} c_{RD,ij} x_{RD,rij} + \sum_{r \in \mathcal{R}} (f_{R,r} Z_{R,r} + \nu_{R,r} y_{R,r}) \\ & + \sum_{t \in \mathcal{T}} \sum_{l \in \mathcal{L}} (f_{T,tl} Z_{T,tl} + \nu_{T,tl} y_{T,tl}) + \sum_{d \in \mathcal{D}} (f_{D,d} Z_{D,d} + \nu_{D,d} y_{D,d}) \\ \min : & \sum_{V' \in \{R,T,D\}} \sum_{h \in \mathcal{H}(i,j) \in \mathcal{A}} p_{HV',\tau_{H,h}jj} x_{HV',hij} + \sum_{V' \in \{R,D\}} \sum_{t \in \mathcal{T}} \sum_{(i,j) \in \mathcal{A}} p_{TV',ij} x_{TV',ij} + \sum_{r \in \mathcal{R}} \sum_{(i,j) \in \mathcal{A}} p_{RD,ij} x_{RD,rij} + \sum_{r \in \mathcal{R}} s_{R,r} y_{R,r} + \sum_{t \in \mathcal{T}} \sum_{l \in \mathcal{L}} s_{T,tl} y_{T,tl} \\ & + \sum_{d \in \mathcal{D}} s_{D,d} y_{D,d} \\ \text{s.t.} \\ & \sum_{r \in \mathcal{R}} y_{HR,hr} = \alpha_{H,h} q_{H,h} \quad \forall h \in \mathcal{H} \\ & (1) \sum_{i,j \in \mathcal{A}} x_{HR,hij} - \sum_{(i,j) \in \mathcal{A}} x_{HR,hji} + \theta_{iR} y_{HR,hi} = \vartheta_{i_{0H,h}} \alpha_{H,h} q_{H,h} \quad \forall h \in \mathcal{H}, \quad i \in \mathcal{V} \\ & (1) \sum_{i \in \mathcal{L}} y_{HT,htl} = \beta_{H,h} q_{H,h} \quad \forall h \in \mathcal{H} \\ & \sum_{i \in \mathcal{D}} y_{HD,hd} = \gamma_{H,h} q_{H,h} \quad \forall h \in \mathcal{H} \\ & \sum_{i \in \mathcal{D}} y_{HD,hd} = \gamma_{H,h} q_{H,h} \quad \forall h \in \mathcal{H} \\ & \sum_{i \in \mathcal{D}} y_{HD,hd} = \gamma_{H,h} q_{H,h} \quad \forall h \in \mathcal{H} \\ & (2) \sum_{i \in \mathcal{D}} x_{HD,hji} + \theta_{iD} y_{HD,hi} = \vartheta_{i_{0H,h}} \beta_{H,h} q_{H,h} \quad \forall h \in \mathcal{H}, \quad i \in \mathcal{V} \\ & \sum_{i \in \mathcal{D}} x_{HD,hji} - \sum_{i \in \mathcal{A}} x_{HD,hji} + \theta_{iD} y_{HD,hi} = \vartheta_{i_{0H,h}} \beta_{H,h} q_{H,h} \quad \forall h \in \mathcal{H}, \quad i \in \mathcal{V} \\ & \sum_{i \in \mathcal{D}} x_{HD,hij} - \sum_{i \in \mathcal{A}} x_{HD,hji} + \theta_{iD} y_{HD,hi} = \vartheta_{i_{0H,h}} \gamma_{H,h} q_{H,h} \quad \forall h \in \mathcal{H}, \quad i \in \mathcal{V} \\ & \sum_{i \in \mathcal{D}} x_{HD,hij} - \sum_{i \in \mathcal{A}} x_{HD,hji} + \theta_{iD} y_{HD,hi} = \vartheta_{i_{0H,h}} \gamma_{H,h} q_{H,h} \quad \forall h \in \mathcal{H}, \quad i \in \mathcal{V} \\ & \sum_{i \in \mathcal{A}} y_{HT,hil} = y_{T,ibw} \quad \forall t \in \mathcal{T}, \quad l \in \mathcal{L}, \quad w \in \mathcal{W} \\ & \sum_{i \in \mathcal{A}} y_{HT,hil} = y_{T,ibw} \quad \forall t \in \mathcal{T}, \quad l \in \mathcal{L}, \quad w \in \mathcal{W} \end{aligned}$$

$$\sum_{w \in \mathcal{W}} y_{\mathrm{T}, l w} = y_{\mathrm{T}, l l} \quad \forall t \in \mathcal{T}, \quad l \in \mathcal{L}$$

$$\tag{10}$$

$$\sum_{r\in\mathcal{R}}^{N} y_{\mathrm{TR},tr} = \sum_{l\in\mathcal{L}} \sum_{w\in\mathcal{W}} \eta_{\mathrm{T},lw} \kappa_{\mathrm{T},lw} y_{\mathrm{T},tlw} \quad \forall t\in\mathcal{T}$$

$$\tag{11}$$

$$\sum_{(i,j)\in\mathcal{A}} x_{\mathrm{TR},tij} - \sum_{(j,i)\in\mathcal{A}} x_{\mathrm{TR},tji} + \theta_{iR} y_{\mathrm{TR},ti} = \vartheta_{it} \sum_{l\in\mathcal{L}} \sum_{w\in\mathcal{W}} \eta_{\mathrm{T},lw} \mathcal{K}_{\mathrm{T},lw} y_{\mathrm{T},tlw} \quad \forall t\in\mathcal{T}, \quad i\in\mathcal{V}$$
(12)

$$\sum_{d \in \mathcal{D}} y_{\text{TD},td} = \sum_{l \in \mathcal{L}} \sum_{w \in \mathcal{W}} \eta_{\text{T},lw} \sigma_{\text{T},lw} y_{\text{T},tlw} \quad \forall t \in \mathcal{T}$$
(13)

$$\sum_{(i,j)\in\mathcal{A}} x_{\text{TD},tij} - \sum_{(j,i)\in\mathcal{A}} x_{\text{TD},tji} + \theta_{iD} y_{\text{TD},ti} = \vartheta_{it} \sum_{l\in\mathcal{L}} \sum_{w\in\mathcal{W}} \eta_{\text{T},lw} \sigma_{\text{T},lw} y_{\text{T},tlw} \quad \forall t\in\mathcal{T}, \quad i\in\mathcal{V}$$
(14)

$$\sum_{h \in \mathcal{H}} y_{\text{HR,hr}} + \sum_{t \in \mathcal{T}} y_{\text{TR,tr}} = y_{\text{R,r}} \quad \forall r \in \mathcal{R}$$
(15)

$$\sum_{d\in\mathcal{D}} y_{\text{RD},rd} = (1 - \lambda_{\text{R},r}) y_{\text{R},r} \quad \forall r \in \mathcal{R}$$
(16)

$$\sum_{(i,j)\in\mathcal{A}} x_{\text{RD},rij} - \sum_{(j,i)\in\mathcal{A}} x_{\text{RD},rji} + \theta_{iD} y_{\text{RD},ri} = \vartheta_{ir} (1 - \lambda_{\text{R},r}) y_{\text{R},r} \quad \forall r \in \mathcal{R}, \quad i \in \mathcal{V}$$

$$\tag{17}$$

$$\sum_{h \in \mathcal{H}} y_{\text{HD},hd} + \sum_{t \in \mathcal{T}} y_{\text{TD},td} + \sum_{r \in \mathcal{R}} y_{\text{RD},rd} = y_{\text{D},d} \quad \forall d \in \mathcal{D}$$
(18)

$$a_{R,r}z_{R,r} \leq y_{R,r} \leq b_{R,r}z_{R,r} \quad \forall r \in \mathcal{R}$$

$$(19)$$

$$a_{R,r}z_{R,r} \leq b_{R,r}z_{R,r} \quad \forall r \in \mathcal{T} \quad l \in \mathcal{L}$$

$$(20)$$

$$\begin{aligned} a_{\mathrm{T},tl} z_{\mathrm{T},tl} &\leq y_{\mathrm{T},tl} \leq b_{\mathrm{T},tl} z_{\mathrm{T},tl} &\forall t \in \mathcal{I} , \quad l \in \mathcal{L} \\ a_{\mathrm{D},d} z_{\mathrm{D},d} &\leq y_{\mathrm{D},d} \leq b_{\mathrm{D},d} z_{\mathrm{D},d} \quad \forall d \in \mathcal{D} \end{aligned}$$
(20)

$$\sum_{V' \in \{\mathbf{R}, \mathsf{T}, \mathsf{D}\}} \sum_{h \in \mathcal{H}} p_{\mathsf{H}V', \tau_{\mathsf{H}, h} ij} \mathbf{x}_{\mathsf{H}V', hij} + \sum_{V' \in \{\mathbf{R}, \mathsf{D}\}} \sum_{t \in \mathcal{T}} p_{\mathsf{T}V', ij} \mathbf{x}_{\mathsf{T}V', tij} + \sum_{r \in \mathcal{R}} p_{\mathsf{R}\mathsf{D}, ij} \mathbf{x}_{\mathsf{R}\mathsf{D}, rij} \leqslant b_{\mathsf{A}, ij} \quad \forall (i, j) \in \mathcal{A}$$

$$\tag{22}$$

$$\begin{aligned} y_{\text{HT,htl}} &\leq b_{\text{T,tl}} \delta_{\text{T,l\tau}_{\text{H,h}}} \quad \forall h \in \mathcal{H}, \quad t \in \mathcal{T}, \quad l \in \mathcal{L} \end{aligned}$$

$$\begin{aligned} y_{\text{T,thv}} &\leq b_{\text{T,tl}} \delta_{\text{T,lv}}, \quad \forall t \in \mathcal{T}, \quad l \in \mathcal{L}, \quad w \in \mathcal{W} \end{aligned}$$

$$\end{aligned}$$

$$\end{aligned}$$

$$\end{aligned}$$

$$\end{aligned}$$

$$\end{aligned}$$

$$\end{aligned}$$

$$\begin{array}{l} (21)\\ \chi_{\text{HR,hij}}, \chi_{\text{HT,hij}}, \chi_{\text{HD,hij}}, y_{\text{HR,hi}}, y_{\text{HD,hi}} > 0 \quad \forall h \in \mathcal{H}, \ (i,j) \in \mathcal{A}, \quad i \in \mathcal{V}, \quad l \in \mathcal{L} \end{array}$$

$$\begin{aligned} x_{\text{TR,tij}}, x_{\text{TD,tij}}, y_{\text{TR,ti}}, y_{\text{TD,ti}} &\ge 0 \quad \forall t \in \mathcal{T}, \ (i,j) \in \mathcal{A}, \quad i \in \mathcal{V} \end{aligned} \tag{26} \\ x_{\text{RD,rij}}, y_{\text{RD,ri}} &\ge 0 \quad \forall r \in \mathcal{R}, \ (i,j) \in \mathcal{A}, \quad i \in \mathcal{V} \end{aligned} \tag{27}$$

$$y_{\mathbf{R},r} \ge 0, \quad z_{\mathbf{R},r} \in \{0,1\} \quad \forall r \in \mathcal{R}$$

$$\begin{array}{ll} y_{\mathrm{R},r} \geq 0, & z_{\mathrm{R},r} \in \{0,1\} & \forall r \in \mathcal{R} \\ y_{\mathrm{T},tlw}, y_{\mathrm{T},tl} \geq 0, & z_{\mathrm{T},tl} \in \{0,1\} & \forall t \in \mathcal{T}, \quad l \in \mathcal{L}, \quad w \in \mathcal{W} \\ y_{\mathrm{D},d} \geq 0, & z_{\mathrm{D},d} \in \{0,1\} & \forall d \in \mathcal{D} \end{array}$$

$$\begin{array}{ll} (28) \\ (29) \\ (30) \end{array}$$

Objective functions (1) and (2) minimize the total cost and total risk respectively in the transportation and location processes. The total cost objective function (1) has six terms in which the first three terms and the last three ones calculate the total transportation cost and total location cost, respectively. The first three terms represent the transportation cost of three bunches of hazardous wastes from generation nodes, two bunches of waste residues from treatment centers, and one bunch of waste residues from recycling centers, respectively. The last three terms express the location cost including the fixed location cost and variable operation cost of recycling, treatment and disposal centers, respectively. The total risk objective function (2) can be understood analogously as the total cost objective function (1), except that the location risk of recycling, treatment and disposal centers only depends on the workloads at the centers.

Constraints (3) and (4) are the mass balance constraints of the recyclable hazardous wastes from generation nodes to recycling centers. These two sets of constraints are created not only to guarantee all recyclable hazardous wastes are recycled at proper recycling centers, but also to determine the allocation plans and associated transportation routes of the recyclable hazardous wastes between generation nodes and recycling centers. Analogously, Constraints (5) and (6) (Constraints (7) and (8)) present the mass balance constraints of the treatable (disposable) hazardous wastes between generation nodes and recycling centers. Analogously, Constraints (5) and (6) (Constraints (7) and (8)) present the mass balance constraints of the treatable (disposable) hazardous wastes between generation nodes and treatment (disposal) centers. Constraints (9)–(14) specify the mass balance constraints of the waste residues at treatment centers by reflecting the mass variation and recyclable possibility of the hazardous wastes after being treated with different technologies. Constraints (9) and (10) count respectively the workloads by treatment technology and waste type and the workloads by treatment technology of treatment centers on the wastes transported from generation nodes. Constraints (11) and (12) (Constraints (13) and (14)) determine the allocation plans and associated transportation routes of the recyclable (disposable) waste residues from treatment centers to recycling (disposal) centers.

Constraints (15)–(17) provide the mass balance constraints of the disposable waste residues at recycling centers. Constraints (15) count the workloads of recycling centers on the wastes transported from generation nodes and treatment centers. Constraints (16) and (17) determine the allocation plans and associated transportation routes of the disposable waste residues between recycling centers and disposal centers. Constraints (18) count the workloads of disposal centers on the wastes transported from generation nodes, treatment centers and recycling centers. Constraints (19)–(21) ensure the minimum waste amount requirement and maximum waste amount capacity constraints of recycling, treatment and disposal centers, respectively. Constraints (22) assure the risk of transporting hazardous wastes and waste residues through each link must be controlled under a prescribed maximum allowable risk tolerance capacity. Constraints (23) and (24) guarantee treatable hazardous wastes can only be allocated to proper treatment centers with comparable technologies. Finally, Constraints (25)–(30) define the non-negative and binary constraints of the decision variables.

The original problem (O) is exactly a multi-objective mixed integer programming problem. It is obviously NP-hard as its single objective version can be reduced to the classical NP-hard capacitated facility location problem. Our model is built on a complex management system with many real-life constraints. It is quite general in that it can nest many existing models as special cases. Typically, if the detailed allocation plans between facilities are not required, the model can be revised as a simpler continuous network flow model as the one proposed in Alumur and Kara (2007). Moreover, if the link capacity constraints are not considered explicitly, the model can be transformed into a location–allocation model as the one developed by Samanlioglu (2013).

Our model is also compact and linear so that it can facilitate the development of efficient solution algorithms. Researchers have developed many multi-objective integer programming algorithms so far. The computational performances of these algorithms significantly depend on the quality of the underlying single objective model. Fortunately, the proposed model is a mixed integer linear programming model. It allows state-of-the-art commercial optimization software to solve large-scale problem instances to optimality within reasonable computation times. Furthermore, the original problem (O) is a strategic decision problem which is insensitive to the computation time. Hence, we customize three existing multi-objective integer programming approaches to solve the proposed problem (O) efficiently.

4. Solution approaches

In this section, the original problem (O) is solved by utilizing multi-objective optimization approaches. As mentioned above, the original problem (O) is a multi-objective integer programming problem with two objectives to minimize the total cost and total risk simultaneously. For such a problem, it is not easy for the decision makers to qualify accurately the relative importance between the cost and risk objectives. A favorable solution method is to firstly compute all or a representative set of highly qualified efficient solutions on the table, and then involve the decision makers to select the most preferred ones based on their own judgements. To this end, the original problem (O) is more suitable to be solved by the a posteriori methods.

Scalarization technique is one of the most typical a posteriori methods to solve the multi-objective integer programming problem till now. The principle of this technique is to transform the original multi-objective problem into a single objective problem by introducing additional parameters, variables or constraints, and then solve repeatedly the single objective problem with different scalarization parameters to produce representative efficient solutions. Three popular scalarization-based multi-objective integer programming approaches in the literature are the weighted-sum approach, Tchebycheff approach and ε -constraint approach. We customize these three approaches to solve the original problem (O) respectively. Before presenting the solution approaches, some basic terminologies are recalled from Ehrgott (2005) and some necessary remarks are provided.

In mathematic terms, a multi-objective optimization problem can be formally stated as: min $z(x) = \{z_1(x), \ldots, z_m(x)\}$ s.t. $x \in X$, where $X \subseteq R^n$ represents the feasible set in the decision space; the image Y of X under vector-valued function $z : X \to R^m$, $z(x) = \{z_1(x), \ldots, z_m(x)\}$ represents the feasible set in the criterion space, i.e. $Y = z(X) = \{y \in R^m : y = z(x) \text{ for some } x \in X\}$; *n* and $m \ge 2$ are the number of decision variables and objective functions, respectively.

A feasible solution $x \in X$ is called *efficient* or *Pareto optimal* (weakly *efficient* or *weakly Pareto optimal*), if there does not exist a $x' \in X$, $x' \neq x$ such that $z_i(x') \leq z_i(x)$ for i = 1, ..., m with at least one strict inequality (such that $z_i(x') < z_i(x)$ for i = 1, ..., m). If x is a/an (weakly) efficient solution in the decision space, then z(x) is called a (weakly) non-dominated solution in the criterion space. The set of all non-dominated solutions $\{z(x) \in R^m : z(x) \text{ is a non-dominated solution}\}$ in the criterion space is called the *non-dominated frontier* (or *Pareto frontier*).

By definition, the efficient solution exists in the decision space, while the non-dominated solution exists in the criterion space. Since several efficient solutions may correspond to only one non-dominated solution, most of the existing a posteriori methods search solutions in the criterion space. Hereafter, for the original problem (O), we focus on the non-dominated solution in the criterion space. Furthermore, an ideal a posteriori method to the original problem (O) should find the non-dominated frontier (i.e. find all non-dominated solutions) within polynomial time. However, its non-dominated frontier generally includes several non-convex and non-continuous line segments. Under such condition, even finding a single non-dominated solution is not easy. Hence, our approaches only find a limited number of non-dominated solutions to approximately visualize the non-dominated frontier.

To facilitate the approach development, the original problem (O) is simply stated as: $\min z(x) = \{z_1(x), z_2(x)\}$ s.t. $x \in X$, where $z_1(x)$ and $z_2(x)$ represent the total cost objective (1) and total risk objective (2), respectively, while the feasible set X is composed of Constraints (3)–(30). Consider that the two objectives may have different scales, a normal method is used to eliminate the dimension of each objective as: $\overline{z}_i(x) = (z_i(x) - z_i^{\min})/(z_i^{\max} - z_i^{\min})$, i = 1, 2, where $\overline{z}_i(x)$ is the *i*th normalized objective, while z_i^{\min} and z_i^{\max} are the lower and upper bounds of the *i*th objective. Let x_1 and x_2 be the optimal solutions to the original problem (O) under objectives (1) and (2), respectively. We call $z^{\min} = (z_1^{\min}, z_2^{\min})$ the utopia point (Ehrgott, 2005). Obviously, we can have $z_1^{\min} = z_1(x_1)$ and $z_2^{\min} = z_2(x_2)$. We further call $z^{\max} = (z_1^{\max}, z_2^{\max})$ the nadir point (Ehrgott, 2005). Note that finding the nadir point is not as straightforward as the utopia point. In this paper, the nadir point is determined by a lexicographic optimization method as follows:

$$z_1^{\max} = \min z_1(x) \quad \text{s.t. } z_2(x) = z_2^{\min}, \ x \in X.$$

$$z_2^{\max} = \min z_2(x) \quad \text{s.t. } z_1(x) = z_1^{\min}, \ x \in X.$$
(31)

4.1. Weighted-sum approach

The weighted-sum approach (Zadeh, 1963) is one of the traditional methods to the multi-objective optimization problem. It can either be employed as an independent approach, or embedded as a component into other advanced approaches. This approach introduces non-negative objective weights to combine the multi-objective problem as a single objective problem. By systematically adjusting the objective weights, representative non-dominated solutions to the multi-objective problem are found sequentially. For the original problem (O), let $\omega_1 \ge 0$ and $\omega_2 \ge 0$ ($\omega_1 + \omega_2 = 1$) be the weights of the total cost objective and total risk objective, respectively. Further, the normal method is utilized to scale these two objectives with different dimensions. Then, the original problem (O) can be transformed into the following single objective optimization problem:

Using the above model (32), our weighted-sum approach to the original problem (O) is implemented as follows. Firstly, we plan to find g + 1 non-dominated solutions by uniformly generating g + 1 objective weight vectors in the closed interval [0, 1]. Then, for each objective weight vector k = 0, ..., g, we set the objective weights $\omega_1 = 1 - k/g$ and $\omega_2 = 1 - \omega_1$, respectively, and then solve the weighted-sum model (32). If model (32) has an optimal solution x^* , we obtain a non-dominated solution $\{z_1(x^*), z_2(x^*)\}$. Note that the objective $\bar{z}(x)$ in model (32) is too small due to the scaling operator. Hence, it is multiplied by a sufficiently large parameter ϕ to avoid the numerical difficulties.

It is known from existing theoretical works, i.e. Koski (1985), Das and Dennis (1997), etc., that utilizing the weighted-sum approach to solve the original problem (O) has positive and negative effects simultaneously. The positive effect is that model (32) has the same computational complexity and just scarifies the same computational effort as the single objective version of the original problem (O). That may explain why many previous studies related to the original problem (O) adopt this approach directly. However, the negative effect is that in the non-convex regions of the Pareto frontier of the original problem (O), the weighted-sum approach cannot find non-dominated solutions, and it could also result in weakly non-dominated solutions. Moreover, this approach often produces poorly distributed non-dominated solutions along the Pareto frontier even

with uniformly distributed objective weights. Thereafter, we adopt two more robust multi-objective integer programming approaches to the original problem (O).

4.2. Augmented weighted Tchebycheff approach

The weighted Tchebycheff approach, or weighted min-max approach, was firstly proposed by Bowman (1976). It searches non-dominated solutions by minimizing the weighted Tchebycheff distance (i.e. weighted l_{∞} norm) between the feasible point and utopia point of a given multi-objective optimization problem. The approach is welcomed in that all non-dominated solutions can be generated by approximately adjusting the objective weights and/or the utopia point. However, an inherit drawback of the approach is that besides non-dominated solutions also weakly non-dominated solutions might be generated at the same time. Steuer and Choo (1983) presented an improved approach, called augmented weighted Tchebycheff approach. The improved approach augments an l_1 norm, weighted by a sufficient small positive parameter $\rho > 0$, into the l_{∞} norm of the objective of the original approach. Previous theoretical works show that the improved approach can efficiently avoid the generation of non-dominated solutions.

Analogously, for the original problem (O), we also define $\omega_1 \ge 0$ and $\omega_2 \ge 0$ ($\omega_1 + \omega_2 = 1$) as the cost and risk objective weights, respectively. According to Steuer and Choo (1983), a standard (linearized) augmented weighted Tchebycheff model for the original problem (O) is formulated as:

$$\begin{array}{ll} \min & \bar{z}(x) = \pi + \rho(z_1(x) - z_1^{\min}) + \rho(z_2(x) - z_2^{\min}) \\ \text{s.t.} & \omega_1(z_1(x) - z_1^{\min}) \leqslant \pi \\ & \omega_2(z_2(x) - z_2^{\min}) \leqslant \pi \\ & x \in X \end{array}$$

$$(33)$$

As the cost and risk objectives have different dimensions, the normal method is also employed to eliminate the objective dimensions. Note that the utopia point should also be normalized so as to maintain the consistence with the feasible point. Then, the standard augmented weighted Tchebycheff model (33) can be reformulated as:

$$\min \quad \overline{z}(x) = \pi + \rho(\overline{z}_1(x) + \overline{z}_2(x))$$
s.t.
$$\omega_1 \overline{z}_1(x) \leqslant \pi$$

$$\omega_2 \overline{z}_2(x) \leqslant \pi$$

$$x \in X$$

$$(34)$$

Based on the above model (34), the implementation procedure of our augmented weighted Tchebycheff approach is the same as that of our weighted-sum approach. More specifically, g + 1 uniformly dispersed objective weight vectors are firstly generated in the closed interval [0, 1]. Then, for each objective weight vector k = 0, ..., g, after respectively setting the objective weights as $\omega_1 = 1 - k/g$ and $\omega_2 = 1 - \omega_1$, we solve the revised augmented weighted Tchebycheff model (34) to seek a non-dominated solution. Once model (34) is solved with an optimal solution x^* , a non-dominated solution $\{z_1(x^*), z_2(x^*)\}$ is found immediately. Note that we also optimize $\phi \bar{z}(x)$ instead of $\bar{z}(x)$ in model (34) so as to perform numerically better, where ϕ is an adequately large parameter.

4.3. Augmented ε -constraint approach

The ε -constraint approach initialized by Haimes et al. (1971) is probably the most theoretically attractive scalarizationbased multi-objective approach. It optimizes only one of the objectives and constrains the other objectives being not inferior to given values. By approximately adjusting the value level (i.e. right hand side) of the constrained objectives, all or a representative set of non-dominated solutions can be identified. As with other traditional scalarization approaches, the original ε -constraint approach also cannot prevent itself from producing weakly non-dominated solutions. Mavrotas and Florios (2013) introduced slack/surplus variables to standardize the original inequality constraints as equality ones, and augmented the original objective with these additional variables multiplied by a small enough positive number $\rho > 0$. The new approach, named as augmented ε -constraint approach, can guarantee that only non-dominated solutions are generated.

For the original problem (O), we retain the total cost objective (1) and introduce the value level e_2 to restrain the total risk objective (2). The non-dominated solution to the original problem (O) can be obtained by solving the following standard ε -constraint model:

$$\begin{array}{ll} \min & \bar{z}(x) = z_1(x) \\ \text{s.t.} & z_2(x) \leqslant e_2 \\ & x \in X \end{array}$$
 (35)

Guided by Mavrotas and Florios (2013), a slack variable μ_2 is introduced to turn the inequality constraint with respect to the total risk objective into an equality constraint. In addition, in order to avoid the scaling problems, the introduced slack variable μ_2 is further divided by the range $v_2 = z_2^{\text{max}} - z_2^{\text{min}}$ of the total risk objective. Then, the standard ε -constraint model (35) is augmented as:

With the above model (36), our augmented ε -constraint approach to the original problem (O) is implemented as follows. Firstly, in order to find g + 1 well dispersed non-dominated solutions, the range v_2 of the total risk objective is divided into g equal intervals using g - 1 equally spaced intermediate grid points. Thus we have g + 1 grid points in total that will be used to adjust the value level e_2 of the total risk objective. Then, for each grid point $k = 0, \ldots, g$, we set the value level e_2 of the total risk objective as $e_2 = z_2^{max} - kv_2/g$, and solve the augmented ε -constraint model (36) to seek a non-dominated solution. Note that our approach starts with the most relaxed value level e_2 of the total risk objective at the first grid point and strengthens the bound at the remaining points gradually.

At the current grid point *k*, if model (36) has an optimal solution $\{x^*, \mu_2^*\}$, a non-dominated solution $\{z_1(x^*), z_2(x^*)\}$ is obtained immediately. At the same time, if $\mu_2^* > v_2/g$, it is implied that at the next one or few grid point(s) the same non-dominated solution only differing in the value of the slack variable μ_2 will be obtained. This solution makes the next one or few grid point(s) redundant so that we can bypass it or them safely and efficiently. Let $\varsigma = \max(\lceil \mu_2^*g/v_2 \rceil, 1)$ be the bypass coefficient, the current grid point is updated as $k = k + \varsigma$ and the approach is continued. If model (36) has no feasible solution, it means that strengthening the value level e_2 of the total risk objective further will also result in infeasible solutions. In such case, it is obvious that continuing the approach cannot make any sense. Hence, the approach is terminated.

5. Case studies

We test the effectiveness and efficiency of the three approaches with case studies. Consider that a small-scale case might contain a limited number of non-dominated solutions. For such a case, a multi-objective approach is easy to produce reduplicate solutions. Meanwhile, a large-scale case could induce a relatively large number of non-dominated solutions. Under such condition, it is also possible for a multi-objective approach to find non-uniform solutions. However, whatever the case scale is, given the upper number of solutions, a good multi-objective approach should identify as many distinct solutions as possible and distribute these solutions as uniformly as possible. Hence, two cases with different scales are studied, one is a small-scale hypothetical case revised from the literature and the other is a large-scale realistic case created from the Sichuan province, China.

The parameters of the approaches are set as follows: the parameter ϕ in the weighted-sum approach and augmented weighted Tchebycheff approach is set as 10^3 , while the parameter ρ in the augmented weighted Tchebycheff approach and augmented ε -constraint approach is set as 10^{-3} . The proposed solution approaches are coded in MathWorks MATLAB R2012b. The underlying single objective optimization problems are solved by IBM ILOG CPLEX 12.5, in which all corresponding parameters are set to their default values. The computations are executed on a personal computer with Intel Core i5-2400 3.10 GHz CPU, 8.00 GB RAM, and Windows 7–64 bits operating system.

5.1. Test cases

5.1.1. Hypothetical case

The prototype of the hypothetical case is derived from Nema and Gupta (1999), but many data are modified or redesigned owing to the problem differences. Specifically, only the transportation network and the hazardous wastes are directly utilized from the source paper. Other data, especially the candidate recycling, treatment and disposal centers, are completely or partially redesigned. One could not compare the approaches in this paper with those in Nema and Gupta (1999) as we actually investigate two different problems.

The hypothetical case has a transportation network composed of 16 nodes and 21 links as shown in Fig. 3. The network has 6 generation nodes producing metal plating wastes, pesticide wastes and petrochemical wastes. The annual waste amount of generation nodes is included in Fig. 3. For the sake of simplicity, the recyclable, treatable and disposable rates of the metal plating waste are assumed as 0.1, 0.8 and 0.1, respectively, and those of the pesticide waste are 0, 1 and 0, respectively, while those of the petrochemical waste are 0.05, 0.90 and 0.05, respectively.

The transportation cost on the link is listed in Table 2. The transportation risk on the link is measured by the traditional risk model (Nema and Gupta, 1999), i.e. transportation risk = waste risk potential \times link risk consequence \times link risk probability. As many types of waste are transported on the same link of the network, the risk potential of a waste on a link is to indicate the potential of the waste to impose risk when transported on the link. This parameter can be estimated by any qualitative/quantitative ranking procedure based on the characteristics of the wastes. Similar to Nema and Gupta (1999), it is determined by the analytic hierarchical process (AHP) method as shown in Table 2. The length, risk consequence and risk probability of links are presented in Fig. 3. The link maximum allowable risk tolerance capacity is assumed as 100 people \cdot ton.

The network has the same 6 candidate sites for the recycling, treatment and disposal centers. Two types of technology including the solidification technology (Tech 1) and incineration technology (Tech 2) are both available at candidate

treatment centers. The metal plating waste and the pesticide waste can only be treated with the solidification and incineration technology, respectively. The petrochemical waste can be simultaneously treated with the two technologies. The location risk at the node is also estimated by the traditional risk model (Nema and Gupta, 1999), i.e. location risk = node risk consequence \times node risk probability. The recycling rate, cost and risk coefficients as well as capacity parameters of candidate recycling centers are listed in Table 3. The mass variation and residue recyclable rates, cost and risk coefficients as well as capacity parameters of candidate treatment centers are included in Table 4. The cost and risk coefficients as well as capacity parameters of candidate disposal centers are provided in Table 5.

5.1.2. Realistic case

The realistic case is created by taking the Sichuan province in China as the background. All data are generated based on the Sichuan statistic yearbook 2013 and the Sichuan transport yearbook 2013. Located in the southwestern China, the Sichuan province by the end of 2012 has an area of 48.6 ten thousand square kilometers and a population of 80.8 million people. It is ranked the fifth and fourth largest provinces in area and population in China, respectively. The province administers 21 (equivalent) cities with the population ranging from 90.7 ten thousand people to 14.2 million people. To make the case setup simple, only the 21 administrative cities are assumed as the hazardous waste origins. These cities are also assumed as the candidate recycling, treatment and disposal centers simultaneously. Moreover, the inter-city wastes can only be transported through the national and provincial highways. The transportation network in the realistic case is illustrated in Fig. 4. The network has 44 nodes, out of which nodes 1–21 are the 21 cities where node 1 is the capital city, and the remaining ones are intermediate nodes. The network has 82 bidirectional links whose length varies from 33.3 km to 381.4 km. To simplify the risk computation, each city is assumed to have a uniformly distributed population as shown in Fig. 4.

The amount of hazardous wastes produced at cities in the province is not available now. The information of existing waste facilities in the province is also not complete. Thus, each city is assumed to produce three types of hazardous waste that can be treated either with the solidification technology (Tech 1) or the incineration technology (Tech 2). The first waste type can only be solidified, whose recyclable, treatable and disposable rates are assumed as 0.1, 0.8 and 0.1, respectively. The second waste type does not have recyclable and disposable compositions, that can only be totally incinerated. The last waste type can either be solidified or incinerated, whose recyclable, treatable and disposable rates are 0.05, 0.90 and 0.05, respectively. In each city, the annual waste amount of the three types of hazardous waste is assumed to be the same. This parameter is further assumed to be proportional to the gross industrial production (GIP) index. The annual waste amount parameter of the capital city is set as 40,000 tons per year, while that of other city is determined by the ratio of the GIP index of the city to that of the capital city.

The transportation cost on the link is included in Table 6. The transportation risk of each type of hazardous waste and waste residue on each link is evaluated by the traditional risk method (Nema and Gupta, 1999), i.e. the transportation risk is equal to the waste risk potential multiplied by the link risk consequence multiplied by the link risk probability. The waste risk potential is also determined by the analytic hierarchical process (AHP) method as provided in Table 6. Similar to Alumur and Kara (2007), the link risk consequence is qualified as the number of people exposed within 800 m width of the link. More specifically, we have link risk consequence = 1.6 (km) × link length (km) × link population density (people/km²). The population density of a link is an average value determined by that of the cities through which the link passes. Following Harwood et al. (1993), the link risk probability is qualified by a conditional probability as $0.4 (\times 10^{-6}/\text{km}) \times 0.9 \times \text{link length (km})$, where the first number is the accident rate of trucks transporting hazardous wastes on inter-city highways, and the second number is the release probability given an accident of a truck with hazardous wastes. The maximum allowable risk tolerance capacity of each link is assumed as 10,000 people · ton.

Each type of waste facility (recycling, treatment and disposal center) in each city is assumed to have the same operation, cost and capacity parameters. The operation parameter, fixed cost, variable cost, lower capacity, upper capacity of the three types of waste facility are listed in Tables 7–9, respectively. The location risk of each type of waste facility in each city is also measured by the traditional risk method (Nema and Gupta, 1999), i.e. the location risk is equal to the city risk consequence multiplied by the city risk probability. The city risk consequence is assumed to be the same for the three types of waste facility. Similar to Alumur and Kara (2007), it is qualified as the number of people exposed within 2.5 km radius of the city. More precisely, we have city risk consequence = 6.25π (km²) × city population density (people/km²). The population density of each city is classified in Fig. 4. The city risk probability is assumed to be the same for each type of waste facility in each city. Based on historical data, it is roughly estimated by the experts in the environmental protection department of the considered province. The risk consequence and risk probability of the three types of waste facility are also included in Tables 7–9, respectively.

5.2. Comparison of approaches

We compare the performance of the weighted-sum approach, augmented weighted Tchebycheff approach and augmented ε -constraint approach in terms of effectiveness and efficiency using the hypothetical and realistic cases simultaneously. The effectiveness is measured by the number and uniformity of obtained distinct solutions under a prescribed upper number of non-dominated solutions. As mentioned, under the same upper number of solutions, a better multi-objective approach should exhibit a larger number of and a more uniform distribution of obtained distinct solutions. Meanwhile,



Fig. 3. Transportation network in the hypothetical case.

Table 2

Transportation cost and risk potential in the hypothetical case.

Waste type	Transportation cost (\$/t/km)	Risk potential
Recyclable hazardous waste at generation node	2.0	0.05
Treatable metal plating waste at generation node	4.0	0.20
Treatable pesticide waste at generation node	5.0	0.20
Treatable petrochemical waste at generation node	5.5	0.20
Disposable hazardous waste at generation node	2.0	0.10
Recyclable waste residual at treatment center	2.0	0.05
Disposable waste residual at treatment center	2.0	0.10
Disposable waste residual at recycling center	2.0	0.10

the efficiency is weighed by the computation time to explore the upper number of non-dominated solutions. For briefness, the three approaches are abbreviated as WSA, AWTA and AECA, respectively.

For each case, all the three approaches are implemented to find 6, 11 and 21 bounded non-dominated solutions, respectively. Note that 21 candidate solutions are large enough to configure the regional hazardous waste management system. Moreover, the 6 and 11 solutions can be directly derived without additional computations from the 21 solutions with increments of 4 and 2, respectively. For each solution, the solution number (sol), the total cost objective weight (wei), the objective function value (obj), the corresponding total cost (cost) and total risk (risk), and the computation time (time) of the underlying model are reported, respectively. Besides, for comparison purpose, the value of the slack variable (slack) in the model of AECA and the average computation time (ave) of all approaches are also reported. The objective function value in the model of AECA is omitted as it is almost equal to the corresponding total cost even if the slack variable is not 0. The computation time is expressed in seconds.

We firstly apply the three approaches to the hypothetical case. The comparison results are summarized in Table 10. The non-dominated frontiers approximated with 6, 11 and 21 bounded solutions are presented in Fig. 5. As shown in Table 10, the objective in the models of WSA and AWTA is quite small so that it is necessary to scale it by a sufficiently large parameter. Meanwhile, the slack variable in the model of AECA can be not 0, which means if its current value is large enough then the next few redundant solutions can be bypassed efficiently. Nevertheless, such efficiency is not reflected here due to the small value of the slack variable. It is further seen from Table 10 that all approaches identify only non-dominated solutions, each of which is found very quickly within 0.5 s. However, they show different performances in the number and uniformity

Table 3					
Information	of recycling	centers in	the	hypothetical	case

Recycling node	Recycling rate	Fixed cost (\$/yr)	Variable cost (\$/t)	Lower capacity (t/yr)	Upper capacity (t/yr)	Risk consequence (×10 ⁴ people)	Risk probability $(\times 10^{-6})$
Fig. 3	0.9	10,000	5	1000	2000	[3.75,4.50]	3

Table 4Information of treatment centers in the hypothetical case.

Treatment node	Technology type	Mass variation rate	Residue recyclable rate	Fixed cost (\$/ yr)	Variable cost (\$/t)	Lower capacity (t/ yr)	Upper capacity (t/ yr)	Risk consequence $(\times 10^4 \text{ people})$	Risk probability (×10 ⁻⁶)
Fig. 3	1	1.3	0.2	60,000	25	2500	5000	[3.75,4.50]	6
Fig. 3	2	0.2	0.0	80,000	30	4000	8000	[3.75,4.50]	7

Table 5

Information of disposal centers in the hypothetical case.

Disposal node	Fixed cost (\$/yr)	Variable cost (\$/t)	Lower capacity (t/yr)	Upper capacity (t/yr)	Risk consequence (×10 ⁴ people)	Risk probability ($\times 10^{-6}$)
Fig. 3	50,000	15	4000	8000	[3.75,4.50]	5

of distinct solutions. The WSA finds only 5, 8 and 11 distinct solutions with 1, 3 and 10 redundant computation(s) for the 6, 11 and 21 bounded solutions, respectively. The AWTA and AECA hold a better performance in that the solutions obtained under each of the three upper numbers of solutions are all distinct. More importantly, as illustrated in Fig. 5, the three approaches approximate non-dominated frontiers with different uniformities. The WSA seems to be able to find only anchor solutions and hence produces poor frontiers regardless of the upper number of solutions. The AWTA and AECA can generate better frontiers under different upper numbers of solutions by distributing the solutions as uniformly as possible.



Fig. 4. Transportation network in the realistic case.

Table 6

Transportation cost and risk potential in the realistic case.

Waste type	Transportation cost (\$/t/km)	Risk potential
Recyclable hazardous waste at generation node	0.50	0.05
Treatable hazardous waste I at generation node	1.00	0.20
Treatable hazardous waste II at generation node	1.00	0.20
Treatable hazardous waste III at generation node	1.00	0.20
Disposable hazardous waste at generation node	0.75	0.10
Recyclable waste residual at treatment center	0.50	0.05
Disposable waste residual at treatment center	0.75	0.10
Disposable waste residual at recycling center	0.75	0.10

Table 7

Information of recycling centers in the realistic case.

Recycling node	Recycling rate	Fixed cost (×10 ⁴ \$/yr)	Variable cost (\$/t)	Lower capacity (t/ yr)	Upper capacity (t/ yr)	Risk consequence (×10 ⁴ people)	Risk probability $(\times 10^{-6})$
Fig. 4	0.9	90	50	5000	10,000	[0.01,2.32]	10

Table 8

Information of treatment centers in the realistic case.

Treatment node	Technology type	Mass variation rate	Residue recyclable rate	Fixed cost (×10 ⁴ \$/yr)	Variable cost (\$/t)	Lower capacity (t/ yr)	Upper capacity (t/ yr)	Risk consequence $(\times 10^4 \text{ people})$	Risk probability (×10 ⁻⁶)
Fig. 4	1	1.3	0.2	1500	200	9000	18,000	[0.01,2.32]	40
Fig. 4	2	0.2	0.0	2000	250	10,000	20,000	[0.01,2.32]	50

Table 9

Information of disposal centers in the realistic case.

Disposal node	Fixed cost (×10 ⁴ \$/yr)	Variable cost (\$/t)	Lower capacity (t/yr)	Upper capacity (t/yr)	Risk consequence (×10 ⁴ people)	Risk probability ($\times 10^{-6}$)
Fig. 4	600	100	12,500	25,000	[0.01,2.32]	20

We then implement the three approaches to solve the realistic case. The comparison results are included in Table 11, and the approximated non-dominated frontiers are visualized in Fig. 6. As can be seen, the three approaches all have a large but different gap between solutions 20 and 21. The existence of the gaps is explained by the non-continuous structure of the true non-dominated frontier. Meanwhile, the difference of the gaps is determined by the identification capability of the three approaches. It is also observed that almost all approaches identify only distinct non-dominated solutions under each upper number of solutions, expect that the WSA obtains 1 reduplicate solution for the 21 bounded solutions, but they differ in the uniformity of the distinct solutions. The WSA looks like only seeking the solutions located at the anchor and knee points of the true frontier even with increased upper numbers of solutions. The AWTA approximates a better frontier under each scenario but it still does not explore the solutions are obviously different from each other and distributed uniformly in the criterion space. Regarding the computation time, the same general trend is that the more important the total cost objective is the more difficult the problem becomes. However, the three approaches present different average computational performances due to the complexity of their underlying model. As expected, the WSA has the fastest speed with an average time of 1236.9 s. The AWTA runs slightly slower with an average time of 1485.5 s. The AECA is the slowest approach whose average time is nearly twice that of the AWTA.

As can be concluded from the case studies, the three approaches have different performances on the regional hazardous waste network design problem. The weighted-sum approach only explores special non-dominated solutions and easily produces redundant solutions. Its theoretical drawbacks are not remedied with uniform objective weights and increased numbers of solutions. The augmented weighted Tchebycheff approach approximates better non-dominated frontiers within slightly longer computation times. However, it still probably ignores some important solutions and leads to non-uniform solutions. The augmented ε -constraint approach provides the best approximated frontiers and avoids the redundant computations efficiently. This approach may be criticized in its relatively long computation time. Recall that the solution quality rather than the computation time is the critical issue. We remark that the weighted-sum approach, even widely used by

Table 10Comparison results of approaches to the hypothetical case.

Sol	Wei	WSA				AWTA				AECA			
		Obj	Cost	Risk	Time	Obj	Cost	Risk	Time	Slack	Cost	Risk	Time
1	1.00	0.000	867625.2	5321.5	0.1	0.001	867625.2	5321.5	0.1	0.0	867625.2	5321.5	0.1
2	0.95	0.050	867625.2	5321.5	0.1	0.047	888318.3	5263.8	0.2	0.0	875413.9	5289.0	0.1
3	0.90	0.100	867625.2	5321.5	0.1	0.088	909334.8	5237.0	0.3	0.0	893348.3	5256.5	0.2
4	0.85	0.150	867625.2	5321.5	0.1	0.128	932274.7	5223.4	0.3	0.0	930697.8	5224.0	0.3
5	0.80	0.200	868465.2	5315.9	0.1	0.157	951599.2	5177.5	0.3	0.0	947873.4	5191.5	0.3
6	0.75	0.249	868825.2	5313.7	0.1	0.179	970117.1	5134.7	0.3	0.0	956771.4	5159.0	0.3
7	0.70	0.298	876025.2	5286.7	0.1	0.205	993166.8	5112.8	0.4	0.0	978759.4	5126.5	0.2
8	0.65	0.343	879025.2	5277.4	0.1	0.222	1014147.1	5081.4	0.3	0.0	1005964.8	5094.0	0.3
9	0.60	0.389	879025.2	5277.4	0.1	0.232	1033704.9	5046.7	0.3	0.0	1025619.1	5061.4	0.3
10	0.55	0.434	879025.2	5277.4	0.2	0.238	1053345.2	5013.4	0.3	0.0	1043498.3	5028.9	0.3
11	0.50	0.438	1125805.2	4852.3	0.2	0.230	1065598.0	4969.7	0.3	6.9	1058185.2	4989.6	0.3
12	0.45	0.422	1127665.2	4849.9	0.2	0.222	1079507.2	4932.6	0.2	0.0	1067754.2	4963.9	0.2
13	0.40	0.382	1271005.2	4679.8	0.1	0.212	1095403.8	4900.2	0.2	0.0	1079944.6	4931.4	0.3
14	0.35	0.336	1271005.2	4679.8	0.1	0.204	1117414.0	4874.0	0.3	0.0	1096489.9	4898.9	0.2
15	0.30	0.290	1271005.2	4679.8	0.1	0.186	1133321.2	4842.9	0.2	0.0	1120275.4	4866.4	0.2
16	0.25	0.243	1271005.2	4679.8	0.1	0.168	1155386.2	4815.9	0.2	0.0	1140702.7	4833.9	0.2
17	0.20	0.197	1271005.2	4679.8	0.0	0.146	1181328.4	4789.5	0.2	0.0	1167222.2	4801.4	0.2
18	0.15	0.150	1297525.2	4671.6	0.0	0.127	1229858.5	4767.7	0.2	0.0	1229409.0	4768.9	0.3
19	0.10	0.100	1297525.2	4671.6	0.0	0.088	1242446.6	4734.1	0.2	0.0	1241599.4	4736.4	0.2
20	0.05	0.050	1299085.2	4671.4	0.0	0.046	1254713.4	4702.1	0.1	0.0	1253990.1	4703.9	0.2
21	0.00	0.000	1299885.2	4671.4	0.0	0.001	1299085.2	4671.4	0.1	0.0	1299085.2	4671.4	0.1
Ave					0.1				0.2				0.2



Fig. 5. Approximated non-dominated frontiers of the hypothetical case.

Table 11					
Comparison	results of ap	proaches to	the	realistic	case.

Sol	Wei	WSA				AWTA				AECA			
		Obj	Cost	Risk	Time	Obj	Cost	Risk	Time	Slack	Cost	Risk	Time
1	1.00	0.000	586722539.7	307852.3	14400.4	0.001	586722539.7	307852.3	14400.1	0.0	586722539.7	307852.3	1993.5
2	0.95	0.050	586744209.1	307519.6	3675.3	0.035	592816573.8	285080.1	4204.4	0.0	587241503.7	304147.6	1538.3
3	0.90	0.094	589740084.8	291255.6	2419.9	0.056	596929749.2	274477.0	4929.3	0.0	588205719.6	300442.9	11509.6
4	0.85	0.130	590996112.6	287417.6	2085.6	0.068	599953452.8	266990.9	903.0	0.0	588840242.6	296738.3	8666.7
5	0.80	0.152	599253936.8	267965.9	1077.0	0.079	603229613.1	263025.2	659.0	0.0	589427877.1	293033.6	4465.8
6	0.75	0.171	599545618.0	267480.3	557.8	0.089	606343420.6	259849.1	558.0	0.0	590630623.9	289328.9	2183.2
7	0.70	0.186	603405216.8	262528.5	398.4	0.097	609743113.3	257567.4	633.5	0.0	592065463.5	285624.3	2277.0
8	0.65	0.200	606439818.5	259765.3	298.5	0.102	612760358.7	255192.2	539.3	0.0	594914991.1	281919.6	11107.9
9	0.60	0.208	611729143.7	255753.1	323.6	0.107	616312442.0	253432.0	599.6	0.0	595592851.9	278214.9	5874.1
10	0.55	0.216	611882797.2	255655.6	306.6	0.109	619819930.4	251689.0	607.8	0.0	596924000.1	274510.3	4693.4
11	0.50	0.212	637367936.8	242666.0	206.2	0.113	624391133.7	250455.1	1263.0	0.0	598070888.9	270805.6	1693.5
12	0.45	0.202	637995973.6	242431.0	71.3	0.109	626974520.8	248356.3	703.0	0.0	599885856.4	267100.9	964.0
13	0.40	0.192	639847961.1	241818.6	29.1	0.105	630506914.5	246696.9	388.5	0.0	603121335.0	263396.3	774.0
14	0.35	0.182	639858246.5	241815.9	13.4	0.099	633697854.8	244970.4	127.9	0.0	606598902.1	259691.6	815.2
15	0.30	0.171	639953588.6	241794.6	13.1	0.090	636374611.1	243190.8	53.2	0.0	611444506.8	255986.9	500.3
16	0.25	0.160	657889548.9	239018.6	13.0	0.081	640649554.7	241726.4	21.3	0.0	618535848.7	252282.2	914.1
17	0.20	0.139	676637432.7	236662.2	12.3	0.078	651702035.0	240959.3	75.4	0.0	626551867.1	248577.6	726.5
18	0.15	0.110	693182971.9	235025.8	13.5	0.064	656958167.5	239252.6	29.1	0.0	633977234.5	244872.9	203.8
19	0.10	0.079	693182971.9	235025.8	16.0	0.052	672603733.4	237988.5	95.2	0.0	651103033.2	241168.2	103.4
20	0.05	0.043	713088641.6	234179.8	21.9	0.032	690036626.6	236169.1	80.3	0.0	673649411.4	237463.6	39.3
21	0.00	0.000	753885911.3	233758.9	22.2	0.001	753885911.3	233758.9	324.0	0.0	753885911.3	233758.9	162.7
Ave					1236.9				1485.5				2914.6

many related works, is actually not a good alternative method to our problem. Moreover, we suggest decision makers to choose either the augmented weighted Tchebycheff approach if they cannot wait too long or the augmented ε -constraint approach if they pursue better solutions.

5.3. Representative efficient solutions

We try to provide decision makers with valuable managerial insights by analyzing some representative efficient solutions of the hypothetical and realistic cases, so as to support them in the design of an economically and environmentally attractive regional hazardous waste management system. For each case and each approach, we analyze at most 3 out of the 21 obtained representative efficient solutions, each of which is obtained with minimal total cost (#1), equal (objective) weights (#11) and minimal total risk (#21), respectively. The efficient solutions with minimal total cost and minimal total risk are both the same for all the three approaches. They are reported to be obtained by a fictitious approach called ALL. To be brief, for each efficient solution, only the result of the location decision is reported, but those of the allocation and transportation decisions are omitted.

Fig. 7 presents four representative efficient solutions of the hypothetical case. The efficient solutions with equal weights of the AWTA and AECA have the same location result and hence they are depicted together in Fig. 7. As can be seen, all approaches with three considered objectives open the same numbers of but different locations of recycling, treatment and disposal centers. Specifically, Fig. 7(a) shows that with the total cost objective, all waste facilities are located at nodes 5 and 10, which are closer to waste origins than other candidate nodes. Besides, Fig. 7(b)–(c) indicates that to obtain compromise solutions, most waste facilities are opened at node 1 with relatively small population, while one incineration treatment center is opened either at node 2 or node 10 with relatively high accessibility. Moreover, Fig. 7(d) illustrates that concerning the total risk objective, all waste facilities are built at nodes 1 and 2, whose population is smaller than other candidate nodes.



Fig. 6. Approximated non-dominated frontiers of the realistic case.

Fig. 8 provides five representative efficient solutions of the realistic case. The efficient solutions with equal weights for all the three approaches are different and therefore they are included separately in Fig. 8. It is seen that all approaches under three considered objectives produce five representative solutions differing significantly in the numbers and locations of opened recycling, treatment and disposal centers. However, some general location patterns can be summarized from these solutions. To be specific, as shown in Fig. 8(a), to obtain the minimal total cost, most of the waste facilities are located at developed cities with high waste output and accessibility. Furthermore, as observed from Fig. 8(b)–(d), in the compromise solution of all approaches, many waste facilities are established at less developed cities where either the accessibility is still high or the population is relatively small. In addition, as illustrated in Fig. 8(e), to suffer the least total risk, the majority of waste facilities are built at the least developed cities with sparse population and low waste output.

Based on the analysis of representative efficient solutions, some instructive managerial insights can be extracted. Specifically, if the decision makers prefer to minimize the total cost, all types of waste facility could be located at or close to areas with high waste output and accessibility. It is strongly recommended to establish integrated facilities at these areas where the recycling, treatment and disposal functions are incorporated simultaneously. Otherwise, if the decision makers pursue the minimal total risk, wastes and residues should be recycled, treated and disposed at areas with sparse population and low industrialization. If possible, integrated facilities with multiple functions could also be built at these areas. Furthermore, if the decision makers require a compromise solution, they could consider locating recycling and disposal centers at highly accessible areas, and operating treatment centers at nearby sparsely populated areas. If allowable, integrated facilities could be established at moderately accessible and populated areas.

5.4. Effect of critical parameters

The regional hazardous waste network design problem defined in this paper contains many critical parameters, such as the cost/risk coefficients, node capacity, and link capacity. Each of these parameters obviously has more or less effects on the objectives and decisions of the problem. Consider that ReVelle et al. (1991) and many subsequent works have already



Fig. 7. Representative efficient solutions of the hypothetical case.



Fig. 8. Representative efficient solutions of the realistic case.

observed that different cost/risk coefficients impact only the resulting objective values but not the decision results. We evaluate the effects of the node capacity and the link capacity on the hypothetical and realistic cases using the sensitivity analysis method. For each case, the analysis is conducted on two representative solutions with minimal total cost and minimal total risk, respectively. To make it brief, in each solution, only the optimized total cost or total risk is reported, but the detailed decisions are omitted. The computation time is not very important and hence it is not provided and analyzed.

5.4.1. Effect of the node capacity

In our problem, each candidate recycling, treatment and disposal node have minimal and maximum capacity parameters. These parameters are introduced to guarantee that each located node is operated within a reasonable workload range. For a candidate node, a capacity ratio (NCR) is defined to be the ratio of its minimal capacity to its maximum capacity. Then, the node capacity effect is evaluated by varying a unified NCR across all candidate nodes and maintaining the initial maximum capacity of each candidate node. Intuitively, NCR balances the objective values of the problem and the economic benefits of operating waste facilities. A small NCR could produce good objective values as large flexibility is available to configure the waste management system. Meanwhile, it might also result in low economic benefits since some waste facilities would be operated under low capacity usage levels. Contrarily, a large NCR could deteriorate the objective values but improve the economic benefits. For each case, NCR is adjusted from 0.0 to 0.9 with 0.1 increment to perform the sensitivity analysis. Note that the NCR of each case is initialized as 0.5.

Fig. 9 depicts the effect of the node capacity on the test cases, where the minimal total cost and minimal total risk of each case under all NCRs are plotted on one double *Y*-axis graph. As can be seen from Fig. 9, the two cases with the two objectives are all a monotonically increasing function but with a unique variation curve with respect to NCR. For the hypothetical case, as given in Fig. 9(a), the total cost objective firstly remains stable when NCR is less than 0.5, followed by a big jump at NCR of 0.6, and then continues stable until NCR is 0.8, followed by a small jump at NCR of 0.9. The curve of the total risk objective is quite similar to that of the total cost objective, expect that the former also increases slowly when NCR is between 0.2 and 0.5. For the realistic case, as provided in Fig. 9(b), the total cost objective firstly maintains non-decreasing until NCR is 0.6, and then increases rapidly when NCR is between 0.7 and 0.9. The total risk objective presents an obviously different curve which is non-decreasing only when NCR is less than 0.2 while strictly increasing when NCR is greater than 0.3. Hence, the node capacity significantly impacts the objectives and decisions of our problem. Decision makers should carefully set this parameter so as to balance the objective values and the economic benefits efficiently.

5.4.2. Effect of the link capacity

In this paper, the link capacity measured by the maximum allowable risk tolerance is introduced to ensure that the transportation risk involved on each link must be controlled under a publicly acceptable value. For each link, a capacity level (LCL) is defined to demonstrate its capacity multiple with respect to its initial capacity. Accordingly, the effect of the link capacity is analyzed by adjusting a unified LCL across all links. LCL obviously compromises the objective values of the problem and the public attitudes for implementing the waste management system. If LCL is small, the problem might become tight resulting in bad objective functions. At the same time, the optimized system could attract public supports due to the low link transportation risk. However, if LCL is large, the objective values could be improved, but the public protests should be emphasized. Recall that the link capacity of the two cases is initialized as 100 and 10,000 people · ton, respectively. To conduct the sensitivity analysis, the initial link capacity of each case is multiplied by a constant ranging from 0.50 to 2.50 with 0.25 increment.

Fig. 10 displays the effect of the link capacity on the test cases, where the total cost and total risk objectives of each case under all LCLs are also included in one double Y-axis graph. As shown in Fig. 10, the two cases under the two objectives are all monotonically decreasing but with different correlations with regard to LCL. For the hypothetical case, as viewed in Fig. 10 (a), the total cost objective decreases rapidly when LCL is less than 1.25 and maintains stable afterward. To obtain the truly minimal total cost, the initial link capacity needs to be increased by at least 25%. Instead, LCL has no significant effect on the total risk objective. The truly minimal total risk can be obtained by even decreasing the initial link capacity by 25%. For the realistic case, as observed in Fig. 10(b), the total cost objective seems to be insensitive to LCL. The truly minimal total cost can



Fig. 9. Effect of the node capacity on the test cases.



Fig. 10. Effect of the link capacity on the test cases.

be received under the initial link capacity. Instead, the total risk objective depends significantly on LCL. The initial link capacity should be enlarged by at least one time to seek the truly minimal total risk. We conclude that the link capacity moderately determines the solution quality of our problem. Decision makers should always control this parameter under a publicly acceptable value to obtain an implementable waste management system.

6. Conclusions

This paper develops improved approaches to the network design problem in the regional hazardous waste management system. The problem is defined as a multi-objective problem on a system with multiple types of hazardous waste, waste facility and treatment technology. Many practical constraints in the management of hazardous wastes including the node capacity, link capacity and comparability requirement are modelled simultaneously. A multi-objective mixed integer linear programming model to minimize jointly the total cost and total risk is formulated to incorporate all location, allocation and routing decisions of the problem. The model is more general than many existing models in the literature. Its single objective version can be solved efficiently by commercial optimization software. A weighted-sum approach, an augmented weighted Tchebycheff approach and an augmented ε -constraint approach are customized to approximate well the non-dominated frontier, respectively. The computational results of two case studies show that the last two approaches, even firstly applied to our problem, perform better than the first approach widely employed by many previous works.

Our paper can be extended in several directions. Firstly, as can be observed from the case studies, some non-dominated solutions of the large-scale realistic case are still identified in a relatively long time. Although the computation time is not a critical issue in our problem, from a practical perspective each non-dominated solution should be found as quickly as possible, since the non-dominated frontier is approximated by many solutions. Hence, one future research will be dedicated to identify a non-dominated solution in a shorter time by developing more compact models or more efficient algorithms. Then, as mentioned, our problem is characterized as a multi-objective problem, of which the solution should be optimized from different perspectives. The proposed model considers two objectives in respect of the cost and risk. Another future research would be to introduce more objectives (i.e. equity) into the problem, and evaluate the applicability of the customized multi-objective optimization approaches. Moreover, from a strategic level perspective, the tactical vehicle routing decision is not considered in our problem. Many related works show that integrating the location and routing decisions in a logistic system can improve the system significantly. The third possible further research is to incorporate the vehicle routing decision into our problem, and develop practically implementable solution models and algorithms.

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