



A simple reservation and allocation model of shared parking lots



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ABSTRACT

With increasing auto demands, efficient parking management is by no means less important than road traffic congestion control. This is due to shortages of parking spaces within the limited land areas of the city centers in many metropolises. The parking problem becomes an integrated part of traffic planning and management. On the other hand, it is a fact that many private parking spots are available during daytime in nearby residential compound because those residents drive their cars out to work. These temporarily vacant parking lots can be efficiently utilized to meet the parking demand of other drivers who are working at nearby locations or drivers who come for shopping or other activities. This paper proposes a framework and a simple model for embracing shared use of residential parking spaces between residents and public users. The proposed shared use is a winning strategy because it maximizes the use of private resources to benefit the community as a whole. It also creates a new business model enabled by the fast-growing mobile apps in our daily lives.

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1. Introduction

Parking in downtown areas (or residential areas) is often a headache for both commuters and traffic managers in most large cities. Finding a parking spot and walking to work often constitutes an appreciable fraction of the total travel time. The desperate scramble for parking spots also adds to the problems of chronic congestion and choking pollution. It is also often the case that parking fees may exceed the total vehicle operating costs. The problem cannot be solved simply by continuing the construction of new parking facilities due to growing population and car ownership, particularly in cities like Hong Kong due to its high density of traffic and limited road and parking capacity. As found in the study by Shoup (2006), 30% of traffic congestion in road networks is caused when people are circulating around to find a parking spot, and about 8.1 min is spent in finding a parking spot. Ayala et al. (2011) find out that every year in Chicago, there is 63 million miles for vehicles to travel in order to find a vacant space to park, which generates 48,000 tons of carbon dioxide to the environment.

Parking management is normally considered as an integrated part of travel demand management. From this perspective, levying some road toll to the vehicle (Glazer and Niskanen, 1992; Verhoef et al., 1995; Arnott and Inci, 2006; Zhang et al., 2008 and Qian et al., 2011) is considered to be effective for simultaneously mitigating traffic congestion and regulating parking demand. Zhang et al. (2011), Yang et al. (2013) and Liu et al. (2014a,b) find out that parking reservation through

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parking permits distribution and trading are efficient in traffic management. Particularly, it is found that an appropriate combination of reserved and unreserved parking spots can temporally relieve traffic congestion at the bottleneck and hence reduce the total system cost, because commuters without a reserved parking spot are compelled to leave home earlier in order to secure a public parking spot. A recent review of parking modeling was given by [Inci \(2015\)](#) and a few latest studies of parking problems in a network context can be found in [Boyles et al. \(2015\)](#) and [Zou et al. \(2015\)](#).

To reduce the wastage of money and time in parking search, various smart parking systems are developed and implemented worldwide to make efficient utilization of these existing parking spots through optimal allocation of the limited parking resources by a management center. [Park et al. \(2008\)](#) and [Panja et al. \(2011\)](#) argued that the available parking spots of each parking lot should be collected to guide the commuters to the low occupancy parking areas which is so called infrastructure-to-vehicle (I2V). In fact, with the fast development of smart mobile and wireless communication techniques, various applications (apps) emerge in recent years, which effectively mitigate information uncertainty on parking availability. In San Francisco, SFPark (<http://sfpark.org/>) puts all the information online for people to park and also uses demand-dependent parking fees to coordinate the traffic congestion in different areas. [Geng and Cassandras \(2013\)](#) propose a novel 'smart parking' system which is based on resource allocation and reservation both on street and off street and show that when informed, people will save quite a lot time and money and the utilization of the parking spots will increase as well, and [Chen et al. \(2015\)](#) discuss a smartphone-based parking reservation system to manage the limited amount of parking spaces located in a downtown area.

Shared parking (also known as gaparking from gap and parking; From Wikipedia, the free encyclopedia) emerged recently as a new notion of making more efficient use of parking facilities like for example, roverparking.com, spot-park.com, just-park.com, parkhound.com.au, parkcirca.com, etc. It uses existing gaps or spaces intended for parking cars when the owner is not using it. Availability of parking gaps for others stems from the fact that most parking spaces are only used part time by a particular driver or owner who lives in one location and works in other, and the utilization and availability patterns follow predictable daily, weekly and annual cycles. By making private parking space publicly available for rent, shared parking not only allows the owner to make additional money but also helps alleviate the aforementioned shortage of parking spaces.

Realizing the above new business opportunity and model particularly in an area with mixed commercial and residential land use developments, parking lot management companies intend to temporarily repurchase some private parking lots and sell them to public users during certain time of a day. [Guo et al. \(2016\)](#) is the first one to develop a simulation-optimization based decision method to determine the repurchase strategy. A Gaussian mixture model is proposed to describe the time-varying arriving/departing behaviors of drivers and meanwhile the stochastic constraints of the profit maximization problem are formulated. The expected optimal repurchase amounts and parking time are estimated via simulation optimization.

With the revolution of information and communications technology and especially the latest rise of the mobile internet, shared parking or gaparking can be simply enabled through an "e-parking platform" that can be added to the fast-growing mobile apps in our daily lives. The e-booking platform can directly connect a private parking space owner who gaparks his property when not in use with people who is searching for a place to park for certain hours. Alternatively, the e-parking platform can serve as an intermediary to collectively gather the daily or weekly availability information of private parking lots through a long-term contract with their owners. These rentable parking lots are then made available through the platform for advanced booking by those who wish to park by completing an e-form with the date and time. It works like "online-to-offline commerce" pattern, which is the most popular trend of entrepreneurship at present called O2O. A business strategy that attracts some potential consumers from online channels to offline stores like "taobao" in mainland China a transaction e-platform operated by Alibaba. Online-to-offline commerce, or O2O, identifies customers when they are browsing a webpage through some internet advertisements or promotions, and then uses a variety of attractions and approaches to solicit the customers to jump to their homepage. This type of strategy incorporates techniques used in online marketing with those used in brick-and-mortar marketing.

Like the abovementioned other intermediaries, gaparking through an e-parking platform offer the following advantages: (1) pre-allocation but booking in advance removes a degree of uncertainty between the private parking lot owners (abbreviated as O-users) and the public users (abbreviated as P-users). P-users can save time as they have a guaranteed reserved parking lot at a certain location and thus do not have to cruise for parking; yet P-users is able to choose a convenient parking lot near their destination. The O-users, on the other hand, have a guarantee of their parking lot availability upon their prescheduled return (of course, the e-parking operator can always set an occupancy buffer of each parking lot in parking allocation and/or set aside a few public parking lots to mitigate the risk of parking conflicts). (2) The P-users can save money in comparison with using commercial car parks and may gain additional discounts by booking ahead of time; By joining and selling their parking lot to the platform, the O-users can earn a certain amount of money at a small cost (e.g., due to inconvenience). (3) By charging a service fee for parking acquisition and booking, the e-parking platform operator can earn operating income (revenue net of all operating costs inclusive of the cost for protection guarantee of both O-users and P-users).

In this paper we consider advanced booking and allocation of shared parking lots. Suppose the e-parking platform operators already acquire a certain number of rentable private parking lots, each with a prespecified available time window of a day (parking supply). The platform receives requests for gaparking, each with a specific entry and leave time (parking demand). We propose a simple binary integer linear programming models to allocate the requests to specific parking lots so as to maximize the parking lot utilization or accommodate as many requests as possible under parking space and time constraints. It can be also regarded as revenue maximization under given demand and supply as well as and preset parking charge.

The rest of the paper is organized as follows. Section 2 presents a binary integer linear programming for an ex-ante optimal parking lot allocation. Section 3 provides a binary integer linear programming under the first-book-first-served basis. In Section 4, some metrics are introduced to evaluate system performance under the proposed parking allocation schemes. A numerical example is provided in Section 5 and conclusions and suggestions for future research are given in Section 6.

2. Parking lot allocation based on global demand information

Suppose the available time windows of a certain number of gaparking lots are provided by the O-users and the requests are submitted by the P-users both in advance (for example, at least one day earlier) through the e-parking platform. Then, based on the global parking demand information gathered beforehand, the platform operator allocates the available parking spots to those P-users who request parking spots by solving a binary integer linear programming problem, taking into account both the profits and the negative effects of request rejection. P-users will be noticed of their requests well ahead of time (for example, the day before the requested parking service) so that those whose requests are rejected have the opportunity to plan for using other alternatives such as commercial parking (we will discuss later how to address the concern of request rejection). We further assume that these parking lots are all in the same residential area, and P-users have no preference for specific parking lots.

For simplicity, the daily planning time period of interest is divided into a number of intervals (for example, 30 min each interval). Let K be the total number of intervals and $k = 1$ be the starting time interval (e.g., 9:00–9:30 AM) and $k = K$ the ending time interval (e.g., 5:30–6:00 PM). Let N be the total number of the resources (parking spots), we introduce a binary indicator, s_{nk} which is defined to be 1 when parking lot n is available in time interval k and 0 otherwise. Thus we have the parking supply matrix $S_{N \times K} = [s_{nk}]$, $n = 1, 2, \dots, N; k = 1, 2, \dots, K$.

Let M be the total number of requests (parking demand) and t_m^s and t_m^e be start and end time interval of request m , $t_m^s, t_m^e \in [1, K]$, $m = 1, 2, \dots, M$. The duration for request m is thus $t_m^e - t_m^s + 1$. We further introduce another binary indicator r_{mk} , which is defined to be 1 if parking request m includes time interval k and 0 otherwise. Thus we have the parking demand request matrix $R_{M \times K} = [r_{mk}]$, $m = 1, 2, \dots, M; k = 1, 2, \dots, K$.

2.1. Parking lots with different available time gaps

We begin with the general case when the available gaps of rentable parking lots can be different. In this case we need to introduce a binary decision variable x_{mn} where $x_{mn} = 1$ if request m is accepted for parking at lot n , and $x_{mn} = 0$ otherwise. Then we have a binary matrix $X_{N \times M} = [x_{nm}]$, $n = 1, 2, \dots, N, m = 1, 2, \dots, M$. From the parking demand request matrix $R_{M \times K}$ and request decision matrix $X_{M \times N}$, we have the following parking lot occupancy matrix, $Z_{N \times K} = [z_{nk}] = X_{N \times M} \times R_{M \times K}$, or in detailed components,

$$z_{nk} = \sum_{m=1}^M x_{nm} \cdot r_{mk}, \quad n = 1, 2, \dots, N, \quad k = 1, 2, \dots, K \tag{1}$$

Clearly, $z_{nk} = 1$ means that parking lot n is occupied in time interval k and $z_{nk} = 0$ means it is vacant, $n = 1, 2, \dots, N; k = 1, 2, \dots, K$.

Let p_s and p_b denote the selling price (charge rate per interval of parking of P-users) and buying price (purchase price of parking lot from O-users), the platform operator aims at maximizing operating revenue while attempting to minimize the loss due to request rejection, which may have long-term negative impact on e-parking demand. For simplicity, we assume that the selling and buying prices are predetermined and constant across all intervals, and we further assume that the e-parking platform involves a certain fixed investment cost but the variable operating cost can be negligible. Since we buy these parking lots in the residential area, the parking lots are in the same location other than scattered. Thus if the users want to park in this area, he/she may have no preference on a certain parking lot. Then, the shared e-parking reservation and allocation problem can be formulated as the following binary integer linear program:

$$\max p_s \cdot \sum_{n=1}^N \sum_{k=1}^K z_{nk} - p_b \cdot \sum_{n=1}^N \sum_{k=1}^K s_{nk} - \mu \cdot \left(M - \sum_{m=1}^M \sum_{n=1}^N x_{mn} \right) \tag{2}$$

subject to

$$\sum_{n=1}^N x_{mn} \leq 1, \quad m = 1, 2, \dots, M \tag{3}$$

$$z_{nk} \leq s_{nk}, \quad n = 1, 2, \dots, N; \quad k = 1, 2, \dots, K \tag{4}$$

$$x_{mn} \in \{0, 1\}, \quad n = 1, 2, \dots, N, \quad m = 1, 2, \dots, M \tag{5}$$

where z_{nk} is defined by Eq. (1).

In the objective function (2), the first term represents the total revenue generated from charging the P-users; the second term is the total cost for purchasing the parking lots from O-users; and the third term represent the penalty that is added to account for the long-term loss due to request rejection, where $(M - \sum_{m,n} x_{mn})$ is the total number of unaccepted parking requests and μ is a preselected positive parameter. In the constraint set, constraint (3) simply indicates that any request should be assigned to at most one parking lot (intermediate parking lot relocation is not allowed or impossible); inequality (4) guarantees that the each parking lot can accommodate only one car in each time interval; constraint (5) simply implies x_{mn} is a binary decision variable.

2.2. Parking lots with identical available time gaps

In the special case when all rentable parking lots are available for the same period, the reservation and allocation problem can be greatly simplified. In this case, we only need to decide whether a parking request of a P-user is accepted or not. Here, with a slight abuse of notation, a binary variable x_m is used, $x_m \in \{0, 1\}$, $m = 1, 2, \dots, M$. $x_m = 1$ if request m is accepted, and 0 otherwise. Because of the homogeneity of all rentable parking lots, it is immaterial how to allocate an available parking lot to a specific, accepted request. The problem can be formulated as the following binary integer linear program:

$$\max p_s \cdot \sum_{m=1}^M \sum_{k=1}^K x_m r_{mk} - p_b \cdot N \cdot K - \mu \cdot \left(M - \sum_{m=1}^M x_m \right) \quad (6)$$

subject to

$$\sum_{m=1}^M x_m r_{mk} \leq N, \quad k = 1, 2, \dots, K \quad (7)$$

$$x_m \in \{0, 1\}, \quad m = 1, 2, \dots, M \quad (8)$$

Inequality (7) simply means that the accepted parking demand cannot exceed the parking supply in each time interval. The objective function value in (6) is contingent on the total parking supply and the final accepted requests. In this case, P-users are only informed of whether their requests are approved or not. P-users with accepted requests can park their cars in (guaranteed) available parking lots upon their arrivals during their prescheduled parking duration.

3. Parking lot allocation based on first-book-first-serve

For the purpose of comparison, we consider a solution on the basis of first-book-first-serve (FBFS) for a given set of rentable parking lots with different available time gaps examined in Section 2.1. P-users send their requests through the e-parking platform and their requests are processed on the basis of FBFS, they will receive confirmation of acceptance immediately but their lot assignments will be noticed later after all accepted requests are processed.

For consistence of comparison, we suppose we have the same number of M potential requests as considered earlier but they reach the e-parking platform randomly. Suppose we already accepted m' requests when request m is received and denote the approved requests by r_{mk} where $\tilde{m} \in [1, m']$. This means that $m - m' - 1$ requests are already rejected and we now determine whether request m should be accepted or not. This can be done by simply checking the availability of parking lots for the requested parking intervals.

$$X_{n \times (m'+1)} \times \begin{pmatrix} R_{mk} \\ r_{mk} \end{pmatrix} \leq S_{nk} \text{ subject to } \sum_{n=1}^N x_{nm} = 1, \quad m = 1, 2, \dots, m' + 1$$

Request m is accepted if the above inequality holds for all its requested time intervals and rejected otherwise. This FBFS-based processing is continued until a full booking is reached and no more requests can be entertained regardless of their parking durations. It should be noted that the above feasibility condition applies for the set of rentable parking lots with or without identical parking gaps. For real-time (first-come-first-serve) e-parking management, the e-parking platform just needs to update supply matrix and provide real-time information on parking lot availability for specific time periods, and a P-user can park his/her car if available upon arrival, without advanced booking through the platform.

In the real world, usually, the combination of global demand and FBFS might be more reasonable. In this case, we can set a proper time period to allocate the parking lots. For example, in every two hours, we make a decision on the requests received during the last period (following global demand) and send notification to the P-users of whether their requests are approved or not (following FBFS).

4. System performance metrics

System performance metrics of interest to the e-parking platform operator include net revenue (profit) and acceptance (or rejection) rates of parking requests. The net revenue is given in the objective function (2) (the total revenue generated from parking charge minus the total acquisition cost of parking lots and minus the long term loss because of rejection).

$$\pi = p_s \cdot \sum_{n=1}^N \sum_{k=1}^K Z_{nk} - p_b \cdot \sum_{n=1}^N \sum_{k=1}^K S_{nk} - \mu \cdot \left(M - \sum_{m=1}^M \sum_{n=1}^N X_{mn} \right) \tag{9}$$

The acceptance rate of parking requests is denoted by θ and given below after final parking allocation.

$$\theta = \frac{1}{M} \sum_{n=1}^N \sum_{m=1}^M X_{nm} \tag{10}$$

where $\sum_{n,m} X_{nm}$ that we have mentioned in the objective function is the total number of requests accepted. Parking utilization rate, ϑ , is also a useful indicator, which is defined as the ratio of the total occupied parking hours to the total supplied or acquired parking hours.

$$\vartheta = \frac{\sum_{n=1}^N \sum_{k=1}^K Z_{nk}}{\sum_{n=1}^N \sum_{k=1}^K S_{nk}} = \frac{\sum_{n=1}^N \sum_{k=1}^K \left(\sum_{m=1}^M X_{nm} \cdot r_{mk} \right)}{\sum_{n=1}^N \sum_{k=1}^K S_{nk}} \tag{11}$$

Clearly, the above performance metrics, particularly the parking utilization rate, depend on the amounts of parking lot supply and parking demand requests and their (start and end) time distribution characteristics. To measure the degree of temporal matching between parking demand and supply, we now define the following temporal parking intensity

$$\rho_k = \frac{D_k}{S_k}, \quad 1, 2, \dots, K \tag{12}$$

where $D_k = \sum_{m=1}^M r_{mk}$ and $S_k = \sum_{n=1}^N S_{nk}$ denote the total numbers of parking requests and rentable parking lots respectively in time interval k . Then mean $E[\rho] = \frac{1}{K} \sum_{k=1}^K \rho_k$ and variance $var[\rho] = \sqrt{\frac{1}{K} \sum_{k=1}^K (\rho_k - \bar{\rho})^2}$ of temporal parking intensity are considered to be rough measures of the degree of matching between the two matrices $R_{M \times K} = [r_{mk}]$ and $S_{N \times K} = [S_{nk}]$ of parking demand and supply. Clearly, $E[\rho] = 1$ and $var[\rho] = 0$ represent the case of perfect matching.

5. Numerical experiments

In this section, we conduct numerical experiments to illustrate the models and results and try to gain some useful insights. Suppose the modeling time interval is 1 h and the modeling period is $K = 8$ (h) starting from 9:00 AM and ending at 5:00 PM on a typical day. In the basic case we suppose a total number of parking lots $N = 100$ are purchased from the O-users, and the available time gap of each parking lot ranges from either one of the first two intervals to either one of the last two intervals. The constant unit parking fare is assumed to be $p_s = 10$ (HK\$/h) per vehicle and the purchase cost is $p_b = 5$ (HK\$/h) per lot, and the penalty factor is $\mu = 5.0$. Furthermore, suppose, in any minute during the whole modeling period, the arrival of P-users follows a Poisson distribution, and parking duration follows a negative exponential distribution, as usually considered in the literature (Richardson, 1974; Cleveland, 1963; Blunden, 1971). The average parking duration time is assumed to be $T = 3.0$ (h). To conduct sensitivity analysis, the parking demand or the number of daily parking requests is varied from 0 to 1000.

From Figs. 1–3, we can see that, with the increase in the number of parking requests, the profit, π , of e-parking platform operator initially increases from a negative value (parking revenue generated falls short of the parking purchase cost) in a linear way at a rate of 29 (HK\$) per lot (approximately equal to the average parking duration $T = 3.0$ (h) multiplied by the parking charge rate of $p_s = 10$ (HK\$/h)), and then reaches a maximum value in a nonlinear way. The nonlinearity stems

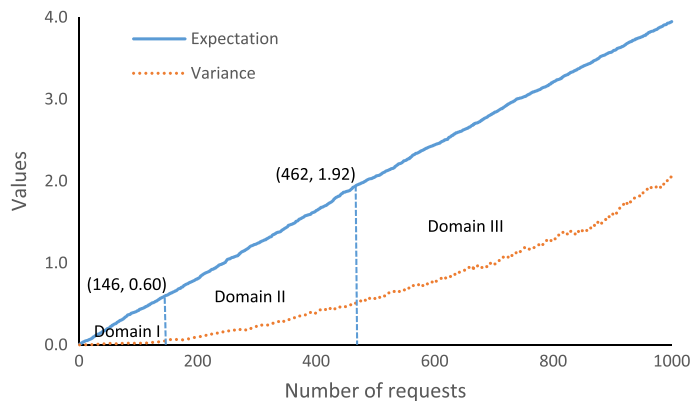


Fig. 1. Change of $E[\rho]$ and $var[\rho]$ with number of parking requests M .

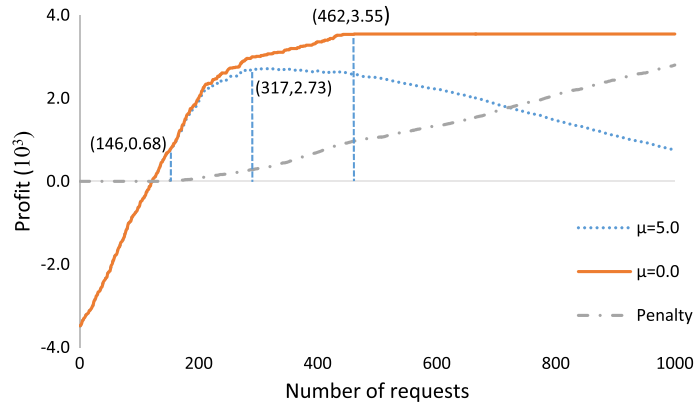


Fig. 2. Change in profit with the number of requests under a fixed supply of 100 lots.

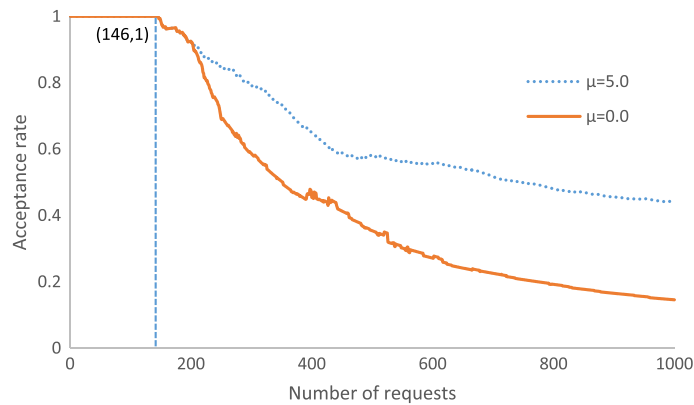


Fig. 3. Change in acceptance rate with the number of requests under a fixed supply of 100 lots.

when the requests reach 146, from the fact that some parking requests are selectively accepted while others are rejected for profit maximization via the optimal allocation model. With exclusion of the penalty term, the maximum profit is achieved at an optimal number of requests of $M^* = 462$. Beyond $M^* = 462$, the profit stays at the maximum value, which means further subsequent parking requests are all rejected. With inclusion of the penalty term, the maximum profit is achieved at a smaller optimal number of requests of $M^* = 317$; and then decreases nonlinearly with M . Beyond a certain number of parking requests, all additional parking requests are rejected, and as a result, the profit decreases linearly with M , with a decreasing rate equal to μ due to the negative effect of request rejection.

Based on the above observations from Figs. 1–3, for a given parking supply, parking demand can be divided into three domains in terms of the change of system performance metrics.

Domain I (under-demand). In this domain, parking demand is much less than parking supply, and parking space constraint is nonbinding. As a result, the acceptance rate of parking requests $\theta = 100(\%)$, and the profit π increases linearly at a rate of $p_s \cdot T$.

Domain II (moderate-demand). In this domain, parking demand is comparable with parking supply. Due to parking lots constraint, a portion of requests are rejected for profit maximization. The total profit fluctuates within this domain, because additional parking requests, on one hand, increase the utilization rate and thus the revenue, and on the other hand, results in more rejected requests and thus more negative impact.

Domain III (over-demand). In this domain, parking demand far exceeds parking supply, and all parking spaces are fully utilized or parking gaps are fully filled up. The profit keeps staying at maximum in the absence of penalty term, and decreases linearly with a slope of μ .

For each combination of demand or supply, we can find the maximum profit via solving the optimal allocation model (2)–(8). Fig. 4(a) and (b) plot the contours of profit in the two-dimensional space of parking demand and supply with and without inclusion of the penalty term, respectively. From the figures, we can find out the optimal ratio of N/M . This optimal ratio is useful because it shows the number of parking lots that the platform operator should purchase for a given or predicted number of parking requests. Comparing Fig. 4(a) and (b), we can see that the optimal ratio changes slightly, which indicates that

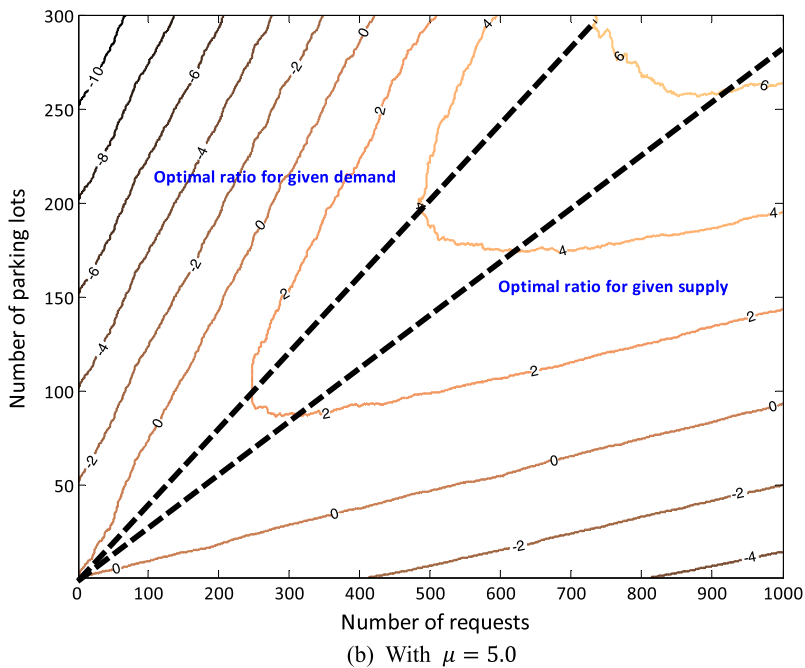
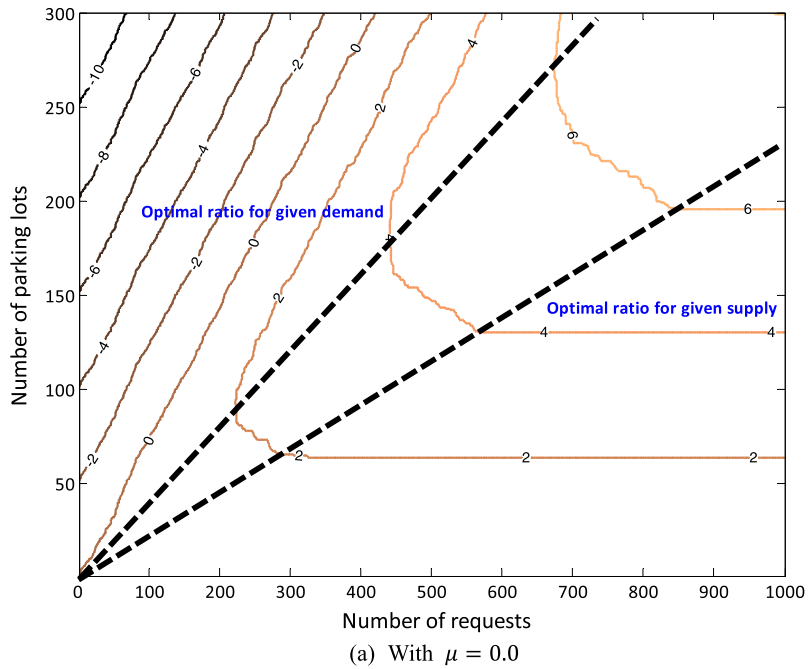


Fig. 4. Change of profit (10^3) in the two-dimensional space of parking demand and supply.

the penalty factor has little impact on the optimal number of parking lots for given demand. However, for a given supply, the slope will be gentle when the penalty is exempted, because the peak point lags behind that with penalty in Fig. 2.

We move to make a comparison between the optimization-based allocation (OA) with the model (2)–(5) and the allocation according to the FBFS. Figs. 5 and 6 portray the profit and parking utilization rate against number of parking requests. In the two figures, the results are extended up to $M = 1200$ parking requests (all other parameter values remain unchanged). For simplicity and clarity of comparison, we ignore the penalty term of parking request rejection ($\mu = 0.0$). Also, the purchase cost of parking spots is set to be zero simply because it is identical under both OA and FBFS allocation.

In Fig. 5, it is obvious that the profit under OA is not lower than that under FBFS without optimization. We also find that the profit reaches its maximum value much earlier by using OA with optimization; it only takes about 411 requests to fully

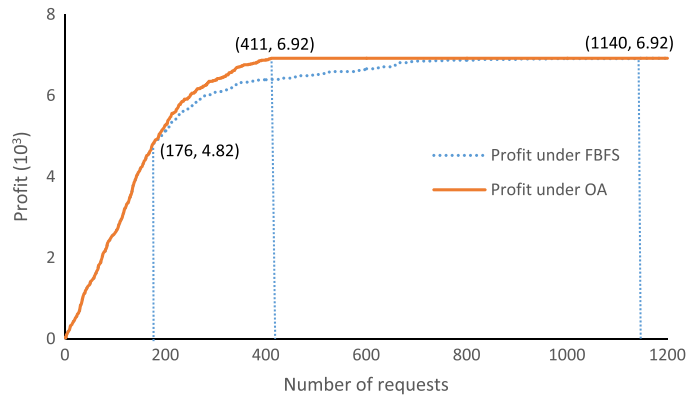


Fig. 5. Comparison of profits between OA and FBFS.

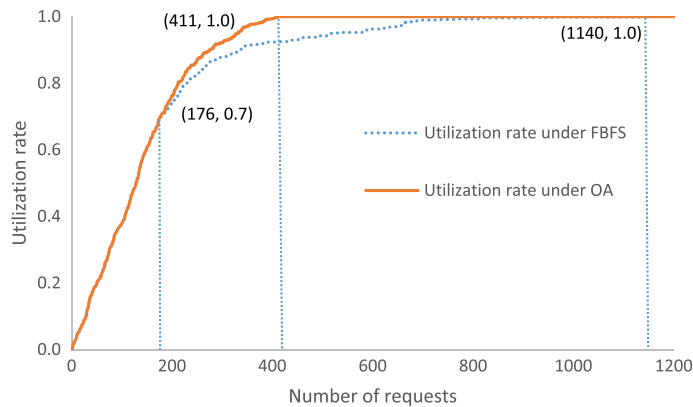


Fig. 6. Comparison of parking utilization rates between OA and FBFS.

utilize all the parking lots. However, on the basis of FBFS, much more requests (1140) need to be collected to make full use of the available parking lots, which implies a much wider Domain II under FBFS. This is because some small parking gaps such as one hour available time slots have to be filled up by appropriate parking requests out of a large number of requests. This is clearly seen from the parking utilization rates plotted in Fig. 6. Moreover, the combination of OA and FBFS falls in between these two lines. As we can see, these are two extreme cases. Under OA, we make final assignment based on perfect and global information. Thus, platform revenue reaches the maximum. However, under FBFS, decisions are made once a request is received, without accounting for the global request information, resulting in the lowest platform revenue.

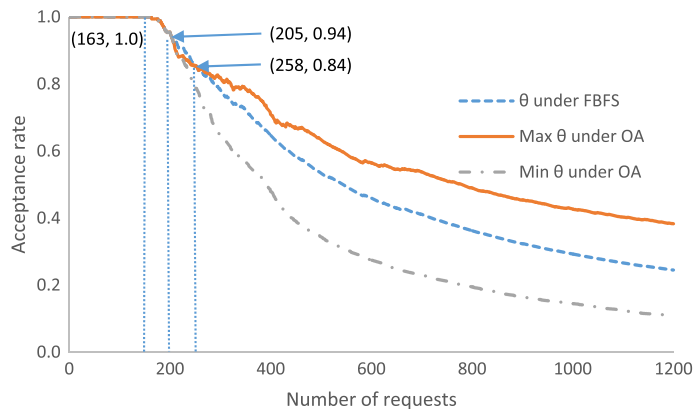


Fig. 7. Comparison of acceptance rates between OA and FBFS.

We further look into acceptance rates of parking requests shown in Fig. 7. With OA method, the solution of accepted parking requests for profit maximization is not unique. For example, accepting one 6-h parking request could be equivalent to accepting two 3-h parking requests but the resulting acceptance rates are different. For a given number of parking requests, we can find out the max (with more requests for short parking duration) and min (with more requests for long parking duration) acceptance rates that give rise to the same profit. As shown in Fig. 7, the acceptance rate equals 1 within Domain I, which has an upper bound of 163. We note that upper bound of Domain I is identical for both OA and FBFS based methods. The reason is simple. If the subsequent request after the upper bound of Domain I cannot be accepted under FBFS, or no appropriate gap is available for whatever allocation adjustment of all accepted requests, then the request cannot be accepted via the OA either without rejecting the former accepted request.

The acceptance rates then drop quickly within Domain II & III. Within domain III, the acceptance rate under FBFS is always between the max and min acceptance rates by the OA. It is interesting to note that within the interval [205, 258], which is located within Domain II, the acceptance rate under the FBFS is even greater than the max value by the OA method. This can be explained below. First, under the FBFS, an accepted request cannot be rejected. Second, the OA based method selects requests after all requests are gathered, which means some requests accepted under FBFS can be rejected by the OA method. Therefore, for profit (or utilization) maximization, the OA method may accept a late request of longer duration by rejecting multiple early requests of shorter durations, resulting in a lower acceptance rate.

Finally, we make a comparison among different average parking durations.

In Fig. 8, the parameters are the same with those above, the only difference is that the average parking duration time varies by taking the three values of 2, 3 and 4, respectively. For simplicity and clarity of comparison, we ignore the penalty term of parking request rejection ($\mu = 0.0$). We can see that under different average parking durations, the profit curves with number of requests diverge before all reaching their maximum value. The profit initially increases at a rate, which is approximately equal to the average parking duration time multiplied by the parking charge rate of $p_s = 10$ (HK\$/h) in domain I. More specifically, the longer the average parking duration is, the earlier the profit achieves the peak point. When the average parking duration is small (e.g., 2), we can see 1000 requests are not enough to reach the maximum profit.

6. Conclusion and future research

This paper examined the shared parking or gaparking problems of private and rentable parking lots through an e-parking platform, a simple binary integer linear programming model is proposed to accept and allocate certain parking requests to specific parking lots for maximizing profit or parking lot utilization under parking space and time constraints. The optimization based allocation method is compared with the first-book (or come)-first-serve based method with numerical experiments. The proposed shared parking strategy makes use of private resources at nearly zero cost to benefit the community as a whole; it also offered a potential business model enabled by the fast-growing mobile apps in our daily lives.

In the future, we plan to further our study in the spirit of a two-sided market with the e-parking platform connecting the O-users and P-users (Caillaud and Jullien, 2003; Rochet and Tirole, 2004; Armstrong, 2006). In this case, we assume that each market side is characterized by a demand function $M = D^s(p_s)$ and $N = D^b(p_b)$, respectively. The platform operator chooses a unit parking time price p_s to charge the P-users for revenue and offer a proper reward p_b to O-users for offering their rentable parking spots. A combination of (p_s, p_b) will be determined to maximize platform profit while balancing the parking demand and supply. In addition, the model can be further extended by considering the spatial effects of parking lots and P-users' preference for their convenient parking lots.

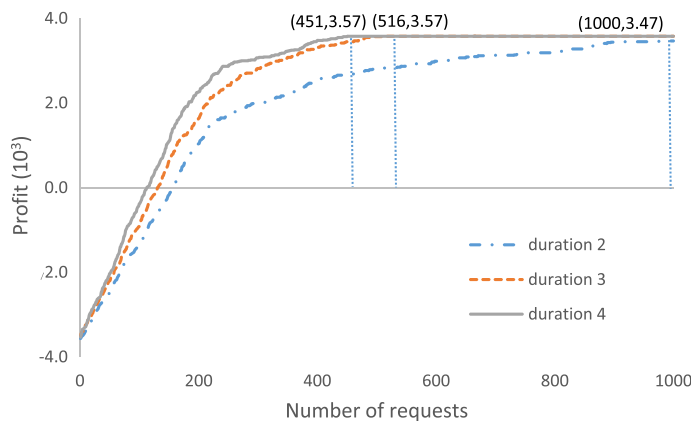


Fig. 8. Comparison of profits among different average parking durations.

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