



# Influence of connected and autonomous vehicles on traffic flow stability and throughput



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## ABSTRACT

The introduction of connected and autonomous vehicles will bring changes to the highway driving environment. Connected vehicle technology provides real-time information about the surrounding traffic condition and the traffic management center's decisions. Such information is expected to improve drivers' efficiency, response, and comfort while enhancing safety and mobility. Connected vehicle technology can also further increase efficiency and reliability of autonomous vehicles, though these vehicles could be operated solely with their on-board sensors, without communication. While several studies have examined the possible effects of connected and autonomous vehicles on the driving environment, most of the modeling approaches in the literature do not distinguish between connectivity and automation, leaving many questions unanswered regarding the implications of different contemplated deployment scenarios. There is need for a comprehensive acceleration framework that distinguishes between these two technologies while modeling the new connected environment. This study presents a framework that utilizes different models with technology-appropriate assumptions to simulate different vehicle types with distinct communication capabilities. The stability analysis of the resulting traffic stream behavior using this framework is presented for different market penetration rates of connected and autonomous vehicles. The analysis reveals that connected and autonomous vehicles can improve string stability. Moreover, automation is found to be more effective in preventing shockwave formation and propagation under the model's assumptions. In addition to stability, the effects of these technologies on throughput are explored, suggesting substantial potential throughput increases under certain penetration scenarios.

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## 1. Introduction

Cities have gone through numerous transformations through time. Transportation systems and technologies have been an integral part of these transformations. The past two decades have seen substantial integration of advances in wireless communication, processing power, and sensing technologies into traffic management systems, with the goal of enhancing mobility, sustainability, safety, and reliability of these systems. The next major wave of technological innovation is seeking to impact the system through vehicle-based innovation. In particular, autonomous vehicles have been prototyped with substantial advances in sensing technologies and associated pattern recognition and control intelligence, while pervasive

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wireless communication technologies provide the opportunity to create an internet of vehicles where individual vehicles can communicate with other vehicles (through Vehicle-to-Vehicle communications) and infrastructure (through Vehicle-to-Infrastructure communications). Consequently, virtually all aspects of drivers' decision making, from strategic to operational decisions, would be impacted and generally enhanced. At the operational level, these technologies are intended to help drivers and vehicles make safe and reliable decisions about acceleration choice and executing lane-changing maneuvers. It is important to note that these two types of communications (Vehicle-to-Vehicle and Vehicle-to-Infrastructure communications) can also improve the efficiency and reliability of operating autonomous vehicles.

The driving environment and associated driver-vehicle behavior are expected to change with the introduction of connected and autonomous vehicles. Human-driven vehicles and autonomous vehicles have different driving logics. Humans have higher reaction time compared to robots; and therefore, to overcome the uncertainty associated with human decisions, they should consider more decision variables (Treiber et al., 2007). In the context of driving, a human driver not only takes into account the behavior of the immediate leader, but he/she also monitors the behavior of several vehicles ahead (possibly the entire traffic stream ahead) (Treiber et al., 2007). This method can result in a more stable car-following behavior (Treiber et al., 2007).

From the modeling standpoint, capturing the effects of these technologies on driving and car-following stability is a challenging task. A major part of driving-related decisions correspond to the acceleration choice. Consequently, acceleration behavior has been studied extensively in the literature and several models with different levels of complexity have been introduced to capture the underlying processes of acceleration decision making (Chandler et al., 1958; Gazis et al., 1959; Gipps, 1981; Hamdar and Mahmassani, 2009; Herman et al., 1959; Talebpour et al., 2011; Yang and Koutsopoulos, 1996). However, previous studies did not clearly present the role of communication in operating connected and autonomous vehicles, and most of the efforts focused on specific applications of connected and autonomous vehicles technologies (e.g. Cooperative Adaptive Cruise Control or Automated Highway Systems). Moreover, these studies did not investigate the interactions between autonomous vehicles, human-driven vehicles with connectivity (these vehicles are called "connected vehicles" in this paper), and human-driven vehicles without connectivity (these vehicles are called "regular vehicles" in this paper) in detail. Therefore, there is a need for a comprehensive acceleration framework to model this new driving environment and capture the interactions between different vehicle types. This study presents such a comprehensive acceleration framework to model this driving environment with regular, connected, and autonomous vehicles. This framework uses different acceleration models with different assumptions to model regular, connected, and autonomous vehicles. A reliable acceleration modeling framework should offer stability. Two types of car-following stability has been identified in the literature (Treiber and Kesting, 2013; Wilson and Ward, 2010): local stability and string stability. Local stability refers to the vehicle's response to its leader's acceleration decisions. It is achieved if a (following) vehicle recovers from a perturbation and retains the original steady-state speed and spacing after deviating from it (this deviation, for instance, can be caused by the leader's sudden break). String stability is defined for a platoon of vehicles and investigates the behavior of the entire platoon in response to its leader's sudden break. If the perturbation decays as it propagates upstream within the platoon, the car-following behavior is called string stable. Since this acceleration framework is based on well-established car-following models, local stability is expected to hold for this framework. Consequently, the main focus of the present study is to investigate the string stability of traffic flow under different market penetration rates of connected and autonomous vehicles. Accordingly, both analytical and simulation-based analyses of string stability of this acceleration framework are performed.

Moreover, through an extensive simulation effort and by investigating the effects of autonomous and connected vehicles on throughput, and on the scatter in the fundamental diagram of traffic flow, this study shows that this framework is capable of capturing the interactions between different vehicle types. These simulations explore possible changes in throughput and structure of the fundamental diagram under different market penetration rates of connected and autonomous vehicles. Consequently, the main contribution of the present study is to utilize these findings to investigate the possible impacts of connected and automated vehicles on traffic flow and string stability.

The remainder of this paper is organized as follows: Section 2 presents a review of the efforts to model connected and autonomous vehicles. Section 3 discusses the possible effects of connected and autonomous vehicles on driving environment. Section 4 presents the acceleration framework and the logic behind selecting each model in this framework. Analytical and simulation-based investigations of string stability under different market penetration rates of connected and autonomous vehicles are offered in Section 5. It is followed by a simulation-based analysis of throughput under different market penetration rates of connected and autonomous vehicles using the proposed acceleration framework in Section 6. The paper is concluded with some summary remarks and future research needs in Section 7.

## 2. Background

Extensive effort has been devoted to model drivers' car-following behavior since the introduction of the General Motor (GM) stimulus-response models (Chandler et al., 1958; Gazis et al., 1959; Herman et al., 1959). However, most of these models are unable to capture driving behavior in the new connected driving environment, and models to capture these new behaviors are very limited in the literature. Early efforts to model this new driving environment focused on Automated Highway Systems (AHS) where fully autonomous vehicles were operated on a set of designated lanes (Varaiya and Shladover, 1991). Long before the "Google car", that pioneering work laid the foundation for understanding and exploring several

critical aspects of automated vehicles systems, and was successfully demonstrated on I-15 in 1991 (Ferlis, 1997). An example of these early works is a study by Varaiya and Shladover (1991) on flow control and congestion management in Automated Highway Systems. They proposed a control scheme with five different levels (physical, vehicle regulation, platoon, link, and network layers), where each level controls a different unit in this system. In another study, Hanebutte et al. (1998) presented a simulation framework to model an AHS in which an early version of Adaptive Cruise Control (ACC) was implemented using a neural network controller. Chien et al. (1997) presented a macroscopic level controller to control density in an automated highway to maximize performance of this system. Using a macroscopic simulation tool, they showed that congestion could be avoided using this controller. Broucke and Varaiya (1996) presented an approach to investigate the performance of an AHS. They showed that the performance of an AHS is a function of vehicle movement strategies (control laws) and decisions of the Traffic Management Center (TMC). They also investigated the causes of congestion in an AHS and proposed a series of actions to prevent/eliminate it. In another study (Broucke and Varaiya, 1997), they presented a design to improve the performance of automated highways (triple the capacity, guarantee a collision-free system, reduce travel time, and reduce emissions).

The concept of Automated Highway Systems has eventually evolved into driver assistance systems. Earlier versions of the driver assistance systems only relied on on-board sensors. ACC is an example of these early versions, which adjusts the vehicle's speed based on the leader's speed. The introduction of V2V and V2I communications can improve the performance of driver assistance technologies. For instance, ACC can be updated to use the information from the V2V communications network. The current flavor of this system is called Cooperative Adaptive Cruise Control (CACC) and automatically adjusts the vehicle's speed based on the behavior of its leaders and followers. Different acceleration logics have been proposed in the literature to control vehicles with CACC. Van Arem et al. (2006) were among the first researchers to propose a car-following logic for CACC. Their model uses safe deceleration, current acceleration, spacing, and relative speed with respect to the immediate leader to calculate the acceleration at the next decision point. Wang et al. (2014) proposed a CACC logic based on model predictive control process in which each vehicle uses the information from its leaders to predict the behavior of the platoon. Zhao and Sun (2013) presented a VISSIM-based simulation framework that considers ACC and CACC vehicles. They investigated the effects of different platoon sizes and market penetration rates of ACC and CACC vehicles on capacity, and confirmed that capacity increases as market penetration rate of CACC vehicles increases.

In addition to the above modeling efforts, several studies have investigated the stability of a platoon of ACC and/or CACC vehicles. Naus et al. (2010) presented a decentralized CACC control logic and derived the necessary and sufficient criteria to achieve string stability. Wang et al. (2013) presented a driver assistance system based on a receding horizon control framework. They used this framework to model ACC and CACC vehicles and derived the necessary conditions to ensure stability in ACC and CACC systems. In another study, Bose and Ioannou (2003) investigated the stability of a mixed traffic consisting of ACC and regular vehicles, and showed that ACC vehicles could improve the stability of traffic flow, reduce emissions, and improve fuel efficiency. The ICC (Ioannou and Xu, 1994) model was used to represent the ACC vehicles.

While these and several other efforts in the literature have established a solid base to study the effects of connected and autonomous vehicles on driving environment, many questions remain to be addressed, including the underlying interactions between these vehicles and the effect of these interactions on throughput and stability of traffic flow. Therefore, there is a need for comprehensive simulation frameworks to model the interactions between regular, connected, and autonomous vehicles. This study presents an effort to develop such a framework using existing technology-appropriate acceleration models, and performs analytical and simulation-based analyses of the stability of the resulting heterogeneous traffic stream.

### 3. Conceptual background

As mentioned previously, change in driving behavior and driving environment can be expected with the introduction of autonomous and connected vehicle technologies. It is important to anticipate these changes, predict their impact, and plan accordingly prior to wide deployment of these technologies. Simulation tools provide the means to predict these changes and investigate different approaches to cope with them. In order to have a reliable simulation tool, clear definition of autonomous, connected, and regular vehicles is required. Moreover, identifying specific characteristics of each group and their driving logic is necessary. Unfortunately, the literature does not offer a clear definition for these types of vehicles, and automation and connectivity are sometimes used interchangeably. This section presents a characterization for autonomous, connected, and regular vehicles in a highway environment.

Driver's decision-making forms the basis of the movement of regular vehicles (or human driven vehicles with no communication capability). Driver's decision-making at the operational level, in this case, is based on his/her perception of surrounding traffic condition including the leaders and followers behavior. The key to understand the driving logic/behavior in this case is the source of this information (perception). This perception mostly comes from driver's visual scanning. The driver is expected to adjust his/her speed not only based on the immediate leader and follower, but also based on the behavior of several vehicles ahead and behind, which results in a more stable, reliable, and comfortable driving experience.

The addition of communication improves drivers' perception of their driving environment; thus, the reliability of driving related decisions is enhanced. To characterize this improvement, it is important to distinguish between different communication types. Two types of communications have been incorporated into the connected vehicles technology, vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications. V2V communications provide detail information about

vehicles' movement and drivers' operational decisions (e.g. speed, acceleration, and location) while V2I communications provide detail information about road conditions, weather condition, and TMC decisions. Consequently, driver behavior can be influenced based on the information received. It is important to note that, regardless of the type of information, drivers are the decision makers in connected vehicles. In light of V2V availability, drivers are not only aware of vehicles in their vicinity, but also receive information from several vehicles upstream and downstream. Therefore, they know about traffic condition downstream ahead of time (e.g. sudden brakes or shockwave formation). This additional information can enhance driver response as having such information can decrease driver's reaction time and improve their confidence in decision-making. On the other hand, availability of V2I communications provides information about breakdown formation downstream (e.g. due to lane closure or a crash), changes in speed limit, workzone condition, weather condition, roadway condition, geometry, etc. The main impact of such information is on drivers' strategic decisions (e.g. lane-selection, route choice, etc.), while it can have some impact on drivers' operational decisions.

The definition of autonomous vehicle provided by USDOT is very general and includes five different levels of automation (National Highway Traffic Safety Administration, 2013). The first level, no-automation, refers to the regular vehicles, as defined previously, where drivers are the only decision makers. The second level, function specific automation, refers to vehicles with at least one vehicle control function. Electronic stability control and lane adjustment are two examples of these control functions. The third level, combined function automation, refers to vehicles with at least two vehicle control functions. These control functions work with each other at the same time and provide more assistance to the driver. At fourth level of automation, limited self-driving automation, vehicles can control safety-critical functions. However, their ability is limited to certain weather, roadway, and traffic conditions. Driver can resume control of the vehicle if necessary and enough time should be provided for this transition. Unfortunately, there are several issues associate with operating autonomous vehicles at this level. Saffarian et al. (2012) pointed out these issues and provided design solutions to overcome them. One of the main issues is overreliance where drivers do not check the performance of autonomous vehicles. Another issue is adaptation with the new system, which requires a tremendous mental workload. Unpredictable mental workload is another issue, which arises when drivers face an unexpected situation. Automation is expected to reduce mental workload in normal driving situations; however, an unpredictable situation can create stress and increase mental workload of the driver and passengers. Finally, skill degradation and reduced situational awareness can occur at this level of automation. Several studies in aviation (Parasuraman et al., 2000) showed that automation can reduce cognitive skills through time. The fifth and the last level of automation, full self-driving automation, refers to fully autonomous vehicles. A vehicle with this level of automation controls entire driving functions in any weather, road, and traffic condition.

Early versions of autonomous vehicles (regardless of automation level) relied only on on-board sensors to collect information about the weather, road, and traffic conditions. Despite the considerable improvement in the quality of these sensors, they still have certain range and accuracy limitations. Consequently, the performance of autonomous vehicles was bounded by these sensor limitations. For instance, maximum speed of an autonomous vehicle is limited by its radar range. A radar sensor has a specific detection range and cannot see beyond that range. Therefore, autonomous vehicles should always assume that there is an obstacle right after the detection range; thus, by considering the maximum deceleration of the vehicle, maximum speed can be calculated.

Connected vehicle technology can overcome the sensor limitations and provide smoother, safer, and more reliable driving experience with autonomous vehicles. In fact, any level of automation can benefit from this technology and some of the applications of this technology (e.g. CACC) require only the third level of automation. However, it is essential to distinguish full self-driving automation, limited self-driving automation, and lower levels of automation while studying the effects of autonomous vehicles and connected vehicles technology on driving environment. The next section presents an acceleration framework that distinguishes between regular, connected, and autonomous vehicles based on the definitions provided in this section.

#### 4. Model formulation

Acceleration behavior has been studied extensively in the literature and several models with different levels of complexity have been introduced to capture the underlying processes of acceleration decision making. Unfortunately, most of these models are designed to capture driving behavior in the absence of communications. Their modeling capabilities are even more limited in a mixed environment where only a portion of vehicles is equipped with the essential communication tools. The addition of autonomous vehicles can further increase the complexity in this environment. Therefore, this study presents an acceleration framework to address the limitations of microscopic simulation models in capturing the changes in driver behavior in such a mixed environment. This section provides an overview of the acceleration framework with a description of the acceleration models.

##### 4.1. Modeling vehicles with no communication capability

The drivers of these vehicles neither receive information from other vehicles nor from the traffic management center (TMC). They only get information from road signs (both VMS and conventional signs). They also have a rough perception

of other drivers' behavior in their vicinity. Moreover, their acceleration behavior has a probabilistic nature and they are uncertain about other drivers' future behavior. This uncertainty may result in crash occurrence.

In general, drivers are seeking to travel at a desired speed while avoiding crashes. Avoiding crashes is an extremely important factor in drivers' decision making because of its severe consequences. Hamdar et al. (2008) presented an acceleration model that avoids (most) crashes by specifying behavioral mechanism based on Kahneman and Tversky's prospect theory (Kahneman and Tversky, 1979). An extension to this model was presented by Talebpour et al. (2011), who recognized that drivers have different perceptions encountering congested versus uncongested regimes. Accordingly, based on prospect theory, they introduced two value functions, one for modeling driver behavior in congested regimes and one for modeling driver behavior in uncongested regimes. The uncongested traffic value function in this model has the following form:

$$U_{PT}^{UC}(a_n) = \frac{\left[ w_m + (1 - w_m) \left( \tanh\left(\frac{a_n}{a_0}\right) + 1 \right) \right]}{2} \left[ \frac{a_n}{(1 + a_n)^{\frac{\gamma-1}{2}}} \right] \tag{1}$$

where  $U_{PT}^{UC}$  denotes the value function for the uncongested traffic conditions.  $\gamma > 0$  and  $w_m$  are parameters to be estimated and  $a_0 = 1 \text{ m/s}^2$  is used to normalize the acceleration. They proposed the following value function for the congested traffic condition:

$$U_{PT}^C(a_n) = \frac{\left[ w'_m + (1 - w'_m) \left( \tanh\left(\frac{a_n}{a_0}\right) + 1 \right) \right]}{2} [a_n]^{\gamma'} \tag{2}$$

where  $U_{PT}^C$  denotes the value function for the congested traffic conditions.  $\gamma' > 0$  and  $w'_m$  are parameters to be estimated. At each evaluation stage, based on drivers' perception of their surrounding traffic condition, drivers employ the corresponding value functions to evaluate the gains from the chosen acceleration. They introduced a binary probabilistic regime selection mechanism into the evaluation stage where drivers use the resulting utility to evaluate each acceleration value, given by:

$$U_{PT}(a_n) = P(C) \cdot U_{PT}^C + P(UC) \cdot U_{PT}^{UC} \tag{3}$$

where  $U_{PT}$ ,  $P(C)$ , and  $P(UC)$  denote the expected value function, the probabilities of driving in a congested traffic condition (C), and the probability of driving in an uncongested traffic condition (UC), respectively. The utility of each choice is calculated using the following equation:

$$Y = \beta K + \varepsilon \tag{4}$$

where  $K$ ,  $\beta$ , and  $\varepsilon$  denote a vector of variables (see Table 1), a vector of unknown parameters to be estimated, and error term with iid Gumbel distribution, respectively. The Gumbel distribution for the error term results in a binary logit expression:

$$P(C) = \frac{e^{Y(C)}}{e^{Y(C)} + e^{Y(UC)}} = \frac{e^{[\beta(C) - \beta(UC)]K}}{1 + e^{[\beta(C) - \beta(UC)]K}} = \frac{e^{\beta'K}}{1 + e^{\beta'K}} \tag{5a}$$

$$P(UC) = 1 - P(C) \tag{5b}$$

Table 1 shows the calibration results for  $\beta'$ . Note that it is assumed that drivers choose the acceleration value function that gives them the higher value for the observed acceleration. Once the expected value function is calculated, total utility function of acceleration can be formulated as follows:

$$U(a_n) = (1 - p_{n,i})U_{PT}(a_n) - p_{n,i}w_c k(v, \Delta v) \tag{6}$$

**Table 1**  
External covariates, their definitions, and their calibrated values (Talebpour et al., 2011).

Data type	Definition	Coefficients	Unites
K0	Model constant	-37.8195	-
K1	Driver's speed	1.7535	m/s
K2	Average headway between driver i and her leaders in all lanes. A value of 9999 is assigned if there is no leader	0.0459	s
K3	Average relative speed between driver i and her leaders in all lanes. A value of 999 is assigned if there is no leader	0.3259	m/s
K4	Average headway between driver i and her followers in all lanes. A value of 9999 is assigned if there is no leader	0.0931	s
K5	Average relative speed between driver i and her followers in all lanes. A value of 999 is assigned if there is no leader	-1.0300	m/s
K6	Driver's average surrounding density. It is defined as the total density (over the number of lanes)	0.5911	veh/km/lane

where  $p_{n,i}$ ,  $w_c$ , and  $k(v, \Delta v)$  denote the crash probability, crash weighting parameter, and crash seriousness term, respectively (see Talebpour et al., 2011 for more details). Finally, to reflect the stochastic response adopted by the drivers, the logistic functional form specified by Hamdar (2009) is used to calculate the probability density function:

$$g(a_n) = \begin{cases} \frac{e^{\beta_{PT} U(a_n)}}{\int_{a_{\min}}^{a_{\max}} e^{\beta_{PT} U(a')} da'} & a_{\min} < a_n < a_{\max} \\ 0 & \text{Otherwise} \end{cases} \quad (7)$$

where  $\beta_{PT}$  reflects the sensitivity of choice to the utility  $U(a_n)$ . Note that this study adopted Talebpour et al.'s (2011) acceleration framework to model car-following behavior in the absence of communication.

#### 4.2. Modeling communication-ready vehicles

These vehicles are expected to have the capability of sending/receiving information to/from other vehicles and infrastructure based equipment. Assuming reliable connectivity in the vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications networks, each vehicle will receive information about other vehicles in this network. The driver also receives real-time updates about the TMC decisions (e.g., real-time changes in speed limit). However, this information may not be available at all times and locations, and drivers' behavior may change according to the amount of information they receive. Accordingly, four scenarios can be defined: Active/Inactive Vehicle-to-Vehicle Communications and Active/Inactive Vehicle-to-Infrastructure Communications.

##### 4.2.1. Active Vehicle-to-Vehicle communications

Considering the flow of information in a V2V/V2I communications network, drivers are certain about other drivers' behaviors. Moreover, they are aware of driving environment, road condition, and weather condition downstream of their current location. Therefore, a deterministic acceleration modeling framework is suitable for modeling this environment. This paper utilizes Intelligent Driver Model (IDM) (Kesting et al., 2010) to model this connected environment. While capturing different congestion dynamics, this model provides greater realism than most of the deterministic acceleration modeling frameworks.

IDM specifies a following vehicle's acceleration as a continuous function of the vehicle's current speed, the ratio of the current spacing to the desired spacing, and the difference between the leading and the following vehicles' velocities. Perceptive parameters such as desired acceleration, desired gap size, and comfortable deceleration are considered in this model (Kesting et al., 2010; Treiber et al., 2000):

$$a_{IDM}^n(s_n, v_n, \Delta v_n) = \bar{a}_n \left[ 1 - \left( \frac{v_n}{v_0^n} \right)^{\delta_n} - \left( \frac{s^*(v_n, \Delta v_n)}{s_n} \right)^2 \right] \quad (8a)$$

$$s^*(v_n, \Delta v_n) = s_0^n + T_n v_n + \frac{v_n \Delta v_n}{2\sqrt{\bar{a}_n b_n}} \quad (8b)$$

where  $\delta_n$ ,  $T_n$ ,  $a_n$ ,  $b_n$ ,  $s_0^n$ , and  $v_0^n$  are parameters to be calibrated. Note that the braking term in the IDM is designed to preclude crashes in the simulation.

##### 4.2.2. Inactive Vehicle-to-Vehicle communications

In this driving environment, no active communication exists between vehicles. In case that V2I communications are unavailable, drivers' only sources of information are road signs and their perception of surrounding traffic condition. Drivers' behavior in this case can be modeled similar to the case that vehicles have no communication capability. In the presence of V2I communications, drivers directly receive information about the TMC decisions. Drivers' behavior in this case can be modeled similar to the case that vehicles have active V2I communications.

##### 4.2.3. Active Vehicle-to-Infrastructure communications

From the TMC point of view, active V2I communications will provide a basis to detect individual vehicle trajectories which can be used as high precision input data to traffic control algorithms. From the driver's standpoint, V2I communications do not directly influence the drivers' acceleration choice. Therefore, the acceleration modeling approach under active V2I communications depends on the availability of V2V communications. However, active V2I communications will provide real-time information about the TMC decisions (e.g. speed limit update in a speed harmonization system) which aim to improve safety and mobility. Note that in this framework, TMC decisions about the speed limit is modeled by updating drivers' desired speed.

##### 4.2.4. Inactive Vehicle-to-Infrastructure communications

In this driving environment, no direct communication exists between vehicles and the TMC. Without V2V communications, drivers' only sources of information are road signs and their perception of surrounding traffic condition. Drivers' behavior in this case can be modeled similar to the case that vehicles have no communication capability. In the presence

of V2V communications, drivers may receive information about the TMC decisions from other vehicles (if at least one vehicle in the V2V communications network receives information from the TMC). Drivers' behavior in this case can be modeled similar to the case that vehicles have active V2I communications.

#### 4.3. Modeling autonomous vehicles

Considering the ability of autonomous vehicles to constantly monitor other vehicles in their vicinity, an autonomous vehicle is certain about other drivers' behavior. Moreover, these vehicles can react almost instantaneously to any changes in the driving environment (the reaction time of these vehicles can be estimated based on the sensing delay and any mechanical delays). Therefore, a deterministic acceleration modeling framework is suitable for modeling this environment. This paper presents an approach to model autonomous vehicles based on the previous studies by [Van Arem et al. \(2006\)](#) and [Reece and Shafer \(1993\)](#). The main contribution of this approach is considering sensor characteristics in the modeling process. In other words, individual sensors are simulated in order to create the input data for the acceleration model. Note that this study assumes that all autonomous vehicles are equipped with similar sensors.

[Fig. 1](#) illustrates the sensor formation on an autonomous vehicle. These sensors are (Smart Micro) Automotive Radar (UMRR-00 Type 30) with  $90\text{ m} \pm 2.5\%$  detection range and  $\pm 35^\circ$  horizontal Field of View (FOV). Each sensor updates the sensing information every 50 ms and can track up to 64 objects.

Since an autonomous vehicle can only observe vehicles that are located in its sensors detection range, it is reasonable to assume that the speed of the autonomous vehicle should be low enough to allow it to stop at the sensors detection range. This is equivalent to assume that there is a vehicle at a complete stop right outside of the sensors detection range, which cannot be spotted by the sensors at the time of decision making. Moreover, if a leader is spotted, it is reasonable to assume that the speed of the autonomous vehicle should be low enough to allow it to stop if its leader decides to decelerate with its maximum deceleration rate and reach a full stop. Considering the maximum possible deceleration for the autonomous vehicle and its leader, maximum safe speed can be calculated using the following equations:

$$\Delta x_n = (x_{n-1} - x_n - l_{n-1}) + v_n \tau + \frac{v_{n-1}^2}{2a_{n-1}^{\text{decc}}} \quad (9)$$

$$\Delta x = \min\{\text{Sensor Detection Range}, \Delta x_n\} \quad (10)$$

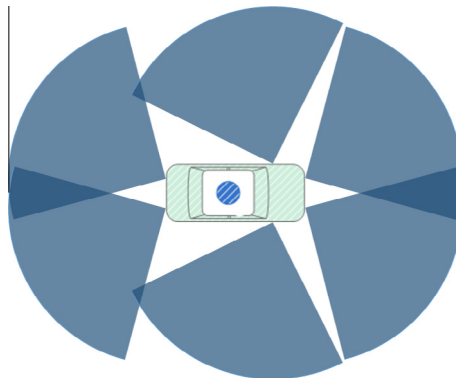
$$v_{\max} = \sqrt{-2a_i^{\text{decc}} \Delta x} \quad (11)$$

where  $n$  and  $n - 1$  present the autonomous vehicle and its leader, respectively.  $x_n$  is the location of vehicle  $n$ ,  $l_n$  is the length of vehicle  $n$ ,  $v_n$  is the speed of vehicle  $n$ ,  $\tau$  is the reaction time of vehicle  $n$ , and  $a_n^{\text{decc}}$  is the maximum deceleration of vehicle  $n$ . [Fig. 2](#) illustrates the concept of maximum safe speed; any speed below the maximum safe speed curve is considered to be safe.

In addition to the safety constraint, the vehicle movement model should be considered. This study adopted the model by [Van Arem et al. \(2006\)](#) to calculate the acceleration of the autonomous vehicle at every decision point:

$$a_n^d(t) = k_a a_{n-1}(t - \tau) + k_v (v_{n-1}(t - \tau) - v_n(t - \tau)) + k_d (s_n(t - \tau) - s_{\text{ref}}) \quad (12)$$

where  $a_n^d$  is the acceleration of vehicle  $i$  and  $k_a$ ,  $k_v$ , and  $k_d$  are model parameters.  $s_n$  is the spacing and  $s_{\text{ref}}$  is the maximum between minimum distance ( $s_{\min}$ ), following distance based on the reaction time ( $s_{\text{system}}$ ), and safe following distance ( $s_{\text{safe}}$ ). In this study, minimum distance is set at 2.0 m and  $s_{\text{system}}$  and  $s_{\text{safe}}$  is calculated as follows:



**Fig. 1.** Radar sensor formation on an autonomous vehicle.

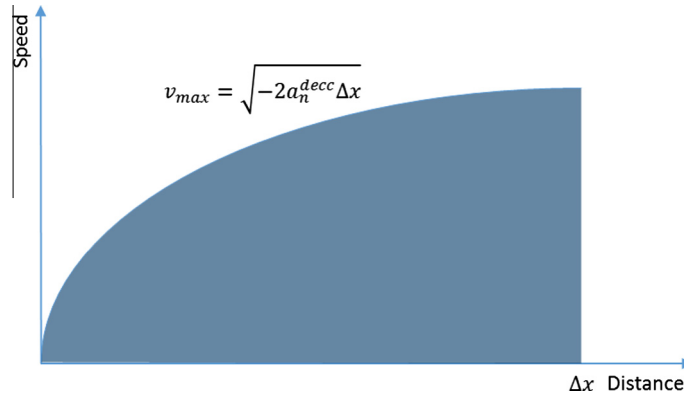


Fig. 2. Maximum safe speed curve (Reece and Shafer, 1993).

$$s_{safe} = \frac{v_{n-1}^2}{2} \left( \frac{1}{a_n^{decc}} - \frac{1}{a_{n-1}^{decc}} \right) \quad (13)$$

$$s_{system} = v_n \tau \quad (14)$$

Finally, the acceleration of the autonomous vehicle can be calculated using the following equation:

$$a_n(t) = \min(a_n^d(t), k(v_{max} - v_n(t))) \quad (15)$$

where  $k$  is a model parameter. In this study, based on the recommendations of Van Arem et al. (2006),  $k = 1.0$ ,  $k_a = 1.0$ ,  $k_v = 0.58$ , and  $k_d = 0.1$ .

## 5. Stability analysis

Stability of simple acceleration models has been studied extensively in the literature (Treiber and Kesting, 2013; Wilson and Ward, 2010). However, most of these studies focused on linear stability of simple acceleration models (Treiber and Kesting, 2013; Wilson and Ward, 2010) and only a few studies investigated the linear stability of mixed traffic flows and/or more complex acceleration models. An example of these works is a study by Ward (2009) in which the string stability of a mixed car-truck traffic flow was investigated. Following Ward's guidelines and based on the presented acceleration framework, this section presents an analytical investigation of linear string stability under different market penetration rates of connected and autonomous vehicles.

It should be noted that car-following dynamics cannot be fully captured through linearization and non-linear stability analysis should be performed. Unfortunately, except for very abstract models, non-linear stability analysis is not analytically tractable. Therefore, in addition to the analytical investigation, this section presents a simulation based approach to investigate string stability under different market penetration rates of connected and autonomous vehicles using the presented acceleration framework.

### 5.1. Analytical investigation of string stability

Stability of car-following models has become a topic of interest in Mathematics and Theoretical Physics since 1990s (Wilson and Ward, 2010). Two types of car-following stability has been defined in the literature (Treiber and Kesting, 2013; Wilson and Ward, 2010): local stability and string stability. Assume that a car-following model is simply formulated by a coupled differential equations (Treiber and Kesting, 2013; Wilson and Ward, 2010):

$$\dot{x}_n = v_n \quad (16)$$

$$\dot{v}_n = f(s_n, \Delta v_n, v_n) \quad (17)$$

Empirical observations suggest that there exists an equilibrium speed-spacing relationship (Treiber et al., 2000). In other words, there exist a function  $v = V(s)$  such that  $f(s^*, 0, V(s^*)) = 0$  for all  $s^* > 0$ .  $s^*$  is called equilibrium spacing and  $V(s^*)$  denotes the speed at equilibrium. local stability is defined based on the vehicle's response to the change in motion of its leader (Zhang and Jarrett, 1997). Consider a finite platoon of vehicles at equilibrium ( $s^*, V(s^*)$ ). If vehicle  $i$  deviates from equilibrium (for instance, by applying a small change in speed), the resulting perturbation will propagate to the upstream traffic. Consequently, vehicles  $i + 1, i + 2, \dots, i + n$  will be forced out of equilibrium to react to this perturbation (this phenomenon is



called shockwave formation and propagation). If the car-following model of interest is locally stable, the perturbation will decay exponentially through time and vehicles will eventually return to the equilibrium condition (Wilson and Ward, 2010). Fig. 3 illustrates this concept, where all vehicles eventually return to the equilibrium condition.

String stability is defined based on the propagation of a fluctuation in one vehicle's motion to the upstream traffic (Zhang and Jarrett, 1997). Consider an infinite platoon of vehicles at equilibrium ( $s^*, V(s^*)$ ). Similar to the local stability, if vehicle  $i$  deviates from equilibrium, the resulting perturbation will propagate to the upstream traffic. If the car-following model of interest is string stable, the perturbation will decay as it propagates upstream (Treiber and Kesting, 2013; Wilson and Ward, 2010). Otherwise, two different regimes can be observed (Treiber et al., 2007): oscillatory regime in which perturbations grow as they propagate upstream but do not lead to a collision and collision regime in which a perturbation leads to crashes. Fig. 3 illustrates these concept, where the fluctuations in spacing decrease or increase as the shockwave propagates upstream. Note that local stability is essential for any acceleration model since driver behavior is locally stable in reality. However, string stability is not always observed in empirical data (depending on the characteristics of drivers and traffic regime).

As mentioned previously, the studies of string stability mostly focused on one simple car-following models and few studies including a study by Ward (2009) investigated the string stability of a heterogeneous traffic flow. This study adopts the finding of Ward's study to investigate the effects of connectivity and automation on the string stability of traffic flow. Considering small perturbations in headway and speed of a vehicle in a platoon of infinite length,  $s_i = s^* + \bar{s}_i$  and  $v_i = V(s^*) + \bar{v}_i$ , and linearizing Eqs. (16) and (17) about the equilibrium, Ward calculated the following instability condition for a heterogeneous traffic flow (Ward, 2009),

$$\sum_n \left[ \frac{f_v^n}{2} - f_{\Delta v}^n f_v^n - f_s^n \right] \left[ \prod_{m \neq n} f_s^m \right]^2 < 0 \tag{18}$$

where  $n$  denotes different vehicle types and the expansion coefficients are

$$f_s^n = \left. \frac{\partial f(s_n, \Delta v_n, v_n)}{\partial s_n} \right|_{(s^*, 0, V(s^*))}, \quad f_{\Delta v}^n = \left. \frac{\partial f(s_n, \Delta v_n, v_n)}{\partial \Delta v_n} \right|_{(s^*, 0, V(s^*))}, \quad f_v^n = \left. \frac{\partial f(s_n, \Delta v_n, v_n)}{\partial v_n} \right|_{(s^*, 0, V(s^*))}$$

Following Eq. (18), the stability of different combinations of regular, connected, and autonomous vehicles are investigated in the following sections. Note that the  $n$  subscript is dropped in the following sections unless otherwise necessary.

### 5.1.1. Homogenous traffic flow

In this section, stability of homogenous platoons of vehicles are investigated for three different vehicle types. In case of a platoon of regular vehicles, for simplicity and tractability of analytical derivations, the acceleration model of Hamdar et al. (2008) is used in this section. This model is simpler than the presented model in Section 4.1 and does not distinguish between congested and uncongested traffic regimes (this model calculates the prospect index based on only Eq. (1)). Due to the probabilistic nature of this acceleration model, calculating the derivatives (i.e.  $f_s^n, f_{\Delta v}^n$ , and  $f_v^n$ ) can be very challenging. Therefore, this section adopts the Wiener process to reflect drivers' estimation error in calculating optimal acceleration. Based on the Wiener process, drivers' acceleration choice can be captured by the following equation,

$$a(t) = a^*(t) + \sigma_a(t)y(t) \tag{19}$$

where  $a^*(t)$  is the optimal acceleration choice at time  $t$ ,  $\sigma_a(t)$  denotes the variance of acceleration choice at time  $t$ , and  $y(t)$  is the standard Wiener process,

$$y(t) = y(t - \Delta t)e^{-\Delta t/\tau} + \sqrt{24\Delta t/\tau}(z - 0.5) \tag{20}$$

where  $z \sim \text{uniform}(0, 1)$ ,  $\tau$  denotes correlation time and  $\Delta t$  is the simulation update time. Note that  $y(t)$  is not a function of  $s$ ,  $\Delta v$ , or  $v$ . Assuming perfect decision makers, who choose optimal acceleration at all times ( $\sigma_a(t)$  equal to zero), simplifies

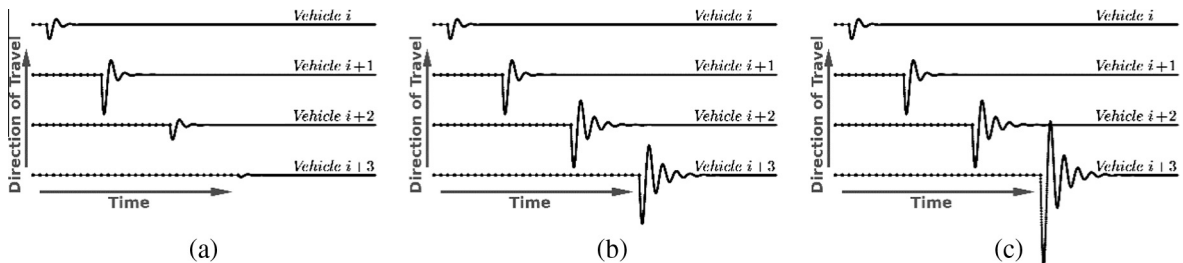


Fig. 3. Acceleration profiles for a platoon of vehicles in a (a) string stable regime, (b) unstable oscillatory regime, and (c) unstable collision regime (Treiber and Kesting, 2013; Wilson and Ward, 2010).

Eq. (19) to finding the optimal acceleration at time  $t$ . Assume  $\gamma = w_m = 1$  in Eq. (1) (linear value function), total utility can be written as follows (Hamdar, 2009),

$$U_{PT}(a|s, \Delta v, v) = a - w_c \Phi(z(a|s, \Delta v, v)) \quad (21)$$

where  $\Phi(z)$  is the density of a Gaussian,  $z(a|s, \Delta v, v) = \frac{\Delta v + \frac{1}{2}\alpha\tau - \frac{s}{v}}{\alpha v}$ , and

$$\tau(s, \Delta v) = \begin{cases} \frac{s}{\Delta v} & \Delta v > \frac{s}{\tau_{\max}} \\ \tau_{\max} & \text{Otherwise} \end{cases}$$

Maximizing the Eq. (21), by taking the derivatives with respect to acceleration, leads to the following equation for the optimal acceleration (Hamdar, 2009),

$$a^* = \frac{2}{\tau_{\max}} \left( \frac{s}{\tau_{\max}} - \Delta v + \alpha v z^* \right) \quad (22)$$

where

$$z^* = \sqrt{2 \ln \left( \frac{w_c \tau_{\max}}{2\sqrt{2\pi\alpha v}} \right)} \quad (23)$$

Combining Eqs. (19), (22), and (23), the partial derivatives at equilibrium can be calculated as follows,

$$f_s = \frac{2}{\tau_{\max}^2} \quad (24)$$

$$f_{\Delta v} = \frac{-2}{\tau_{\max}} \quad (25)$$

$$f_v = \frac{2\alpha z_e^*}{\tau_{\max}} + \frac{2\alpha v_e}{\tau_{\max}} \left[ \frac{1}{\sqrt{2} v_e} \left( \ln \left( \frac{w_c \tau_{\max}}{2\sqrt{2\pi\alpha v_e}} \right) \right)^{-1/2} \right] \quad (26)$$

Through calculating the partial derivatives, the stability of a platoon of regular vehicles can be evaluated based on Eq. (18). Fig. 4a presents such a stability analysis for a platoon of regular vehicles with infinite length (infinite number of vehicles). In this figure, typical values of model parameters (see Table 2) are used to plot Eq. (18) against equilibrium speed. Fig. 4a reveals that a platoon of regular vehicles (with parameters values of Table 2) is string stable for equilibrium speeds below 3.5 m/s. For any speed above 3.5 m/s the platoon becomes string unstable.

In case of a platoon of connected vehicles, the partial derivatives of the Intelligent Driver Model (IDM) can be calculated as follows:

$$f_s = \frac{2\bar{a}}{s_e} \left( \frac{s_0 + T v_e}{s_e} \right)^2 \quad (27)$$

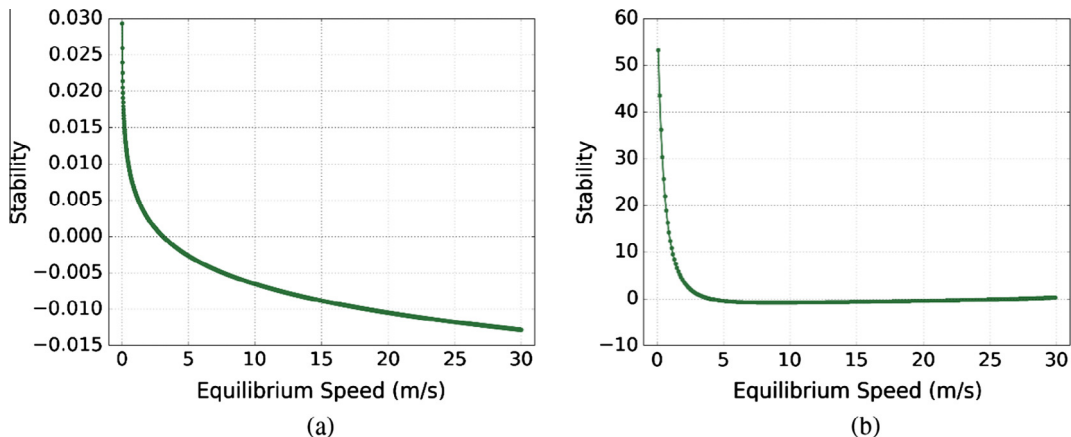


Fig. 4. Stability of a platoon of (a) regular vehicles and (b) connected vehicles with infinite length (values below zero indicates the instability).

**Table 2**  
Acceleration model parameters and their typical values for the simplified car-following model of Hamdar (2009).

Parameters	Typical value
Velocity uncertainty variation coefficient	$\alpha = 0.08$
Weighing factor for accidents	$w_c = 100000.0$
Maximum anticipation time horizon	$\tau_{max} = 4.0$ s

$$f_{\Delta v} = -\frac{v_e}{s_e} \sqrt{\frac{\bar{a}}{b}} \left( \frac{s_0 + T v_e}{s_e} \right) \tag{28}$$

$$f_v = \frac{-\bar{a}\delta}{v_0} \left( \frac{v}{v_0} \right)^{\delta-1} - \left( \frac{2\bar{a}T}{s_e} \right) \left( \frac{s_0 + T v_e}{s_e} \right) \tag{29}$$

Note that relative speed is set to zero at equilibrium (i.e.  $\Delta v_e = 0$ ). Fig. 3b presents the stability analysis results for a platoon of connected vehicles with infinite length. Typical values of IDM model parameters (see Table 3) are used to plot Eq. (18) against equilibrium speed. Note that the partial derivatives of IDM (Eqs. (27), (28), and (29)) are functions of vehicle speed and gap at equilibrium; therefore, the following relationship between speed and equilibrium gap is used to simplify the stability analysis (Treiber and Kesting, 2013):

$$s_e(v) = \frac{s_0 + vT}{\sqrt{1 - \left(\frac{v}{v_0}\right)^\delta}} \tag{30}$$

Fig. 4b reveals that a platoon of connected vehicles (with parameter values of Table 3) is string stable for equilibrium speeds below 3.5 m/s and string unstable for speeds above 3.5 m/s.

In case of a platoon of autonomous vehicles, it is assumed that the acceleration of the leader is zero during the estimation time. This is a limiting assumption, however, it simplifies the calculation of partial derivatives. Considering this assumption, the partial derivatives become constant and no longer functions of equilibrium speed or equilibrium gap:

$$f_s = k_d \tag{31}$$

$$f_{\Delta v} = k_v \tag{32}$$

$$f_v = -k_d \tau \tag{33}$$

Therefore, the model is always string stable for the parameter values presented in the previous section (i.e.  $k_a = 1.0$ ,  $k_v = 0.58$ , and  $k_d = 0.1$ ).

### 5.1.2. Heterogeneous traffic flow

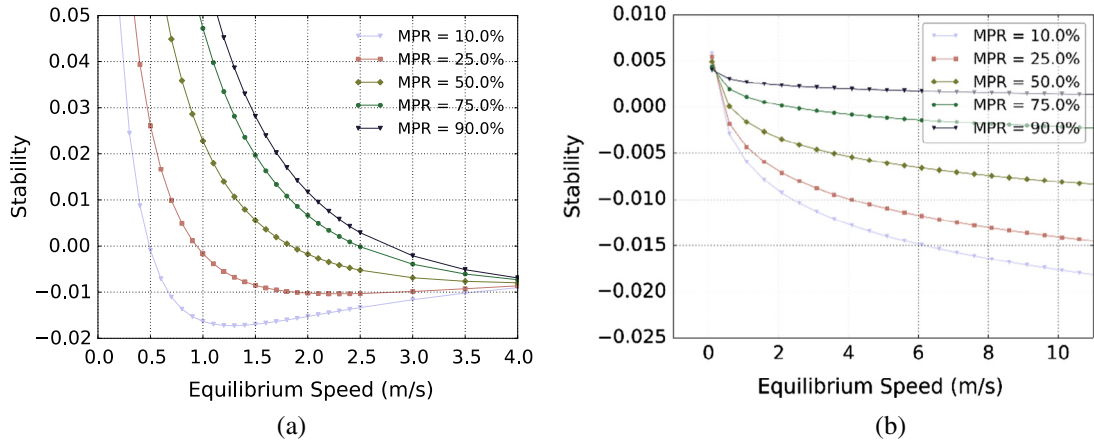
In this section, stability of heterogeneous platoons of vehicles with infinite length are investigated for different combinations of regular, connected, and autonomous vehicles and different market penetration rates of connected and autonomous vehicles. To magnify the effects of connectivity and automation on stability of traffic flow, parameters of regular vehicles are adjusted ( $w_c$  is reduced to 10000.0) to create a very unstable traffic flow in a platoon of regular vehicles. The first case investigates the driving environment consists of regular and connected vehicles. Following the Ward’s approach in Eq. (18), the instability condition can be written as follows:

$$(1 - \psi_c)(f_s^R)^2 \left[ \frac{f_v^{R2}}{2} - f_{\Delta v}^R f_v^R - f_s^R \right] + \psi_c (f_s^C)^2 \left[ \frac{f_v^{C2}}{2} - f_{\Delta v}^C f_v^C - f_s^C \right] < 0 \tag{34}$$

where  $\psi_c$  denotes the market penetration rate of connected vehicles and R and C stand for regular and connected vehicles, respectively. Fig. 5a shows the stability analysis results for a platoon of regular and connected vehicles under different

**Table 3**  
Acceleration model parameters and their values for the Intelligent Driver Model.

Parameters	Typical value
Free acceleration exponent	$\delta = 4.0$
Desired time gap	$T = 2.0$ s
Jam distance	$s_0 = 2.0$ m
Maximum acceleration	$\bar{a} = 4.0$ m/s <sup>2</sup>
Desired deceleration	$b = 2.0$ m/s <sup>2</sup>



**Fig. 5.** Stability of a platoon of (a) regular and connected vehicles and (b) regular and autonomous vehicles (values below zero indicates the instability).

market penetration rates of connected vehicles. This figure clearly reveals that higher market penetration rate of connected vehicles improves the stability of traffic flow and increases the speed threshold in which traffic becomes unstable (critical speed). Similar to the first case, in a driving environment consists of regular and autonomous vehicles, the instability condition can be written as follows:

$$(1 - \psi_A)(f_s^A)^2 \left[ \frac{f_v^{R^2}}{2} - f_{\Delta v}^R f_v^R - f_s^R \right] + \psi_A (f_s^R)^2 \left[ \frac{f_v^{A^2}}{2} - f_{\Delta v}^A f_v^A - f_s^A \right] < 0 \quad (35)$$

where  $\psi_A$  denotes the market penetration rate of autonomous vehicles and A stands for autonomous vehicles. Fig. 5b shows the stability analysis result for a platoon of regular and autonomous vehicles under different market penetration rates of autonomous vehicles. Similar to Fig. 5a, higher market penetration rate of autonomous vehicles improves the stability of traffic flow. Comparing Fig. 5a and b, it is clear that autonomous vehicles are more effective than connected vehicles in increasing the stability of traffic flow (at the same market penetration rate).

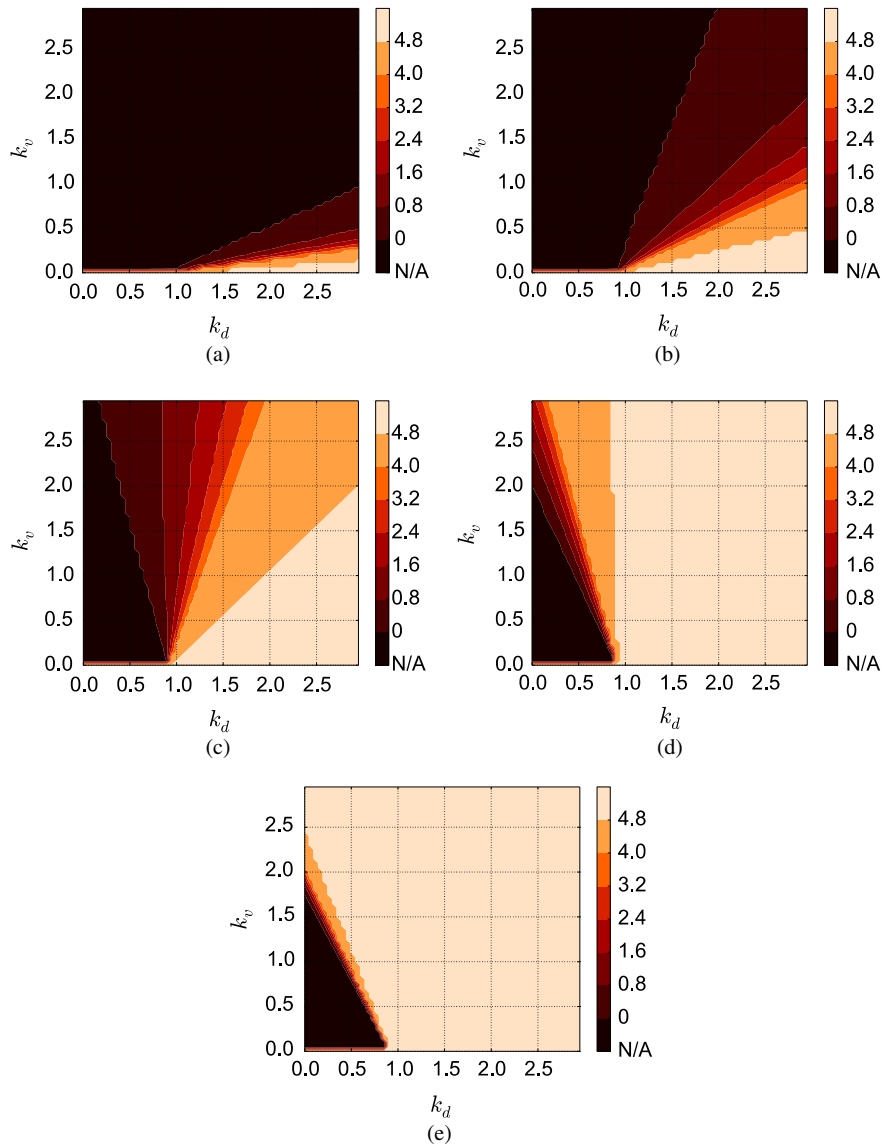
To extend the above discussion, a sensitivity analysis on model parameters is presented in Figs. 6–8. Fig. 6 presents the results of sensitivity analysis for different parameters of automated vehicles at different market penetration rates of automated vehicles. The sensitivity analysis results indicate that, at low market penetration rates, lower values of  $k_v$  and higher values of  $k_d$  result in a more stable system. On the other hand, at high market penetration rate, the system is not very sensitive to the value of  $k_d$ , while stability increases as  $k_v$  increases. In other words, at low market penetration rates, automated vehicles need to be more aggressive in response to distance and less aggressive to speed difference to enhance stability. At high market penetration rates, automated vehicles need to be only aggressive to speed difference to improve stability.

Figs. 7 and 8 present the results of sensitivity analysis for different parameters of connected vehicles at different market penetration rates of connected vehicles. Fig. 7 shows the critical speed for different values of  $s_0^n$  and  $\delta_n$  at different market penetration rates. The results indicate that, in general, the system is more stable at low values of  $\delta_n$ . Particularly, at low penetration rates, the system is not stable except for very low values of  $\delta_n$ . As the market penetration rate increases, the system becomes more stable and the effect of  $s_0^n$  is more obvious. At low penetration rate, stability does not depend on the value of  $s_0^n$ , while at high market penetration rates, the system becomes less stable as  $s_0^n$  increases. Fig. 8 indicates the critical speed for different values of  $s_0^n$  and  $T_n$ . This figure also confirms that stability does not depend on the value of  $s_0^n$  at low market penetration rates. The results indicate that as  $T_n$  increases, critical speed increases (except for very large values of  $T_n$ ).

Finally, the third case investigates the driving environment consists of regular, connected, and autonomous vehicles. Fig. 9 shows the speed threshold in which traffic becomes unstable for all combinations of market penetration rates of connected and autonomous vehicles. This figure reveals that low market penetration rates of autonomous vehicles do not result in significant stability improvements, whereas stability is improved even at low market penetration rates of connected vehicles. On the other hand, high market penetration rates of autonomous vehicles result in more stable traffic flows compare to high market penetration rates of connected vehicles. Moreover, at low market penetration rates of autonomous vehicles, critical speed increases linearly with the increase in the market penetration rate of connected vehicles.

## 5.2. Simulation-based investigation of string stability

The simulation-based investigation of string stability is performed by adapting the methodology proposed by Treiber et al. (2007). Following this methodology, the occurrence of different stability regimes (stable, oscillatory, and collision) is investigated for different platoon sizes, reaction times, and market penetration rates of connected and autonomous vehicles.

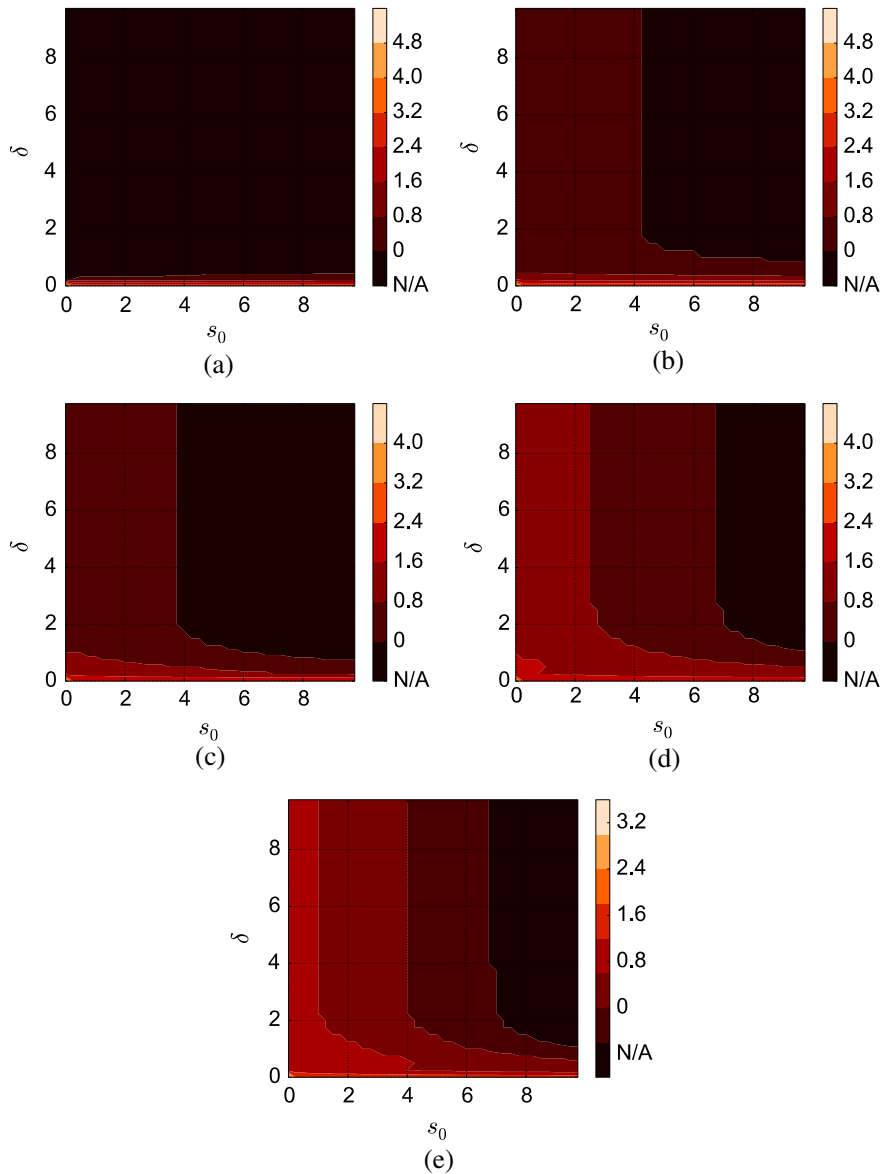


**Fig. 6.** Critical speed (m/s) for different values of  $k_d$  and  $k_v$  at different market penetration rates of automated vehicles, (a) 10%, (b) 25%, (c) 50%, (d) 75%, and 100%.

Treiber et al. (2007) proposed some criteria to categorize stable regime, oscillatory regime, and collision regime. Based on these criteria, if  $a < 3 \text{ m/s}^2$  at all times and the platoon reaches the steady-state condition with  $a = 0 \text{ m/s}^2$  at some point after the perturbation, the acceleration behavior is considered to be string stable. On the other hand, collision regime is identified if perturbations lead to a crash ( $\text{Spacing} < 0 \text{ m}$ ) at some point in the simulation. Finally, oscillatory regime is identified if neither of these cases is recognized.

Platoons with 100, 80, 60, 40, and 20 vehicles are simulated on a one-lane highway (no lane-changing) with an infinite length. Initial headway between the vehicles is set to 1 s and desired speed is set to 25 m/s. To create a perturbation, once the steady state condition is reached, the leader of the platoon is slowed down at the rate of  $-2 \text{ m/s}^2$  for 10 s. Note that for these simulations, the typical values of model parameters are used (see Tables 4 and 5) and no driver heterogeneity is considered. Moreover, based on the findings of Talebpour and Mahmassani (2014), it is assumed that reaction time of connected vehicles is 50% less than regular vehicles. Note that reaction time is considered as the delay in the driver's response to the changes in the leader's acceleration/speed. A rolling horizon approach is implemented in the simulations to consider this delay. The time steps in the simulation experiments are set to 100 ms.

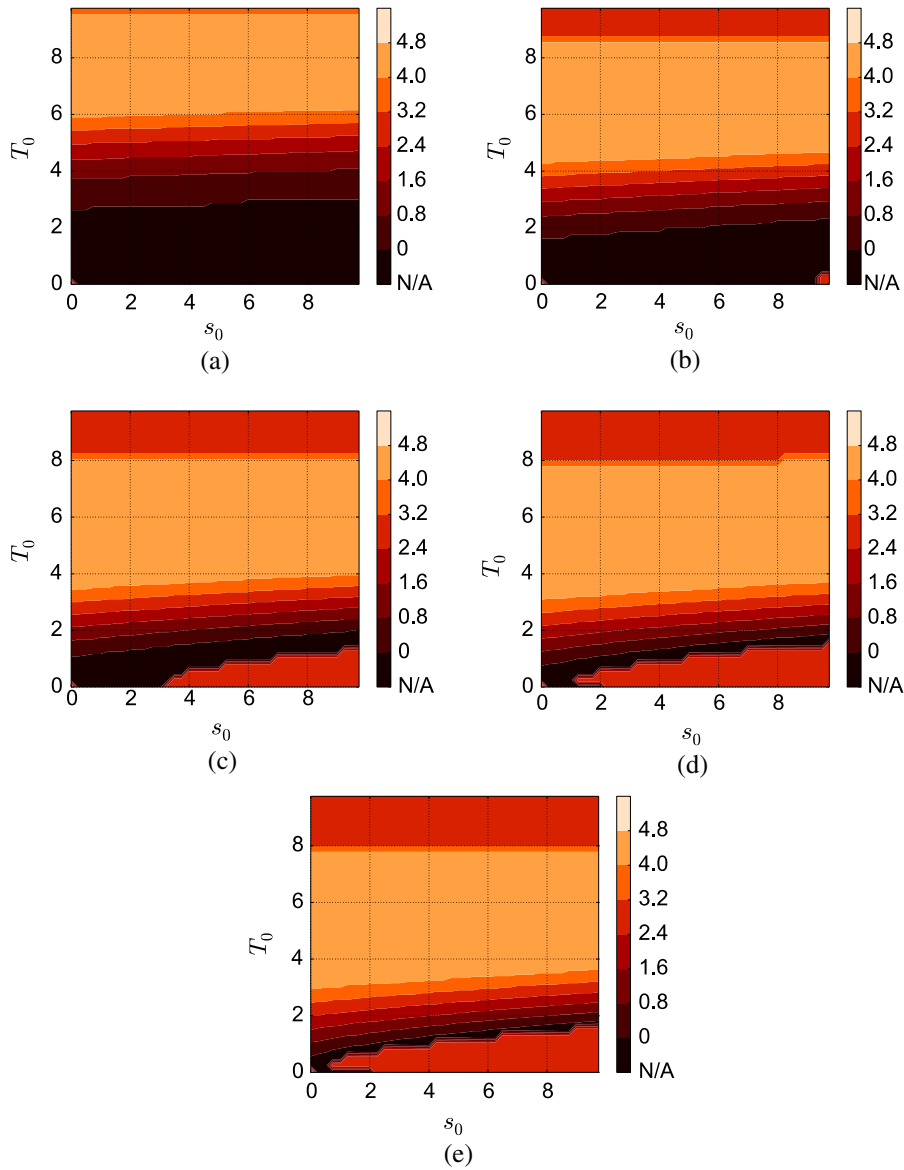
Fig. 10 shows the string stability as a function of reaction time and platoon size for different market penetration rates of connected and autonomous vehicles. It should be noted that investigating string stability requires infinite number of



**Fig. 7.** Critical speed (m/s) for different values of  $s_0^0$  and  $\delta_n$  at different market penetration rates of automated vehicles, (a) 10%, (b) 25%, (c) 50%, (d) 75%, and 100%.

vehicles. Consequently, the reaction time for string stability is given by the vertical asymptotes of the thresholds in Fig. 10. As a general observation, regardless of vehicle types in the platoon, stability threshold increases as platoon size decreases. Fig. 10a presents the thresholds in which stable regime becomes oscillatory and oscillatory regime turns into collision regime in a platoon of regular vehicles. For instance, a platoon of 60 vehicles is stable at reaction times  $R_t < R_t^{Cr,1} = 0.6$  s. Oscillatory regime started after this point and small perturbations started to grow and propagate upstream. At reaction times  $R_t > R_t^{Cr,2} = 1.1$  s, this instability leads to crashes and collision regime is identified. It is important to note that these thresholds increase (stability increase) as the number of vehicles in a platoon decreases.

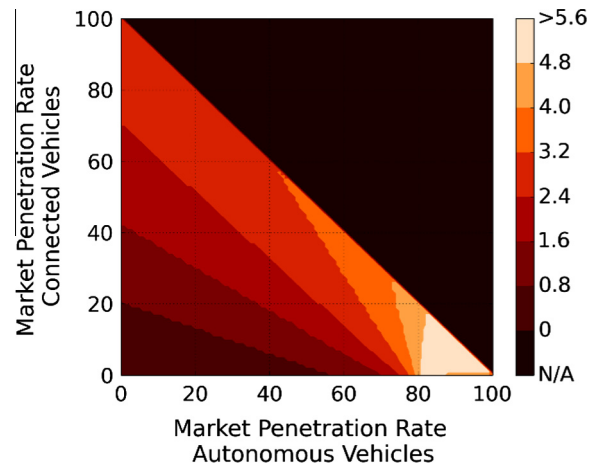
Fig. 10b–e investigate the impact of different market penetration rates of connected and autonomous vehicles on the stability of traffic flow. Fig. 10b presents the simulation results for 10% market penetration rate of connected vehicles. Comparing this figure with Fig. 10a reveals that connected vehicles can improve the stability of traffic flow even at low market penetration rates ( $R_t^{Cr,1}$  is higher in this case). The effect is more obvious for small and large platoon sizes. On the other hand, the threshold for collision regime ( $R_t^{Cr,2}$ ) is not improved at this market penetration rate and the threshold values are similar to the values in Fig. 10a.



**Fig. 8.** Critical speed (m/s) for different values of  $s_0$  and  $T_n$  at different market penetration rates of automated vehicles, (a) 10%, (b) 25%, (c) 50%, (d) 75%, and 100%.

Fig. 10c shows the simulation results for 90% market penetration rate of connected vehicles. A comparison of this figure with Fig. 10a and b indicates some improvements in both oscillatory and collision regimes thresholds, especially for small platoon sizes. For instance, oscillatory regime threshold is improved by 93% for a platoon of 20 vehicles, while this threshold is only improved by 18% for a platoon of 100 vehicles (see Fig. 10a–c).

Autonomous vehicles have similar effects on oscillatory regime threshold in low market penetration rates (except for very small platoons in which more improvements are observed with connected vehicles). For instance, for a platoon of 40 vehicles and 10% market penetration rate, the oscillatory regime threshold is 0.7 s for both connected and autonomous vehicles. However, the impact is higher at 90% market penetration rate (compare 0.7 s for connected vehicle with 1.0 s for autonomous vehicles). Lower reaction time and less uncertainty are the main reasons for this difference. On other hand, the impact of autonomous vehicles on collision threshold is significant even at very low market penetration rates. In other words, since autonomous vehicles are certain about other vehicles' movements and have a very low (0.1 s) reaction time, they can damp small perturbations in traffic and prevent shockwaves from propagating upstream at the onset of shockwave formation. At low penetration rates, the impact is minimal, however, at high penetration rates huge improvements can be observed (see Fig. 10d and e).



**Fig. 9.** Critical speed (m/s) at different market penetration rates of connected and autonomous vehicles for a platoon of regular, connected, and autonomous vehicles with infinite length (any speed value above the critical speed results in an unstable traffic flow).

**Table 4**

Acceleration model parameters and their typical values for car-following model of Talebpour et al. (2011).

Parameters	Typical value
Sensitivity exponents of the generalized utility	$\gamma = \gamma' = 0.2$
Asymmetry factor for negative utilities	$w_m = w'_m = 2.0$
Velocity uncertainty variation coefficient	$\alpha = 0.08$
Weighing factor for accidents	$w_c = 100000.0$
Maximum anticipation time horizon	$\tau_{\max} = 4.0$ s
Logit uncertainty parameter (intra-driver variability)	$\beta = 5.0$
Maximum acceleration	$a_{\max} = 4$ m/s <sup>2</sup>
Minimum acceleration	$a_{\min} = -8$ m/s <sup>2</sup>

**Table 5**

Acceleration model parameters and their typical values for IDM (Kesting et al., 2010).

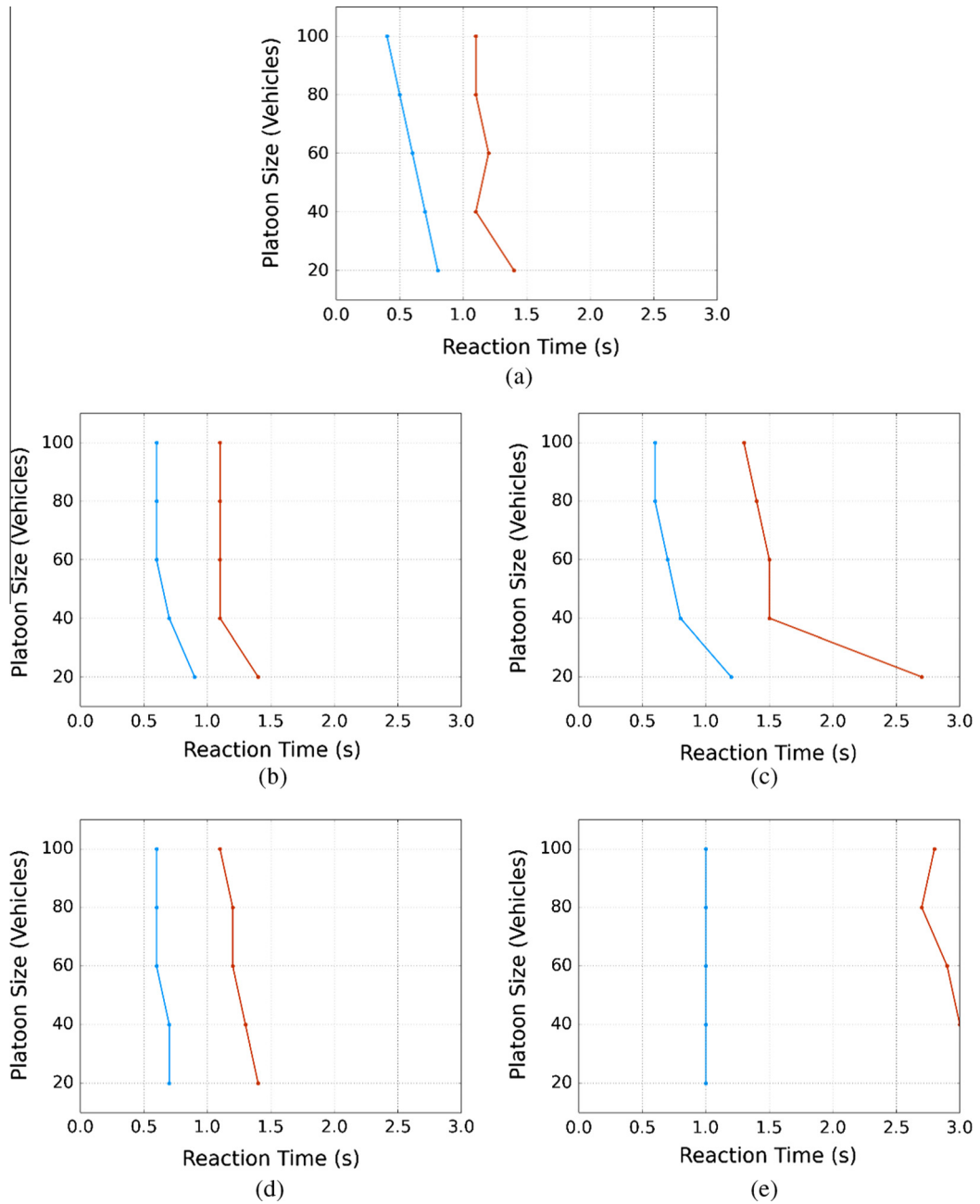
Parameters	Typical value
Free acceleration exponent	$\delta = 4.0$
Desired time gap	$T = 1.5$ s
Jam distance	$s_0 = 2.0$ m
Maximum acceleration	$a = 1.4$ m/s <sup>2</sup>
Desired deceleration	$b = 2.0$ m/s <sup>2</sup>

## 6. Throughput analysis

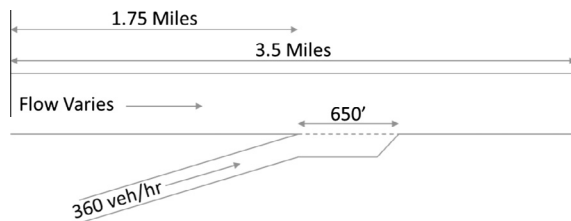
In addition to the string stability of traffic flow, investigating the throughput improvements is also an essential element in assessing the effects of connected and autonomous vehicles on traffic flow. Accordingly, this section presents an investigation of the throughput improvements under different market penetration rates of connected and autonomous vehicles. All of the simulations in this section are performed on a hypothetical one-lane highway with an on-ramp located in the middle of the segment. Fig. 11 shows the geometric characteristics of this hypothetical segment. Note that a gap-acceptance based lane-changing model is selected for merging maneuvers.

The first set of simulations investigates the effects of connected vehicles on throughput and scatter in fundamental diagram of traffic flow. Note that the main-line flow is set to 1800 veh/h/lane in the first and second sets of simulations. Fig. 12 shows the fundamental diagrams for 6 different market penetration rates of connected vehicles. It is obvious that throughput (or breakdown flow rate) increases as market penetration rate increases. Moreover, fundamental diagrams show that scatter increases as market penetration rate increases from 0% to 50%. After this point, the scatter starts to decrease and at 90% market penetration rate, the entire inflow (main-line plus ramp) is accommodated and no breakdown and/or scatter in fundamental diagram is observed.

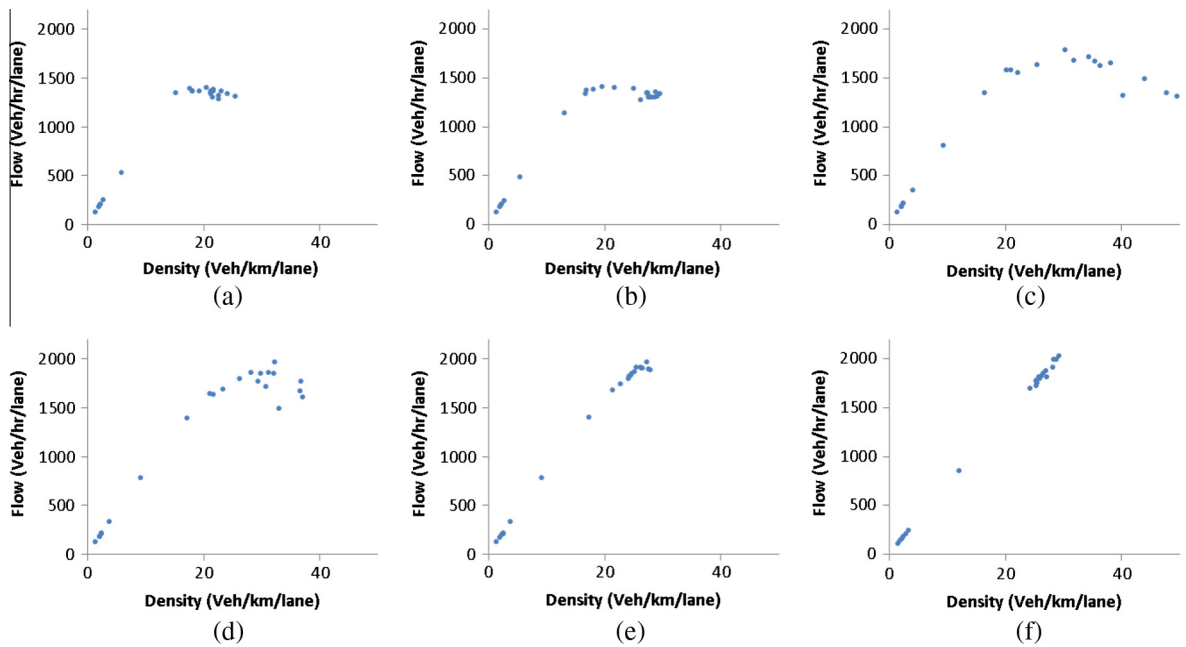




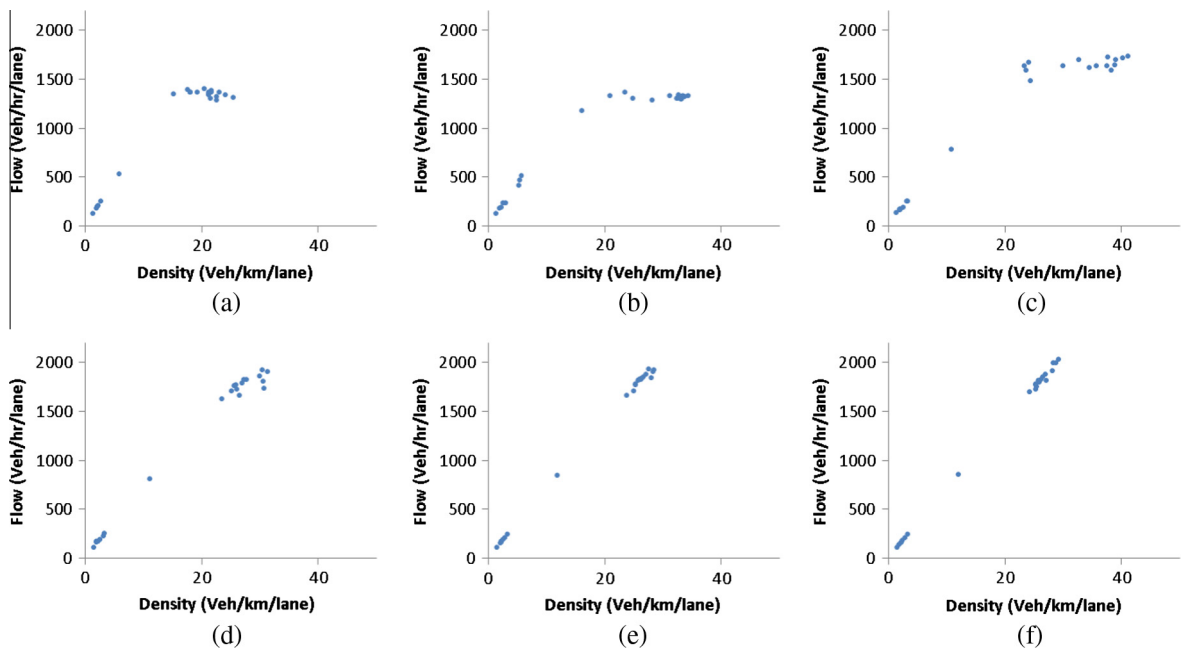
**Fig. 10.** String stability regimes as a function of reaction time and platoon size for different market penetration rates of connected and autonomous vehicles, (a) only regular vehicles, (b) 10% connected vehicles, (c) 90% connected vehicles, (d) 10% autonomous vehicles, and (e) 90% autonomous vehicles (the blue line indicates the oscillation threshold and the red line indicates the collision threshold). (For interpretation of the references to color in this figure legend, the reader is referred to the web version of this article.)



**Fig. 11.** Geometric characteristics of the hypothetical simulation segment.



**Fig. 12.** Fundamental diagrams from simulating a platoon of regular and connected vehicles for different market penetration rates of connected vehicles: (a) 0% connected vehicles, (b) 10% connected vehicles, (c) 50% connected vehicles, (d) 70% connected vehicles, (e) 90% connected vehicles, and (f) 100% connected vehicles.



**Fig. 13.** Fundamental diagrams from simulating a platoon of regular and autonomous vehicles for different market penetration rates of autonomous vehicles: (a) 0% autonomous vehicles, (b) 10% autonomous vehicles, (c) 50% autonomous vehicles, (d) 70% autonomous vehicles, (e) 90% autonomous vehicles, and (f) 100% autonomous vehicles.

The second set of simulations investigates the effects of autonomous vehicles on throughput and scatter in fundamental diagrams. Fig. 13 shows the fundamental diagrams for 6 different market penetration rates of autonomous vehicles. Similar to Fig. 12, scatter in fundamental diagrams increases as market penetration rate increases from 0% to 50% and decreases after this point. Moreover, at high market penetration rates, no scatter is observed in fundamental diagrams. While Figs. 2 and 13 result in similar conclusions, comparing them side by side reveal that autonomous vehicles are more effective in increasing throughput and reducing scatter in fundamental diagram.

Finally, the third set of simulations investigates the simultaneous effects of autonomous and connected vehicles on throughput and scatter in fundamental diagram. More precisely, while the number of regular vehicles is kept at 10% (of the total vehicles), six combinations of different market penetration rates of connected and autonomous vehicles are simulated. Note that the main-line flow is set to 2200 veh/h/lane in these simulations. Fig. 14 illustrates the fundamental diagrams from these simulations. This figure suggests that scatter in fundamental diagram is negligible as long as the number of autonomous vehicles is more than connected vehicles (see Fig. 14a–c). The scatter in fundamental diagram increases dramatically once the number of connected vehicles increases and becomes more than autonomous vehicles (compare Fig. 14c and d). However, scatter is negligible at very high market penetration rates of connected vehicles (see Fig. 14f). Moreover, Fig. 14 suggests that throughput is higher in a system dominated by autonomous vehicles (compare 2500 veh/h/lane in Fig. 14a and b with 2200 veh/h/lane in Fig. 14f).

Finally, Fig. 15 shows the maximum throughput for all combinations of market penetration rates of connected and autonomous vehicles. This figure reveals that high market penetration rates of autonomous vehicles result in higher throughput

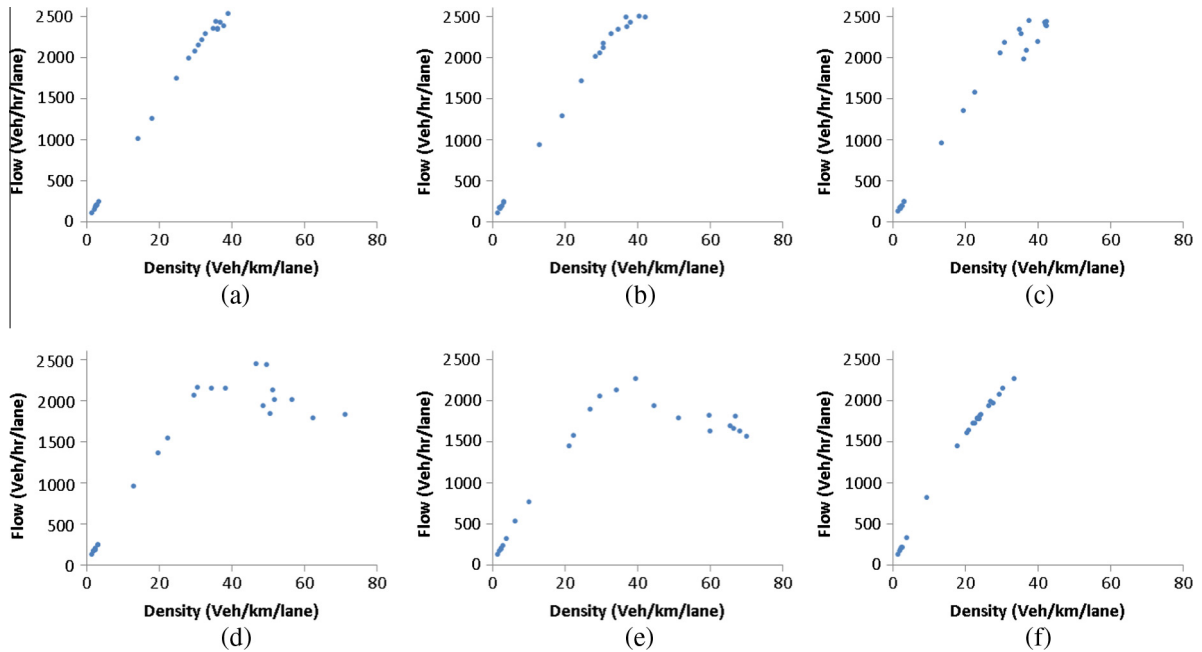


Fig. 14. Fundamental diagrams from simulating a platoon of regular, connected, and autonomous vehicles for different market penetration rates of connected and autonomous vehicles: (a) 0% connected vehicles and 90% autonomous vehicles, (b) 20% connected vehicles and 70% autonomous vehicles, (c) 40% connected vehicles and 50% autonomous vehicles, (d) 50% connected vehicles and 40% autonomous vehicles, (e) 70% connected vehicles and 20% autonomous vehicles, and (f) 90% connected vehicles and 0% autonomous vehicles.

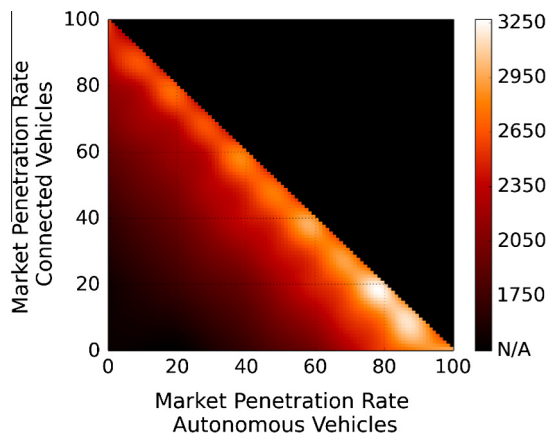


Fig. 15. Maximum throughput (veh/h/lane) at different market penetration rates of connected and autonomous vehicles for a platoon of regular, connected, and autonomous vehicles with infinite length.

compare to high market penetration rates of connected vehicles. Moreover, at low market penetration rates of autonomous vehicles, throughput increases linearly with the increase in market penetration rates of connected vehicles. In overall, this figure reveals that these technologies have the potential to improve the throughput by more than 100%.

## 7. Conclusion

Connected and autonomous vehicles will shape the future of the road transportation system. These technologies are intended to enhance mobility, safety, comfort, and fuel consumption, while reducing emissions. However, the amount of this improvement is unknown and despite the extensive efforts in the literature to analyze their impacts on driving environment, there is still a need for more comprehensive studies. This paper presents an effort to investigate the effects of connected and autonomous vehicles on driving environment. Accordingly, a microscopic simulation framework is presented, which recognizes different vehicle types and uses different existing models to capture the interactions between regular, connected (at different levels of communication), and autonomous vehicles.

By employing the presented framework, this study offers analytical and simulation-based investigations of the string stability of mixed traffic streams with varying percentages of the three types of vehicles. Such mixed traffic scenarios are especially important because they correspond to likely evolutionary paths for the introduction and market penetration of these vehicle capabilities. The analytical studies revealed that connected and autonomous vehicles can improve the string stability of traffic flow. Automation is likely to be more effective than connectivity alone in preventing shockwave formation and propagation, which is also confirmed by simulation results. Simulation results also revealed that oscillation and collision thresholds increase as platoon size decreases/market penetration rate increases.

Finally, the effects of connected and autonomous vehicles on throughput are investigated through a series of simulations. The simulation results revealed that scatter in fundamental diagrams increases as market penetration rate of connected/autonomous vehicles increases from 0% to 50% and decreases after this point. However, the throughput increases as market penetration rate increases. The simulation results also showed that autonomous vehicles result in higher throughput compare to connected vehicles at similar market penetration rates. Substantial potential throughput increases are possible under certain market penetration scenarios. Extending the acceleration framework to include driver's anticipation is the subject of future research. Note that several models could be potentially used to model regular, connected, and automated vehicles. The model selection in this paper was based on the current state-of-the-art in modeling these vehicles. Therefore, the results presented in this paper are illustrative and explanatory, and accurate modeling requires actual observations of the real-world implementation of these systems.

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