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# Optimal recharging strategies for electric vehicle fleets with duration constraints ☆

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ABSTRACT

Electrical vehicles (EVs) have become a popular green transportation means recently because they have lower energy consumption costs and produce less pollution. The success of EVs relies on technologies to extend their driving range, which can be achieved by the good deployment of EV recharging stations. This paper considers a special EV network composed of fixed routes for an EV fleet, where each EV moves along its own cyclic tour of depots. By setting up a recharging station on a depot, an EV can recharge its battery for no longer than a pre-specified duration constraint. We seek an optimal deployment of recharging stations and an optimal recharging schedule for each EV such that all EVs can continue their tours in the planning horizon with minimum total costs. To solve this difficult location problem, we first propose a mixed integer program (MIP) formulation and then derive four new valid inequalities to shorten the solution time. Eight MIP models, which were created by adding different combinations of the four valid inequalities to the basic model, have been implemented to test their individual effectiveness and synergy over twelve randomly generated EV networks. Valuable managerial insights into the usage of valid inequalities and the relations between the battery capacity and the total costs, number of recharging facilities to be installed, and running time are analyzed.



## 1. Introduction

According to greenhouse gas emission statistics by the European Environment Agency (EEA), transportation has contributed 22% of the total EU-28 greenhouse gas emissions (or 1030 million tons of $CO_2$-equivalents) in 2012. It is believed that the global warming caused by conventional fueled-based vehicles may be remedied by the adoption and popularity of electrical vehicles (EVs) or other alternative-fuel vehicles (AFVs) because these vehicles consume less energy and produce less pollution. More counties have set new environmental regulations or begun to provide incentives to push the greater use of EVs to replace fuel-based vehicles. Some companies such as FedEx (FedEx, 2010, 2014), Frito Lay (Morris, 2012), and Staples (Ramsey, 2010) have also introduced EV fleets for freight shipments.

There are two major obstacles that hinder the popularity of EVs: (1) the lack of recharging infrastructures due to high installation costs, which in turn reduces the interest for customers to purchase EVs (MirHassani and Ebrazi, 2012; Kuby and Lim, 2005), and (2) the limited driving range of EVs due to their lower energy efficiency and weight limitations

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(Lim and Kuby, 2010; Wang and Lin, 2009). These difficulties may be resolved, to some extent, by an effective deployment of recharging stations and good recharging schedules for EVs. In particular, by setting up the right number of recharging facilities in the right places, all EVs can be recharged whenever and wherever necessary, with the minimum total costs, including facility setup and recharging operational costs. The cost savings would be even more significant for a transportation network composed of fixed routes, as commonly seen in some logistics or bus companies, where each EV periodically moves along a pre-specified sequence of depots or stops to collect or drop off cargo or people.

The optimization problem considered in this paper covers two difficult subproblems at the same time: (1) a special facility location problem that seeks optimal locations to install recharging stations and (2) an optimal recharging schedule for an EV fleet. The transportation network is composed of a set of EV routes, where each route is run by an EV periodically. When an EV visits a depot on its route for conducting a transport operation or rest, it must obey a duration constraint (e.g., no longer than 1 h for a daytime transport task or no longer than 6 h for an overnight rest). This duration constraint is necessary in our problem. Without considering the duration constraint, an EV can be recharged for as long as necessary to minimize the total costs, which is not realistic. For example, an electric bus that picks up and drops off passengers should not stop by an intermediate station for too long, or the on-board passengers will have a long waiting time to their destinations. In addition, the length of the duration would also affect the location decisions for setting up recharging facilities.

If the depot is equipped with a recharging facility, the EV can be recharged to gain more battery power. A battery can, at most, be recharged to its capacity, and an EV must maintain a sufficient battery level to reach its next depot at all times. As a result, the optimal recharging schedule must be considered in conjunction with the locations to install recharging stations. To this end, this paper first provides a mixed integer linear program to address both subproblems and then investigates how the solution time can be further shortened by our proposed valid inequalities.

The paper is organized as follows: Section 2 reviews the related literature. In Section 3, we present a basic mixed integer programming formulation for the problem, propose new valid inequalities, and demonstrate our formulation by a small illustrative example. Extensive computational tests and analyses are reported in Section 4, where the performance of the basic MIP model in conjunction with different combinations of added valid inequalities is compared. Section 5 summarizes our findings and suggests several possible topics for future research.

## 2. Review of related literature

Our paper investigates a special facility location that has been extensively studied for decades. Here, we only review the literature related to EV or AFV fleets and the literature regarding refueling scheduling problems.

Based on given potential locations, there are many studies examining the AFV refueling facility-location problem. Kuby and Lim (2005) extended the flow capturing location model (FCLM) of Hodgson (1990) to give a flow refueling location model (FRLM) that considers range-limited vehicles. In particular, FRLM calculates the optimal deployment for a given number of refueling stations to maximize the total refueled flow volume for given round-trip paths. Based on FRLM, Lim and Kuby (2010) developed three heuristic algorithms to eliminate the pre-generation of all combinations of candidate locations required in FRLM such that the model could be used for larger realistic networks. Capar and Kuby (2012) also proposed a new mixed binary integer programming model to solve larger FRLM problems, which resolves the difficulty of having too many generated candidate sites in the previous model. To the best of our knowledge, Capar and Kuby (2012) has tested the largest network (1000 nodes, 80 O–D pairs) considered to date in the related literature for solving a relaxed FRLM formulation called MBIP. Their model requires many new types of variables and constraints to ensure the same covering rules for round trips. Capar et al. (2013) presented a generalized FRLM formulation based on covering the arcs that comprise each path. Their new formulation is more efficient, compact, and flexible and can thus account for different fuel types and geographic scales.

Lam et al. (2014) studied an EV charging station placement problem (EVCSPP) for seeking the best locations to construct charging stations in smart city planning. In order to guarantee the charging station network spans the entire city, they gave a mathematical programming model with quadratic constraints. They also proposed four solution methods (iterative MILP, greedy approach, effective MILP, and Chemical Reaction Optimization heuristics).

Wang (2007) proposed a set covering model with nonlinear constraints to address the e-scooter recharging facility location problem. Their model considers candidate sites located on a given route/path of a single origin and destination (O–D) and is validated on an island to recharge the recreation-oriented e-scooters for tourists. Based on Wang (2007), Wang (2008) formulated a battery exchange station location model on a single path by a mixed integer program that can address multiple O–D routing demands on that path. Then, Wang and Lin (2009) further extended the works of Wang (2008) to calculate a minimum cost recharging facility location problem that considers intercity O–D round-trips based on a given 2-dimensional distance matrix between all candidate locations. A hybrid model with dual objectives of minimum cost and maximum coverage was formulated by Wang and Wang (2010). Their hybrid model could account for the travel of both long (inter-city) and short (intra-city) distances. However, both models (i.e., Wang and Lin, 2009 and Wang and Wang, 2010) still require the recharging capacity constraints at recharging sites to be eliminated such that an exact algorithm, such as branch-and-bound, can work properly.

MirHassani and Ebrazi (2012) formulated a flexible MIP model based on the same assumptions of FRLM (Kuby and Lim, 2005) and similar assumptions of Wang and Lin (2009). Their model was more efficient than previous set covering models and showed satisfactory performance in solving the flow-based maximum coverage form. Chung and Kwon (2015) investigated a multi-period charging station location problem by extending FRLM. They have tested on the Korean Expressway network of 324 nodes, 880 directed arcs, 104,652 paths, over 6 periods. Hosseini and MirHassani (2015b) considered the traffic flow uncertainty in determining the locations for both permanent and portable refueling stations.

Lin et al. (2008) defined the fuel accessibility as the ease of accessing a refueling station whenever necessary and proposed an MIP model to calculate optimal locations for installing hydrogen stations. They aimed to minimize the total time of fuel-travel-back time (i.e., the total travel time for the fuel to travel from where it is burned back to the nearest station) and assumed any point along the road network rather than the home or workplace to be the possible origin of the refueling trip, which was more realistic.

Recently, Hosseini and MirHassani (2015a) considered the waiting time at capacitated recharging stations for given locations to build recharging stations. In particular, if the number of EVs exceeded the station capacity, a queue formed. Furthermore, if the waiting time was greater than some threshold, the model located one or more additional recharging stations.

Most of the literature reviewed above reserved an initial state for the fuel tank or battery at the origin of a path to one of two possible settings: (1) a half-full tank (Kuby and Lim, 2005; Capar and Kuby, 2012; MirHassani and Ebrazi, 2012) or (2) a full tank (Wang, 2007, 2008; Wang and Lin, 2009; Wang and Wang, 2010; Hosseini and MirHassani, 2015a). Such a fixed setting is relaxed to be flexible (i.e., any value between 0 and capacity) in our model and Capar et al. (2013) because any stop in a cyclic tour can be viewed as an origin, provided that a vehicle has sufficient fuel/battery to reach its next stop.

We also reviewed locomotive refueling literature related to our problem, but these works have not been noted in the literature concerning EV facility locations. In particular, locomotives usually move along a fixed route periodically, and a refueling operation is similar to a recharging operation. The locomotive refueling problem raised in the problem solving competition (RAS, 2010) held by the Railway Application Section (RAS) of INFORMS in 2010 has detailed the problem description, and Nourbakhsh and Ouyang (2010), Raviv and Kaspi (2012), and Kumar and Bierlaire (2015) solved this problem by using MIP formulations and valid inequality techniques. In general, previous locomotive refueling research works have focused more on an optimal refueling plan that minimizes the total costs of facility installation and fuel consumption. Their formulation can more accurately capture fuel consumption than most EV facility location models. This is especially useful when the refueling costs vary at different geographical sites. In addition, keeping track of fuel consumption can help estimate the time of refueling. In other words, this formulation can help more accurately estimate the recharging time for EVs, which is crucial for real-world applications in logistics or bus companies.

Based on these observations, this paper provides a new formulation that takes advantage of the conventional locomotive refueling models and follows the duration constraints that are encountered more frequently in EV applications but rarely seen in EV related literature. We believe by considering the tactical or operational recharging planning and duration constraints into the strategic locational decisions (Berger et al., 2007; Derigs et al., 2013; Lin et al., 2011), it would be more beneficial in practice. Although we derive our own valid inequalities using techniques similar to those used by Raviv and Kaspi (2012) and Kumar and Bierlaire (2015), our work is the first to employ such techniques in the EV facility location literature. Moreover, our comparative analyses on the effectiveness of valid inequalities are new, useful, and more complete than the locomotive refueling literature. Finally, we add the duration constraints that are commonly seen in EV practices but not in railroad fields. Duration constraints are important because they regulate the upper bound of recharging time at a station. Previous EV facility location models usually assumed the battery to be full or half-full upon departure. Such assumptions are somewhat oversimplified such that the time required for recharging is usually neglected in the related EV facility location literature.

In summary, our paper differs from previous works in the following aspects: First, we calculate detailed optimal recharging plans (i.e., battery level increase and time spent at each station) in addition to the optimal location decisions. This is more realistic and commonly seen in logistics or bus companies but is often ignored in the EV-related literature. Our formulation also resolves the full or half-full departure tank assumption, which is unrealistic but commonly used in most EV facility location literature. Second, to meet the needs of contracted shipping of freight or people in practice, we first introduce the duration constraints associated with each stop for each EV and then derive its possible extension to the time window constraints. Third, we attempt to address networks of larger scales (with a size up to 187 nodes, 224 arcs, and 200 O–D pairs), whereas most previous studies conducted experiments on networks of smaller sizes (e.g., fewer than 50 nodes and O–D pairs) or simpler structures (e.g., on a set of fewer than 20 paths). Finally, to further shorten the solution time, we introduce several classes of valid inequalities and analyze their effectiveness, which is new to the EV facility location literature.

## 3. Mathematical programming models

### 3.1. Definitions and notations

We consider a transportation network constructed by several fixed routes, where each route is periodically run by an EV during the planning horizon (e.g., 2 weeks). Let $Q$ represent the set of fixed routes run by $|Q|$ EVs. These routes form an EV transportation network $G = (N, A)$, where $N$ and $A$ denote the depot and arc sets passed by all EVs, respectively. Suppose that an EV route $q$ passes $n_q$ stops ($k = 1, 2, \ldots, n_q$) and returns to its origin ($k = 1$) during the planning horizon (e.g., 2 weeks). Let $g(q, k)$ represent the index of the depot that corresponds to the $k$th stop of an EV route $q$. In other words, the fixed cyclic route

$q$ is composed of $n_q$ arcs: $(g(q,1),g(q,2)) - (g(q,2),g(q,3)) - \cdots - ((g(q,n_{q-1}),g(q,n_q)) - (g(q,n_q),g(q,1))$. Note that some stops in a route may correspond to the same depots geographically.

For each EV $q$ arrives at its $k$th stop (or, depot $g(q,k)$), let $x^a_{q,k}$ and $x^d_{q,k}$ denote the battery level of arrival and departure; $y_{q,k}$ is the amount of electricity to be recharged with unit cost $C_{g(q,k)}$ at that stop, which cannot exceed $R_{q,k}$ , the electricity upper bound to be supplied; $E_{q,k}$ is the minimum battery power required to move to the next (i.e., the $k+1$th) stop. Let $U \geqslant E_{q,k}$ be the battery capacity for each EV. If we install a recharging facility with a unit-period fixed cost, $S$, at depot $i \in N$, we set $v_i = 1$; otherwise, $v_i = 0$. To simplify the problem, we assume a recharging facility has unlimited charging outlets. To illustrate our settings, suppose a recharging facility costs 260,000 to be installed and is expected to work for 5 years, whereas the duration of a planning horizon is 2 weeks. Then there will be 5 ∗ (52/2) = 130 periods for using a recharging facility, and thus S = 260,000/130 = 2000 here in our formulation.

### 3.2. A basic MIP formulation (M0) and an illustrative example

We can give a mixed integer program, denoted by (M0), for our problem as follows:

$$\text{Minimize} \;\; Z = \sum_{q \in Q} \sum_{k=1}^{n_q} C_{g(q,k)} y_{q,k} + S \sum_{i \in N} v_i \tag{M0}$$

Subject to

$$x^d_{q,k} - x^a_{q,k} - y_{q,k} = 0 \quad \forall q \in Q, k = 1,2,3,\ldots,n_q \tag{1}$$
$$x^d_{q,k} - x^a_{q,k+1} = E_{q,k} \quad \forall q \in Q, k = 1,2,3,\ldots,n_q - 1 \tag{2}$$
$$x^d_{q,n_q} - E_{q,n_q} = x^a_{q,1} \quad \forall q \in Q \tag{3}$$
$$y_{q,k} \leqslant R_{q,k} v_{g(q,k)} \quad \forall q \in Q, k = 1,2,3,\ldots,n_q \tag{4}$$
$$x^d_{q,k} \leqslant U \quad \forall q \in Q, k = 1,2,3,\ldots,n_q \tag{5}$$
$$x^d_{q,k}, x^a_{q,k}, y_{q,k} \in R^+ \quad \forall q \in Q, k = 1,2,3,\ldots,n_q \tag{6}$$
$$v_i \in \{0,1\} \quad \forall i \in N \tag{7}$$

The objective value $Z$ is composed of two parts: the operational costs of recharging, denoted by $\sum_{q \in Q} \sum_{k=1}^{n_q} C_{g(q,k)} y_{q,k}$ , and the fixed setup costs $S \sum_{i \in I} v_i$. Constraints (1)–(3) model the relation of flow balance. In particular, constraint (1) indicates the battery level of departure $x^d_{q,k}$ equal to its initial level of arrival $x^a_{q,k}$ plus $y_{q,k}$, the amount of electricity recharged. Constraints (2) and (3) regulate the difference in battery level change between consequent depots $x^d_{q,k} - x^a_{q,k+1}$ to be the required battery level consumption $E_{q,k}$.

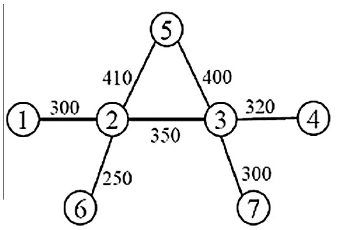
Constraint (4) can be viewed as the duration constraint, which determines whether a recharging operation is possible, depending on whether a recharging facility is installed. In particular, an EV $q$ can be recharged at the $k$th stop (i.e., $y_{q,k} > 0$), only if a recharging facility has been installed at depot $i = g(q,k)$ (i.e., $v_i = 1$). In addition, the amount of recharged electricity $y_{q,k}$ cannot exceed the available upper bound $R_{q,k}$, which has been pre-specified, depending on the duration of that visit. For example, if an EV $q$ stays at its $k$th and $k'$th stops for, at most, 30 min and 60 min, respectively, then $R_{q,k}$ should be approximately half of $R_{q,k'}$. Similarly, an EV that stays overnight at a depot equipped with a recharging facility might very likely recharge its battery to its capacity $U$.

Constraint (4) can be extended to conventional time window constraints. In particular, we can define new variables $t^a_{q,k}$ and $t^d_{q,k}$ to represent the arrival and departure time for an EV $q$ that visits its $k$th stop. Then, $t^d_{q,k} - t^a_{q,k}$ becomes the duration for its stay. If we further define a constant recharging rate, $\alpha_{q,k}$, then the amount of recharged battery level can be calculated by $y_{q,k} = \alpha_{q,k}(t^d_{q,k} - t^a_{q,k})$, which should be less than or equal to $R_{q,k}$, for a recharging operation.

The upper and lower bound constraints (5) and (6) force an EV to be recharged to a level that is no greater than its battery capacity $U$ but is still sufficient to reach each next stop (such that $x^a_{q,k} \geqslant 0$).

To illustrate our MIP model $M_0$, we use a small EV network of 3 EV routes of $Q = \{q_1, q_2, q_3\}$, whose parameters are shown in Fig. 1, to explain our formulation.

In this example, depots 1 and 5 can be viewed as overnight depots, where an EV can be recharged to its battery capacity. The objective is to minimize $Z = 3.4y_{1,1} + 3.5y_{1,2} + 3.1y_{1,3} + 3.04y_{1,4} + 3.1y_{1,5} + 3.5y_{1,6} + 3.1y_{2,1} + 3.5y_{2,2} + 3.18y_{2,3} + 3.5y_{2,4} + 3.1y_{3,1} + 3.1y_{3,2} + 3.14y_{3,3} + 3.1y_{3,4} + 3.1y_{3,5} + 3.1y_{3,6} + 3.14y_{3,7} + 3.1y_{3,8} + 500\sum_{i=1}^{7} v_i$. For the case of $q = 1$, $k = 5$, the flow balance constraints (1)–(3) can be written as $x^d_{1,5} - x^a_{1,5} - y_{1,5} = 0$ , $x^d_{1,5} - x^a_{1,6} = 350$, and $x^d_{1,6} - 300 = x^a_{1,1}$; the corresponding depot has an index $i = g(1,5) = 3$. Thus, constraint (4) is expressed by $y_{1,5} \leqslant 350 v_3$, and constraint (5) is expressed by $x^d_{1,5} \leqslant 700$.




$N = \{1,2,3,4,5,6\}$; $U = 700$; $S = 500$;
$q_1$: 1-2-3-4-3-2-1, $q_2$: 5-2-6-2-5,
$q_3$: 5-3-7-3-5-3-7-3-5;
$n_1 = 6$, $n_2 = 4$, $n_3 = 8$

$[g(1,1),\ldots,g(1,6)] = [1,2,3,4,3,2]$
$[g(2,1),\ldots,g(2,4)] = [5,2,6,2]$
$[g(3,1),\ldots,g(3,8)] = [5,3,7,3,5,3,7,3,]$
$[C_1,\ldots,C_6] = [3.4, 3.5, 3.1, 3.04, 3.1, 3.18, 3.14]$
$[E_{1,1},\ldots,E_{1,6}] = [300, 350, 320, 320, 350, 300]$
$[E_{2,1},\ldots,E_{2,4}] = [410, 250, 250, 410]$
$[E_{3,1},\ldots,E_{3,8}] = [400,300,300,400, 400,300,300,400]$
$[R_{1,1},\ldots,R_{1,6}] = [700, 350, 350, 700, 350, 350]$
$[R_{2,1},\ldots,R_{2,4}] = [700, 350, 700, 350]$
$[R_{3,1},\ldots,R_{3,4}] = [700,350,700,350,700,350,700,350]$

**Fig. 1.** A small illustrative EV network.

### 3.3. New MIP formulations by adding new classes of valid inequalities

The basic MIP formulation M0 is in a compact form, namely, removing any of constraints (1)–(8) would lead to an incorrect solution. In our preliminary computational tests, we found that M0 may require a longer running time for larger or more difficult cases. To further shorten the solution time, we may define new variables or derive new constraints in the hope of cutting off the fractional extreme points when solving for the linear programming relaxation problems in the branch-and-bound scheme. To this end, we introduce a new binary variable $z_{q,k}$ for each EV $q$ at its $k$th stop to indicate whether a recharging operation is occuring ($z_{q,k} = 1$) or not ($z_{q,k} = 0$). Then, we can relate $z_{q,k}$ and $y_{q,k}$ by constraint (8) and (7′), which are similar to constraint (4) and (7), respectively.

$$y_{q,k} \leqslant R_{q,k} z_{q,k} \quad \forall q \in Q, k = 1,2,3,\ldots,n_q \tag{8}$$

$$z_{q,k} \in \{0,1\} \quad \forall q \in Q, k = 1,2,3,\ldots,n_q \tag{7′}$$

Adding constraint (8) alone to M0 is not helpful for shortening the solution time because thus far, $z_{q,k}$ has no relation to the objective function or other constraints. Therefore, we derive additional constraints to regulate $z_{q,k}$ and other variables. In particular, we have derived four classes of new valid inequalities based on the following 3 observations.

#### 3.3.1. Observation 1

If the accumulated electricity levels required to move EV $q$ along contiguous segments $g(q,k_s) - \cdots - g(q,k_t)$ have exceeded the battery capacity $U$, then we must recharge the battery at least once on an intermediate depot selected from $\{g(q,k_s+1), g(q,k_s+2), \ldots, g(q,k_t-2), g(q,k_t-1)\}$ during this trip.

Based on this observation, we can derive our first class of valid inequalities from the EV point of view, as constraints (9) illustrated in Table 1:

The left column of Table 1 lists four conditions where a recharging operation is necessary with respect to an EV. These four conditions differ from one another depending on the boundaries of the route segments to be checked. Take the EV route

**Table 1**
Proposed first class of valid inequalities.

| | Cyclic EV route segments and conditions | Constraints (9) |
|---|---|---|
| (a) | Segments: $g(q,k_s) - \cdots - g(q,k_t)$<br>$1 \leqslant k_s \leqslant k_t - 2; k_t \leqslant n_q$<br>$U \in \left[\sum_{k=k_s}^{k_t-2} E_{q,k}, \sum_{k=k_s}^{k_t-1} E_{q,k}\right]$ | $\sum_{k=k_s+1}^{k_t-1} z_{q,k} \geqslant 1$ |
| (b) | Segments: $g(q,k_s) - \cdots - g(q,n_q) - g(q,1)$<br>$k_s \in [1, n_q - 1], k_t = 1$<br>$U \in \left[\sum_{k=k_s}^{n_q-1} E_{q,k}, \sum_{k=k_s}^{n_q} E_{q,k}\right]$ | $\sum_{k=k_s+1}^{n_q} z_{q,k} \geqslant 1$ |
| (c) | Segments: $g(q,k_s) - \cdots - g(q,n_q) - \cdots - g(q,2)$<br>$k_s \in [1, n_q], k_t = 2$<br>$U \in \left[\sum_{k=k_s}^{n_q} E_{q,k}, \sum_{k=k_s}^{n_q} E_{q,k} + E_{q,1}\right]$ | $\sum_{k=1}^{k_t-1} z_{q,k} \geqslant 1$ |
| (d) | Segments: $g(q,k_s) - \cdots - g(q,n_q) - g(q,1) - \cdots - g(q,k_t)$<br>$k_s \in [1, n_q], k_t \in [3, n_q]$<br>$U \in \left[\sum_{k=k_s}^{n_q} E_{q,k} + \sum_{k=1}^{k_t-2} E_{q,k}, \sum_{k=k_s}^{n_q} E_{q,k} + \sum_{k=1}^{k_t-1} E_{q,k}\right]$ | $\sum_{k=k_s+1}^{n_q} z_{q,k} + \sum_{k=1}^{k_t-1} z_{q,k} \geqslant 1$ |

**Table 2**
Proposed second class of valid inequalities.

| | Cyclic EV route segments and conditions | Constraints (10) |
|---|---|---|
| (a) | Segments: $g(q,k_s) - \cdots - g(q,k_t)$<br>$1 \leqslant k_s \leqslant k_t - 2; k_t \leqslant n_q$<br>$U \in \left[\sum_{k=k_s}^{k_t-2} E_{q,k}, \sum_{k=k_s}^{k_t-1} E_{q,k}\right]$ | $\sum_{k=k_s+1}^{k_t-1} v_{g(q,k)} \geqslant 1$ |
| (b) | Segments: $g(q,k_s) - \cdots - g(q,n_q) - g(q,1)$<br>$k_s \in [1, n_q - 1], k_t = 1$<br>$U \in \left[\sum_{k=k_s}^{n_q-1} E_{q,k}, \sum_{k=k_s}^{n_q} E_{q,k}\right]$ | $\sum_{k=k_s+1}^{n_q} v_{g(q,k)} \geqslant 1$ |
| (c) | Segments: $g(q,k_s) - \cdots - g(q,n_q) - \cdots - g(q,2)$<br>$k_s \in [1, n_q], k_t = 2$<br>$U \in \left[\sum_{k=k_s}^{n_q} E_{q,k}, \sum_{k=k_s}^{n_q} E_{q,k} + E_{q,1}\right]$ | $\sum_{k=1}^{k_t-1} v_{g(q,k)} \geqslant 1$ |
| (d) | Segments: $g(q,k_s) - \cdots - g(q,n_q) - g(q,1) - \cdots - g(q,k_t)$<br>$k_s \in [1, n_q], k_t \in [3, n_q]$<br>$U \in \left[\sum_{k=k_s}^{n_q} E_{q,k} + \sum_{k=1}^{k_t-2} E_{q,k}, \sum_{k=k_s}^{n_q} E_{q,k} + \sum_{k=1}^{k_t-1} E_{q,k}\right]$ | $\sum_{k=k_s+1}^{n_q} v_{g(q,k)} + \sum_{k=1}^{k_t-1} v_{g(q,k)} \geqslant 1$ |

$q_1$ in Fig. 1 as an example: an EV with full battery departing from depot 1 can, at most, reach depot 3 because $E_{1,1} + E_{1,2} \leqslant U \leqslant E_{1,1} + E_{1,2} + E_{1,3}$. This implies $z_{1,2} + z_{1,3} \geqslant 1$. Similarly, $z_{1,3} + z_{1,4} \geqslant 1$, $z_{1,4} + z_{1,5} \geqslant 1$, $z_{1,5} + z_{1,6} \geqslant 1$, $z_{1,6} + z_{1,1} \geqslant 1$, and $z_{1,1} + z_{1,2} \geqslant 1$. Note that the last two inequalities are raised for the sake of periodical movement for EVs. Based on these cyclic constraints, we do not need to force an EV to have full battery power from its origin.

Based on the same observation, but from the point of view of the depot, we can also formulate another class of valid inequalities, as constraints (10) illustrated in Table 2.

The conditions of constraint (10) are exactly the same as those of constraint (9). To ensure that a recharging operation is feasible, we have to install recharging facilities on some of those candidate depots. Take the EV route $q_1$ in Fig. 1 as an example: $z_{1,2} + z_{1,3} \geqslant 1$ implies $v_2 + v_3 \geqslant 1$. Similarly, $v_3 + v_4 \geqslant 1$, $v_4 + v_3 \geqslant 1$, $v_3 + v_2 \geqslant 1$, $v_2 + v_1 \geqslant 1$, and $v_1 + v_2 \geqslant 1$. Note that some constraints may be identical if the route segments cover the same set of depots.

*3.3.2. Observation 2*

For each EV $q$, the total electricity required to finish the route equals $\sum_{k=1}^{n_q} E_{q,k}$ , which implies it has to be recharged at least $\lceil \sum_{k=1}^{n_q} E_{q,k}/U \rceil$ times.

This observation gives the following valid inequality:

$$\sum_{k=1}^{n_q} z_{q,k} \geqslant \left\lceil \sum_{k=1}^{n_q} E_{q,k}/U \right\rceil \quad \forall q \in Q \tag{11}$$

Take EV route $q_1$ in Fig. 1 as an example: the entire route requires 300 + 350 + 320 + 320 + 350 + 300 = 1940 units of electricity, which indicates that it has to be recharged at least $\lceil 1940/700 \rceil = 3$ times. Thus, we add a new constraint $\sum_{k=1}^{6} z_{1,k} \geqslant 3$. Similar, for EV route $q_3$, it requires a total of 2800 units of electricity. Thus, we add a new constraint of $\sum_{k=1}^{8} z_{3,k} \geqslant \lceil 2800/700 \rceil = 4$.

*3.3.3. Observation 3*

For each EV $q$, it can only be recharged at a depot that has been equipped with a recharging facility.

In particular, this gives the following valid inequality:

$$z_{q,k} \leqslant v_{g(q,k)} \quad \forall q \in Q, k = 1,2,3,\ldots,n_q \tag{12}$$

Take EV route $q_1$ in Fig. 1 as an example: we can add the following new constraints: $z_{1,2} + z_{1,3} \geqslant 1$, $z_{1,4} \leqslant v_4$ , $z_{1,5} \leqslant v_3$ , and $z_{1,6} \leqslant v_2$ . Although constraint (12) seems intuitive, it turns out to be extremely effective based on our computational tests in next section, where more extensive tests are conducted and analyzed.

## 4. Computational experiments and analyses

*4.1. Settings for our computational experiments*

To evaluate the performance of different valid inequalities, we set seven new MIP models, i.e., M1, M2, ..., and M7, of the same objective function but with different composition of constraints, as listed in Table 3.

By comparing the performance of M0 and M1, we can learn the effectiveness caused by introducing constraint (10). Similarly, we can evaluate the performance of constraint (9), (11) and (12) by comparing the solution times of M2, M3, and M4 to

**Table 3**
MIP models and their composition.

| Models | Constraints | Comments |
|---|---|---|
| M0 | (1)–(7) | The original basic MIP model. |
| M1 | (1)–(7) + (10) | Adding (10) to M0. (related with $v_i$ only) |
| M2 | (1)–(7′), (8) + (9) | Adding (7′), (8), (9) to M0 ($z_{q,k}$ related) |
| M3 | (1)–(7′), (8) + (11) | Adding (7′), (8), (11) to M0 ($z_{q,k}$ related) |
| M4 | (1)–(7′), (8) + (12) | Adding (7′), (8), (12) to M0 ($z_{q,k}$ related) |
| M5 | M4 + (10) | Adding (10) to M4 |
| M6 | M5 + (9) | Adding (9) to M5 |
| M7 | M6 + (11) | Adding (11) to M6 |

that of M0. After learning the effectiveness of individual constraints (9)–(12), we combine several of them in decreasing order of their individual effectiveness to construct the hybrid models M5, M6, and M7. This is based on the intuition to seek better combinations of effective constraints. Finally, by checking the performance of the hybrid models M5, M6, and M7, we can identify the most effective combination of valid inequalities to be applied to calculate the optimal recharging facility locations and recharging schedules for EVs.

We tested all 8 MIP formulations (i.e., M0, M1, ..., M7) using Gurobi 6.0.3 on a personal computer with UBUNTU 13.10 OS, 8 GB RAM, and Intel Core i7-920 CPU of 2.67 GHz. Four families (i.e., SP1, SP2, SP3, and SP4) of 12 random transportation networks, modified from the INFORMS RAS 2010 Problem Solving Competition (RAS, 2010), were generated for our tests, where each family contains 3 random networks of similar size and number of routes. The base network is shown in Fig. 2. Details of the network parameters such as the number of nodes $|N|$, arcs $|A|$, and EV routes $|Q|$ are listed in Table 4. In particular, we randomly select $|Q| \in \{50, 100, 150, 200\}$ routes from the base network, whose layout may differ for different combinations of selected routes. Among those many random networks generated by the same $|Q|$, we select 3 networks for that family. Note that large networks may not necessarily take more time to solve because the network layout may be a more important factor than the actual running time. Thus, we generated 12 random networks of different layouts and sizes for our testing. We set the planning horizon to 12 days. During this period, an EV may have run for 1, 2 or 3 round trips. The duration constraint for each EV to stay at each stop on its route was previously specified.

### 4.2. Comparative running time analyses on the effectiveness of valid inequalities

Table 5 records the running time (s) taken by M0, M1, ..., M7 for each of the 12 random test networks. Because there was no clear connection between the running time and network size, we focused our analysis on the comparative performance of

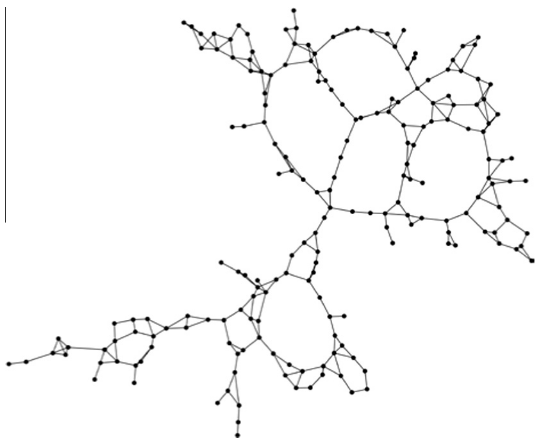


**Fig. 2.** The base network.

**Table 4**
Parameter settings of the 12 random networks.

| | SP 11 | SP12 | SP13 | SP21 | SP22 | SP23 | SP31 | SP32 | SP33 | SP41 | SP42 | SP43 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| $\|N\|$ | 142 | 148 | 153 | 169 | 169 | 175 | 180 | 184 | 184 | 181 | 181 | 187 |
| $\|A\|$ | 168 | 172 | 179 | 193 | 202 | 204 | 214 | 216 | 220 | 214 | 214 | 224 |
| $\|Q\|$ | 50 | 50 | 50 | 100 | 100 | 100 | 150 | 150 | 150 | 200 | 200 | 200 |
| $S = 7000$ $U = 5500$ | | | | | | | | | | | | |
| $R_{q,k} \in \{1375, 5500\}$ $C_{g(q,k)} \in [2.96, 3.56]$ $E_{q,k} \in [170, 1115]$ | | | | | | | | | | | | |

**Table 5**
Running time (s) of 8 MIP models on 12 test networks.

| | M0 | M1 | M2 | M3 | M4 | M5 | M6 | M7 |
|---|---|---|---|---|---|---|---|---|
| SP11 | 399 | 287 | 399 | 399 | 394 | 240[a] | 323 | 272 |
| SP12 | 115 | 136 | 115 | 115 | 98 | 95[a] | 99 | 119 |
| SP13 | 356 | 392 | 356 | 356 | 328 | 316[a] | 368 | 363 |
| SP21 | 938 | 531 | 937 | 938 | 383[a] | 432 | 632 | 601 |
| SP22 | 1008 | 1311 | 1654 | 1007 | 967 | 822[a] | 997 | 1122 |
| SP23 | 138 | 119 | 132 | 138 | 96[a] | 114 | 117 | 105 |
| SP31 | 228 | 251 | 297 | 355 | 519 | 217[a] | 349 | 294 |
| SP32 | 1216 | 699 | 843 | 1218 | 1064 | 672 | 854 | 417[a] |
| SP33 | 820 | 472 | 554 | 784 | 1591 | 411[a] | 460 | 1285 |
| SP41 | 977 | 486 | 1618 | 1618 | 582 | 579 | 442[a] | 1082 |
| SP42 | 189 | 172 | 152 | 161 | 148 | 190 | 142[a] | 153 |
| SP43 | 132 | 134 | 132 | 132 | 131[a] | 134 | 142 | 155 |

[a] To represent the minimum computational time among all 8 models.

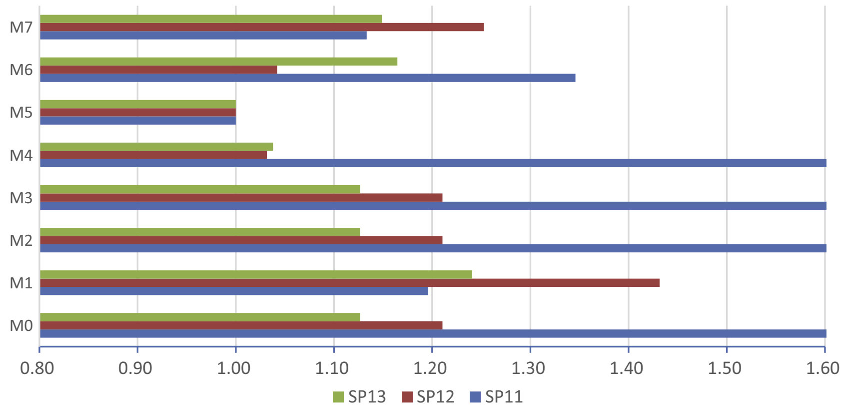



**Fig. 3.** Comparison of the normalized running time for SP11, SP12, and SP13.

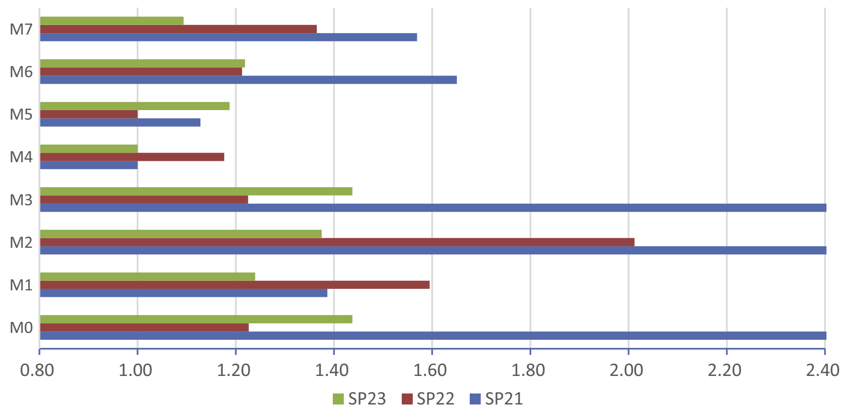



**Fig. 4.** Comparison of the normalized running time for SP21, SP22, and SP23.

different models. To this end, we normalized the running time for each category of 3 test networks, as shown in Figs. 3–6, with respect to the minimum time taken by the 8 models.

Fig. 3 shows that M5 is the most efficient and stable model. M4 performs the second best for SP12 and SP13 but performs poorly for SP11. M7 perform the third best in the sense that it can address SP11 better than the other models can, except for M5. Among the 4 models that add individual valid inequalities (i.e., M1, M2, M3, and M4), M1 and M4 show better performance. Based on these results, we make the following observations:

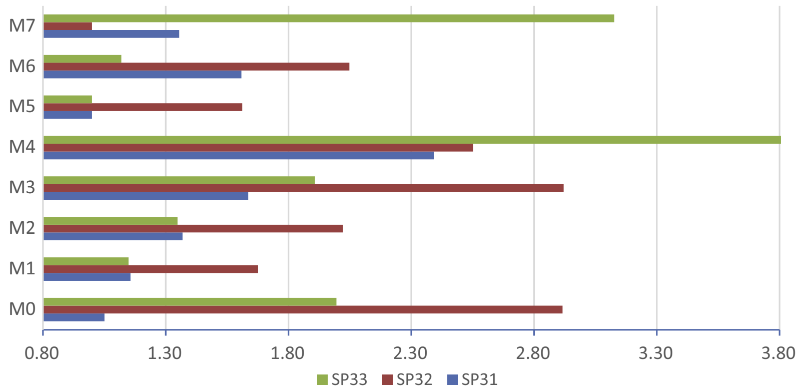



**Fig. 5.** Comparison of the normalized running time for SP31, SP32, and SP33.

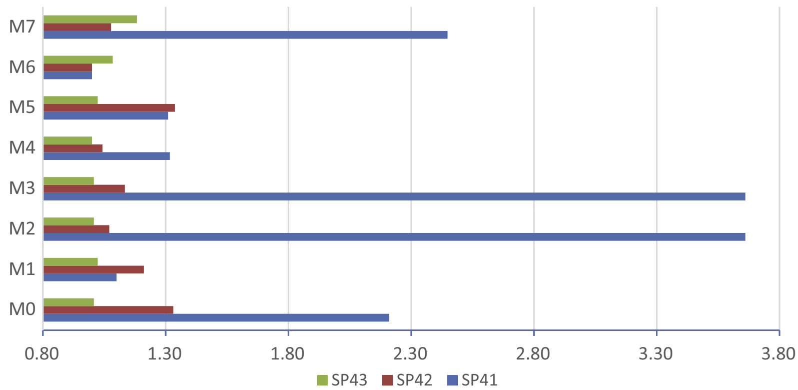



**Fig. 6.** Comparison of the normalized running time for SP41, SP42, and SP43.

- Hybrid models (M5–M7) seem to perform better than individual models (M1–M4), but this result is mainly caused by constraints (10) and (12).
- Constraints (10) and (12) are better valid inequalities in general, and combining the two would work the best. More specifically, constraint (10) is effective for SP11, and constraint (12) is effective for SP12 and SP13.
- Constraint (9) and (11) are not helpful for the SP1 family.
- Adding valid inequalities may not necessarily help for all cases. For example, although constraint (10) shortened the solution time for SP11, it slowed down the model for SP12 and SP13.

Similar to the results shown in Fig. 3, all of the hybrid models show better performances than do most of the individual models, except for M4, when solving the SP2 family. Nevertheless, by comparing the performances of M6 and M7 and of M2 and M3, as shown in Fig. 4, we learn that adding constraints (9) and (11) will cause even longer solution times. Constraint (10) slightly increases (and also decreases) the efficiency in some cases. Constraint (12) is the major key to the time saving of hybrid models in solving for the SP2 family.

For solving the SP3 family, Fig. 5 suggests constraint (10) is the most effective and can even be improved in conjunction with constraint (12). On the contrary, adding constraint (12) alone actually causes the most harm to performance. Constraint (11) has similar characteristics to constraint (12). In particular, adding constraint (11) alone worsens the performance, but it may save some time in some cases (e.g., SP31, SP32). Constraint (9) does not have a substantial impact on the efficiency.

When solving for the SP4 family, M6 performs the best. M1 and M4 perform better than the other models do, i.e., constraints (10), (12), and (9) perform better than (11), in this order. In Fig. 6, most models show similar performance for solving SP43 and SP42. When solving SP41, constraints (9) and (11) even slow down the basic model M0, but the effect of constraint (9) can be remedied by constraints (10) and (12).

In summary, based on our tests, the effectiveness of individual valid inequalities are ranked as follows: (12), (10), (9), and (11). Compared with constraints (9) and (11), which define the lower bound for $z_{q,k}$, constraint (12) defines the upper bound for $z_{q,k}$. This may explain why constraints (9) and (11) are not as effective. We suspect that the effectiveness of constraint (10) might be caused by the fact that constraints over nodes are more effective than constraints over EVs. In particular, a node may be passed by several EV routes. As a result, whether to install a recharging facility over a single node will affect more

EV routes at the same time, which makes constraint (10) stronger than constraints (9) and (11). In addition, constraint (10) employed in conjunction with (12) should have better synergy effects, which explains the best performance of M5.

Adding more valid inequalities may not necessarily guarantee improved performance, but adding more effective valid inequalities according to their effectiveness rank does lead to better performances. This observation is useful, especially for more difficult cases that require solutions within limited time windows.

### 4.3. Sensitivity analyses

To have more managerial insights, we conducted several analyses on the performance change in number of recharging stations to be installed (Figs. 7–9), and total cost (Figs. 10–12) incurred by varying the value of the battery capacity ($U$), upper bound on the amount of recharged electricity ($R$), and the fixed cost for installing a recharging facility ($S$). Without loss of generality, we choose the test network SP43 (187 nodes, 224 arcs, and 200 routes) as described in Table 4, for conducting sensitivity analysis. For convenience, we denote $R_0$ and $S_0$ to represent the vectors of the original $R_{q,k}$s and $S$ for test network SP43, respectively.

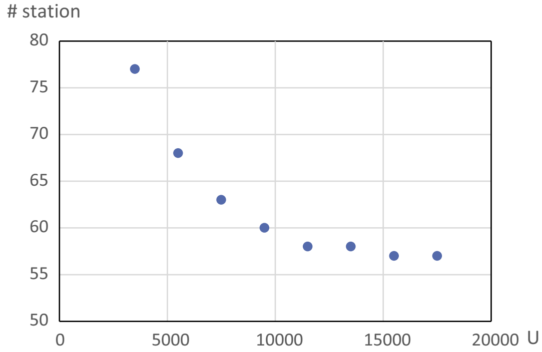



**Fig. 7.** Changes in the number of recharging stations installed by varying $U$.

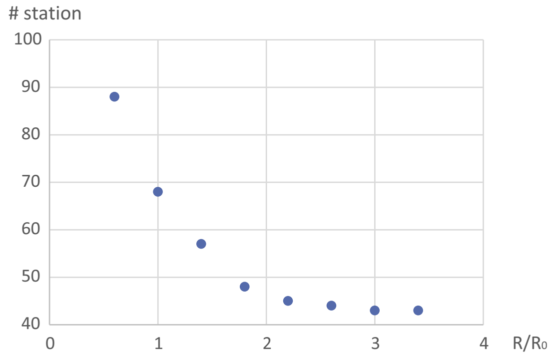



**Fig. 8.** Changes in the number of recharging stations installed by varying $R$.

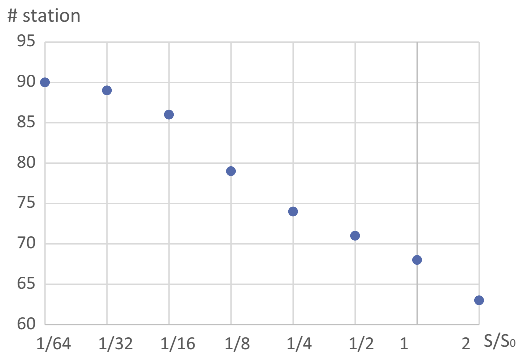



**Fig. 9.** Changes in the number of recharging stations installed by varying $S$.

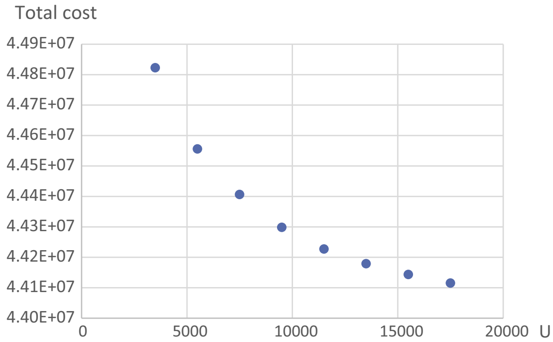



**Fig. 10.** Changes in the total cost caused by varying *U*.

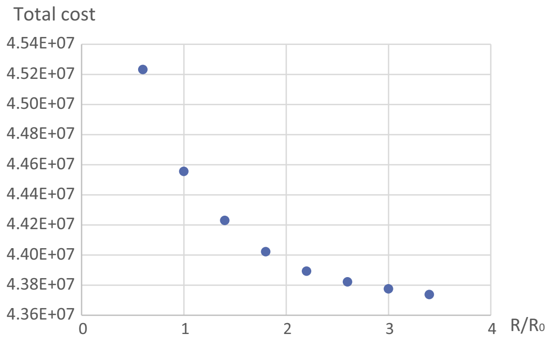



**Fig. 11.** Changes in the total cost caused by varying *R*.

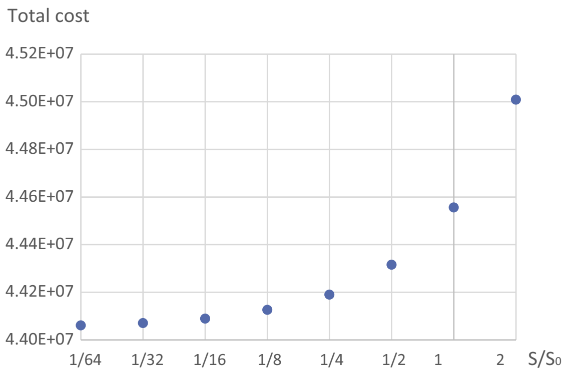



**Fig. 12.** Changes in the total cost caused by varying *S*.

In our experiments, we observe that larger battery capacity, smaller upper bound for non-overnight recharging, or larger fixed cost may lead to longer solution time. Since the difficulty of the MIP mainly caused by the integral decisions, and larger *U* and *S*, as well as smaller *R*, make the locational integral decisions to select better recharging sites more important. Otherwise (i.e., smaller *U* and *S* , and larger *R*), the locational integral decisions might become less important than the linear recharging decisions (i.e., how much electricity to be recharged), which makes the problem to be simpler to solve.

Figs. 7–9 indicate larger *U* , *R* , and *S* lead to install fewer recharging stations. Indeed, if the battery has larger capacity, or it can be recharged more electricity in each station, then we will need fewer recharging stations. Moreover, since if we consider the unit recharging cost $C_{g(q,k)}$ may vary at different location, by reducing the fixed cost for installing a recharging station, we may have better chance to install more recharging stations on places with cheaper recharging unit costs.

Since the fixed cost is usually more significant than the recharging cost, larger *U* and *R* lead to the decisions to install more recharging facilities (see Figs. 7 and 8), which in turn incur larger total cost, as shown in Figs. 10 and 11. On the other hand, even if increasing *U* might discourage the installation of new recharging facilities (see Fig. 9), recharging facilities are still necessary to keep all EVs on-road. Considering the system requires at least some minimum number of recharging facilities, increasing the fixed cost *U* would still incur larger total cost.

Note that the change in R can provide some managerial insights as follows: First, if we consider R to be set by some kind of contract (e.g., an electric bus that can be recharged at a station for no longer than 30 min; or an electric truck that stops by a convenient store but has to departure within 30 min to head for its next stop), then by negotiating a better value of R one may lead to more savings in the total costs. Second, from another point of view, the value of R also depends on the effectiveness of recharging equipment's. For example, if we employ a faster recharging equipment, more electricity can be recharged within the same duration, which would lead to the same effects of larger $R$ and thus decrease the number of recharging stations.

From these sensitivity analysis, we can roughly assess the benefits of adopting new battery technology to determine whether doing so is worthwhile in terms of the savings in total cost or number of recharging facilities.

## 5. Conclusions and discussions

More logistics and transport companies are expected to use EV fleets in the near future for transporting materials or people. This paper considers a special transport network where each EV moves along the same route periodically in a planning horizon. Each EV route is a round trip composed of a cyclic sequence of depots, where an EV can stop to transport materials (or people), rest, or recharge its battery within a pre-specified duration limit. Our model can be easily extended to account for conventional time window constraints associated with a visit made by an EV $q$ at its $k$th stop. We first propose a basic compact MIP model (M0) to calculate an optimal deployment for recharging stations and an optimal recharging schedule for the entire EV fleet. Then, we derive four valid inequalities based on insights observed from some necessary conditions to recharge the battery. Although these new valid inequalities are derived in the hope of accelerating the solution procedure, how they can be used in conjunction with each other to achieve better performance is not clear. To this end, we implemented 8 different MIP models by adding one or a few valid inequalities to M0 and tested their performance on 4 families of random networks. The results suggest that combining valid inequalities according to the order of their effectiveness would produce a better MIP model. In our case, adding constraints (12) and (10) to M0 would provide the most effective MIP model for all cases. However, adding all or some of the new valid inequalities to M0 might not necessarily help improve the performance and can even increase the solution time. Our comparative analyses and suggestions are important, especially when there is not sufficient time to determine the optimal combination of constraints. In addition, our sensitivity analyses can help understand the impact of the battery technology to the total costs, facility location plan, and solution time.

The contributions of this paper are threefold: First, we employ the techniques of valid inequalities (rarely seen in the EV facility location literature), and conducted comprehensive computational experiments in comparing the performance of the basic MIP model in conjunction with different combinations of added valid inequalities is compared. Some of our findings may not be trivial. For example, introducing additional variable $z_{q,k}$ , and right additional constraints (12) would enlarge the problem site but in fact boost the performance. Our results can serve as a good start for future study in designing better constraints. Second, our proposed mathematical models, based on the locomotive refueling literature, can deal with partial recharging, and calculate accurate battery level for each EV. This leads to more accurate estimate on the battery consumption and recharging, so that it would give a more accurate recharging planning as well as facility location decisions, compared with previous work. We think similar techniques may be employed to deal with more difficult problem such as the location routing problem for an EV fleet (EVLRP). Third, we introduce the duration constraint, which is crucial to resolve the range anxiety in practice, to make better and more practical decisions. In addition, we also explain how to extend our model to deal with general time windows constraints.

For future research, we suggest investigating the topics of the optimal deployment for battery swap stations or the EVLRP. The former differs from this paper in that the duration constraints for recharging can be neglected, but it would induce new challenges on optimal battery inventory for each battery swap station; the latter is a more fundamental and challenging problem than that presented in this paper. However, we suspect our techniques in modeling the detailed battery level might be applicable to deal with EVLRP. In particular, an EVLRP also seeks best sites for installing recharging facilities, so that a fleet of EVs could depart from a depot, visit customers to pick up or deliver shipments, be recharged on some sites, and return the depot after completing all the shipments. Besides the variables (usually some binary variable like $X^k_{ij}$ to represent whether vehicle $k$ travels directly from node $i$ to $j$) and constraints (flow balance, set packing) in conventional LRP models, we can associate with each vehicle $k$ a battery power level $\widehat{\beta}^k_i$ (and $\breve{\beta}^k_i$) to enter (and leave) node $i$ . Suppose traveling on arc $(i,j)$ consumes $E^k_{ij}$ units of electricity. Also, amount of recharged electricity on node $i$ equals to $w^k_i$ . Then, we can give the following constraints: $\breve{\beta}^k_j = \widehat{\beta}^k_i + w^k_i$ , $\breve{\beta}^k_j - E^k_{ij}X^k_{ij} - M\left(1 - X^k_{ij}\right) \leqslant \widehat{\beta}^k_j \leqslant \breve{\beta}^k_j - E^k_{ij}X^k_{ij} + M\left(1 - X^k_{ij}\right)$ , and $w^k_i \leqslant U v_i$ , similar to our constraints (1), (2), and (4). Note that we can also easily to employ the duration or time windows constraints on EVLRP, and these timing constraints can help avoid the subtour elimination constraints commonly appeared in LRP models. However, how to design good valid inequalities for EVLP remains challenging.

## Acknowledgement

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