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Path-constrained traffic assignment: A trip chain analysis under range anxiety

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ABSTRACT

This paper proposes and analyzes a distance-constrained traffic assignment problem with trip chains embedded in equilibrium network flows. The purpose of studying this problem is to develop an appropriate modeling tool for characterizing traffic flow patterns in emerging transportation networks that serve a massive adoption of plug-in electric vehicles. This need arises from the facts that electric vehicles suffer from the "range anxiety" issue caused by the unavailability or insufficiency of public electricity-charging infrastructures and the far-below-expectation battery capacity. It is suggested that if range anxiety makes any impact on travel behaviors, it more likely occurs on the trip chain level rather than the trip level, where a trip chain here is defined as a series of trips between two possible charging opportunities (Tamor et al., 2013). The focus of this paper is thus given to the development of the modeling and solution methods for the proposed traffic assignment problem. In this modeling paradigm, given that trip chains are the basic modeling unit for individual decision making, any traveler's combined travel route and activity location choices under the distance limit results in a distance-constrained, node-sequenced shortest path problem. A cascading labeling algorithm is developed for this shortest path problem and embedded into a linear approximation framework for equilibrium network solutions. The numerical result derived from an illustrative example clearly shows the mechanism and magnitude of the distance limit and trip chain settings in reshaping network flows from the simple case characterized merely by user equilibrium.

1. Introduction

Electric vehicles offer an evidently promising approach to reducing greenhouse emissions and air pollution, mitigating risks associated with the shortage of fossil fuels, and utilizing excess energy from various renewable sources. The recent decade observes a fast penetration of electric vehicles of different technologies in many cities and regions worldwide. Despite the anticipated environmental and economic benefits to both individual drivers and the society, however, a massive adoption of electric vehicles is still an ambitious goal that may not be achieved in a short period. It is generally believed that the

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major barriers to wide acceptance and use of electric vehicles, especially battery electric vehicles, are the unavailability or insufficiency of public electricity-charging stations (Marrow et al., 2008; Dong et al., 2014) and limited driving ranges due to the current electricity charging and storage technologies (Pearre et al., 2011). As circulated in the general public, these factors poses the well-known *range anxiety* issue: The mental distress or fear of being stranded on roads because the battery runs out of charge.

How to incorporate range anxiety into the conventional travel demand modeling and transportation planning processes poses a series of interesting research questions. As an initial attempt, Jiang et al. (2012, 2014) and Jiang and Xie (2014) studied distance-constrained traffic assignment problems for predicting traffic network flows when a large number of plug-in electric vehicles prevail in congested networks while suffering from range anxiety. These researchers assumed that the distance constraint caused by range anxiety sets a restriction on trips and accordingly developed a set of trip-based models. However, most electric vehicles in the current market are of a driving range of 60 miles or higher if their onboard batteries are fully charged (Borden and Boske, 2013; NREL, 2015). Even if the batteries are not fully charged, the effective driving range is often well beyond the distance of a typical commuting trip or a trip of other purposes. Obviously, electric vehicle drivers typically consider the range anxiety concern (Tamor et al., 2013), if they do, more probably on the trip chain level than the trip level, since in most cases they have more charging opportunities at the destinations of their tours, such as home or workplace, rather than at some intermediate parking places. In other words, what these drivers are really concerned about is whether the electricity in the batteries is sufficient for completing an entire home-based or workplace-based tour.

Following such a behavioral speculation, we present in this paper a distance-constrained traffic assignment problem that incorporates trip chains, as a more realistic modeling tool to the ones proposed by Jiang et al. (2012, 2014). It is more reasonable to assume that the distance constraint caused by range anxiety enforces a restriction on the length of an entire trip chain than that of a single trip (Tamor et al., 2013). In the former case, individual travelers face a series of choices on activity sites and travel routes subject to the distance limit. Travelers visit one or more activity sites along a trip chain to satisfy a variety of predetermined socioeconomic purposes, such as shopping, dining, entertainment, education, religious activities, and so on.

For modeling convenience and solution tractability, we make the following modeling assumptions in this paper: (1) as similar to Jiang et al. (2012, 2014), a common distance limit is applied to the entire network or all electric vehicles in the population, which implies the battery capacity of all electric vehicles and the electricity consumption rate of these vehicles on roadways are constant; (2) the types of activity sites and the *order or sequence of activities* along a trip chain are exogenously given, although visiting which location in each type of activity sites is a decision making under equilibrium in the model; (3) activity disutility is simply modeled as a function of activity flow, while other explanatory factors are assumed to be exogenously determined and appear as fixed coefficients in our model; (4) activity and travel dynamics and time constraints on them are ignored. When needed, the first assumption can be readily relaxed, so as to take into account the heterogeneity of actual and perceived distance limits in the driving population. The second assumption can be eliminated as well, to accommodate a more general case, so that the types of activity sites and their orders along trip chains are endogenously determined. Either of the relaxations, however, will significantly increase the modeling and solution complexity. As for the last two assumptions, they are made here just for simplicity, which leads to a model as simple as what we introduce below. Moreover, in addition to the distance limit and trip chain settings, we also require network flow patterns to be still characterized by the classic Wardropian equilibrium principle (1952), under which all used trip chains connecting an origin–destination pair are of the same combined activity-travel disutility and no individual traveler can improve his or her combined disutility by unilaterally switching to any alternative activity site or alternative travel path. To this end, the major contribution of this paper is on the development and evaluation of modeling and solution methods for the proposed distance-constrained, trip chain-based traffic assignment problem.

The remaining part of this paper is organized as follows. The relevant literature is reviewed in the next section, including traffic assignment problems with trip chains, traffic assignment problems with combined choices, and traffic assignment problems with path constraints. Then we discuss the equilibrium conditions, problem formulation and equivalency and uniqueness of this formulation. A linear approximation algorithm is adopted for problem solutions, in which a newly developed cascading labeling algorithm is embedded for solving the linearized subproblem—a distance-constrained, node-sequenced shortest path problem. To our best knowledge, this algorithm has not appeared previously in the literature. We then present and analyze the numerical results from applying the solution algorithm for an illustrative problem. Finally, we conclude this paper with highlighting a few modeling and solution comments and some suggestions for future research, such as the relaxation of activity sequences and introduction of stochastic distance limits.

2. Relevant research

As we claimed earlier, this paper focuses on the development of a mathematical programming model and method for the proposed traffic assignment problem that encapsulates trip chains and distance limits. The combination of these extra modeling elements invites a greater deal of modeling complexity than the basic traffic assignment problem such as the one defined by Beckmann et al. (1956). Various forms of traffic assignment problems have been studied since the birth of the basic traffic assignment model. In many of these problems, extra constraints imposed on nodes, links, paths, or origin–destination pairs to restrict traffic flows were considered, which pose the so-called traffic assignment problems with side constraints. Example traffic assignment problems with side constraints include capacity-constrained problems (Daganzo, 1977a, 1977b; Larsson and Patriksson, 1995; Cheng et al., 2003; Nie et al., 2004), speed-constrained problems (Yang et al., 2012), time-constrained problems (Jahn et al., 2005; Schulz and Stier-Moses, 2006), and distance-constrained problems (Jiang et al., 2012, 2014; He et al., 2014; Xie and Jiang, 2016). By incorporating a distance constraint imposed on trip chains, obviously, the proposed traffic assignment problem in this paper poses a new member in this family.

From the earlier discussion, it is clear that in the proposed traffic assignment problem three parts of extra modeling components are taken into account, namely, trip chain structures, combined travel choices, and path-based constraints. Due to this reason, we carry out a literature review below by tracing relevant research along the three lines.

2.1. Traffic assignment problems with trip chains

A number of researchers (Adler and Ben-Akiva, 1979; Kitamura, 1984, 1988; Bowman and Ben-Akiva, 2000; Shiftan, 1998; Recker, 1995, 2001) early recognized that different types of trips in a chain are generally spatially and temporally interrelated and it is necessary to incorporate trip chain structures into traffic network flow and travel demand forecasting models if one wants to properly capture mutual effects among interrelated trips in an individual's travel itinerary. However, casting trip chaining effects in an analytical traffic assignment or travel demand model is not a trivial task. Most of existing traffic assignment models except the following ones use trips as the basic modeling and analysis unit, due to their simple model structures and appealing solution tractability. In particular, previous traffic assignment problems embracing trip chains are of two types, namely, static and dynamic (including quasi-dynamic) problems, depending on whether or not time is included as a modeling dimension. Maruyama and Harata (2005, 2006) first presented a set of convex programming models for static traffic assignment with different types of trip chains and applied these models for evaluating tolled networks. Higuchi et al. (2011) developed a variational inequality model for static traffic assignment with trip chains for mixed traffic and transit networks. In the dynamic network paradigm, a set of variational inequality models for quasi-dynamic and dynamic traffic assignment problems with embedded trip chains appeared in, for example, Lam and Yin (2001), Lam and Huang (2002, 2003), Ouyang et al. (2011), and Fu and Lam (2014), in which time-dependent activity disutilities are explicitly modeled as part of trip chain costs.

2.2. Traffic assignment problems with combined choices

Initially, traffic assignment models with combined choices were developed for overcoming the inherent inconsistency of different travel choices that are included by a sequential procedure. Florian et al. (1975) and Evans (1976) both proposed convex programming formulation for the combined trip distribution and traffic assignment problem, as an extension of Beckmann et al.'s (1956) transformation for the prime traffic assignment problem. To capture the congestion effect at destinations, Oppenheim (1993) added the endogenous destination cost into the above model, as a function of arriving flows at destinations. Erlander (1990) derived an alternative convex optimization model for the combined trip distribution and traffic assignment problem, on the basis of stochastic user equilibrium. Florian and Nguyen (1978) developed a convex programming model for the combined trip distribution, modal split and trip assignment problem, which avoids the utilization of asymmetric Jacobian elements by using separate traffic and transit subnetworks. Friesz (1981) presented an equivalent optimization model for a combined multiclass trip distribution, traffic assignment and modal split problem, which eliminates the symmetry restriction on cost functions by expressing the Wardropian equilibrium as a set of nonlinear constraints. However, this model is not convex and requires route enumeration for its solution. Lam and Huang (1992) then proposed a convex formulation for the multiclass combined trip distribution and traffic assignment problem, which uses symmetric "normalized" link cost functions. Recently, Wong et al. (2004) presented a combined trip distribution, modal split, and traffic assignment model with multiple user and mode classes. In the same year, Boyce and Bar-Gera (2004) reviewed progress in traffic assignment models with combined travel choices and discusses the implementation and application issues of multiclass models of this type. When the need arises for activity-based travel demand modeling, traffic assignment models with combined activity and travel choices were placed in the agenda. An activity cost function is typically used at each activity site, as a function of activity flow, reflecting the congestion effect at each site. Such traffic assignment models with combined activity and travel choices appeared in Lam and Yin (2001), Lam and Huang (2002, 2003), Maruyama and Harata (2005, 2006), Higuchi et al. (2011), Ouyang et al. (2011), and Fu and Lam (2014), as described in the last paragraph.

2.3. Traffic assignment problems with path constraints

Adding side constraints into a traffic assignment problem often increases the solution intractability significantly. In the literature, research efforts on network equilibrium problems with side constraints were dominantly focused on link-level side constraints, which resulted in capacity-constrained as well as speed-constrained problems, as aforementioned. In contrast, few studies were devoted to traffic assignment problems with path-level constraints except a few studies as follows. Jahn et al. (2005) proposed a system-optimum traffic assignment problem with an upper bound on path travel times for designing a route guidance system that simultaneously promotes system optimum and user fairness. Jiang et al. (2012, 2014) and Jiang and Xie (2014) presented a user-equilibrium traffic assignment problem with an upper bound on path lengths for the need for predicting network flows of electric vehicles, the routing behavior of which is subject to range anxiety. This problem was extended by He et al. (2014) and Xie and Jiang (2016) to embrace the relay requirement of electric vehicles in their long-haul trips, where the trip length is typically beyond the driving distance limit. He et al. (2015) further formulated a tour-based user equilibrium problem with range constraints and recharging opportunities in their charging station location study, where the locations of intermediate destinations in the problem are exogenously determined.

3. Problem formulation and properties

3.1. Trip chains and activity sequences

The core modeling component in the proposed problem is the distance limit of electric vehicles imposed on the length of trip chains. In a network where its flow constitutes individual trip chains, a traveler needs to make a set of discrete decisions in choosing activity nodes and travel paths so as to satisfy his or her economic or social demands while minimizing the total activity-travel cost. As a minimum requirement for modeling the cost composition of a trip chain, two types of costs, activity costs occurring at activity nodes and travel costs spent on traffic links are included.

In our setting, both of activity costs and travel costs are flow-dependent variables. For the mathematical modeling requirement, we presume that both the activity cost and travel cost functions are convex, increasing, and continuously differentiable, with respect to their corresponding traffic flow rates. Without loss of generality, we also set that activity and travel costs along a trip chain are both additive and mutually commensurable; in other words, the total activity-travel cost along a trip chain is the sum of all at-node activity costs and on-link travel costs along the chain. On the other hand, when evaluating the impact from the distance limit, we only take into account the physical length of traffic links in calculating the total length of a trip chain, since in general no activity needs to consume electricity from vehicle batteries. That is to say, the total length of a trip chain is the sum of lengths of all traffic links along the trip chain.

As a simple and convenient treatment for accommodating trip chains, we classify the activity-travel demand between any origin–destination pair in terms of the order or sequence of activities, such as "school-shopping-dinning", or "dinning-shop ping-entertainment". In this regard, the activity-travel demand over the network is distinguished by both origin–destination pairs and activity sequences. The combination of an origin–destination pair and an activity sequence specifies a complete feasible choice set of activity nodes and travel paths for its corresponding demand.

A toy network shown in Fig. 1 can be used to illustrate the feasibility of trip chains constrained by the activity sequence and distance limit. Suppose that for simplicity there are only two types of activities, "dining" and "entertainment", in the network, in which nodes 2, 5 and 6 are "dining" activity nodes, while nodes 4 and 7 are "entertainment" activity nodes (where "dining" is denoted by d and "entertainment" denoted by e). Now we intend to identify which trip chains connecting origin node 1 and destination node 8 in this network satisfy a simple "dining-entertainment" activity sequence and a distance limit of $\leqslant 20$. Obviously, three trip chains, for example, trip chains $1(r) \to 2(d) \to 4(e) \to 7 \to 8(s)$ and $1(r) \to 2(d) \to 4 \to 7(e) \to 8(s)$ with their length of 19, and trip chain $1(r) \to 6(d) \to 7(e) \to 8(s)$ with its length of 18, are feasible, while other three trip chains, trip chain $1(r) \to 2(d) \to 4(e) \to 5(d) \to 8(s)$ with its length of 21, trip chain $1(r) \to 2(d) \to 3 \to 5(d) \to 8(s)$ with its length of 22, and trip chain $1(r) \to 4(e) \to 5(d) \to 8(s)$ with its length of 18, are not feasible. The infeasibility of the three latter ones is due to the violation of distance limit, the absence of activity sequence, or both of them. From this result, we can see that the infeasibility caused by the distance limit and activity sequence may exclude a large number of trip chains from carrying traffic flows. Among all the described trip chains above, we should also note that the first two share the same set of traffic links, but contain different activity nodes (i.e., node 4 vs. node 7 for the "entertainment" activity). As a result, the two trip chains comprise different sets of trips, i.e., trip chain $1(r) \to 2(d) \to 4(e) \to 7 \to 8(s)$ includes three trips, $1 \to 2$, $2 \to 4$, and $4 \to 7 \to 8$, while trip chain $1(r) \to 2(d) \to 4 \to 7(e) \to 8(s)$ includes $1 \to 2$, $2 \to 4 \to 7$, and $7 \to 8$.

It must be acknowledged that such an exogenous specification of activity sequences for individual travelers is a quite strict behavioral assumption on the proposed problem. A more flexible setting that allows the model to endogenously determine individual activity sequences can be readily realized, which we leave aside until the last section of this paper. Furthermore, for a discussion regarding various individual location and route choice behaviors embedded in activity-based trip chains, interested readers may refer to a recent research paper by Chow and Liu (2012).

On the basis of the above settings, we shall start our discussion on the proposed traffic assignment problem with trip chains from its equilibrium conditions and mathematical formulation. For discussion convenience, the notation used in the formulation is presented first.

3.2. Notation

Please refer to Table 1 for the notation used throughout the paper.

3.3. Equilibrium conditions

Given the user equilibrium principle and distance limit constraint, the equilibrium conditions for the proposed problem can then be described as follows: For a certain amount of given activity-travel demand that moves between a specific origin–destination pair and through a specific activity sequence, if the total length of a trip chain is no longer than the distance limit

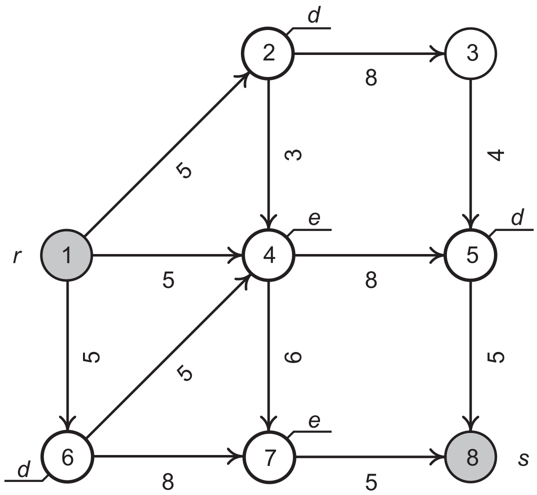

Fig. 1. Trip chains in an illustrative network.

Table 1
A notation list for the problem formulation.

Sets	
N	Set of nodes, where $N = \{n\}$
A	Set of links, where $A = \{a\}$
R	Set of origin nodes, where $R = \{r\}$
S	Set of destination nodes, where $S = \{s\}$
P	Set of activity nodes, where $P = \{p\}$
$K_{r,s}^{m}$	Set of trip chains used by the activity-travel demand of the mth activity sequence from origin r and destination s, where $K_{r,s}^{m} = \{k\}$
Parameters	
d_a	Physical length of link a
c_a	Nominal capacity of link a
D	Distance limit
t_a^0	Free-flow travel cost of traffic link a
c_p	Nominal capacity of activity node p
υ_p^0	Free-flow activity cost of node p
$q_m^{r,s}$	Activity-travel demand rate of the mth activity sequence from origin r to destination s
$\delta_{k,a,m}^{r,s}$	Link-chain incidence indicator, indicating the number of times a traffic link is incident upon a trip chain, where $\delta_{k,a,m}^{r,s} = n$ and $n \geqslant 1$ is an integer number, if traffic link a is used by trip chain k for n times for the demand of the mth activity sequence from origin r to destination s, and $\delta_{k,a,m}^{r,s} = 0$ if traffic link a is not used by trip chain k
$\delta_{k,p,m}^{r,s}$	Node-chain incidence indicator, where $\delta_{k,p,m}^{r,s} = 1$ if activity node p is on trip chain k for the demand of the mth activity sequence from origin r to destination s, and $\delta_{k,p,m}^{r,s} = 0$ otherwise
θ	Activity-travel cost conversion factor
Variables	
t_a	Travel cost on traffic link a
υ_p	Activity cost at activity site p
$t_{k,m,r,s}^{i,i+1}$	Travel cost on a trip from activity node of the ith type to another activity node of the $(i+1)$th type, where this trip is part of trip chain k for the demand of the mth activity sequence from origin r to destination s
$t_{k,m}^{r,s}$	Activity-travel cost on trip chain k for the demand of the mth activity sequence from origin r to destination s, where $t_{k,m}^{r,s} = \sum_a t_a \delta_{k,a,m}^{r,s} + \frac{1}{\theta}\sum_p \upsilon_p \delta_{k,n,m}^{r,s}$
$\mu_{m,r,s}^{i,i+1}$	Minimum activity-travel cost along all trips from an activity node of the ith type to another activity node of the $(i+1)$th type for the demand of the mth activity sequence from origin r to destination s
$\mu_m^{r,s}$	Minimum activity-travel cost along all trip chains for the demand of the mth activity sequence from origin r to destination s
$l_{k,m}^{r,s}$	Physical length of trip chain k for the demand of the mth activity sequence from origin r to destination s, where $l_{k,m}^{r,s} = \sum_a d_a \delta_{k,a,m}^{r,s}$
$f_{k,m,r,s}^{i,i+1}$	Traffic flow rate on a trip from an activity node of the ith type to another activity node of the $(i+1)$th type, where this trip is part of trip chain k for the demand of the mth activity sequence from origin r to destination s
$f_{k,m}^{r,s}$	Traffic flow rate on trip chain k for the demand of the mth activity sequence from origin r to destination s
x_a	Traffic flow rate on traffic link a
y_p	Traffic flow rate on activity site p

and the total cost of this trip chain is equal to the minimum cost of all trip chains, this trip chain may carry a positive amount of traffic flow generated from this demand; otherwise, i.e., either the total length of a trip chain is longer than the distance limit or the total cost of this trip chain is higher than the minimum one, this trip chain must carry no flow. Mathematically, the equilibrium conditions can be expressed as:

$$\begin{cases} l_{k,m}^{r,s} \leqslant D \text{ and } t_{k,m}^{r,s*} = \mu_m^{r,s*} \Rightarrow f_{k,m}^{r,s*} \geqslant 0 \\ l_{k,m}^{r,s} > D \text{ or } t_{k,m}^{r,s*} > \mu_m^{r,s*} \Rightarrow f_{k,m}^{r,s*} = 0 \end{cases} \quad \forall\, r,s,m,k \in K_{r,s}^m \tag{1}$$

where $t_{k,m}^{r,s}$, $l_{k,m}^{r,s}$, $f_{k,m}^{r,s}$ and $\mu_m^{r,s}$ are the total cost, total length, traffic flow rate and minimum cost of trip chain k, experienced or induced by the activity-travel demand of the mth activity sequence between origin r and destination s. Here we use "*" to indicate the values of decision variables in the equilibrium solution. Meanwhile, traffic flows in the network must satisfy the following constraints, where two (sub)path flow variables, $f_{k,m}^{r,s}$ and $f_{k,m,r,s}^{i,i+1}$, for the trip chain and trip levels are used:

$$f_{k,m,r,s}^{i,i+1} \geqslant 0 \quad \forall\, r,s,m,i,k \in K_{r,s}^m \tag{2}$$

$$f_{k,m}^{r,s} = f_{k,m,r,s}^{i,i+1} \quad \forall\, r,s,m,i,k \in K_{r,s}^m \tag{3}$$

$$\sum_k f_{k,m}^{r,s} = q_m^{r,s} \quad \forall\, r,s,m,k \in K_{r,s}^m \tag{4}$$

To understand the path-subpath flow relationship specified by the above constraints, one should keep in mind that in a network under our setting, a trip chain is represented by a path, and the trips on this trip chain are represented by subpaths of this path. Constraint (2) simply indicates the nonnegativity of the traffic flow rate on any trip that connects two activity nodes, which belong to two consecutive activity types. Constraint (3) establishes an equivalency relationship of traffic flows between the trip chain and trip levels; it also means that a traffic stream on any trip chain can be spatially decomposed into a set of consecutive traffic streams on individual trips included in the trip chain. Activity type indexes i and $i+1$ here are two consecutive ones implied in activity sequence m connecting origin node r and destination node s. Origin r and destination s can be virtually regarded as two special activity types at the start and end of a trip chain, where no cost is incurred at these two dummy activity nodes and only a single activity node exists in each of the two activity types. To this end, we may regard r as the first index i in an activity sequence and s as the last index $i+1$. Constraint (4) indicates the flow conservation maintained between an origin–destination pair.

3.4. Problem formulation

Now we attempt to construct an optimization problem, which gives rise to its optimal solution equivalent to the above equilibrium conditions. Starting from Beckmann's transformation (1956), we propose the following problem formulation:

$$\min \quad z(\mathbf{x},\mathbf{y}) = \sum_a \int_0^{x_a} t_a(w)dw + \frac{1}{\theta}\sum_p \int_0^{y_p} v_p(w)dw \tag{5}$$

$$\text{subject to} \quad \sum_k f_{k,m}^{r,s} = q_m^{r,s} \quad \forall\, r,s,m,k \in K_{r,s}^m \tag{6}$$

$$f_{k,m}^{r,s} = f_{k,m,r,s}^{i,i+1} \quad \forall\, r,s,m,i,k \in K_{r,s}^m \tag{7}$$

$$f_{k,m,r,s}^{i,i+1} \geqslant 0 \quad \forall\, m,i,k \in K_{r,s}^m \tag{8}$$

$$(D - l_{k,m}^{r,s})f_{k,m}^{r,s} \geqslant 0 \quad \forall\, r,s,m,k \in K_{r,s}^m \tag{9}$$

$$\text{where} \quad x_a = \sum_{rs}\sum_m\sum_k f_{k,m}^{r,s}\delta_{k,a,m}^{r,s} \quad \forall\, a \tag{10}$$

$$y_p = \sum_{rs}\sum_m\sum_k f_{k,m}^{r,s}\delta_{k,p,m}^{r,s} \quad \forall\, p \tag{11}$$

where the two cost functions, $t_a(\cdot)$ and $v_p(\cdot)$, are both convex, increasing, continuously differentiable functions of traffic link flow rate x_a and activity node flow rate y_p, respectively, and the two path flow variables, $f_{k,m}^{r,s}$ and $f_{k,m,r,s}^{i,i+1}$, on the trip chain and trip levels, are defined exactly the same as the definition in Beckmann's transformation, in which a path between an origin–destination pair is a combination of a consecutive series of links connecting the origin and destination. It is noted that constraints (6–8) are the same as those presented in the equilibrium conditions. Constraint (9) presents a complementarity relationship between $D - l_{k,m}^{r,s}$ and $f_{k,m}^{r,s}$, implying that if $D \geqslant l_{k,m}^{r,s}$, then $f_{k,m}^{r,s} \geqslant 0$, and if $D < l_{k,m}^{r,s}$, then $f_{k,m}^{r,s} = 0$. Constraints (10) and (11) are two definitional constraints for the traffic flow rates on traffic links and activity nodes, in which $\delta_{k,a,m}^{r,s}$ and $\delta_{k,p,m}^{r,s}$ are link-chain and node-chain incidence indicators, respectively. Please be noted here that link-chain incidence indicator $\delta_{k,a,m}^{r,s}$ may be an integer number greater than 1, indicating link a may be used multiple times by trip chain k for the demand of the mth activity sequence from origin r to destination s. In contrast, node-chain incidence indicator $\delta_{k,p,m}^{r,s}$ are still a 0–1 integer number, implying that an activity node is at most visited once by a traveler in his or her trip chain.

The distance-constrained traffic assignment problem with trip chains can also be formulated into other mathematical forms. For example, if we write it into a variational inequality, then the equilibrium solution $(\mathbf{x}^*, \mathbf{y}^*)$ of the problem is determined by the following inequality:

$$\mathbf{t}^*(\mathbf{x} - \mathbf{x}^*) + \frac{1}{\theta}\mathbf{v}^*(\mathbf{y} - \mathbf{y}^*) = \sum_a t_a(x_a^*)(x_a - x_a^*) + \frac{1}{\theta}\sum_p v_p(y_p^*)(y_p - y_p^*) \geqslant 0 \tag{12}$$

where $\mathbf{x}, \mathbf{y}$ subject to (6–11).

3.5. Solution equivalency and uniqueness

Then we prove the equivalency of the solution of the proposed optimization problem (5–11) and the equilibrium conditions defined in (1–4). The relevant Lagrangian problem to the optimization problem is, if we relax constraints (6) and (9),

$$\min \quad z + \sum_{rs}\sum_{m}\sum_{i}\mu^{i,i+1}_{m,r,s}\left(q^{rs}_{m} - \sum_{k} f^{i,i+1}_{k,m,r,s}\right) - \sum_{rs}\sum_{m}\sum_{k}\lambda^{rs}_{k,m}(D - l^{r,s}_{k,m})f^{r,s}_{k,m} \tag{13}$$

$$\text{subject to} \quad f^{r,s}_{k,m} \geqslant 0 \quad \forall\, r,s,m,k \in K^{m}_{r,s}$$

By making use of the optimality conditions of the Lagrangian problem and the flow equivalency or definitional relationships in constraints (7), (10) and (11), we obtain the following system of equations and inequalities:

$$[t^{r,s*}_{k,m} - \mu^{r,s*}_{m} - \lambda^{rs}_{k,m}(D - l^{r,s}_{k,m})]f^{r,s*}_{k,m} = 0 \quad \forall\, r,s,m,k \in K^{m}_{r,s} \tag{14}$$

$$t^{r,s*}_{k,m} - \mu^{r,s*}_{m} - \lambda^{rs}_{k,m}(D - l^{r,s}_{k,m}) \geqslant 0 \quad \forall\, r,s,m,k \in K^{m}_{r,s} \tag{15}$$

$$f^{r,s*}_{k,m} \geqslant 0 \quad \forall\, r,s,m,k \in K^{m}_{r,s} \tag{16}$$

$$(D - l^{r,s}_{k,m})f^{r,s*}_{k,m}\lambda^{rs}_{k,m} = 0 \quad \forall\, r,s,m,k \in K^{m}_{r,s} \tag{17}$$

$$(D - l^{r,s}_{k,m})f^{r,s*}_{k,m} \geqslant 0 \quad \forall\, r,s,m,k \in K^{m}_{r,s} \tag{18}$$

$$\lambda^{rs}_{k,m} \geqslant 0 \quad \forall\, r,s,m,k \in K^{m}_{r,s} \tag{19}$$

$$q^{r,s}_{m} - \sum_{k} f^{r,s*}_{k,m} = 0 \quad \forall\, r,s,m \tag{20}$$

where the following definitional relationships are used:

$$t^{r,s*}_{k,m} = \sum_{a} t^{*}_{a}\delta^{r,s}_{k,a,m} + \frac{1}{\theta}\sum_{p} v^{*}_{p}\delta^{r,s}_{k,n,m} \quad \forall\, r,s,m,k \in K^{m}_{r,s} \tag{21}$$

$$\mu^{r,s*}_{m} = \sum_{i}\mu^{i,i+1*}_{m} \quad \forall\, r,s,m \tag{22}$$

Please be noted first that inequality (16) is an equivalent replacement of constraints (7) and (8) in the original formulation. From observing the system of equations and inequalities in (14–20), we derive the following conditions: (1) If $l^{r,s}_{k,m} \leqslant D$, then $\lambda^{rs}_{k,m} = 0$ (according to Eq. (17) and inequality (19)) and then $\lambda^{rs}_{k,m}(D - l^{r,s}_{k,m}) = 0$. Under this condition, if $t^{r,s*}_{k,m} = \mu^{r,s*}_{m}$, then we readily know $t^{r,s*}_{k,m} - \mu^{r,s*}_{m} - \lambda^{rs}_{k,m}(D - l^{r,s}_{k,m}) = 0$ and hence $f^{r,s*}_{k,m} \geqslant 0$ (according to Eq. (14)). (2) If $l^{r,s}_{k,m} > D$, we readily know $f^{r,s*}_{k,m} = 0$ (according to inequality (16) and Eq. (18)). (3) If $t^{r,s*}_{k,m} > \mu^{r,s*}_{m}$, we get $f^{r,s*}_{k,m} = 0$ as well. This conclusion can be proved by using the following procedure.

Let us assume $f^{r,s*}_{k,m} > 0$ under this condition. Then we readily know $(D - l^{r,s}_{k,m})\lambda^{rs}_{k,m} = 0$ (according to Eq. (17)), and then we can conclude $f^{r,s*}_{k,m} = 0$ (according to Eq. (14)). This result contradicts our assumption. Thus we must have $f^{r,s*}_{k,m} = 0$. Combing all these conditions, we establish an equivalency relationship between the optimality conditions characterized by (14–20) and the equilibrium conditions defined in (1–4).

As for the solution uniqueness (for traffic link and activity node flow solutions) of the optimization problem (5–11), it can be readily proved by observing the following facts: The objective function in (5) is convex, given that t_a and v_p are both convex functions; all constraints in (6–11) have a linear form.

As for the alternative variational inequality problem formulation in (12), the solution equivalency and uniqueness can be proved by the variation inequality theory.

4. Solution method

Given the convex objective function and linear constraint sets, we readily know that the proposed problem could be tackled by the classic linear approximation algorithm, or the so-called Frank-Wolfe algorithm (Frank and Wolfe, 1956), in an alternative name, as long as the solution procedure for its linearized subproblem is computationally acceptable. In fact, in most applications of the linear approximation method, solving the linearized subproblem poses the computational bottleneck of the entire algorithmic procedure. In our case, the linearized subproblem at any iteration n of the algorithmic procedure can be written as:

$$\min \quad \sum_{rs}\sum_{m}\sum_{k}\left(\sum_{a} t^{(n)}_{a}\delta^{r,s}_{k,a,m} + \frac{1}{\theta}\sum_{p} v^{(n)}_{p}\delta^{r,s}_{k,p,m}\right)f^{rs}_{k,m} \tag{23}$$

$$\text{subject to constraints } (6-11)$$

where $t_a^{(n)}$ and $v_p^{(n)}$ are the travel cost on link a and activity cost at node p in the nth iteration, respectively, which are given constants in the above linearized subproblem.

Obviously, this subproblem can be further decomposed by origin–destination pairs and activity sequences. Specifically, the decomposed subproblem for origin–destination pair $r-s$ and each activity sequence m reads:

$$\min \quad \left(\sum_a t_a^{(n)} \delta_{k,a,m}^{r,s} + \frac{1}{\theta}\sum_p v_p^{(n)} \delta_{k,p,m}^{r,s}\right) f_{k,m}^{rs} \tag{24}$$

$$\text{subject to} \quad \sum_k f_{k,m,r,s}^{i,i+1} = q_m^{r,s} \quad \forall\, i \tag{25}$$

$$f_{k,m,r,s}^{i,i+1} = f_{k,m}^{r,s} \quad \forall\, i,k \tag{26}$$

$$f_{k,m}^{r,s} \geqslant 0 \quad \forall\, k \tag{27}$$

$$(D - l_{k,m}^{r,s}) f_{k,m}^{r,s} \geqslant 0 \quad \forall\, k \tag{28}$$

where it is noted that constraints (25–28) are identical to (6–9) except the superscripts and subscripts of those relevant variables. This decomposed problem poses a new shortest path problem, which we name in this paper the *distance-constrained, node-sequenced shortest path problem*, given that the problem is defined as finding a shortest path between a given origin–destination pair and through a given activity node sequence, the length of which must be no more than a given distance limit. If no distance limit is imposed, then this problem collapses to the *activity selection problem* described by Chow and Liu (2012).

In the remaining part of this section, we describe a labeling method that can efficiently solve the distance-constrained, node-sequenced shortest path problem in a cascading manner and then illustrate how this method is embedded into the linear approximation procedure for the proposed traffic assignment problem.

4.1. Solving the distance-constrained, node-sequenced shortest path problem

It is well known that the distance-constrained shortest path problem can be well solved by bi-objective label-correcting algorithms. In the following, we extend this algorithmic idea to address the increasing problem complexity from the activity sequence requirement in the distance-constrained, node-sequenced problem. The resulting modified label-correcting algorithm is implemented in a cascading manner over the activity sequence. For discussion convenience, additional notation used by the algorithm is accordingly presented below (see Table 2).

The core operations in this label-correcting process are to, whenever a new label set is generated, check the violation condition of the distance constraint and make domination comparisons between different label sets in terms of activity-travel cost and travel distance. For conducting these operations, one or more label sets are constructed, maintained, or removed over the algorithmic process. In our design, each label set is such a quadruple, including four elements: Accumulative activity-travel cost, accumulative travel distance, preceding node and label set information, and status indicator. The specifications of these terms can be referred to in Table 2. By using the additional notation, we then write the label-correcting procedure as follows:

Step 0 (Initialization): For each node $i \in N$, set its first quadruple label set as $g_i^1 = [c_i^1 = +\infty, d_i^1 = +\infty, pre_i^1 = (-,-), \pi_i^1 = 0]$ and put it into the list of label sets of this node. For origin r, set its first quadruple label set as $g_r^1 = [c_r^1 = 0, d_r^1 = 0, pred_r^1 = (r,1), \pi_r^1 = 0]$. Set $I = r$, $q = 0$.

Step 1 (Labeling): While activity type $q \leqslant Q_{r,s}^m + 1$, do the following:

Step 1.1: While $I \neq \emptyset$, remove the first node i from I. For each label set h associated with node i, if $\pi_i^h = 0$, do the following:

Step 1.1.1: For each link $(i,j) \in O(i)$, create a new label set m for node j by setting $g_j^m = [c_j^m = c_i^h + c_{ij}, d_j^m = d_i^h + d_{ij}, pre_j^m = (i,h), \pi_j^m = 0]$.

Step 1.1.2: If $d_j^m \leqslant D$ and g_j^m is not dominated by any other existing label set associated with node j in terms of $c(j)$ and $d(j)$, insert g_j^m into the list of label sets of node j, and add node j into I as its last element if $j \notin I$.

Step 1.1.3: If any existing label set associated with node j is dominated by g_j^m in terms of $c(j)$ and $d(j)$, delete this existing label set from the list of node j.

Step 1.1.4: Set $\pi_i^h = 1$.

Step 1.2: For each activity node $p \in N_{r,s}^{m,q}$, scan each label set h associated with node p with setting $c_p^h = c_p^h + v_p$ and $\pi_p^h = 0$. Save the label sets of all nodes over the network.

Step 1.3: For each node $i \in N \setminus N_{r,s}^{m,q}$, empty its list of label sets, and then set its first label set as $g_i^1 = [c_i^1 = +\infty, d_i^1 = +\infty, pre_i^1 = (-,-), \pi_i^1 = 0]$ and put this set into the list of label sets. Set $I = N_{r,s}^{m,q}$ and $q = q + 1$.

Table 2
A supplementary notation list for the labeling algorithm.

Indexes	
i, j	Index of a node
p	Index of an activity node
h, m	Index of a label set with a node
(i, j)	Index of a traffic link from node i to j
q	Index of the type of activity nodes, where $q = 0, 1, 2, \ldots, \lvert N_{r,s}^{m,q}\rvert + 1$
Sets	
I	Set of active nodes in the algorithmic process
$O(i)$	Set of traffic links emanating from node i
$N_{r,s}^{m,q}$	Set of activity nodes of the qth type visited by the activity-travel demand of the mth activity sequence from origin r and destination s
Parameters and labels	
c_{ij}	Travel cost of links (i, j)
d_{ij}	Physical length of links (i, j)
$Q_{r,s}^{m}$	Number of activity types involved in the activity-travel demand of the mth activity sequence from origin r to destination s
c_i^h	Accumulative activity-travel cost from origin r to node i, as part of label set h of this node
d_i^h	Accumulative travel distance from origin r to node i, as part of label set h of this node
pre_i^h	Preceding node and label set to label set h of node i
π_j^h	Status indicator for label set h of node j, where $\pi_j^h = 1$, if label set h of node j has been used to update its downstream nodes, and $\pi_j^h = 0$, if otherwise

Step 2 (Backtracking): Choose the minimum c_s^h among all existing nondominated label sets of destination s and backtrack its preceding nodes and labels until reaching origin r so as to obtain the optimal solution.

From the above algorithmic procedure, it is readily observed that this algorithm follows such a cascading process over the activity sequence (refer to step 1): It first finds all possible distance-constrained nondominated subpaths from origin r to all activity nodes of the first type, then all possible subpaths from origin r to all activity nodes of the second type through the activity nodes of the first type, and so on, until all possible paths from origin r to destination s through the activity nodes of the first type, the second type, ..., and finally the $Q_{r,s}^{m}$th type. That is to say, from any activity node of the qth type, the algorithm continues to find all distance-constrained nondominated subpaths through this node to all activity nodes of the $(q + 1)$ th type. At the end, the algorithm selects the path with the minimum activity-travel cost as the optimal solution from the set of all distance-constrained nondominated paths from origin r to destination s (refer to step 2).

4.2. An application of the linear approximation method

Given that the subproblem—the distance-constrained, node-sequenced shortest path problem—can be well tackled by the above cascading procedure, we can readily construct a linear approximation algorithm for solving the proposed traffic assignment problem. Since the procedure is well known and widely used in the literature, we only present below a compact form of the algorithmic steps without elaborating its underlying mathematical principles.

Step 0 (Feasibility test): For each origin–destination pair and activity sequence, find the node-sequenced shortest path in terms of travel distance. If the length of this shortest path is greater than the distance limit D, then there does not exist any feasible trip chain that can carry traffic flow for this origin–destination pair and activity sequence.
Step 1 (Initialization): For each origin–destination pair r - s and activity sequence m for which there is at least one feasible trip chain, find the distance-constrained minimum-cost trip chain based on free-flow travel costs $t_a^{(0)}$, $\forall a$ and free-flow activity costs $\upsilon_p^{(0)}$, $\forall p$ in the network and assign the corresponding activity-travel demand $q_{r,s}^{m}$ to this trip chain. Aggregating activity-travel flows from all origin–destination pairs and activity sequences yields the initial solution $\{x_a^{(1)}, y_p^{(1)}\}$. Set iteration counter $n = 1$.
Step 2 (Update): Update the travel costs and activity costs in the network by calculating $t_a^{(n)} = t_a(x_a^{(n)})$, $\forall a$ and $\upsilon_p^{(n)} = \upsilon_p(y_p^{(n)})$, $\forall p$, respectively.
Step 3 (Direction finding): For each origin–destination pair $r - s$ and activity sequence m, find the distance-constrained minimum-cost trip chain based on updated travel costs $t_a^{(n)}$, $\forall a$ and updated activity costs $\upsilon_p^{(n)}$, $\forall p$ and assign the corresponding activity-travel demand $q_m^{r,s}$ to this trip chain. This yields an auxiliary solution $\{x_a^{(n)'}, y_p^{(n)'}\}$.
Step 4 (Line search): Apple the bisection method to obtain the optimal move size ρ^* by solving the following one-dimensional optimization problem:

$$\min_{0 \leqslant \theta \leqslant 1} \sum_a \sum_a \int_0^{x_a^{(n)} + \rho(x_a^{(n)'} - x_a^{(n)})} t_a(w)dw + \frac{1}{\theta} \sum_p \int_0^{y_p^{(n)} + \rho(y_p^{(n)'} - y_p^{(n)})} \upsilon_p(w)dw$$

Step 5 (Move): Set $x_a^{(n+1)} = x_a^{(n)} + \rho^*(x_a^{(n)'} - x_a^{(n)})$, $\forall a$ and $y_p^{(n+1)} = y_p^{(n)} + \rho^*(y_p^{(n)'} - y_p^{(n)})$, $\forall p$.
Step 6 (Convergence test): If the convergence criterion is not met, set $n = n + 1$; otherwise, stop and use the latest solution $\{x_a^{(n+1)}, y_p^{(n+1)}\}$ as the final solution.

5. Numerical analysis

To illustrate the effectiveness of the proposed model and solution method and study the impacts of distance limit on network flow patterns, we coded the algorithmic procedure in C++ and applied the code to solve an example network problem. The network is fully synthetic, simply because of the unavailability of trip chain-related travel demand data. The purpose of this numerical analysis is rather methodologically illustrative than for an empirical investigation or policy analysis.

The hypothetical network owns a grid topology, as shown in Fig. 2, consisting 24 nodes, 86 links. There are three types of nodes in the network: Origin or destination nodes (representing residential areas or workplaces), activity nodes (representing dining places, shopping malls, entertainment centers, or other activity sites), and other intermediate nodes (representing interchanges or intersections). To distinguish them, specifically, we use the following capital letters to label different types of nodes:

- H: Residential areas (including nodes 1, 5 and 10).
- W: Workplaces (including nodes 15 and 24).
- D: Dinning places (including nodes 4, 7, 9, 18 and 19).
- S: Shopping malls (including nodes 16, 21 and 22).
- E: Entertainment centers (including nodes 12 and 13).

Each line segment connecting a pair of nodes in the network represents a pair of links with counter traffic directions, where any pair of links are assumed to own the same link attributes. The numbers beside each line segment indicate free-flow travel cost, physical length, and capacity, respectively.

A travel demand pattern is further hypothesized for the afternoon peak period. The travel demand pattern is specified by the following combinations of origin–destination pairs and activity sequences with their travel demand rates:

- $H_1 - D - S - H_1$: 300.
- $H_1 - D - E - H_1$: 450.
- $H_2 - D - H_2$: 300.
- $H_2 - D - S - H_2$: 450.
- $H_3 - D - S - E - H_3$: 150.
- $H_3 - S - D - E - H_3$: 300.
- $W_1 - D - S - H_1$: 600.
- $W_1 - D - H_1$: 450.
- $W_1 - S - D - H_2$: 300.
- $W_1 - D - S - H_3$: 300.
- $W_2 - D - S - H_1$: 750.
- $W_2 - D - E - H_3$: 600 s.

where $D = \{D_1, D_2, D_3, D_4, D_5\}$, $S = \{S_1, S_2, S_3\}$, and $E = \{E_1, E_2\}$ are the three given sets of activity nodes. Obviously, the list of origin–destination pairs includes two types of trip chains for the afternoon peak period, home-based and work-to-home trip chains. This demand setting is given in such an electric-charging availability background that electric vehicle drivers may charge their vehicles at either home or workplace, but not other places in the network.

The network supply characteristics are then specified by the on-link travel cost function,

$$t_a = t_a^0\left[1 + \alpha\left(\frac{x_a}{c_a}\right)^{\beta}\right] \quad \forall\, a$$

where $\alpha = 0.15$ and $\beta = 4$, and at-node activity cost function,

$$v_p = v_p^0\left[1 + \delta\left(\frac{x_p}{c_p}\right)^{\gamma}\right] \quad \forall\, p$$

where $\delta = 0.1$ and $\gamma = 5$. Moreover, the following activity-related parameter values are applied to different types of activity nodes: $v_p^0 = 20$ and $c_p = 400$ for dinning places D_2, D_3, and D_5 (i.e., nodes 7, 9 and 19); $v_p^0 = 30$ and $c_p = 300$ for dinning places D_1 and D_4 (i.e., nodes 4 and 18); $v_p^0 = 30$ and $c_p = 600$ for all shopping malls; $v_p^0 = 20$ and $c_p = 800$ for entertainment center E_1 (i.e., node 12); $v_p^0 = 30$ and $c_p = 1000$ for entertainment center E_2 (i.e., node 13). Finally, we arbitrarily set the activity-travel cost conversion factor equal to 1.0.

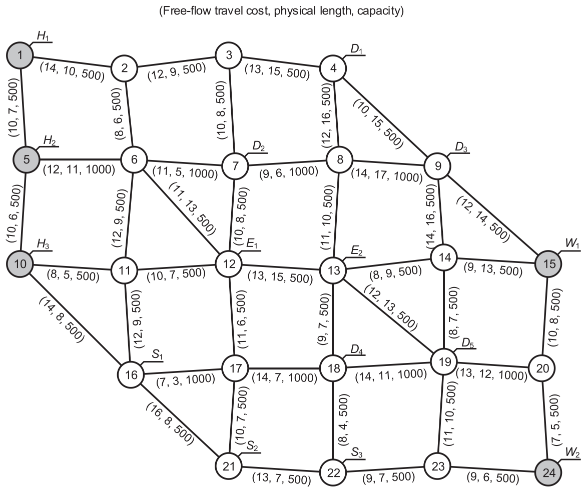

Fig. 2. A test network for numerical analysis.

Table 3
Numbers of used paths under different distance limit values.

Distance limit	Combination of origin–destination pairs and activity sequences											
	$H_1-D-S-H_1$	$H_1-D-E-H_1$	H_2-D-H_2	$H_2-D-S-H_2$	$H_3-D-S-E-H_3$	$H_3-S-D-E-H_3$	$W_1-D-S-H_1$	W_1-D-H_1	$W_1-S-D-H_2$	$W_1-D-S-H_3$	$W_2-D-S-H_1$	$W_2-D-E-H_3$
75	14	10	4	10	9	16	13	4	6	8	28	7
85	29	29	4	36	22	32	39	6	27	40	104	18
95	30	15	4	31	24	35	91	7	23	80	128	34
$+\infty$	63	26	5	47	26	38	180	9	22	105	165	55

One of our analysis interests is on the impacts of distance limit on route choices and network flow patterns. For this purpose, we solved the traffic assignment problem under different distance limits (including $D = 75$, 85, 95 and infinity) by using the computer code and presented the resulting path numbers, link flows and activity flows in Table 3, Fig. 3 and Table 4, respectively.

It is readily known that the solution of proposed traffic assignment problem neither owns a unique path flow pattern nor a unique used path set. However, by checking the number of used paths affiliated with each combination of origin–destination pairs and activity sequences in equilibrium solutions, we can approximately assess how distance limit impacts the path usage. The number of used paths between each combination of origin–destination pairs and activity sequences is a result determined by both the number of all feasible paths and the resulting path flow pattern, where the latter is affected by the algorithmic process. As an illustration, we listed in Table 3 the number of used paths generated by the linear approximation algorithm under different distance limit values. The numbers clearly exhibit that for all combinations of origin–destination pairs and activity sequences except two (i.e., $H_1-D-E-H_1$ and $W_1-S-D-H_2$), the number of used paths increases with the increase of distance limit. This result is highly consistent with our anticipation that a lower distance limit value will lead to a less number of feasible paths and tend to concentrate traffic flows to a less number of used paths.

Then we turn our attention to the traffic link and activity node levels. What we can observe from these two levels is that the network flow patterns are considerably reshaped by the distance limit with different tightness degrees. Although the link and node flow rates are merely aggregated results of path flow rates, which implies that some path flow changes would be offset or compromised on the link and node levels, they still pose significant variations over different distance limit values. In particular, we see that a number of links, which carry a considerable amount of traffic flow when no distance limit is imposed, carry no flow or very little flow when distance limit is set as 75, e.g., links $3 \to 7$, $4 \to 8$, and $8 \to 13$ (see Fig. 3). The phenomenon of link flows decreasing to zero indicates that some feasible paths going through those links may be no longer feasible after a certain distance limit value is imposed to the network. Of course, on the other hand, some links carry less flow when the distance limit value increases, e.g., link $6 \to 12$. After all, any link flow change is an equilibrium result networkwide. As for activity nodes, although we do not see such a dramatic flow decrease or increase, the impact from distance limit on activity node flow rates is still evident and remarkable (see Table 4).

If focusing our observation on the variation of individual link flow rates over different distance limit values, we can see a convergence tendency over an increase of distance limit values. Fig. 4 depicts the relationship of link flow rates and distance

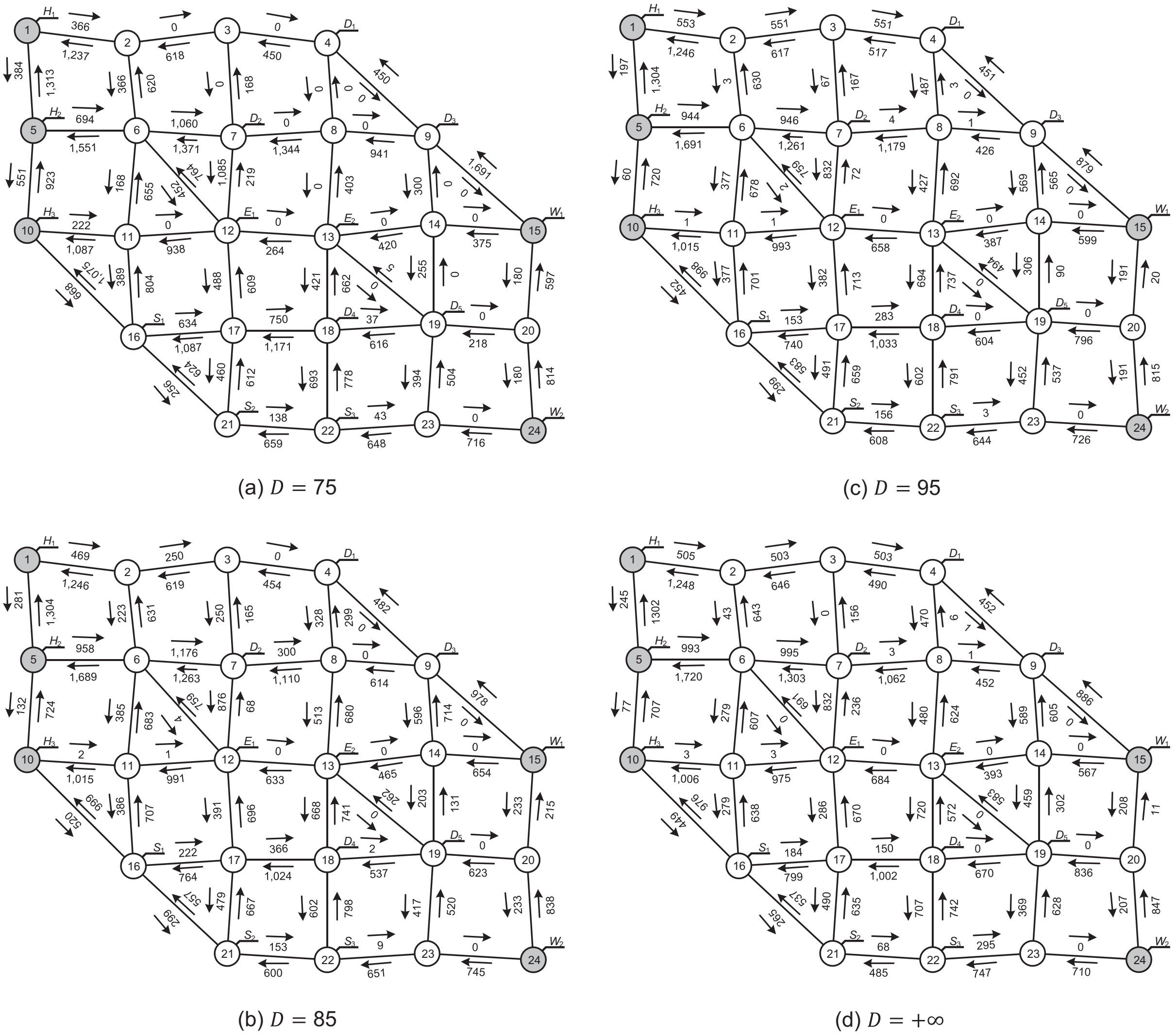

Fig. 3. Link flow rates under different distance limit values.

Table 4
Node flow rates under different distance limit values.

Distance limit	Activity node									
	D_1	D_2	D_3	D_4	D_5	E_1	E_2	S_1	S_2	S_3
75	450	1359	1242	885	1014	1128	372	1168	991	991
85	780	1234	1211	800	925	1104	396	1120	1015	1015
95	786	1230	1213	797	923	1104	396	1115	1017	1017
$+\infty$	786	1229	1214	798	923	1104	396	1112	1019	1019

limit values for some select links. From this figure, although no monotone tendency of variations of link flow rates is found with increasing distance limit values, it can be clearly seen that all link flow rates gradually converge to their respective stable points when the distance limit value approaches to the infinity. A similar result can also be observed from some representative activity node flows in Table 4, e.g., dinning nodes D_1, D_3 and D_5. This table records the distribution of dining, entertainment and shopping demands among different activity nodes in the network. When distance limit $D = 75$, we can see significantly different activity node flow compositions from the results under no distance limit; in contrast, when distance limit $D = 105$, the activity node flow compositions are pretty close to the ones under no distance limit.

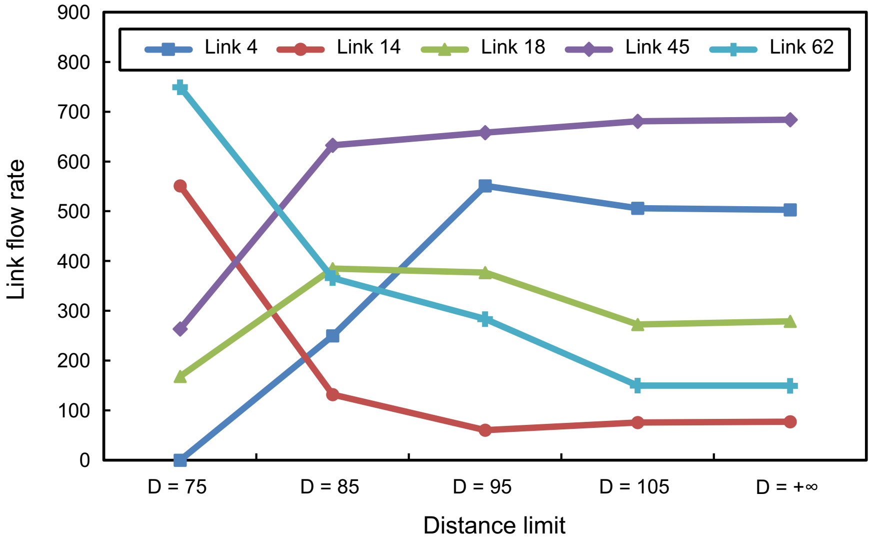

Fig. 4. Variation of link flows over different distance limit values.

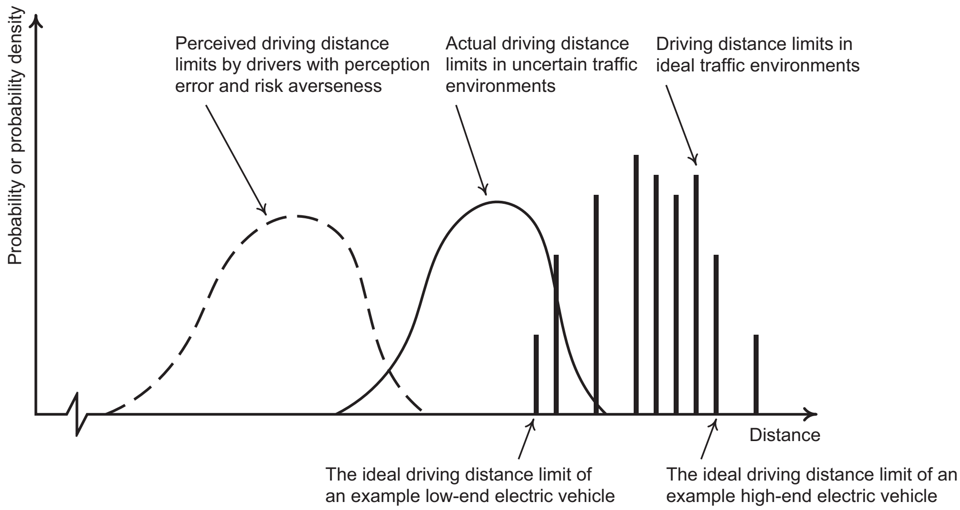

Fig. 5. Heterogeneous distance limit values.

6. Concluding remarks and future research

In this paper, we asserted a behavioral argument on electric vehicle drivers that if range anxiety makes any impact on travel behaviors, it more likely occurs on the trip chain level rather than the trip level. For modeling range anxiety, a simple yet effective treatment is to introduce an upper bound on driving distances, although the value of this bound may depend on many factors. Starting from this behavioral speculation and modeling treatment, we formulated and solved a distance-constrained traffic assignment problem that encapsulates trip chains as its basic analysis unit. While the problem does not embrace a more flexible and realistic situation that each individual determines activity locations and orders subject to traffic and activity congestion conditions, the developed modeling and solution tool may be used to more reasonably evaluate the impacts of distance limits on network equilibria than previous work.

It should be noted that almost all existing solution algorithms for the basic traffic assignment problem, including link-based, path-based, and origin-based ones, can be applied directly or transplanted with minor modifications for solving the proposed traffic assignment problem, if the cascading labeling procedure for the distance-constrained, node-sequenced shortest path problem is inserted. This is obvious, since all those traffic assignment algorithms virtually includes only two main algorithmic tasks: Path generation and path equilibration. The cascading procedure helps accomplish the path generation task in any of those algorithms, which is typically separate from the path equilibration process. The reason that we did not choose in this paper a more advanced algorithm than the linear approximation is simply due to the latter's mathematical simplicity and ease of implementation.

As we discussed earlier, a more flexible and probably more realistic setting for modeling activity-travel choices is to prespecify only activity patterns (i.e., the combination of types of activities) but not activity sequences in the model. If only the activity pattern to be accomplished along each individual's trip chain is given, the resulting model will allow his or her activity sequence to be endogenously determined. Following this relaxation of activity sequence, an alternative problem formulation could be formulated and this problem could be solved by the linear approximation algorithm, as well as other path-based or bush-based traffic assignment algorithms. No matter which algorithmic framework is employed for its solutions, however, finding a minimum cost trip chain subject to the given distance limit and activity purposes is an essential task, which poses a key subproblem to be solved in the framework. Mathematically, this subproblem becomes a distance-constrained version of the *generalized traveling salesman problem* through n sets of nodes. The generalized traveling salesman problem were studied by Srivastava et al. (1969), Laporte and Nobert (1983), and Laporte et al. (1987). Developing a solution algorithm for this subproblem as well as accordingly constructing a complete solution procedure for the alternative problem is one of our future research tasks along the direction.

In this paper, range anxiety is evaluated in a traffic assignment problem by setting a distance limit. For the sake of modeling tractability, we hold a strong assumption that a single distance limit value is imposed on all trip chains in a network. This assumption, obviously, ignores the heterogeneity of perceived or estimated distance limits by individuals in a real traffic environment. The heterogeneity may be due to multiple factors, including the variation of electric vehicles' onboard battery capacity, electricity storage level, electricity consumption rate, and the difference of vehicle drivers' risk-taking behavior and their perception on the remaining electricity quantity or mileage values read from the dash boards. For an arbitrary trip, the electricity consumption rate is heavily dependent on the driving behavior of the driver and the traffic flow conditions experienced by the vehicle, in addition to the vehicle's passenger and cargo loads and its electric motor's efficiency. In face of such uncertain traffic conditions, drivers tend to estimate the distance limits of their vehicles in a risk-averse manner, simply for avoiding as much as possible the pessimistic situation that their vehicles are stranded roadside due to the battery depletion. Such a speculation was identified and examined by a recent survey conducted by Franke and Krems (2013). As an illustration, Fig. 5 shows how heterogeneous distance limit values could be derived from different sources. Obviously, how to incorporate stochastic distance limits into a traffic assignment problem poses a very interesting yet challenging modeling task.

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