



A general corridor model for designing plug-in electric vehicle charging infrastructure to support intercity travel [☆]



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ABSTRACT

This paper proposes to optimally configure plug-in electric vehicle (PEV) charging infrastructure for supporting long-distance intercity travel using a general corridor model that aims to minimize a total system cost inclusive of infrastructure investment, battery cost and user cost. Compared to the previous work, the proposed model not only allows realistic patterns of origin–destination demands, but also considers flow-dependent charging delay induced by congestion at charging stations. With these extensions, the model is better suited to performing a sketchy design of charging infrastructure along highway corridors. The proposed model is formulated as a mixed integer program with nonlinear constraints and solved by a specialized metaheuristic algorithm based on Simulated Annealing. Our numerical experiments show that the metaheuristic produces satisfactory solutions in comparison with benchmark solutions obtained by a mainstream commercial solver, but is more computationally tractable for larger problems. Noteworthy findings from numerical results are: (1) ignoring queuing delay induced by charging congestion could lead to suboptimal configuration of charging infrastructure, and its effect is expected to be more significant when the market share of PEVs rises; (2) in the absence of the battery cost, it is important to consider the trade-off between the costs of charging delay and the infrastructure; and (3) building long-range PEVs with the current generation of battery technology may not be cost effective from the societal point of view.

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1. Introduction

Successful transition to alternative fuel vehicles (AFV) demands well planned supporting infrastructure, especially a network of refueling stations. The problem of optimally designing the refueling network has been studied separately for different AFVs, see e.g. [Stephens-Romero et al. \(2010\)](#) and [Nicholas et al. \(2004\)](#) for hydrogen vehicles, [Frick et al. \(2007\)](#) for compressed natural gas vehicles, and [Frade et al. \(2011\)](#), [Dashora et al. \(2010\)](#) and [Sweda and Klabjan \(2011\)](#) for PEVs, including plug-in hybrid electric vehicles (PHEV) such as Chevrolet Volt and battery electric vehicles (BEV) such as Nissan Leaf. We shall focus on PEVs in this paper because of their high energy efficiency ([Romm, 2006](#); [Eberhard and Tarpenning, 2006](#)), the ability to substitute electricity for petroleum and the potential to reduce the carbon footprint ([Samaras and Meisterling, 2008](#); [Crist, 2012](#)).

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The design of charging infrastructure is a facility location problem, which may be classified as “covering point demand” (e.g., Toregas et al., 1971; Daskin, 1995), “capturing origin–destination (O–D) demand” (Ghosh and McLafferty, 1987; Hodgson, 1990; Ghosh, 1991) and considering both types of demand (so-called hybrid models) (Goodchild and Noronha, 1987; Hodgson and Rosing, 1992). The point demand approach has been a popular choice in the context of locating charging stations for PEVs. The idea is to place these stations near the urban activity centers (e.g. home, shopping malls and workplaces) so as to minimize the access cost of PEV owners. With this approach the charging design problem is typically formulated as a set covering or P-median facility location problem (e.g. Dashora et al., 2010; Pan et al., 2010; Frade et al., 2011; Chen et al., 2013; Sweda and Klabjan, 2011; He et al., 2013; Huang et al., 2015; Ghamami et al., 2016). The point demand approach does not typically address intercity trips traditionally made using passenger vehicles. The premise of our study is that making charging facilities available along the corridors where these long-distance trips concentrate is important to resolving the range anxiety issue that has been considered a critical obstacle to PEV adoption (Hidrué et al., 2011; Shiau et al., 2009).

Flow capturing facility location models (FCLM) (e.g. Hodgson, 1990) are better suited to tackle intercity trips. Kuby and Lim (2005) and Kuby and Lim (2007) are among the early efforts to apply the FCLM in the context of the refueling problem for range-limited vehicles. The objective of these refueling location models is to locate refueling facilities to maximize the total vehicle flows refueled. Lim and Kuby (2010) propose a few efficient heuristic algorithms for solving this type of problems. The refueling station location problem studied in Wang and Lin (2009) and Wang and Wang (2010) also considers O–D demands. Yet, instead of trying to maximize flow being captured, their model minimizes the total facility cost while ensuring all flows are properly served according to a “refueling logic”. Nie and Ghamami (2013) propose a conceptual model to analyze travel by PEVs along a long corridor. The objective of their model is to select the battery size and charging capacity to meet a given level of service in such a way that the total social cost is minimized. In a similar spirit, Sathaye and Kelley (2013) develop a continuous facility location model (Daganzo, 2005) for the optimization of PEV charging facility deployment for highway corridors. Unlike Nie and Ghamami (2013), their model does not consider the battery cost. Rather, the focus is to complement private charging infrastructure by publicly funded charging stations, while considering demand uncertainty. Mak et al. (2013) propose a robust location model of battery swapping stations, which also considers demand uncertainty explicitly.

The analysis of Nie and Ghamami (2013) reveals the interesting tradeoff between charging capacity, battery size and the level of service experienced by PEV drivers (measured by the extra time spent on charging). Yet, a couple of simplifying assumptions make their model unsuitable even for sketchy design in a practical setting. These include: (1) trips take place between a single origin–destination pair with a fixed refueling logic; (2) each station is equipped with as many charging capacities as required to accommodate all PEVs as if they would use the facility simultaneously. The first assumption restricts the analysis to trips connecting two ends of a single corridor, and the second leads to potential overbuilding of charging capacities. This study aims to operationalize Nie and Ghamami (2013)’s corridor model by relaxing the above assumptions. Notably, the proposed general model will consider congestion at the charging stations (i.e., the fact that PEV drivers may wait in the line during peak periods), and how the *flow-dependent queuing delay* affects the optimal configuration of infrastructure and optimal refueling decisions. To the best of our knowledge, few have endogenized the waiting cost at charging stations in the context of modeling PEVs. A notable exception, De Weerd et al. (2013), considers queuing delay at charging stations but their focus was on optimal routing instead of infrastructure planning. While the proposed model is labeled as “general”, our focus is still on tandem linear corridors, which implies that the general route choice is not explicitly considered. The reason for this simplification is twofold. First, for the intercity travel that motivates this study, it is not uncommon that only one viable route is available for a given O–D pair. Second, the model proposed herein can be readily extended to cases involving multiple routes between O–D pairs, which could elevate computational challenges (because of the need to partially enumerate routes) but only add modest analytical complexities.

Because the general corridor model is formulated as a mixed integer nonlinear program, solving it poses a significant challenge. While off-the-shelf solvers for such problems do exist, we note that these general purpose tools often scale poorly. To address this limitation, a specialized metaheuristic solution method based on Simulated Annealing (SA) is developed and compared against a popular commercial solver in several numerical experiments. Note that the SA algorithm has been successfully employed to solve challenging optimization problems in general (e.g. Boston and Bettinger, 1999; Baskent and Jordan, 2002; McKendall et al., 2006; Dong et al., 2009), and facility location problems in particular (e.g. Murray and Church, 1996; Arostegui et al., 2006; Paik and Soni, 2007; Davari et al., 2011; Zockaie et al., 2016).

For the remainder, Section 2 presents the model formulation, followed by the development of a specialized solution algorithm in Section 3. Section 4 presents the setting and results of numerical experiments. Section 5 concludes the study with remarks on directions for future research.

2. Model formulation

Consider a set of tandem linear highway corridors that are divided into N segments with uniform length h (see Fig. 1). The corridors consist of a set of nodes denoted as $\Omega = \{0, 1, \dots, N\}$, with 0 and N being the first and last nodes, respectively. Each highway segment is identified by its end node – that is, the index of the segment from node $n - 1$ to n is n . Without loss of generality, we assume that the length of each corridor is the multiple of h . Thus, the starting and ending nodes of any

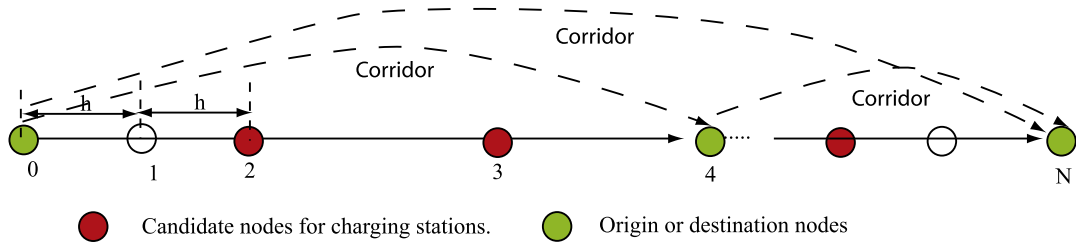


Fig. 1. Illustration of tandem linear highway corridors.

corridor belong to Ω , and the union of these end nodes is denoted as $R \subset \Omega$. All trips are assumed to occur between a pair of nodes r and s in R ($0 \leq r < s \leq N$). $\Pi \subset \Omega$ denotes a set of nodes at which charging facilities can be built. Let v_{rs} and d_{rs} be the free flow speed and the potential demand (i.e. number of PEVs owned) between O–D pair rs . Assume all PEV trips take place in a fixed time period T_0 and be uniformly distributed within T_0 . f^{rs} denotes the average trip frequency of the PEV trips taking place between r and s during T_0 (measured as trips/hour). Thus, the total number of PEV trips between r and s within T_0 can be written as $d^{rs}f^{rs}T_0$.

A binary variable $x_l, \forall l \in \Pi$ is used to represent the location decision at candidate node l . That is, $x_l = 1$ if a charging facility is built, and 0 otherwise. Accordingly, $z_l, \forall l \in \Pi$ represents the total number of chargers installed at l , which is modeled as a discrete variable. The charging power of each charger is denoted as P . Thus, at each charging facility, the power supply is given by $z_l P$. Furthermore, let y_{rs}^{ij} be the PEV flow from O–D pair rs that travel from node i to node j without stopping at any node $l \in \{i + 1, \dots, j - 1\}$. This variable represents the demand traveling from origin node r to destination node s that charge at nodes i and j but not at any other nodes l between i and j . The flow from O–D pair rs that stops at node l can be represented by $\sum_{i=r}^{l-1} y_{rs}^{il}$ ¹. We further define the total number of cars charging at station l , denoted as η_l , as

$$\eta_l = \sum_{rs \in \Theta} \sum_{i=r}^{l-1} y_{rs}^{il} \tag{1}$$

where Θ is the set of all valid O–D pairs.

Following Nie and Ghamami (2013), we introduce a parameter β to represent the battery performance in terms of mile driven in each unit of battery energy. Assuming PEVs would recharge the battery to full every time they stop, the amount of energy needed by all PEVs from O–D pair rs at node l is

$$g_{rs}^l = \sum_{i=r}^{l-1} y_{rs}^{il} \frac{h(l-i)}{\beta} \tag{2}$$

Since the unit of g_{rs}^l is energy consumed in T_0 , it is called *power demand* of O–D pair rs at node l . The average time taken to charge g_{rs}^l is estimated as

$$\tau_{rs}^l = \alpha \frac{g_{rs}^l}{z_l P} \tag{3}$$

where $\alpha > 1$ represents battery’s charging efficiency (see Nie and Ghamami, 2013). The total power demand at node l is given

$$G_l = \sum_{rs \in \Theta} g_{rs}^l \tag{4}$$

When $G_l < z_l P$, i.e. the total power demand is smaller than the total supply, there is no waiting time for an available charger and thus, delay only includes the charging time as $\sum_{rs} \tau_{rs}^l$. When $G_l > z_l P$, i.e. the total power demand is larger than the total supply, PEV drivers will have to wait in a queue to be charged at the station l . Using the deterministic queuing theory, the average waiting time can be computed as (see Fig. 2)

$$w_l = 0.5T_0 \left(\frac{G_l}{z_l P} - 1 \right) \tag{5}$$

¹ For example, suppose that, in Fig. 1, the demand between the origin node 0 and the destination node 4 needs to charge at least once and they can charge either at node 2 or 3. Let us assume that there are 100 PEV drivers, traveling from node 0 to node 4, of which 40% charges at node 2 while the rest charge at node 3. More specifically, 40 drivers stop only at node 2 to charge before reaching node 4, while the other 60 drivers stop only at node 3. Using the proposed notation, the flows can be represented as $y_{04}^{01} = 0, y_{04}^{02} = 40, y_{04}^{03} = 60, y_{04}^{12} = 0, y_{04}^{24} = 40, y_{04}^{34} = 60$.

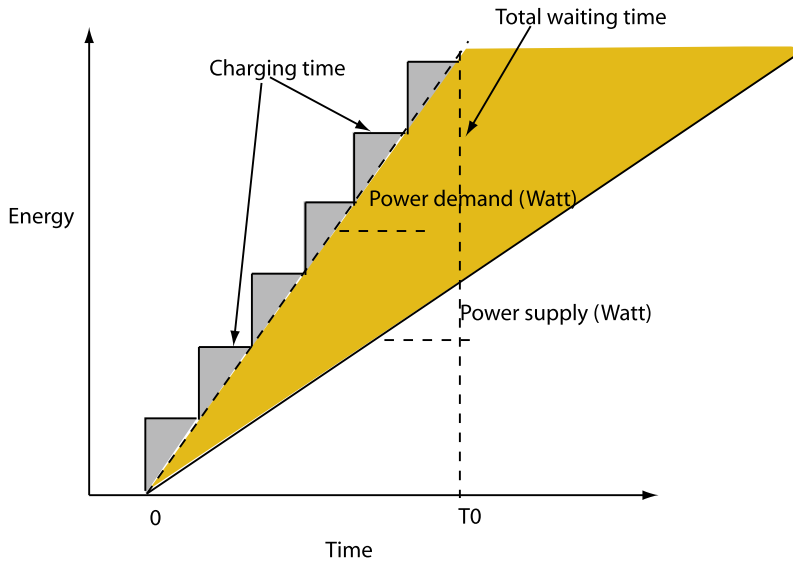


Fig. 2. Estimation of charging and waiting time at a charging facility.

All PEVs are assumed to be equipped with the same type of battery that contains energy E . The anxiety-free range of the battery is given as $\theta\beta E$, where $\theta \in (0, 1)$ is the range tolerance – that is, travelers would recharge when the battery is depleted 100% of its capacity. We note again that all vehicles are assumed to be fully charged when they depart from the origin and every charging station.

Now we are ready to formulate the PEV charging location and battery choice problem as the following mixed integer non-linear program.

$$\min_{x,y,z,\mu,E} \sum_{l \in \Pi} (C_p^l x_l + z_l PC_s + \gamma \pi_l) + \sum_{rs \in \Theta} d^{rs} C_e E \tag{6a}$$

subject to

$$\sum_{j=n+1}^s y_{rs}^{nj} - \sum_{i=r}^{n-1} y_{rs}^{in} = \begin{cases} d^{rs} f^{rs} T_0 & n = r \\ -d^{rs} f^{rs} T_0 & n = s \\ 0 & \text{otherwise} \end{cases}, \quad \forall n \in \Omega, rs \in \Theta \tag{6b}$$

$$y_{rs}^{il} \leq x_l M, \quad \forall l \in \Pi, rs \in \Theta \tag{6c}$$

$$z_l \leq x_l M, \quad \forall l \in \Pi \tag{6d}$$

$$\beta\theta E - (j - i)h \leq \mu_{ij} M, \quad \forall 0 \leq i < j \leq N \tag{6e}$$

$$\beta\theta E - (j - i)h \geq (\mu_{ij} - 1)M, \quad \forall 0 \leq i < j \leq N \tag{6f}$$

$$y_{rs}^{ij} \leq \mu_{ij} M, \quad \forall 0 \leq i < j \leq N, rs \in \Theta \tag{6g}$$

$$\sum_{rs \in \Theta} \tau_{rs}^l + \frac{0.5T_0(G_l - z_l P)}{z_l P} \eta_l \leq \pi_l, \quad \forall l \in \Pi \tag{6h}$$

$$\sum_{rs \in \Theta} \tau_{rs}^l \leq \pi_l, \quad \forall l \in \Pi \tag{6i}$$

$$x_l \in \{0, 1\}, \quad \forall l \in \Pi \tag{6j}$$

$$y_{rs}^{ij} \geq 0, \quad \forall 0 \leq i < j \leq N, rs \in \Theta \tag{6k}$$

$$\mu_{ij} \in \{0, 1\}, \quad \forall 0 \leq i < j \leq N \tag{6l}$$

$$z_l \text{ is integer}, \pi_l \geq 0, \quad \forall l \in \Pi \tag{6m}$$

The objective function consists of (1) the infrastructure investment on the charging facilities, (2) the monetary value of total time spent on charging the battery and waiting in the queue at the charging stations, and (3) the total battery cost of the PEV fleet. In (6a), C_p^l denotes the cost of infrastructure investment on building a charging station at node $l \in \Pi$ (measured in \$ per station), C_s is the unit cost of building the charging capacity (measured in \$ per kW), γ is the value of time (\$ per hour), and C_e is the unit battery cost (\$ per kW h). Clearly, the main tradeoff here is between investment in charging infrastructure (x_l, z_l), cost of manufacturing batteries (E), and the user cost (π_l).

Constraints (6b) state the flow conservation conditions at an intermediate stop l for each O–D pair. Constraint (6c) dictates that only when a charging facility is built at node l , the node can be used by a PEV as an intermediate stop. Note that M here is a large number. Constraint (6d) dictates that only nodes with charging stations can accommodate charging spots. Constraints (6e)–(6g) state that no flow is allowed between node pair ij unless the distance between i and j is smaller than the anxiety-free range. Note that μ_{ij} is an auxiliary binary variable used to test range feasibility. That is, if the distance between i and j is smaller than the anxiety-free range, μ_{ij} is equal to one; otherwise it is equal to zero. Constraint (6g) states that the flow between i and j , y_{rs}^{ij} is equal to zero, if the distance between i and j is larger than the anxiety-free range.

The combination of Eqs. (6h) and (6i) defines the delay (π_l). If the supply is larger than the demand, $G_l - z_l P$ would be negative. Thus, Eq. (6h) will become redundant and the delay will be defined by Eq. (6i), as just the charging time. On the other hand, if the demand is larger than the supply, $G_l - z_l P$ would be positive, which makes Eq. (6i) redundant, and as a result Eq. (6h) will be used to calculate the delay. Constraint (6i) is introduced to avoid negative values of $\frac{G_l - z_l P}{z_l P}$ to affect the amount of charging time delay. It is worth noting that Constraint (6h) is nonlinear and, hence, is difficult to deal with. Finally, Constraints (6j)–(6m) identify integer variables and non-negativity.

Table 1 summarizes the number of the constraints and variables of the presented optimization model, as a function of network size (i.e., number of O–D pairs, nodes and candidate nodes for building charging stations).

Note that the optimal solution to this Mixed Integer non-linear problem is not unique. For a simple explanation, consider the example illustrated in Fig. 1 with nodes 1 and 4 being the origin and destination respectively. Suppose (1) that the range of the PEVs is such that users would have to charge either at node 2 or node 3 to complete their trip, (2) that the cost of building charging stations is not location specific and (3) that the demand is low enough so that all users may be served simultaneously by one charging station with the minimum capacity. In this case, building a minimum capacity charging station at either node 2 or 3 would be optimal, which shows the optimal solution to this problem is not unique in general.

3. Solution algorithm

The optimization model presented in the previous section is a mixed integer problem with non-linear constraints, which is known to be NP-hard. Commercial solvers such as Knitro (Waltz and Plantenga, 2009) can solve such problems via branch-and-bound techniques. Yet, because these solvers may not fully exploit the special structure of the problem, their performance typically degrades quickly as the problem size grows (see numerical experiments for test results based on Knitro). Tackling large-scale NP-hard problems often calls for metaheuristics that are designed to find good approximate solutions quickly. In light of this, a metaheuristic based on Simulated Annealing (SA) is developed in this section. We shall validate its computational promise by comparing it with Knitro.

The proposed algorithm decomposes the charging station design and battery choice problem into two subproblems: (1) configure charging stations and battery size using a modified Simulated Annealing (SA) algorithm, (2) determine the optimal flow and the total delay for a given configuration of charging station and battery size.

In the first subproblem, the battery size E is discretized into a set of energy levels. One can simply add E into the set of integer variables and search the optimal combination using SA. Because the discrete set of E is typically small and it has a distinctive physical meaning (all other integer variables are related to charging), a simpler approach is adopted here: we solve multiple instances of the original problem, each corresponding to a fixed discrete value of E , and then find the optimum from all solutions. The complexity of each instance is lower because it does not have to consider the choice of battery size.

The second subproblem is a linear program that can be viewed as a traffic assignment problem over a special linear network with multiple O–D pairs. Let us first explain how this network is constructed, given O–D demands, battery size E , and the choice of charging infrastructure. Suppose we start from the set of discrete nodes shown in Fig. 1. At first, feasible links, i.e., those that are no longer than the anxiety-free travel distance, are added into the network (see Fig. 3(a)). Specifically, a link should be added from i to j if

$$(j - i)h \leq \beta\theta E \tag{7}$$

Table 1
Number of constraints and variables.

<i>Constraints</i>	
Constraint (6b)	$ RS (N + 1)$
Constraint (6c)	$ RS \frac{N(N-1)}{2}$
Constraint (6e)–(6g)	$\frac{N(N-1)}{2}$
Constraint (6h) and (6i)	$N - 1$
<i>Variables</i>	
y	$ RS \frac{N(N-1)}{2}$
μ	$\frac{N(N-1)}{2}$
z	Z
x, π	$N - 1$
E	1

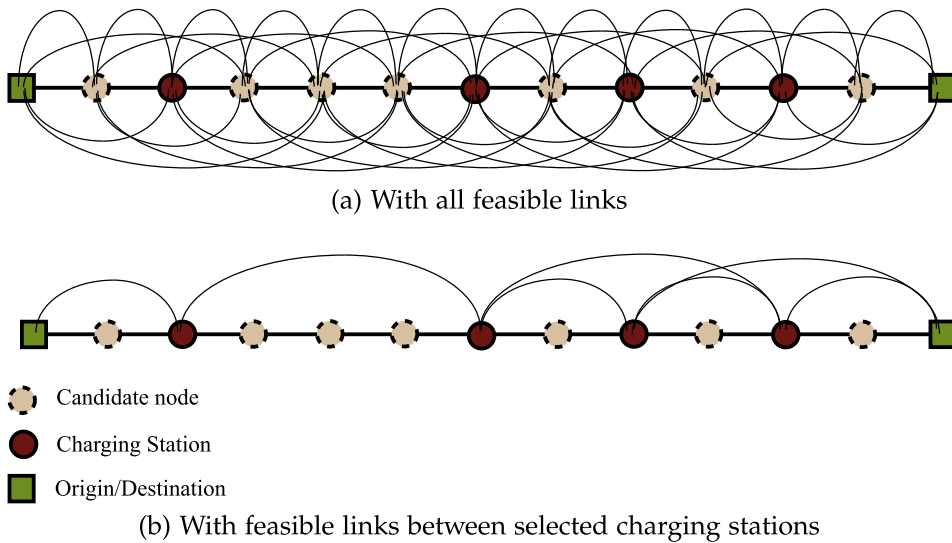


Fig. 3. Illustration of network construction process.

In the second step, once the location of charging stations is fixed, the set of feasible links is reduced to those no longer than $\beta\theta E$ and with a charging station or an origin or a destination node at each end (see Fig. 3). Consequently, an optimal assignment of PEV flows will be performed over a simplified network $G(M, A)$ where $M = R \cap \Pi$, and

$$A = \{ij | 0 \leq i < j \leq N, i, j \in M, (j - i)h \leq \beta\theta E\}$$

The solution of this assignment problem is the focus of the next subsection.

3.1. The assignment problem

Once the network is constructed, the original problem is reduced to assigning all O–D flows so that the total travel delay associated with charging is minimized. This problem can be formulated as follows:

$$\min_y \sum_{l \in \Pi} \gamma \pi_l \tag{8a}$$

subject to

$$\sum_{j: nj \in A} y_{rs}^{nj} - \sum_{i: in \in A} y_{rs}^{in} = \begin{cases} d^{rs} f^{rs} T_0 & n = r \\ -d^{rs} f^{rs} T_0 & n = s \\ 0 & \text{otherwise} \end{cases}, \quad \forall n \in M, rs \in \Theta \tag{8b}$$

$$y_{rs}^l \leq Mx_l, \quad \forall l \in \Pi, il \in A, rs \in \Theta \tag{8c}$$

$$z_l \leq x_l M, \quad \forall l \in \Pi \tag{8d}$$

$$\sum_{rs \in \Theta} \tau_{rs}^l + \frac{0.5T_0(G_l - z_l P)}{z_l P} \eta_l \leq \pi_l, \quad \forall l \in \Pi \tag{8e}$$

$$\sum_{rs \in \Theta} \tau_{rs}^l \leq \pi_l, \quad \forall l \in \Pi \tag{8f}$$

$$y_{rs}^{ij} \geq 0, \quad \forall ij \in A, rs \in \Theta \tag{8g}$$

$$\pi_l \geq 0, \quad \forall l \in \Pi \tag{8h}$$

Note that the number of chargers (z), the location of charging facilities (x), the battery size (E), and the auxiliary variable (μ) are all treated as inputs. Accordingly, the objective function contains only the “flow-dependent” travel delay and the only decision variable is the assigned flow vector y , while π is a state variable. Using the definition of τ_{rs}^l in Eq. (3), we have

$$\sum_{rs \in \Theta} \tau_{rs}^l = \sum_{rs \in \Theta} \alpha \frac{g_{rs}^l}{z_l P} = \frac{\alpha}{z_l P} \sum_{rs \in \Theta} g_{rs}^l = \frac{\alpha}{z_l P} G_l \tag{9}$$

Thus, Constraints (8f) and (8e) can be simplified as

$$\alpha G_l \leq \pi_l z_l P, \quad \forall l \in \Pi \tag{10}$$

$$(\alpha + 0.5T_0\eta_l)G_l - 0.5T_0z_lP\eta_l \leq \pi_l z_l P, \quad \forall l \in \Pi \tag{11}$$

respectively. Note that (11) is still a nonlinear constraint because of the definition of η_l .

The above assignment problem (8) has only one nonlinear constraint and no integer variables, and can be solved by commercial NLP solvers such as Knitro (Byrd et al., 2005). Specialized algorithms for solving the assignment problem (see e.g. Patriksson, 1994) may be adapted to solve the problem efficiently. Yet, we shall explore this possibility in a follow-up study, since the focus of the present study is on the metaheuristic for searching the optimal combination of x and z . We now turn our attention to this effort.

3.2. A Simulated Annealing (SA) algorithm

Metaheuristics based on Simulated Annealing is inspired by annealing in metallurgy (see Metropolis et al., 1953; Zockaie et al., 2016). Its iterative process resembles the heating and controlled cooling of a solid material to increase the size of its crystals and reduce their defects. At the cooling stage, an equilibrium state should be achieved at each temperature before moving to a lower temperature. The final solution is achieved at the minimum or final temperature.

An SA-based algorithm typically has two main steps (Zockaie et al., 2016). In the first, it searches over the feasible set of the integer solutions, starting from a current feasible solution and then moving to a neighbor feasible solution. The second step compares the objective functions of the current and the new solutions, and based on the difference, replaces the current solution with the new one with a probability. The probability is gradually reduced as the solution process proceeds. SA schemes allow larger objective function values (worse solutions) relative to the current solution be accepted, which offers a mechanism to avoid getting trapped in local optimum solutions. This feature is very useful when the problem is known to have multiple local optima. A recent paper by Zockaie et al. (2016) shows that the SA algorithm is able to solve flow capturing mixed integer programs (MIPs) efficiently.

The SA scheme is adapted to solve Problem (6) in this paper as follows. To find an initial solution (x and z), the basic corridor model (Nie and Ghamami, 2013) is adopted because it gives closed form approximate solutions. Since the basic corridor model can only handle a single O–D, we aggregate demands from multiple O–D pairs so that the model only considers O–D pairs between neighboring nodes in Θ . For instance, let $\Theta = \{A - C, B - C\}$, and the corresponding demands are d_{AC} and d_{BC} . In the initialization, the demand pattern of the corridor will be viewed as between $A - B$ and $B - C$, and the demand between $A - B$ is set to d_{AC} and the demand for $B - C$ is set to $d_{AC} + d_{BC}$. The travel frequency for each segment is taken as the average of all O–D pairs passing through the segment. We set the value of z also based on the assumption of the basic corridor model, which requires that the number of chargers is enough to avoid waiting at the stations.

We proceed to explain how to obtain a neighbor solution. The basic idea is as follows. According to the type of perturbation, each location is associated with a weight factor (e.g., total flows, total delays). Then, the location is picked randomly, but those with larger weight factors will have a higher probability of being selected. More specifically, we define

$$\Phi(l) = \sum_{i=1}^l \phi_i \tag{12}$$

where ϕ_i is the weight factor and $\Phi(l)$ is the cumulative weight factor. Then the location l is selected if it satisfies the following

$$\frac{\Phi(l-1)}{\Phi(N)} \leq \rho \leq \frac{\Phi(l)}{\Phi(N)} \tag{13}$$

where ρ is a random number drawn from a uniform distribution between 0 and 1, written as $\rho = U[0, 1]$.

In addition, the following rules will be used to guide the perturbation process.

- Rule 1 When a new station is to be added or new chargers are to be added to a station, each location l will be weighted based on the total delay, i.e. $\phi_l = \pi_l$. This means the stations with higher delay will receive priority.
- Rule 2 When a new station is to be removed, each location l will be weighted based on the inverse of total flow, i.e., $\phi_l = 1/\sum_i \sum_r \sum_s y_{rs}^{il}$. This implies that the stations with small flows will receive priority.
- Rule 3 When chargers are to be removed from a station, each location l will be weighted based on the infrastructure cost, i.e. $\phi_l = C_p^l x_l + z_l PC_s$. This implies that the station with high infrastructure cost will get priority.

If the perturbed solution improves the objective function value, it will be accepted. Otherwise, the probability of acceptance depends on how much worse it is compared to the current solution and the current annealing “temperature”. Finally, note that there are two sets of integer variables x and z . Each time only one set will be perturbed, and we switch between the two sets whenever a perturbed solution is accepted.

Algorithm 1. Simulated Annealing Algorithm

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1: Input: Maximum number of temperature changes  $K_0$ , Maximum number of inner iterations at each temperature  $K_1$ , battery energy level  $E$ .
2: Output:  $x^*, z^*$  for the given  $E$ .
3: Initialize:
4: Set a state variable  $\psi = 0$ , which indicates that the perturbation will be made to  $x$ .  $\psi = 1$  when the perturbation is for  $z$ .
5: Set the current temperature stage  $t = 0$ , choose the initial temperature  $u_t$ .
6: Initialize  $x^t$  and  $z^t$  based on the basic corridor model of Nie and Ghamami (2013), as described above. Set  $X^0 = \sum_i x_i^i$ , the total number of stations built in the initial solution.
7: while  $t < K_0$  do
8:   Set inner iteration index  $k = 0$ . Set  $x^k = x^t, z^k = z^t$ .
9:   while  $k < K_1$  do
10:    Perturbation.
11:    if  $\psi = 0$  then
12:      if  $\sum_i x_i^k \leq X^0$  then
13:        Add a station at a selected location by invoking Rule 1.
14:      else
15:        Set a random number  $\gamma = U[0, 1]$ .
16:        if  $\gamma > 0.5$  then add a station at a selected location by invoking Rule 1.
17:        else remove a station at a selected location by invoking Rule 2.
18:      end if
19:    end if
20:    else
21:      Set a random number  $\gamma = U[0, 1]$ .
22:      if  $\gamma > 0.5$  then
23:        add one charger at a selected location by involving Rule 1, if possible.
24:      else
25:        remove one charger at a selected location by involving Rule 3, if possible.
26:      end if
27:    end if
28:    Set  $k = k + 1$  and the perturbed solution as  $x^k, z^k$ . Draw a random number  $\gamma \in U[0, 1]$ .
29:    Set  $\Gamma^k$  and  $\Gamma^{k-1}$  be the objective function values associated with the perturbed and current solutions (the assignment problem has to be solved for each solution).
30:    if  $\Gamma^k \geq \Gamma^{k-1}$  and  $\exp\left(\frac{\Gamma^k - \Gamma^{k-1}}{u_t}\right) > \gamma$  then
31:      Discard the perturbed solution, i.e. set  $x^k = x^{k-1}, z^k = z^{k-1}$ .
32:    end if
33:  end while
34:  Set  $t = t + 1, x^t = x^k, z^t = z^k$ , and  $u_t = \theta u_0$ , where  $\theta = 0.85$  (see Zockaie et al., 2016).
35: end while
36: Set  $x^* = x^t, z^* = z^t$ .

```

Details of the SA procedure developed in this paper are summarized in Algorithm 1. Note that Algorithm 1 assumes the battery energy level E is given. In order to search for the optimal E , one has to call the algorithm once for each discrete value of E .

4. Numerical experiments

Numerical experiments conducted in this section are designed to (1) examine the computational performance of the proposed metaheuristic, compared to that of using Knitro directly to solve (6a), (2) understand how the planning of charging station is affected by queuing delay at the charging stations and more realistic representation of O–D demands; and (3) identify the sensitivity of the results to critical input parameters. The SA algorithm is coded in MATLAB and all experiments are conducted on a 64-bit WINDOWS desktop computer with an Intel(R) Core(TM) i5-2400 CPU@3.10 GHz and 8 GB RAM. The Knitro solver used in this study is called using AMPL (IBM, 2010).

4.1. Default parameters

Default values of the parameters used in the model are listed in Table 2, which are mainly adopted from the basic corridor model (Nie and Ghamami, 2013). For simplicity all parameters are assumed not to vary with location or O–D pairs, even

Table 2
Parameters definitions and default values.

Parameter	Description	Unit	Value
α	Energy efficiency (converting energy/power ratio to charging time)	–	1.3
β	Battery performance	$\frac{\text{mile}}{\text{kW h}}$	2.5
γ	Value of time	$\frac{\$}{\text{h}}$	18
T_0	Time period	h	1
C_p	Fixed construction cost of charging stations	k\$	520
C_e	Unit manufacturing cost of battery	$\frac{\$}{\text{kW h}}$	650
C_s	Per spot construction cost of recharging outlet	$\frac{\$}{\text{kW}}$	500
θ	Range tolerance (confident range)	–	0.8

though the model can capture such variations. We note that all capital costs (charging facility and battery) are later amortized based on the life of the facility.

The Chicago–Madison–Minneapolis corridor (see Fig. 4) is used in our case study to analyze intercity travel. The spatial demand profile is estimated based on the 1995 American Travel Survey (ATS) data. Table 3 shows the origin–destination pairs, the distance, demand and frequency of travel between each O–D pair. In the base case all electric vehicles are assumed to have an average range of 100 miles with a 40 kW h battery. Nie and Ghamami (2013) show that a reasonable level of service can only be achieved with Type III (fast) chargers. We thus assume all installed chargers will have a power of 50 kW, a typical value for Type III chargers.

Unless otherwise specified, the parameter values presented in this section will be used throughout this section.

4.2. Algorithm performance

We first examine how the performance of the SA algorithm may vary with the number of main and inner iterations (K_0 and K_1), and then compare the performance of the SA algorithm with that of Knitro.

Sensitivity analysis for different combinations of inner and main iterations shows that for the tested combinations, the best objective function value achieved seems not highly sensitive to the number of iterations. There is no clear indication that a higher number of iterations would help reduce the objective function value, as one might expect. On the other hand, the computational time does increase quickly with the number of iterations. The results suggest that after the number of inner and main iteration each reach 25, the final objective function no longer improves. We thus choose $K_0 = K_1 = 25$ for the remaining experiments.

Table 4 compares the results given by the SA algorithm and Knitro for three different demand levels of the Chicago–Madison corridor (the small problem with one O–D pair connected by a short corridor). Note that the low demand level in the table is the level reported in Table 3 demand, whereas the medium and high demand levels are five and ten times of the base value, respectively. The results show that directly solving the problem using Knitro is faster for the small problem than the proposed SA algorithm, but it leads to loss of accuracy ranging from about 2% for the high demand case to about 15% for the low demand case. For larger problems, the Knitro solver used in this study failed because it was unable to find any feasible solutions to Problem (6). Yet, the proposed SA-based solution method succeeded in finding a solution in all cases. These results indicate that the proposed SA algorithm scales better. More importantly, it can get comparable (if not consistently better) solutions than commercial MIP solvers, which we believe offer a reasonable guarantee of the solution quality. A more rigorous assessment of the solution quality (e.g. by developing a tight lower bound for the original problem) is beyond the scope of this paper and will be left to a future study.

4.3. Main results

Fig. 5 reports the optimal configuration of charging stations for the Chicago–Madison–Minneapolis corridor with current demand levels (solved by the SA method). The solution places five charging stations along the corridor, with two medium sized stations (more chargers) and the other three small charging stations.

We compare the above solution with that of the basic corridor model (Nie and Ghamami, 2013), which assumes that each station holds enough chargers to serve all potential PEV users. We recall that the general corridor model proposed in this paper allows the queuing delay at the charging station and so the number of chargers may be optimally determined.

Fig. 6 compares the optimal configurations obtained by the two models. In the plots, the circles represent the charging stations and the size of the circle visualizes the number of chargers in that station. As expected, the basic model produces an identical number of chargers at each station. On the contrary, the general corridor model is able to differentiate the size of charging station based on the demand pattern and the tradeoff between infrastructure and user costs.

Table 5 further compares the two models under different levels of demand. We expect that, as the demand level rises, more PEV users would need to charge, and therefore, the impact of queuing delay would become more prominent. The results reported in Table 5 confirms exactly that expectation. In all cases, we found that the basic model overestimates

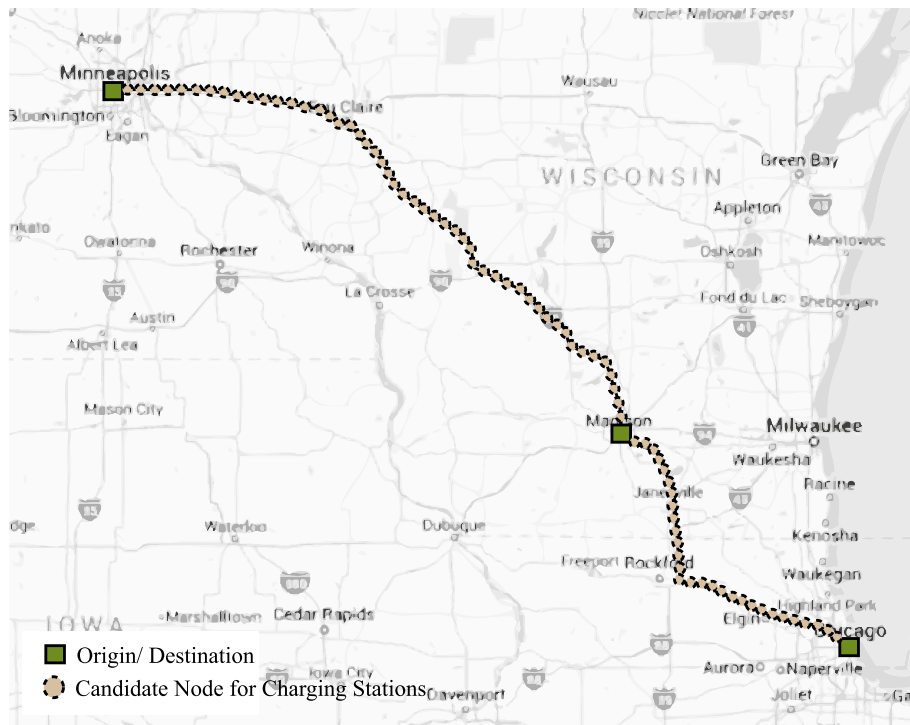


Fig. 4. Chicago–Madison–Minneapolis corridor origin–destination pairs and candidate nodes for building charging stations.

Table 3

Origin–destination pairs along Chicago–Minneapolis corridor.

Origin–destination pairs	Chicago–Madison	Madison–Minneapolis	Chicago–Minneapolis
Distance (mile)	150	270	420
Electric vehicle demand ^a	937	487	1588
Frequency (trip per year)	2.99	1.72	1.84

^a Considering 0.5 percent penetration rate (see: <http://www.hybridcars.com/December-2012-dashboard>, Last visited: 02-17-2015).

Table 4

Comparison of algorithm performance with commercial solvers for Chicago–Madison corridor.

Demand level		Low	Medium	High
Objective function (\$)	Heuristic	6872.03	33,804.91	67,554.65
	Solver	8167.49	35,194.30	68,916.29
Relative error (%)		15.9	3.95	1.98
Solution time (s)	Heuristic	129.86	141.52	147.57
	Solver	28.45	30.27	33.79
Number of solution variables		932		
Number of constraints		1858		
Number of O–D pairs		1		

the number of charging spots and the total system cost, even though it avoids queuing delay completely. *These findings indicate that ignoring queuing delay could lead to suboptimal configuration of charging infrastructure, and this effect will be more significant as PEVs gain more market share in the future.*

4.4. Sensitivity to battery size

The battery size is an important factor affecting the location of charging stations. Larger batteries are more expensive to manufacture but they typically require a sparser charging network (hence lower infrastructure cost). In all experiments

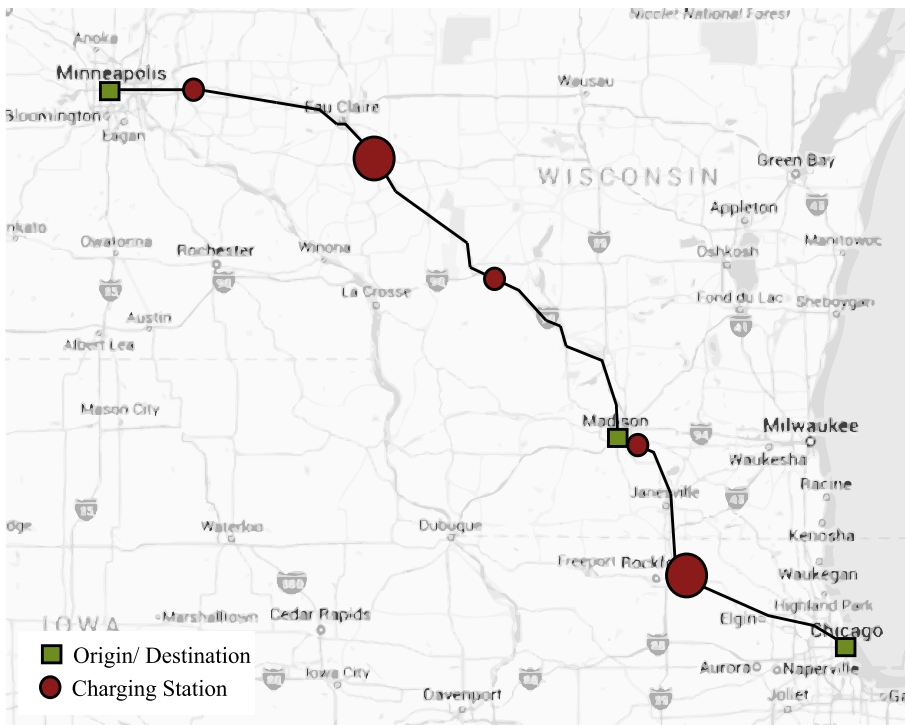
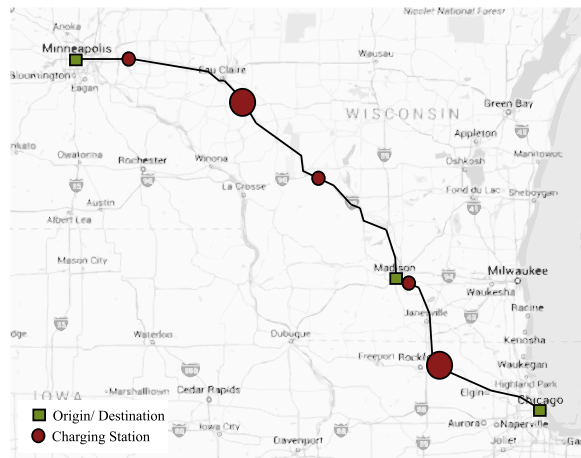


Fig. 5. Optimal configuration of charging stations on Chicago–Madison–Minneapolis corridor.



(a) Basic corridor model



(b) General corridor model

Fig. 6. Configuration of charging stations in basic vs. general corridor models.

Table 5

Comparison of basic corridor model and general corridor model considering different demand levels.

Demand level		Low	Medium	High
Objective function	Basic corridor model	22,653.7	112,129.0	223,973.0
	General corridor model	22,532.8	110,397.1	221,332.6
Total number of spots	Basic corridor model	90	450	900
	General corridor model	35	130	275

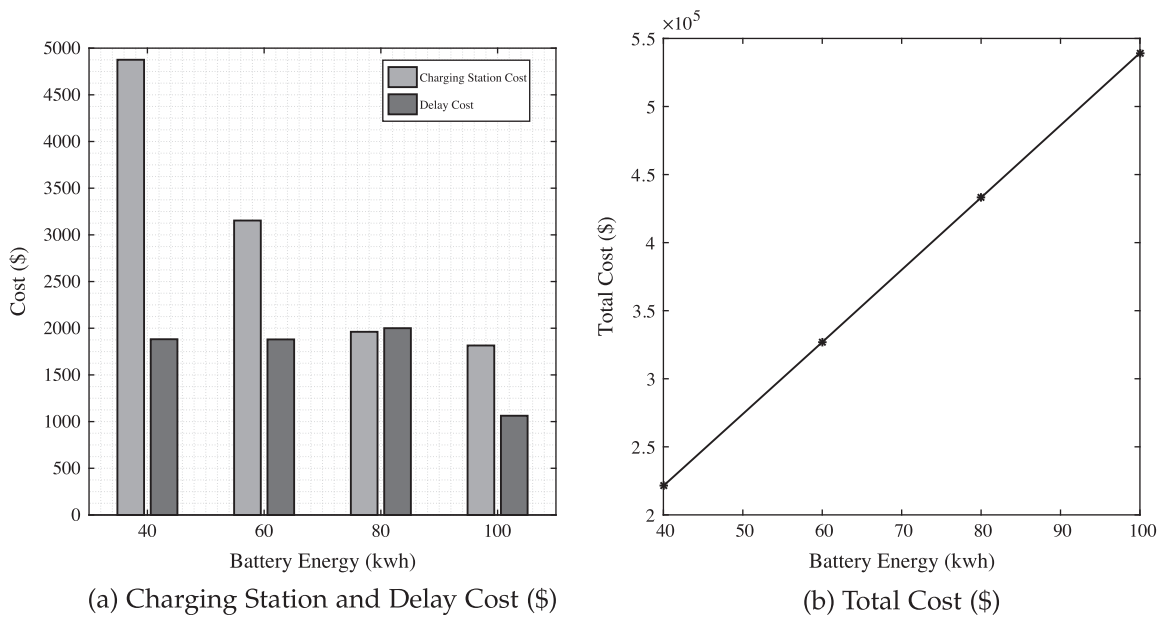


Fig. 7. Various cost at optima for different types of batteries (Chicago–Madison–Minneapolis Corridor).

conducted so far, the battery size is fixed at 40 kW h. This section explores the sensitivity of the model to the battery size. The demand level is set to the “high” level, i.e. ten times of the default value given in Table 3.

Fig. 7(a) reports the charging infrastructure cost and delay cost for different battery sizes. As expected, the infrastructure cost decreases as the battery size (hence the range) grows. However, as the battery size increases, the total charging delay costs remain at roughly the same level until the battery capacity reaches 100 kW h. A close look reveals that the benefit of cutting the infrastructure investment is initially greater than reducing the charging delay, because the initial increases in the battery sizes enables the elimination of some charging stations. When the battery size increases from 80 kW h to 100 kW h, that change is not sufficient to eliminate another station. In this case, it is more effective to reduce the queuing delay than to reduce the number of chargers. This finding suggests that at this demand level (ten times of the current value), installing more chargers at existing stations is more cost effective than allowing drivers to wait for charging. However, reducing charging delays may not be sufficient to justify the investment on new stations.

Fig. 7(b) shows how the total cost changes with the battery size. It is clear that the total battery cost is about 20 times of the combined infrastructure and delay costs when the battery size is 40 kW h. As the battery size increases, the percentage of the battery cost in the total cost grows even greater. Thus, with the current battery technology, minimizing social cost seems to require small batteries being used with dense charging stations and potentially high charging delays. In other words, *building long-range PEVs may not be cost effective from the societal point of view until the unit battery cost is reduced significantly*. We note that, however, excessive charging in long-distance trips may slow down the transition to PEVs, which in turn may increase the long-term environmental costs associated with GHG and air pollutant emission.

5. Conclusions

We proposed and solved a general corridor model for optimally configuring charging infrastructure to support long-distance intercity travel. The model aims at minimizing the total system cost, inclusive of infrastructure investment, battery cost and user cost. By “general” we mean the proposed model generalizes a previous model developed by Nie and Ghamami (2013) by (1) allowing realistic patterns of O–D demands, and (2) considering flow-dependent charging delay induced by congestion at charging stations. With these extensions, the model is better suited to performing a sketchy design of charging infrastructure along highway corridors.

The general corridor model is formulated as a mixed integer program with nonlinear constraint and solved by a specialized metaheuristic algorithm based on Simulated Annealing. We found that the algorithm produces satisfactory solutions in comparison with benchmark solutions obtained by Knitro, a widely used commercial solver for this type of problem. Yet, the metaheuristic algorithm scales much better and hence holds promises for large-scale applications in practice. Important findings from our numerical experiments are summarized in the following.

- Ignoring queuing delay inducted by charging congestion could lead to suboptimal configuration of charging infrastructure, and this effect can become more significant as PEVs gain more market share in the future. Because it tends to over-build charging capacity, the basic corridor model of Nie and Ghamami (2013) leads to higher system costs compared to the general model, even it eliminates queuing delays.
- The magnitude of the charging delay cost is comparable to that of the infrastructure cost, while the battery cost is almost an order of magnitude higher. Thus, in the absence of the battery cost, it is important to consider the trade-off between the costs of charging delay and the infrastructure.
- Building long-range PEVs with the current generation of battery technology may not be optimal from the societal point of view. Smaller batteries with dense charging networks may be more cost effective.

The natural next step is to further generalize the corridor model to allow route choices between a given O–D pair. As mentioned in the introduction, we believe that this can be achieved without further complicating the modeling and solution process much, provided that the number of routes is few and hence enumeration would not be prohibitively expensive. In this paper we solve the assignment problem using Knitro. However, many efficient solution algorithms exist that can take advantage of the special structure of the problem (see e.g. Nie, 2010, for a recent review). A future computation study can explore how much efficiency can be gained by applying one of the specialized traffic assignment algorithms.

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