# A multi-period optimization model for the deployment of public electric vehicle charging stations on network 

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#### Abstract

A multi-period multipath refueling location model is developed to expand public electric vehicle (EV) charging network to dynamically satisfy origin-destination (O-D) trips with the growth of EV market. The model captures the dynamics in the topological structure of network and determines the cost-effective station rollout scheme on both spatial and temporal dimensions. The multi-period location problem is formulated as a mixed integer linear program and solved by a heuristic based on genetic algorithm. The model and heuristic are justified using the benchmark Sioux Falls road network and implemented in a case study of South Carolina. The results indicate that the charging station rollout scheme is subject to a number of major factors, including geographic distributions of cities, vehicle range, and deviation choice, and is sensitive to the types of charging station sites.


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## 1. Introduction

Plug-in electric vehicles (PEVs), including plug-in hybrid electric vehicles (PHEVs) and battery electric vehicles (BEVs), have long been recognized as one of the promising alternatives to supplant internal combustion engines (ICE) powered vehicles and as an effective way to alleviate the dependency on petroleum and to reduce greenhouse gas emissions. PEVs cost as little as $2-3$ cents per mile (compared to 13 cents per mile for ICE powered vehicles) and help reduce $50 \% \mathrm{CO}_{2}$ emissions per mile than ICE powered vehicles (Mitchell, 2010). However, limited travel range and high life-cycle cost of ownership (U.S. Department of Energy, 2014) have been identified as major barriers for massive market adoptions (Huang and Zhou, 2015; Dong et al., 2014). This situation has been exacerbated because of the lack of sufficient public charging stations, especially fast charging stations to facilitate intercity trips by PEVs. In this study, we develop a multi-period optimization model that gradually rolls out fast charging stations along highway corridors to dynamically satisfy intercity trips. These trips are under geographical expansion with the growth of PEV market over time.

The question about where to locate PEV public charging stations stems from the general facility location problems (Daskin, 1995), in which a central planner allocates supplies or services to satisfy demand based on nodes in a spatial network, for instance, locations of electric taxi recharging stations (Jung et al., 2014) and EV charging stations (Ip et al., 2010; Momtazpour et al., 2014). However, these node-based facility location models are not well suited for charging station location problems, because in such problems demands are flows of goods or services in a network and thus models that can capture the flows are a better fit. The recognition of this feature has led to the prosperous growth of flow-based facility location problems. First developed by Hodgson (1990) and Berman et al. (1992), the flow capturing location model (FCLM) is the first

[^0]generation of flow-based location problem, in which a traffic flow is considered as captured if the flow passes through a facility. However, the model neglects an important feature of PEVs or other types of alternative fuel vehicles - limited vehicle range, i.e., a vehicle can only run a limited distance per refuel or recharge. For example, a PEV may need to stop multiple times for refueling before it can complete an origin-destination (O-D) trip. To take account of this effect, the FCLM was extended to the flow refueling location model (FRLM) by Kuby and Lim (2005), which maximizes the total covered traffic flows by strategically locating a predefined number of stations in a network. Instead of maximizing the traffic flow coverage, another stream of research studies, seeks a minimum number of stations to satisfy all travel demand, which are essentially set covering problems (Wang and Lin, 2009, 2013). These two distinct types of flow-based location models were recently reformulated as a flexible refueling station location problem by Mirhassani and Ebrazi (2013). In all those models, travelers were assumed to only take a shortest path between O-D pairs. Recently, this assumption was relaxed by Huang et al. (2015) in their new, Multipath Refueling Location Model (MPRLM), which allows travelers to complete a trip with single/multiple refueling stops via at least one path, which can be either a shortest path or a path with a reasonable deviation. The MPRLM is a mixed integer linear program (MILP), which can be intractable even for medium sized problems due to the exponential increase of number of decision variables and constraints with the deviation paths. A greedy-adding heuristic was thereby developed for the implementation of the MPRLM in real-world sized case studies (Li and Huang, 2014). The model was implemented with the heuristic on a case study of South Carolina (Li and Huang, 2015). With the growing popularity of EV problems in transportation research community, the traffic-flow based models have been rapidly emerging to explicitly integrate the effects of traffic flows into the charging-location decisions on the network, including studies (Nie and Ghamami, 2013; He et al., 2013, 2014, 2015).

All aforementioned models are capably of capturing the spatiality of travel trips and charging stations in a network, but they are static and not adaptive to dynamic evolvement of demand, especially in an emerging PEV market (Electric Drive Transportation Association, 2015; Tamor et al., 2015). To factor the inherent dynamics into an expansion of a charging network, a multi-period location model is desired.

Multi-period or dynamic facility location problems are not particularly new, which have been extensively studied in the past few decades. However, similar to the static flow-based location models, the majority of the multi-period/dynamic models are node-based and there are two general categories models: location and location-relocation models. The first category assumes that once a facility is in service, it will not be relocated (Wesolowsky, 1973; Van Roy and Erlenkotter, 1982). This type of models is well suited for capital intense infrastructure planning, such as refineries (Huang et al., 2010a,b). The second category allows for facility's relocation after location (Wesolowsky and Truscott, 1975), which is particular suitable for mobile facilities, such as ambulance (Carson and Batta, 1990) and public service facilities (Gregg et al., 1988). Interested readers can refer to recent survey papers (Arabani and Farahani, 2012; Nickel and da Gama, 2015; Melo et al., 2009) for detailed reviews.

These dynamic node-based facility location models are not well suited for dynamic location of charging stations for the same reason as for the static counterparts. Compared to the static flow-based location models, the literature on dynamic flow-based location problems is scarce. To our knowledge, the multi-period location model in Chung and Kwon (2015), perhaps, is the only dynamic flow-based location model. The model was extended from the FRLM, in which a pre-specified number of charging stations were sequentially placed on a freeway network in Korea for a finitely many time stages. The goal is to maximize the total traffic flow covered over time. In their problem, the topological structure of freeway network is given with a fixed set of O-D pairs while traffic flows are time-dependent and increase with the growth of PEV market. Distinct from this study, we tackle a different problem. First, our proposed model will take into account the topological dynamics of network, in which the origins and destinations of the network will be undertaken a sequential expansion with more cities becoming PEV markets. Second, rather than seeking a maximum-flow-coverage solution with a given number of stations, we aim to find a least-cost solution that can dynamically satisfy all $0-D$ trips over time. Third, an integrated view is taken to incorporate deviation paths and limited vehicle range into the model and further allow location and relocation of charging stations for the sake of lower cost. From the modeling perspective, our proposed model belongs to the second category of location-relocation models.

We propose a multi-period multi-path refueling location model ( $M^{2}$ PRLM) , which is built upon the MPRLM (Huang et al., 2015). The model expands a PEV charging network to serve growing intercity trips. The objective is to minimize the total cost of installations of new stations and relocations of existing ones while satisfying every O-D trip via at least one path between the O-D pair. The path can be either a shortest path or a path that is deviated away from a pre-defined path within a reasonable tolerance (called deviation path). The $\mathrm{M}^{2}$ PRLM is formulated as a mixed integer linear program. We adopt a heuristic based on the genetic algorithm (Vose, 1999) and justify the model and heuristic using a benchmark network - the Sioux Falls network (LeBlanc et al., 1975). With the success of numerical experiments, we demonstrate the model with a real-world case study based on the geographic settings of South Carolina and explore the interplay between major factors, including geographic distributions of cities, vehicle range, and deviation choice.

The remainder of this paper is organized as follows. The formulation of the $M^{2}$ PRLM is presented in Section 2 and the heuristic is reported in Section 3. In Section 4, we first justify the model and heuristic using the Sioux Falls network and then demonstrate the model with the case study of South Carolina followed by result discussions. We draw conclusions in Section 5 and briefly outline the directions of future work.

## 2. Mathematical formulation

The M ${ }^{2}$ PRLM is extended from the MPRLM (Huang et al., 2015) to make sequential decisions considering both spatial and temporal distributions of intercity trips. The model simultaneously considers multiple paths, limited vehicle range, and dynamic topologies of network in the decision process of installing new charging stations and possibly relocating existing ones whenever advantageous. The multiple paths in this study are comprised of both shortest and deviation paths. We deem a traveler willing to take either a shortest path (a pre-planned path) or a path that is detoured from a pre-planned path within their deviation tolerances. The concepts and generations of deviation paths have been explicitly discussed in details in the recent study (Huang et al., 2015).

In this study, there are two trade-offs: (i) installing new charging stations and/or relocating existing stations, and (ii) doing it now or later. The future costs of installations and relocations will be discounted to the present worth by using the engineering economics equation: $P=F(P / F, i, n)$, in which all future values $(F)$ are converted to the equivalent present worth ( $P$ ), $i$ is the annual discount rate, and $n$ is the number of years from now. The maintenance cost, between $\$ 1000$ and $\$ 2000$ per year (NYC Taxi and Limousine Commission, 2013), is less than $1 \%$ of the installation cost and too trivial to be included in the model.

PEV sales have been rising with more cities becoming PEV adopters (Electric Drive Transportation Association, 2015; Tamor et al., 2015), from which new intercity trips are generated. From a network modeling perspective, new O-D pairs will be sequentially added to a transportation network, which requires the charging infrastructure network to be expanded and adaptive to this topological dynamics. Cities will be ranked and selected to be the next EV adopters according to the result of multivariate statistical analysis. We report the details of such selection process on the case study of South Carolina in Section 4.2 as an example. In the model section, we focus on the developing a multi-period optimization model.

Let $(N, A)$ be a transportation network, where $N$ and $A$ are the sets of nodes and links, respectively. Let $\widetilde{N}$ be the set of candidate charging station locations, $\widetilde{N} \subseteq N$ and this set is assumed to fixed and unchanged over time. For example, they can be the rest areas on highway network and junctions of highways. Cities are both origins and destinations on the network and they increase over time. Let $R_{t} \subseteq N$, index $r$, be a set of origin nodes, and $S_{t} \subseteq N$, index $s$, be a set of destination nodes, where $t$ is the index of time stages (or periods) $t \in T$. Let $K^{r s}$ be a predefined maximum number of deviation paths for $0-\mathrm{D}$ pair $r-s$, which are exogenously generated (Huang et al., 2015). ${ }^{1}$ We denote by $P^{r s, k}$ a sequence of nodes on the $k$ th path for O-D pair $r-s$, where $k=1,2, \ldots, K^{r s}$. Denote a link by $a \in A$ or a pair of ending nodes, i.e., $a=(i, j) \in A$.

We made the following assumptions to simplify the modeling without the loss of generality: (1) the number of time stages is predetermined and each time stage has an equal length; (2) vehicles are homogeneous and fully charged at origins; (3) energy consumed is unified in terms of travel distance; (4) vehicle range is known and homogenous for all EVs and for all time; and (5) charging stations are uncapacitated. In addition, we simplified the modeling and assumed that the future demand is known exactly and a priori. In reality, stochasticity is inevitably inherent, especially in a multistage decision process. This concern is highlighted in our future research discussion and planned to be addressed in our future study.

The notation used in the model is first presented, and followed is the complete mathematical formulation of the $\mathrm{M}^{2}$ PRLM in (1)-(13).

```
Indices
\(i \quad\) index of candidate sites for charging stations in the network, \(i \in \widetilde{N} \subseteq N\), where \(\widetilde{N}\) is the set of candidate sites and \(N\)
    is the node set,
\(t \quad\) index of time stages, \(t \in T\)
\(r \quad\) index of origin in the network, \(r \in R_{t} \subset N\)
\(s \quad\) index of destination in the network, \(s \in S_{t} \subset N\),
\(k \quad\) index of the paths for an O-D pair, \(k=1,2, \ldots, K^{r s}\), where \(K^{r s}\) is maximum number of deviation path allowed
    between O-D pair \(r-s\),
\(a \quad\) index of arc set \(A, a=(i, j) \in A\),
Parameters
\(c_{i t}^{b} \quad\) cost of building a new charging station at node \(i\) in time stage \(t\),
\(c_{i j t}^{r} \quad\) fixed cost of relocating an existing charging station from node \(i\) to \(j\) in time stage \(t\),
\(\beta\) battery capacity unified in travel distance, i.e., vehicle range,
\(M\) a sufficiently large number,
\(P^{r s, k}\) the sequence of nodes on the \(k\) th path between O-D pair \(r-s\),
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[^1]$d_{i j} \quad$ distance between node $i$ and $j$,
$p_{i j}^{r} \quad$ variable cost of relocating charging stations from node $i$ to $j$ (including both distance- and time-transportation cost). Assume that the cost is invariant with time stages,
$\delta_{i}^{r s, k}=1$ if node $i$ is on $k$ th path between O-D pair $r-s, i \in P^{r s, k} ; 0$ otherwise.
$\omega_{i} \quad$ weighting factor that differentiates candidate sites, $i \in \widetilde{N}$,
Variables
$X_{i t}=1$ if a charging station is available at node $i$ in time stage $t ; 0$ otherwise,
$Z_{i t} \quad=1$ if a charging station is newly built at node $i$ in time stage $t ; 0$ otherwise,
$\bar{Z}_{i j t} \quad=1$ if a charging station is relocated from node $i$ to $j$ in time stage $t ; 0$ otherwise,
$Y^{r s, k}=1$ if the $k$ th path between $r$ and $s$ is taken; 0 otherwise,
$B_{i}^{r s, k}$ remaining power on a PEV at node $i$ on the $k$ th path of an O-D pair $r-s$,
$l_{i}^{r, k}$ amount of power recharged to an PEV at node $i$ on the $k$ th path of an O-D pair $r-s$.
\[

$$
\begin{equation*}
\text { Minimize } \sum_{i \in \widetilde{N}} \sum_{t \in T} \omega_{i} c_{i t}^{b} z_{i t}+\sum_{i \in \widetilde{N}} \sum_{j \in \widetilde{N}} \sum_{t \in T}\left(c_{i j t}^{r}+p_{i j}^{r}\right) \bar{Z}_{i j t} \tag{1}
\end{equation*}
$$

\]

Subject to :

$$
\begin{align*}
& B_{i}^{r s, k}+l_{i}^{r s, k} \leqslant M\left(1-Y^{r s, k}\right)+\beta, \forall r \in R_{t}, s \in S_{t} ; i \in P^{r s, k} ; t \in T ; k=1, \ldots, K^{r s}  \tag{2}\\
& B_{i}^{r s, k}+l_{i}^{r s, k}-d_{i j}-B_{j}^{r s, k} \leqslant M\left(1-Y^{r s, k}\right), \forall r \in R_{t}, s \in S_{t} ;(i, j) \in A ; i, j \in P^{r s, k} ; t \in T ; k=1, \ldots, K^{r s}  \tag{3}\\
& -\left(B_{i}^{r s, k}+l_{i}^{r s, k}-d_{i j}-B_{j}^{r s, k}\right) \leqslant M\left(1-Y^{r s, k}\right), \forall r \in R_{t}, s \in S_{t} ;(i, j) \in A ; i, j \in P^{r s, k} ; t \in T ; k=1, \ldots, K^{r s}  \tag{4}\\
& \sum_{r \in R_{t}} \sum_{s \in S_{t}} \sum_{k} l_{i}^{r s, k} \delta_{i}^{r s, k} \leqslant M X_{i t}, \forall t \in T ; i \in \widetilde{N}  \tag{5}\\
& \sum_{k=1}^{K^{s s}} Y^{r s, k} \geqslant 1, \forall r \in R_{t}, s \in S_{t} ; t \in T  \tag{6}\\
& B_{r}^{r s, k}=\beta, \forall r \in R_{t}, s \in S_{t} ; t \in T ; k=1, \ldots, K^{r s}  \tag{7}\\
& X_{i t}=X_{i, t-1}+Z_{i t}-\sum_{j} \bar{Z}_{i j t}+\sum_{j} \bar{Z}_{j i t}, \forall t \in T \backslash 1 ; i \in \widetilde{N}  \tag{8}\\
& X_{i 1}=Z_{i 1}, \forall i \in \widetilde{N}  \tag{9}\\
& X_{i t}, Z_{i t} \in\{0,1\}, \forall t \in T ; i \in \widetilde{N}  \tag{10}\\
& \bar{Z}_{i j t} \in\{0,1\}, \forall t \in T \backslash 1 ; i, j \in \widetilde{N}  \tag{11}\\
& Y^{r s, k} \in\{0,1\}, \forall r \in R_{t}, s \in S_{t} ; t \in T ; k=1, \ldots, K^{r s}  \tag{12}\\
& B_{i}^{r s, k} \geqslant 0, l_{i}^{r s, k} \geqslant 0, \forall r \in R_{t}, s \in S_{t} ; i \in P_{t}^{r s, k} ; t \in T ; k=1, \ldots, K^{r s} \tag{13}
\end{align*}
$$

The objective is to minimize the total cost of new charging stations and relocations for a finite planning horizon. The cost of installation of new charging stations may be location specific and varies with site conditions (e.g., pre-wired). The relocation cost depends on the relocation distance and site conditions. The inequalities (2)-(7) are constraints for each time stage and describe the spatiality of the network and constraints (8), (9) capture the temporality of the problem for a sequential expansion of charging network.

Constraint set (2) assures that the total onboard energy does not exceed battery capacity $\left(B_{i}^{r s, k}+l_{i}^{r s, k} \leqslant \beta\right)$ on paths that are taken (i.e., $Y^{r s, k}=1$ ); otherwise no restriction is applied (i.e., $Y^{r s, k}=0$ ), simply because no traveler will use that route. Constraints (3) and (4) concur to ensure that the energy consumption conservation (i.e., $B_{i}^{r s, k}+l_{i}^{r s, k}-d_{i j}-B_{j}^{r s, k}=0$ ) holds for all links on the $k$ th path if the path is taken (i.e., $Y^{r s, k}=1$ ). Otherwise, when $Y^{r s, k}=0$, the inequality becomes $B_{i}^{r s, k}+l_{i}^{r s, k}-d_{i j}-B_{j}^{r s, k} \leqslant M$, i.e., no restraining effects. Constraint set (5) is a logic constraint, stating that recharging is only available at node $i$ if a charging station is open. Constraint set (6) states that there is at least one path, either shortest or deviation paths, available between an O-D pair. Constraint set (7) realizes the assumption that all PEVs are fully charged at origins. Constraint set (8) describes an adaptive relationship, at node $i \in \widetilde{N}$ and in time stage $t$, between "status" variable (i.e., $X_{i t}$ ), which indicates the availability of charging station, and "activity" variables (i.e., $Z_{i t}$ and $\bar{Z}_{i j t}$ ), which indicate if a new station is built or an existing station is relocated. In particular, availability of a charging station involves both spatial and temporal interactions: station available from time stage $t-1$, (i.e., $X_{i, t-1}$ ), new station installed in time stage $t$, (i.e., $Z_{i t}$ ), and the stations relocated in time stage $t$ (i.e., $\sum_{j} \bar{Z}_{j i t}-\sum_{j} \bar{Z}_{i j t}$ ). The boundary condition of the charging network is given in constraint set (9), which states that all charging stations available in the first period are newly built at the beginning of the planning horizon. Constraints (10)-(13) are binary and nonnegativity constraints.

Remark 1. Both charging and travel costs of paths are not included in the objective of the model. The $M^{2}$ PRLM is a spatial economics model, which determines the locations of charging stations only based on the spatial relationships between O-D pairs and roadway networks. Computing both costs is in need of the exact number of trips between O-D pairs. We can use crowd sourced data, such as call detailed records (CRDs), to extract traces and estimate the distributions of trips, other than simulation as used in a recent study (Jung et al., 2014). The inclusion of traffic flow or intercity trips will lead to a new study.

The $M^{2}$ PRLM is a MILP. The number of decision variables and constraints increase exponentially with the number of deviation paths and time stages. The problem is NP-hard, because it reduces to the well-known set-covering location problem (Daskin, 1995), when the planning horizon shrinks to one single period. Without an effective solution, this model is intractable even for a moderate sized problem.

Remark 2. The binary relocation variable $\bar{Z}_{i j t}$ can be relaxed and it will not affect the solution of the model. This is because the $\bar{Z}_{i j t}$ will naturally converge to binary due to the binary variables $X_{i t}$ and $Z_{i t}$ in constraint set (8). In this study, all numerical results obtained by CPLEX are result of relaxing variable $\bar{Z}_{i j t}$.

## 3. A heuristic based on genetic algorithm

There exist various solution methods, both exact and heuristic, for solving multi-period location problems, which in many cases are modeled as mixed-integer programs. The initial attempt to overcome the computational difficulties, perhaps, involves a "myopic" approach, which consists of solving the first-period problem without taking into account futureperiod demands, and then solving the second-period problem given the optimum facility locations identified in the firstperiod problem, and so forth. Chung and Kwon (2015) extended it to consider both forward-myopic and backwardmyopic methods, in which they demonstrated the resulting solutions were suboptimal compared with the optimum solutions from the multistage optimization model solved by CPELX. In this paper, we compare our optimization model solutions to the myopic solutions to elaborate the effects of taking into account the future demands in a multistage model and temporal trade-offs in terms of investment and deferral (see details in Section 4.2). Other solution methods that solve the problem as a whole could be dynamic programming method, which is naturally suitable for the multi-period problems by taking the advantage of its adaptive solution process (Ballou, 1968; Erlenkotter, 1981; Canel et al., 2001). However, implementation of this method can be problematic if each single-period subproblem is already difficult to solve (e.g., the set covering location problem). In addition, the branch-and-bound approach, though solving (mixed) integer programs, is often limited to handling small sized problems (Chow and Regan, 2011a), especially when it comes to solving multi-period location problems.

It is not of a surprise to note that there is a vast pool of studies developing heuristic methods to solve multistage location problems while balancing the solution efficiency and quality. One of them is the Lagrangian relaxation based heuristic solutions, which have been widely used in the location and inventory planning problems, such as the integration with branch-and-bound method (Contreras et al., 2011), coupling with heuristics and subgradient optimization method for obtaining lower bounds (Kim and Kim, 2000; Hinojosa et al., 2000), and solving for the dynamic facility location problems (Chardaire et al., 1996). However, the success of implementing the Lagrangian relaxation method for a particular problem depends on several factors, such as the constraint(s) identified to be relaxed, the goodness of bounds, and the solution efficiency of the relaxed problem. These are something that we readily understood after we tried and failed to solve our model. There are other heuristics that have been proven effective, especially for real-world large-scale problems involving hundreds of nodes and arcs, such as the Tabu-search (Kaku and Mazzola, 1997), the Simulated Annealing (SA) algorithm (Baykasoğlu and Gindy, 2001; Antunes and Peeters, 2001) and Genetic Algorithm (GA) (Balakrishnan and Cheng, 2000). We tried the SA method when we realized that it would be relatively simple to apply this method to our model and understood that our efforts to develop Lagrangian relaxation based heuristic were unlikely to be successful. However, the tests on our complex, large-scale model failed to meet expectations, mainly due to unsatisfactory neighborhood search within a reasonable time.

To a broader extent, new methods can be elucidated by the solution methods applied to solving inventory routing problems that are inherently multi-period (Campbell and Savelsbergh, 2004). In particular, a two-phase approach decomposed the set of decisions into a delivery schedule first, followed by the construction of a set of delivery routes. As relating to our multi-period charging location problems, the first phase can utilize integer programming to identify the locations to be placed over time, whereas the second phase solves a linear program to identify optimal routes between O-D pairs. This method will be investigated in our future study.

In this study, we adopt a heuristic based on GA, which was developed by Vose (1999) for solving single-stage set-covering problems. We modify the operations to tune up the solution performance and add a procedure of feasibility check and solution refining. For completeness, the major procedures of the algorithm are reported and explained in the context of our problem formulation, which are representation and fitness function, parent selection, crossover operator and mutation, and feasibility check and solution refining. The notation used in solution is adopted from Section 2.

### 3.1. Representation and fitness function

A 0-1 matrix $X_{|N| \times|T|}$ is used to show the charging stations deployed on a spatiotemporal network, in which each row represents availability of charging stations for a time stage and each column represents how a charging station is used over
time. For example, a cell $X_{i t}=1$ indicates that a charging station is available for service at node $i$ in time stage $t$. We call a matrix $X_{|N| \times|T|}$ a feasible solution only if charging stations deployed over space and time can satisfy all O-D trips. With the solution matrix of "status" variables, the "activity" decisions, i.e., $Z_{i t}$ and $\bar{Z}_{i j t}$, and routing decision $Y^{r s, k}$ can be readily retrieved. In this study, the fitness of each candidate solution (also called, individual) is defined as the value of the objective function (1) of the $\mathrm{M}^{2}$ PRLM. As a cost minimization problem, a lower objective value indicates a better fitness, and vice versa.

### 3.2. Parent selection

The binary tournament selection (Beasley and Chu, 1996) is used for parent selections. In particular, we initialize a large population of feasible solutions (e.g., 100), and randomly pick four individuals, to form two pools, each of which contains two individuals.

### 3.3. Crossover operator and mutation

The individual with better fitness in each pool will be selected as a parent to breed children based on fitness-based crossover operator (Beasley and Chu, 1996). Let $f_{P 1}$ and $f_{P 2}$ be the values of objective function for parents $P 1$ and $P 2$ respectively and let $C$ be child matrix. Given $i=1, \ldots,|N|, t=1, \ldots,|T|$ :
(1) if $P 1_{i t}=P 2_{i t}$, then set $C_{i t}:=P 1_{i t}$ or $P 2_{i t}$;
(2) if $P 1_{i t} \neq P 2_{i t}$, then set $C_{i t}:=P 1_{i t}$ with probability $p=\frac{f_{p 2}}{f_{P_{1}+f_{2}}}$, and $C_{i t}:=P 2_{i t}$ with probability $1-p$.

Once a child matrix is formed, each cell in the matrix will be inverted based on a mechanism, called mutation (Beasley and Chu, 1996). In this study, we invert a cell (i.e., invert the value of the cell from zero to one, or vice versa) if a randomly generated probability is less than a pre-defined threshold (e.g., 10\%); otherwise the cell remains unchanged.

### 3.4. Feasibility check and solution refining

The crossover and mutation processes may inevitably cause infeasibility. A feasibility check is thus developed to examine if the resulting charging stations in every time stage can cover trips between every $0-D$ pair, given a fixed vehicle range. In any time stage $t$, if the deployed charging stations cannot cover all $O-D$ trips, this deployment solution will be replaced by another solution, which is randomly selected from any later period $\left(t^{\prime}=t+1, \ldots,|T|\right)$ of the initial set of feasible solutions. Such replacement warrants feasibility. This is because the set of O-D pairs of time stage $t$ is a subset of a later period $t^{\prime}$, as the network expands over time, so that a solution that is feasible in period $t^{\prime}$ must also be feasible in period $t$.

On the other hand, this feasibility remedy may introduce redundant charging stations. We develop a refining procedure to eliminate the redundancy as follows: randomly eliminate a node from a solution set $V$ (for a single period). If the elimination does not cause infeasibility, the subset $V \backslash\{i\}$ preserves; otherwise continue on to the next node. It terminates when all the nodes in the set have been examined. If the solution set is ever reduced, the same refining process will be executed, which repeats until there is no more new solution set produced. This process is applied for all the periods.

### 3.5. Population replacement

If a child solution is identical to any of the solutions in the initial population, this child solution will be neglected; otherwise it replaces the solution in the initial population with the worst fitness.

The GA procedure terminates when a maximum predetermined number of iterations $M$ (e.g., 100) is reached. The final solution is the one with the best fitness in the population.

The procedure of the heuristic is also summarized as follows:

Initialization: Randomly generate $n$ solutions as the initial population;
For iteration $i=1,2, \ldots, M$
Step 1. Pick four solutions from the population to form two pools, each of which contains two solutions;
Step 2. Select the solution with better fitness in each pool as one of the parents then do crossover to form a temp solution (may be infeasible);
Step 3. Mutation is applied to the temp solution with a mutation rate of $\alpha$;
Step 4. Check whether the temp solution is feasible. If yes, go to step 5; otherwise, use the feasibility remedy procedure to generate a feasible solution;
Step 5. Remove redundant stations in the temp solution to generate a child solution;
Step 6. If the child solution is not identical to any solutions in the population, replace the solution in the population with the worst fitness. If this is the last iteration, then terminate, otherwise, update the index of iteration and go to Step 1.

## 4. Numerical experiments and South Carolina case study

We first demonstrate the model and heuristic using the Sioux-Falls road network (LeBlanc et al., 1975), which is widely used for transportation network analysis test. With successful numerical experiments, we implement the model for a realworld case study based on geographic settings of South Carolina. All deviation paths are exogenously generated using MATLAB (Li and Huang, 2014) and the run times are not included. The exact solutions were obtained by using the CPLEX solver 12.6. All numerical implementations were programmed in AMPL (Fourer et al., 2011) and ran on a desktop with 8 GB RAM and Intel Core i5-2500@3.30 GHz processor under Windows 7 environment.

### 4.1. The Sioux Falls road network

The Sioux Falls network is shown in Fig. 1, which consists of 24 nodes and 76 directed links. The numbers in the circles represent the node indices. The numbers on the links denote the test distances in miles. All nodes are candidate sites for charging stations, i.e., $|\widetilde{N}|=|N|=24$. There are six time stages with equal time intervals (e.g., one year). The O-D pairs are gradually added following the procedure: starts with four nodes that are randomly selected from the 24 nodes, resulting in $4 \times 3=12$ O-D pairs, and add another four new nodes in each time stage until all the 24 nodes are used up. In the end, there are a total of 552 O-D pairs. The vehicle range (VR) is assumed to be 100 miles in this case study. We assume that weighting factor on each candidate site is identical to unity, i.e., $w_{i}=1, \forall i \in \widetilde{N}$.

The average costs of new fast charging stations and fixed relocation cost are $\$ 122,000$ and $\$ 38,000$, respectively, according to National Research Council (2013). The new fast charging station cost includes the costs of equipment, installation, utility interconnection, as well as host-site identification, analysis, screening and leases while the fixed relocation cost includes


Fig. 1. Sioux Falls network.

Table 1
$\mathrm{M}^{2}$ PRLM solution ( $K=1$ ).

| Time stage | Number of origins and destinations (O-D pairs) | Number of total available stations (station location IDs) | Number of new stations | Stations relocated |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4 (12) | 4 (\#5, 8, 13, 18) | 4 | - |
| 2 | 8 (56) | 8 (\#5, 8, 10, 11, 13, 14, 18, 19) | 4 | - |
| 3 | 12 (132) | 9 (\#5, 8, 10, 11, 13, 14, 18, 19, 22) | 1 | - |
| 4 | 16 (240) | 11 (\#2,3,5,8,10,11,13,14,18, 19,22) | 2 | - |
| 5 | 20 (380) | 12 (\#2,3,5, , 10,11, 13, 14, 18, 19, 21,22) | 1 | - |
| 6 | 24 (552) | 12 (\#2,3,5,8,10,11,13,14,18,19,21,22) | 0 | - |
| Total present worth of cost |  | \$1.37M |  |  |

every component except for the equipment cost. The variable cost of relocation mainly occurred in transportation is $\$ 1.38$ per mile (The Truckers Report, 2015). A 5\% annual discount rate is used for calculating the present worth of costs. To simplify the numerical tests, we assume that installation cost is identical to all sites.

### 4.1.1. Baseline case

We implement the $\mathrm{M}^{2}$ PRLM for both the shortest path $(K=1)$ and multi-path ( $K=3$ ) scenarios for all O-D pairs and results are presented in Tables 1 and 2, respectively. The results were obtained by using CPLEX. The location IDs of new charging stations are highlighted in the tables.

Both tables show that there is no station relocation as the anticipation of future O-D pairs is incorporated in the M ${ }^{2}$ PRLM. By comparing the results between the two deviation scenarios (i.e., $K=1$ to $K=3$ ), the higher deviation reduces the total number of stations needed from 12 to 7 or a $(1.37-0.84) / 1.37=38 \%$ reduction in the total cost. This is because the deviation paths allows trips between an O-D pair to be completed via more than a shortest path (i.e., $K=1$ ). Note also that in both cases there are more stations deployed in earlier stages, even though there are more O-D pairs introduced to the system in the later stages, which implies that paths used by the O-D trips in later stages largely overlap with the ones in early stages.

We test the performance of the heuristic and compare the solution quality and solving times to the counterparts of exact solutions obtained by CPLEX. As the heuristic may be sensitive to the choices of parameters in the heuristic, we conduct a series of numerical experiments of varying a few major parameters. In particular, we consider 18 different combinations from three different population size (i.e., 30, 50 and 100), three different mutation rates (i.e., $5 \%, 10 \%$ and $15 \%$ ), and two different numbers of iterations (i.e., 50 and 100) for the deviation scenario $K=1$. We run the heuristic for 200 times and report the results of the 95 percentiles of objective values and solving times in Table 3. From the table, we observe that as population size increases, the objective value decreases or solution quality improves. Mutation rate is crucial in controlling the solution quality and efficiency. A low rate may not suffice while a high rate may increase the solution time and make the problem too random. When the population size and mutation rate are fixed, more iterations generally improve quality but in the meantime result in longer solving time. Within the 18 combinations, we pick the combination: 100 population, $10 \%$ mutation rate, and 100 iterations, for implementing the heuristic.

The results and computational performances of heuristic are reported in Table 4, compared with the counterparts of the exact solution (CPLEX). The size of problem dramatically increases with deviation paths. In particular, there are 10,792 variables (including 3168 binary variables) and 6890 constraints when $K=1$. When $K=3$, the numbers of variables and constraints increase to 40,746 (including 7284 binary variables) and 52,178 , respectively. From Table 4 , the heuristic can yield high quality solutions, both within an average $3 \%$ gap of optimality. The solution is efficient especially when the problem is getting complex with deviations $(K=3)$. The results lend us confidence in implementing the heuristic for solving reallife case study of South Carolina.

### 4.1.2. Comparisons with myopic solutions

The model solutions are compared to myopic solutions. The myopic method is a so-called "shortsighted" approach in the sense that the method only does the best for now but neglects the future. This method is popular in engineering practice for its easy implementation. In this paper, although the complete information about future demand is assumed to be available, we compare our optimization solution to the myopic solution to highlight the differences in staged decision-making processes. In particular, the multistage optimization model takes into account the whole trajectory of future demand while the myopic method solves single-period MPRLM for each period successively. For illustration purpose, we only report the results of myopic solution based on $K=1$ in Table 5.

The table presents a different sequence of locating and relocating stations compared to our optimization model solutions in Table 3. Although both myopic and optimization solutions result in the same total of 12 charging stations, station relocations occur in the last four time stages if myopic solution is adopted while there is no relocation by the optimization solution. Because of the relocations, the myopic solution yields a higher total cost by (1.59-1.37)/1.37 $=16 \%$. We run another test, in which no relocation is allowed. The results in Table 6 show that the total cost is even higher by ( $1.69-1.37$ )/1.37 $=23 \%$ from the optimization solution, because of the increased total number of stations from 12 to 15 .

### 4.2. South Carolina case study

We use the case study of developing a public fast charging network in South Carolina to demonstrate the real-world application of $\mathrm{M}^{2}$ PRLM. The following data are processed: roadway network, locations of cities, and candidate sites for fast charging stations. In this study, we use Geographic Information System (GIS) software packages (e.g., ArcGIS) to integrate location data with transportation network data (South Carolina Department of Transportation, 2014). An aggregate roadway network is shown in Fig. 2, which consists of 519 nodes and 876 bidirectional links. The 519 nodes include cities, highway junctions, and rest areas, which are candidate sites for charging stations, and the links are interstate highway, US and state routes. We adopt the cost data of new charging stations and relocation from the report (National Research Council, 2013) and assume that the cost of installing new charging stations is identical to all candidate sites.

A total of 94 cities with population great than 5000 are considered as demand cities in this study, which almost represent the entire population of South Carolina. We assume that the fast charging network will be undertaken in three phases until O-D trips between all the 94 cities will be covered. For each phase, the cities are selected based on the probabilities of cities

Table 2
$\mathrm{M}^{2}$ PRLM solution ( $K=3$ ).

| Time <br> stage | Number of origins and destinations (O-D <br> pairs) | Number of total available stations (station <br> location IDs) | Number of new <br> stations |
| :--- | :--- | :--- | :--- |
| 1 | $4(12)$ | $4(\# \mathbf{1 1 , 1 2 , 1 8 , 2 1})$ | Stations <br> relocated |
| 2 | $8(56)$ | $7(\# \mathbf{5}, 11,12,15,18,21)$ | - |
| 3 | $12(132)$ | $7(\# 5,8,11,12,15,18,21)$ | 3 |
| 4 | $16(240)$ | $7(\# 5,8,11,12,15,18,21)$ | - |
| 5 | $20(380)$ | $7(\# 5,8,11,12,15,18,21)$ | 0 |
| 6 | $24(552)$ | $7(\# 5,8,11,12,15,18,21)$ | - |
| Total present worth of cost | $\$ 0.84 \mathrm{M}$ | - |  |

Table 3
Heuristic parameters and performance.

| Population size | Mutation rate (\%) | Number of iterations | Objective value (\$M) | Solving time (CPU seconds) |
| :---: | :---: | :---: | :---: | :---: |
| 30 | 5 | 50 | 1.45 | 21.5 |
| 30 | 5 | 100 | 1.43 | 43.3 |
| 30 | 10 | 50 | 1.44 | 23.3 |
| 30 | 10 | 100 | 1.43 | 45.0 |
| 30 | 15 | 50 | 1.45 | 22.9 |
| 30 | 15 | 100 | 1.44 | 44.1 |
| 50 | 5 | 50 | 1.42 | 20.2 |
| 50 | 5 | 100 | 1.40 | 38.4 |
| 50 | 10 | 50 | 1.42 | 22.6 |
| 50 | 10 | 100 | 1.42 | 43.8 |
| 50 | 15 | 50 | 1.42 | 22.2 |
| 50 | 15 | 100 | 1.42 | 44.3 |
| 100 | 5 | 50 | 1.41 | 21.6 |
| 100 | 5 | 100 | 1.40 | 42.2 |
| 100 | 10 | 50 | 1.41 | 21.5 |
| 100 | 10 | 100 | 1.40 | 41.7 |
| 100 | 15 | 50 | 1.42 | 21.7 |
| 100 | 15 | 100 | 1.41 | 42.5 |

Table 4
Comparisons between heuristic and CPLEX.

|  |  | Heuristic |  |
| :--- | :--- | :--- | :--- |
| Total present worth of cost $(\$ M)$ | $K=1$ | $\$ 1.40$ | $\$ 1.37$ |
|  | $K=3$ | $\$ 0.85$ | $\$ 0.84$ |
| Solving time (CPU seconds) | $K=1$ | 33 | 42 |

Table 5
Myopic solution with relocation ( $K=1$ ).

| Time stage | Number of origins and destinations (O-D pairs) | Number of total available stations (station location IDs) | Number of new stations | Stations relocated |
| :---: | :---: | :---: | :---: | :---: |
| 1 | 4 (12) | 3 (\#6, 13, 18) | 3 | - |
| 2 | 8 (56) | $8(\# 4,6,12,13,14,16,18,19)$ |  | - |
| 3 | 12 (132) | 9 (\#4,6, 10, 12, 13, 14, 15, 16, 18) | 1 | \#19 $\rightarrow$ \#15 |
| 4 | 16 (240) | 11 (\#2,3,4,6,10,12,13,14, 17, 18,22) | 2 | $\begin{aligned} & \# 16 \rightarrow \# 17 \\ & \# 15 \rightarrow \# 22 \end{aligned}$ |
| 5 | 20 (380) | 12 (\#2, 3, 4, 6, 10, 11, 13, 14, 17, 18, 21, 22) | 1 | \#12 $\rightarrow$ \#21 |
| 6 | 24 (552) | 12 (\#2,3,5,8,10,11,13,14,18,19,21,22) | 0 | $\begin{aligned} & \# 6 \rightarrow \# 8 \\ & \# 4 \rightarrow \text { \#5 } \\ & \# 17 \rightarrow \text { \#19 } \end{aligned}$ |
| Total present worth of cost |  | \$1.59M |  |  |

becoming EV adopters or markets, which are obtained by using the logistic regression analysis (Hosmer et al., 2000) on multivariate socioeconomic data, including household size, homeownership, and population density of cities (U.S. Census Bureau, 2014). The resultant probabilities are in a range from zero to $100 \%$.

Table 6
Myopic solution without relocation ( $K=1$ ).

| Time stage | Number of origins and destinations (O-D pairs) | Number of total available stations (station location IDs) | Number of new stations |
| :--- | :--- | :--- | :--- |
| 1 | $4(12)$ | $3(\# \mathbf{6 , 1 3 , 1 8})$ | 3 |
| 2 | $8(56)$ | $8(\# \mathbf{4}, 6, \mathbf{1 0 , 1 2 , 1 3 , 1 4 , 1 8 , 1 9 )}$ | 5 |
| 3 | $12(132)$ | $10(\# 4,6,10,12,13,14, \mathbf{1 5}, \mathbf{1 6}, 18,19)$ | 2 |
| 4 | $16(240)$ | $13(\# \mathbf{2}, 4,6,10,12,13,14,15,16,18, \mathbf{2 0})$ | 3 |
| 5 | $20(380)$ | $15(\# 2,3,4,6,10,11,13,14,15,16,18,19,20, \mathbf{2 1})$ | 2 |
| 6 | $24(552)$ | $15(\# 2,3,4,6,10,11,13,14,15,16,18,19,20,21)$ | 0 |
| Total present worth of cost | $\$ 1.69 \mathrm{M}$ |  |  |



Fig. 2. The South Carolina network.

Cities of $100 \%$ probability already have PEV users and public charging stations in place within city boundaries. In South Carolina, there are 15 such cities (see Fig. 2), which accounts for $33 \%$ of the total population statewide. We set the goal of Phase I to make PEV drivers freely travel between these cities. For Phase II, the goal is set to cover $50 \%$ of the total population, which makes 13 additional cities with the highest probabilities added to the network, resulting in a total of 28 cities. In Phase III, all remaining cities of the 94 cities will be considered. Each phase is assumed to last five years.

### 4.2.1. Baseline case

We consider both shortest path $(K=1)$ and multi-path $(K=3)$ scenarios for all O-D pairs and two different vehicle ranges (VRs) of 100 and 150 miles. No optimal solutions can be attained within a reasonable amount of computing time by using CPLEX. We set the upper bound of run time to be four CPU hours and report the heuristic results and the best solution obtained by CPLEX in Table 7. In the case study, the heuristic uses population size of 100 and mutation rate of $10 \%$ and terminates after 50 iterations in each run. The heuristic results in Table 7 are of 95 percentiles of results of 50 runs. From Table 7, the heuristic attains lower total system cost with shorter solving times than the counterparts of the exact solutions (within four CPU hours) for all cases.

The results of this case study presented in this section are result of the heuristic. From the table, the extension of vehicle range by 50 miles reduces the total system cost by about $2 / 3$ for both for $K=1$ and $K=3$. A deviation results in a lower total cost, but by a trivial extent. This is mainly because many cities are interconnected through freeways and alternative paths through secondary highways are not in favor. In other words, though available, the deviation paths do not render as much flexibility as we have seen in the case of the Sioux Falls network.

We illustrate the layouts of charging stations over the three phases based on the heuristic solutions for $K=3$ with vehicle range of 150 miles in Fig. 4. There are nine charging stations installed in Phase I, as shown in Fig. 3(a), which are geograph-

Table 7
Comparison between heuristic and CPLEX.

|  |  | $\mathrm{VR}=100$ |  | $\mathrm{VR}=150$ |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | GA | CPLEX ${ }^{\text {a }}$ | GA | CPLEX ${ }^{\text {a }}$ |
| Total present worth of cost (\$M) | $K=1$ | \$3.87 | \$5.27 | \$1.29 | \$1.36 |
|  | $K=3$ | \$3.44 | \$4.39 | \$1.24 | \$1.27 |
| Solving time (CPU seconds) | $K=1$ | 321 | 14,400 | 287 | 14,400 |
|  | $K=3$ | 1662 | 14,400 | 2,155 | 14,400 |

${ }^{\text {a }}$ Best solutions obtained within four CPU hours.

(c) Phase III (11 stations)

Fig. 3. Charging station deployment over time ( $K=3$ and VR $=150$ miles $)$.
ically dispersed and located along the freeways. The same nine stations remain to serve the $0-\mathrm{D}$ trips expanded for 13 new cities in Phase II (see Fig. 3(b)). This is because the additional O-D pairs generated by the 13 cities largely overlap with the existing O-D pairs between the 15 cities in Phase I. In Phase III, all 94 cities are considered, which are clustered around a few major cities, including Greenville, Rock Hill, Columbia, Charleston and Myrtle Beach. Only two new charging stations are placed as highlighted in Fig. 3(c), together with the existing nine stations, to serve the 8742 O-D trips. Throughout the planning horizon, no charging station is relocated.

Table 8
Comparisons between $M^{2}$ PRLM and myopic solutions.

| Phase |  | I |  | II |  | III |  | Overall cost |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Cost | Deployment ${ }^{\text {a }}$ | Cost | Deployment ${ }^{\text {a }}$ | Cost | Deployment ${ }^{\text {a }}$ |  |
| Net Present Value (\$M) | Myopic | \$0.976 | $8+0$ | \$0.124 | $1+1$ | \$0.169 | $2+1$ | \$1.269 |
|  | $\mathrm{M}^{2}$ PRLM | \$1.098 | $9+0$ | \$0 | $0+0$ | \$0.146 | $2+0$ | \$1.244 |

${ }^{\text {a }}$ Deployment strategy consists of the number of new stations installed (the number before " + ") and number of stations relocated (the number after " + "). For example, $8+0$ means that there are eight new stations installed and no station relocated.

As a multi-stage decision making process, it is of interest to understand how adding or relocating a station on a network may result in a different performance in the current period and the expected performance in future periods. We measure the effects of "invest" or "defer" options (Chow and Regan, 2011b) by comparing our model solution to the myopic solution. The decomposed costs in each phase and corresponding deployment strategy are displayed in Table 8 in comparison with the results of the $\mathrm{M}^{2}$ PRLM. Note that all the costs in the table stated in net present values. In Phase I, in relative to myopic solution, one more charging station is invested, which costs $\$ 0.122 \mathrm{M}(=1.098-0.976)$ to the current period but saves $\$ 0.147 \mathrm{M}$ $(=0.124-0+0.169-0.146)$ for the future periods or about $\$ 25,000$ less in the overall cost. In terms of deployment strategy, the one more new station installed in the first phase helps eliminate the one new station and one station relocation in phase II and one station relocation in phase III.

The results also indicate that there are more stations to be deployed earlier than later periods, largely because of the demand city distributions. This insight is consistent with the findings in Chung and Kwon (2015) that their optimization model emphasized more on earlier locations and associated flow coverages and consequently selected charging station locations to cover most frequently used paths in earlier time periods while considering the increase of traffic volume in the later time periods.

### 4.2.2. Evaluation of charging station deployment strategy under demand uncertainty

We evaluate the performance by the deployed charging stations from the baseline case under demand uncertainty in terms of the number of $\mathrm{O}-\mathrm{D}$ pairs completed or covered. A higher coverage implies a more robust station deployment strategy under uncertainty. The uncertainty mainly refers to the randomness that cities become demand cities (or EV adopters), which may be due to the factors, such as economic and population growth. In this analysis, the uncertainty only emerges in the second phase, whereas the 15 cities in first phase and 94 cities in the last (third) phase of planning process are fixed and given. We randomly select cities (except those already having EV charging stations) until the total population of the selected cities is at least $50 \%$ of the statewide population. The study is based on the same, fixed five-year phase. Note that the phase length can be uncertain as well. However, the variations of the phase length will neither affect the location strategy nor the O-D pair coverage, but only change the present worth of the cost. In this sensitivity analysis, we conduct an analysis of 20 random sets of different cities in the Phase II for each of the four combinations of deviation choices (i.e., $K=1$ and $K=3$ ) and vehicle ranges (i.e., $V R=100$ and $V R=150$ ), a total of 80 scenarios. Within the 20 random sets, the number of cities selected is ranged from 27 to 39 and the population coverage is between $50.04 \%$ and $53.72 \%$.

The results are reported in Fig. 4, in which the horizontal axis denotes O-D pair coverage achieved in percentages while the vertical axis is the cumulative probabilities following each of the four combinations of deviation choices and vehicle ranges. For example, following $K=1$ and VR=100, the minimum and maximum coverages are about $90.5 \%$ and $98.5 \%$, respectively, and there is about $48 \%$ of the chance that the coverage is between $90.5 \%$ and $94 \%$. It is also observed that other combinations result in overall higher coverage, due to the increased flexibility that helps achieve a higher coverage, given a station deployment. When the vehicle range is extended to 150 miles, regardless the deviation choice, the coverage is at least


Fig. 4. Cumulative distribution of percentage of O-D pairs covered under random tests.
$98 \%$ and there is a high chance to have all O-D pairs covered. Even for the same vehicle range of 100 miles, the deviation can substantially increase the coverage as shown in Fig. 4. Further investigation reveals that the average O-D pair coverages are $94.58 \%$ and $99.34 \%$ for combinations of $K=1$ and $\mathrm{VR}=100$ and $K=3$ and $\mathrm{VR}=150$, respectively. These results indicate that the charging station deployment from baseline is quite robust in providing high coverage of $0-D$ pairs under demand uncertainty, mainly due to the clustered cities along major highways in the case study of South Carolina. This conclusion may not hold for another different geographic setting.

Table 9
Effects of weighting factors on rollout schemes.

|  |  | Weighting factors $\left(\omega_{i}\right)$ |  |  |  |  |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
|  |  | 0.2 | 0.4 | 0.6 | 0.8 | 1.0 |
| Number of stations located at rest areas and cities/total number of stations | Phase I | $8 / 10$ | $9 / 10$ | $10 / 11$ | $7 / 9$ | $4 / 9$ |
|  | Phase II | $10 / 12$ | $10 / 11$ | $10 / 12$ | $8 / 11$ | $4 / 9$ |
|  | Phase II | $9 / 12$ | $8 / 11$ | $8 / 12$ | $7 / 12$ | $5 / 11$ |


(a) Phase I (10 stations)

(b) Phase II (12 stations)

(c) Phase III (12 stations)

Fig. 5. Charging station deployment with differentiation of the types of sites ( $K=3$ and $\mathrm{VR}=150$ ).

### 4.2.3. Differentiations of types of candidate sites

In the baseline study, all candidate locations are assumed to be identical, which may not always reflect the reality. For example, rest areas and cities, because of being prewired, would be cheaper than highway junctions to install new charging stations. There could be other societal concerns. For example, rest areas and cities may be more preferable than highway junctions for siting charging stations as travelers may be more familiar with them.

To differentiate the types of candidate sites, we set five different weighting factors ( $\omega_{i}$ in the model formulation) for rest areas and cities between 0.2 and 1 with an increment of 0.2 while fixing the weighting factor for junctions at 1 . The correspondent rollout schemes are presented in Table 9. Compared to the baseline (in the last column), the total number of stations located at rest areas and cities decreases with the increase of weighting factors, which implies that the junctions are more geographically favorable. Except for the baseline, relocations occur in Phase III, which is indicated by the reduced number of stations placed at rest areas and cities from Phases II to III. This is because for a low weighting factor, a relocation is as equivalently costly as a new installation.

We plot the layouts of stations of weighting factor $\omega_{i}=0.2$ as an example in Fig. 5 to demonstrate the effects of weighting factor on system planning. To be comparable with baseline case, the same combination of $K=3$ and $V R=150$ is used. The new station rollout scheme is noticeably different from baseline. In Phase I, there are ten stations installed, eight of which are placed at rest areas and cities (Fig. 5(a)). In contrast, there are only four out of nine stations placed at rest areas and cities in the baseline case (Fig. 3(a)). In Phase II, two more new stations (see Fig. 5(b)) are added to serve increased O-D pairs. In the last phase, instead of adding new stations, three stations are relocated (Fig. 5(c)).

## 5. Conclusions and future work

We developed the $\mathrm{M}^{2}$ PRLM for strategically expanding public charging network to facilitate intercity trips by PEVs. The model successfully captured the topological dynamics of network with the emerging PEV markets and integrally considered the effects of limited vehicle range and flexibility in selecting deviation paths. To make the model tractable for large-scale problems, we solve the problem by adopting a heuristic based on genetic algorithm.

We justified the model and heuristic using the benchmark Sioux Falls network by comparing the numerical results and solutions performance with the exact solutions obtained by CPLEX. With the success of numerical experiments on Sioux Falls network, the model was demonstrated on a real-world South Carolina case study, which provided us with the following major insights. First, the charging station rollout scheme was subject to the geographic distributions of cities, vehicle range, and deviation choice. Second, the anticipation of the increase of future demands in our multistage optimization model could help reduce the overall cost in a long run, although it could result in poorer performance for the current period. Third, our charging location strategy was robust in providing high coverage of $0-D$ pairs under demand uncertainty. Last, the highway junctions were more geographically favored than the rest areas or cities, and relocation could be a cost-effective alternative to new installation when installing new stations at rest areas or cities are cheaper.

We also outline several extensions that could enrich the context of the study. First, the probability of each city becoming an EV adopter is assumed to be static and fixed throughout the planning horizon. A more realistic, time-dependent probability assessment of future demand would be essential for determining best possible phasing intervals. The results will offer rich insights on public policies, such as the critical timing to cease the tax incentives on PEV. Second, this study simplifies the modeling by assuming that we know exactly the trajectory of expansion of future demand cities, based on the projected probabilities as a result of statistical analysis. In reality, the major issue would stem from the stochasticity embedded in future demand. In other words, we may face multiple possible trajectories depending on the multivariate socioeconomic data used. How to incorporate this stochasticity into modeling of infrastructure system expansion is a challenge in both modeling and solution and has been well noted in prior studies (Snyder, 2006; Ballou, 1968; Daskin et al., 1992; Baron et al., 2011; Arabani and Farahani, 2012; Owen and Daskin, 1998; Farahani et al., 2009). Depending on the nature, the problem may be formulated as a multistage stochastic program and solved by a nested decomposition method or reformulated as a dynamic programming problem and solved by approximate dynamic programing method (Powell, 2007). Lastly, when a massive adoption of PEV is realized and PEV flow on the roadway network can be readily predicted, the EV traffic flows can be incorporated in the future modeling practice and explicit station capacity design (e.g., number of chargers to be placed at each station) can be included in the model as an integer variable. A bi-level optimization framework can be used to incorporate traffic flow at lower level while the upper-level will be locational decisions. It leads to a mathematical problem with equilibrium constraints (MPEC) problem if traffic equilibrium is sought in the lower level. In terms of solution methods, we will investigate a new method to decompose the set of decisions to a set of charging station locations first, followed by the construction of routes between O-D pairs.

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[^1]:    ${ }^{1}$ The deviation paths are exogenously generated using either the $K$-shortest path loopless algorithm Yen (1971). Finding the K Shortest Loopless Paths in a Network. Management Science, 17, 712-716. or the K-shortest path algorithm with deviation cap. The second algorithm is described as follows: For each O-D pair ( $r, s$ ), find the shortest path using any efficient shortest path algorithm, such as Dijkstra's algorithm Dijkstra (1959). A note on two problems in connexion with graphs. Numerische Mathematik, 1, 269-271. for ( $r, s$ ) (the length denoted as $L^{r s, 1}$ ) and set $k=1$. Then, compare the length of the $k$ th path (denoted as $L^{r s, k}$ ) with $(1+p) L^{r s, 1}$, where $p$ is a predefined deviation cap. If $L^{r s, k} \geqslant(1+p) L^{r s, 1}$, then stop; otherwise, set $k=k+1$, and find the $k$ th shortest path using Yen's algorithm and go back to last step.

