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# Bus priority at signalized intersections with single-lane approaches: A novel pre-signal strategy

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ABSTRACT

Signalized intersections often represent a major source of bus delays in urban environments. One strategy to mitigate this problem is to dedicate an existing car lane for bus-use only and use an additional signal to help minimize interactions between cars and buses. However, this is physically impossible at approaches where only a single travel lane is available for each direction. To this end, this research explores a novel method to provide priority to buses at signalized intersections with single-lane approaches that (nearly) eliminates bus delays while minimizing the negative impacts imparted to cars. Using additional signals to stop cars on the opposing travel lane, the bus can jump a portion of the car queue using the travel lane in the opposite direction.

This paper theoretically quantifies the delay savings buses can achieve, and the negative impacts imparted onto cars when this pre-signal strategy is applied. The negative impacts are measured as the additional car delays experienced when the intersection signal is under-saturated, and the reduction in car-discharge capacity when the intersection signal is over-saturated. In the under-saturated case, the results show that moderate average bus delay savings (~5–7 s per vehicle, equivalent to about 25% of the average delay expected at the intersection) are achieved if the pre-signal is always in operation and the total passenger delay is decreased only if bus occupancies are very high. However, if the pre-signal operation is targeted to only provide priority to the buses that would benefit the most, bus delay savings can be more than doubled while reducing the total passenger delay, even if the ratio of bus to car occupancy is relatively modest (greater than about 20). In the over-saturated case, bus delay savings can be much more significant (greater than 30 s per bus), and this delay saving can increase further for longer block lengths (greater than 100 m). However, the capacity of the intersection decreases by up to 25% during each cycle in which a bus arrives to the intersection. Simulation tests confirmed that the general trends and magnitudes of bus delay savings and negative impacts to cars hold for more realistic behaviors. The overall benefits are slightly smaller in the simulations, but nevertheless the strategy seems promising as a bus priority strategy at intersections with single-lane approaches in the field.

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## 1. Introduction

Public transportation is a viable way to combat urban traffic congestion as public transport vehicles (e.g., buses) are able to use urban roadspace more efficiently compared to private vehicles (e.g., cars). However, when buses and cars interact, the operation of both modes could be impaired. This is very evident at signalized intersections, which constitute one of the major

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sources of vehicular delays in urban environments. For example, buses dwelling at stops near a signalized intersection could significantly reduce discharge capacity and this could lead to increased car queues and delays (Gu et al., 2013, 2014). More worrying to public transportation operations is that buses can also get delayed by car queues, leading to more unreliable transit service. Mitigating the impacts of these harmful interactions on transit vehicles is essential to promote public transportation as a solution to urban traffic congestion.

One strategy to minimize negative bus–car interactions at signalized intersections has been to install an additional traffic signal upstream of the intersection to help manage conflicts between the two vehicle types. These additional signals, called pre-signals or bus gates, are used on approaches with dedicated bus lanes. The pre-signals are typically installed at the location where the bus lane ends to help buses transition out of the dedicated bus lane with minimal interruption. The pre-signal stops cars on the mixed-used lanes, which allows a bus approaching on the dedicated lane to bypass any car queues it might otherwise encounter and travel unencumbered by car traffic. The pre-signal intermittently changes the *allocation* of the lane downstream of the pre-signal from mixed-use to bus-use only and only impacts vehicles traveling in the same direction as the bus. Both analytical investigations and field implementations of pre-signals provide evidence that they can yield significant delay savings to buses while adding only modest travel delays to cars (Wu and Hounsell, 1998; Guler and Cassidy, 2012; Guler and Menendez, 2014a,b; He et al., 2015). This strategy has been implemented throughout the world, including in the U.K., Switzerland and Germany.

However, this type of pre-signal operation is only possible at locations with dedicated bus lanes. Unfortunately, bus lanes do not currently exist in many urban environments and expanding the roadway to add a dedicated bus lane is not feasible either due to cost or space restrictions. While an existing lane could be taken away from car traffic to implement a dedicated bus lane, doing so is not viable in situations where only a single travel lane is present. Such single-lane roadways and intersection approaches are very common in urban environments. The only current option available to provide priority for transit vehicles at intersections with single-lane approaches is to implement Transit Signal Priority (TSP) at the main intersection signal, which can provide signal timing benefits to approaches with an arriving bus (Rakha and Zhang, 2004; Stevanovic et al., 2008; Koehler and Kraus, 2010; Christofa and Skabardonis, 2011; He et al., 2014; Ma et al., 2014; AHmed and Hawas, 2015). While this can provide some benefits, TSP still forces buses to mix with cars, which can result in significant bus delays at the intersection.

To this end, this research explores a novel method to provide priority to buses at signalized intersections with single-lane approaches using pre-signals in a way that (nearly) eliminates bus delays while minimizing the negative impacts imparted to cars. The pre-signal that we propose operates differently than traditional pre-signals in several ways: (1) it does not require the existence of a dedicated bus lane; (2) it intermittently changes both the *direction* of vehicle movement and the *allocation* of a travel lane; and, (3) it impacts vehicles traveling in both directions. The idea is to clear cars out of the opposite direction travel lane ahead of an arriving bus, allowing the bus to use this lane to jump a portion of the car queue. To do this, pre-signals are required on travel lanes in both directions. The pre-signal on the opposite direction travel lane allows that lane to clear of cars so that a bus can safely use it as an intermittent bus lane. The pre-signal on same direction travel lane stops cars and allows the bus to merge back onto its original lane without conflict. The details of this dual pre-signal operation are explained in the following section.

The goals of this paper are twofold: (1) to define this new pre-signal operating strategy, and (2) to theoretically evaluate its potential impacts, including both the benefits to buses and the corresponding negative impacts to cars. Only through an exhaustive analysis can a complete understanding of the operation and the bounds of application of this strategy be determined. The benefits to buses are quantified by the delay savings when compared to a no priority strategy. The negative impacts to cars due to the pre-signals are measured differently based on the operating conditions expected at the main signal. If the intersection signal is expected to be under-saturated, the negative impacts are quantified by the additional delays imparted onto cars. In this case, there exists a clear metric that can be used to assess the overall impact of this strategy—change in overall delay of all users in the system. On the other hand, when the intersection signal is expected to be over-saturated, the negative impacts are quantified as the reduction in total throughput (i.e., capacity) at the main signal. Here, the overall impacts are harder to quantify due to the trade-offs between reductions in bus delays vs. reduction in throughput. These trade-offs are best quantified by the relevant transportation agency based on how they value car-moving capacity and transit delays. The theoretical results are then confirmed with tests performed through a micro-simulation.

The remainder of the paper is organized as follows. The details of the pre-signal operation are described in Section 2. Next, theoretical models to analyze the delays and the capacity changes associated with the pre-signals for under-saturated and over-saturated intersections are developed in Sections 3 and 4, respectively. The results of simulation tests and comparisons with theory are presented in Section 5. Finally, some concluding remarks and discussion are presented in Section 6.

## 2. Description of problem and strategy

We consider here an isolated intersection with single-lane approaches, as illustrated in Fig. 1a. The intersection is assumed to be signalized with a fixed cycle length, $L_C$ [h], and red time, $R$ [h], in both the subject direction (i.e., the direction of bus movement) and the opposite direction. We assume here that the subject and opposite direction share a green phase, however the results can be generalized to cases when different phasing schemes are used (e.g., leading or lagging greens in the opposite direction). Traffic on all approaches is assumed to obey Kinematic Wave Theory (Lighthill and Whitham, 1955;

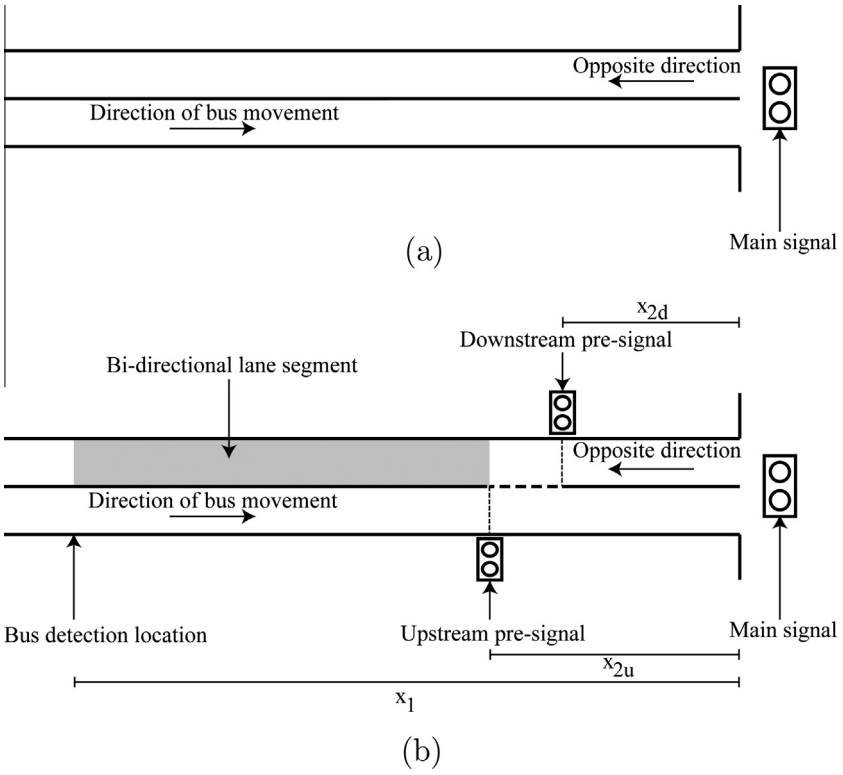


**Fig. 1.** (a) Intersection with single-lane approaches considered in this paper; (b) example layout of pre-signal strategy to provide priority on a single-lane approach.

Richards, 1956) with a given fundamental diagram that relates flow, $q$, to density, $k$. We assume that the functional form of the fundamental diagram is triangular as depicted in Fig. 2a. From this figure, the capacity of the roadway is $q_{max}$ [veh/h] depicted as state C, car arrival rate is $q_A$ [veh/h] depicted as state A, and the jam density is $k_j$ [veh/km] depicted as state J. The free flow speed is $v$ [km/h], the queue grows at a speed $u$ [km/h] and dissipates at a speed $w$ [km/h].

The basic idea of how priority is provided to buses on single-lane intersection approaches is illustrated in Fig. 1b. Two additional signals (called pre-signals) are placed upstream of the main signal in the direction of bus travel. The upstream pre-signal, located at a distance $x_{2u}$ [km] from the main signal, affects cars in the subject direction. The downstream pre-signal, located at a distance $x_{2d}$ [km] from the main signal, affects cars in the opposite direction. Although they are separated by a marginal distance, the two signals operate jointly to create an intermittent bus priority lane as will be described now.

When no bus is present in the subject direction, both pre-signals remain green to allow cars to discharge through the intersection as if the pre-signals did not exist. Whenever an approaching bus from the subject direction reaches a distance $x_1$ [km] from the main signal (referred to as the detection location), the pre-signal turns red at both $x_{2u}$ and $x_{2d}$ to stop cars traveling in both directions.[1] This would allow the bi-directional lane segment (shaded in Fig. 1b) to temporarily clear of vehicles. Once the bi-directional lane segment is clear, the bus is free to maneuver onto the opposite lane and travel unimpeded until it can merge back onto its original lane between $x_{2u}$ and $x_{2d}$. Once the bus re-enters its original lane, the green phase is resumed at both pre-signals. Notice that the bus does not need to use the full length of the bi-directional lane segment. It can change into the opposite lane at any location between $x_1$ and $x_{2u}$ and will likely merge into the bi-directional lane segment when it encounters the back of the car queue in the subject direction. The distance between $x_{2u}$ and $x_{2d}$ would need to be large enough so that the bus has room to maneuver back onto its original lane. However, to facilitate the theoretical analysis, in the next two sections we assume that the distance between $x_{2u}$ and $x_{2d}$ is negligible; i.e., that both are located at the same distance, $x_2$ [km], from the main signal (referred to as the pre-signal location). This assumption is relaxed in the simulation tests to obtain a more realistic estimate of the potential impacts. For simplicity of exposition, we also assume in the body of the paper that buses do not arrive within the same, or consecutive cycles. This implies that in general the proposed strategy would be applicable when bus headways are greater than 2–3 min. This is a realistic assumption since smaller bus headways would only be expected on very busy urban areas, where intersections most likely would have more than one approach lane. A discussion on extending this strategy to multiple bus arrivals within a cycle is provided in Section 6.

[1] The constraint that the two pre-signals must turn red at the same time can be relaxed to reduce delays imparted onto cars and/or to reduce the extent of the queue in the travel direction of the bus. However, we keep this constraint here as this provides a worst-case analysis of the proposed pre-signal strategy.

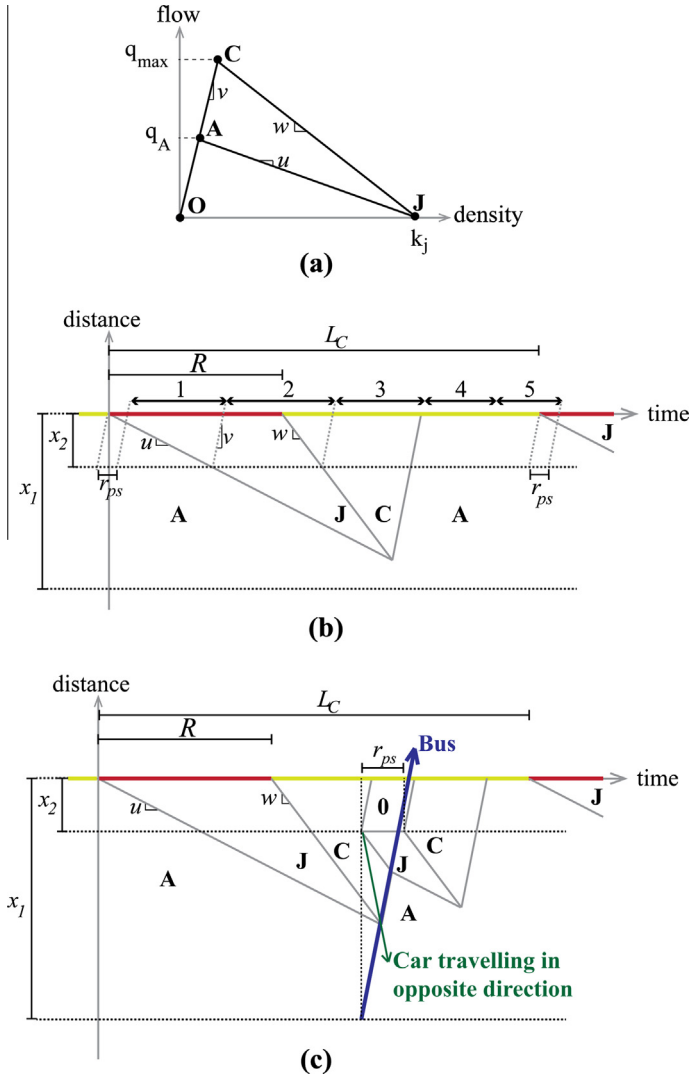


**Fig. 2.** (a) Sample fundamental diagram, and (b) time–space diagram of the intersection approach in the direction of bus travel for an under-saturated intersection.

The pre-signal location would have significant impacts on both bus delay savings at the intersection and car traffic operations. Pre-signals placed closer to the intersection would allow buses to skip over longer queues, providing them with more priority. However, they would also result in a longer red duration at the pre-signal and increased car delays. In addition, they could increase the probability of queue spill-overs in the opposite direction, resulting in significant capacity loss if vehicles queued at the pre-signal block the main intersection. The increased car delays and the probability of a queue spill-over can be reduced by placing the pre-signals further away from the intersection. This, however, comes at the expense of decreased bus delay savings. Therefore, trade-offs exist between the amount of priority afforded to the bus and the negative impacts imparted onto cars. The goal of this work is to systematically study these trade-offs and identify the domains under which such a priority strategy would be beneficial at isolated single-lane intersection approaches.

## 3. Operation and theoretical analysis of car delays at under-saturated intersections

When the intersection signal is under-saturated and the pre-signal is not in operation, cars and buses approaching the intersection in the subject direction stop at most once before proceeding through the intersection; see the sample time–space diagram provided in Fig. 2b. Furthermore, buses would approach the intersection traveling at free flow speed, $v_b$ [km/h], before encountering the car queue at the signal, see Fig. 2c. The goal of the pre-signal in this case would be to allow an approaching bus to skip all or a portion of the queue, defined by the triangular regions labeled J in Fig. 2c. Notice that when the pre-signal is triggered, a secondary queue will also grow at the pre-signal.

Using this time–space diagram, a minimum distance between the detection location and the pre-signal location ($x_1 - x_2$) can be calculated to ensure that the bus is unimpeded by cars both when traveling on its own lane and on the bi-directional lane segment. This minimum distance is determined such that the bi-directional lane segment ahead of the bus clears at the exact moment that the bus reaches the back of the queue at its maximum length (i.e., such that the opposite direction travel lane is cleared only when and where the bus would be required to leave its own lane); see the time–space diagram provided in Fig. 2c. The maximum length of the queue is chosen to calculate the minimum distance between the detection location and the pre-signal location since this represents the earliest time a bus would be required to leave its own lane. This ensures that for any other (shorter) queue length the bus could encounter, the opposite travel lane would be clear of cars before the bus would be required to leave its own lane. While a secondary queue could also grow when a pre-signal turns red, this is not a concern since opposite direction cars always travel faster than the back of the queue. The distance $x_1$ can be expressed as this minimum required distance, plus a margin of safety, $\delta$, to account for variability in travel speeds:

$$x_1 = \frac{q_{max} \times R}{q_{max} - q_A} \times \frac{(v + v_b) \times v_b}{v} \times \frac{u}{v_b + u} - x_2 \times \frac{v_b}{v} + \delta. \tag{1}$$

The duration of the red phase at the pre-signal, $r_{ps}$, would need to be at least as long as the time it takes the bus to travel unimpeded between $x_1$ and $x_2$. A margin of safety, $\epsilon$, can be added to account for the variability in travel time, the time the bi-directional segment requires to clear, and the time required for the bus to merge back onto its original travel lane. Thus, this red time is:

$$r_{ps} = \frac{x_1 - x_2}{v_b} + \epsilon. \tag{2}$$

Note that (2) implies that moving $x_1$ further away from $x_2$ would require the pre-signal to stop car traffic for a longer duration. However, doing so provides no additional benefit to the bus since it would travel at free-flow speed while upstream of the queue formed at $x_2$ regardless of the location of $x_1$. Since longer red durations would be associated with larger delays imparted onto cars, the optimal location of $x_1$ for a chosen location $x_2$ would be given by (1). We will see that this optimal location no longer holds when the main signal is over-saturated in Section 4. Notice that the combination of (1) and (2) implies that the duration of the red phase at the pre-signal decreases as $x_2$ increases if $x_1$ is always chosen in this optimal way.

To facilitate the discussion of bus and car delays when the pre-signal is in operation, we divide a signal cycle into five cases based on how the pre-signal would impact operations at the main signal in the subject direction.[2] For illustration, the origin of the time axis is set at the start of the red phase for the cycle in which the bus would arrive to the intersection if the pre-signal was not in operation. The cases are defined based on the virtual arrival time of a bus to the main signal, $t_m$. As illustrated in Fig. 2b, the five cases are as follows:

- The first case represents the time period when the pre-signal would provide priority to buses that would otherwise queue between the main signal and the pre-signal locations. The bounds for this case are:

$$r_{ps} < t_m < x_2 \frac{k_j}{q_A}$$

- The second case represents the time period when the queue in the absence of the pre-signal would have engulfed the pre-signal location, $x_2$. It starts immediately after the conclusion of case 1 and concludes when the last car queued between the pre-signal and main signal discharges through the main signal. The bounds for this case are:

$$x_2 \frac{k_j}{q_A} < t_m < R + x_2 \frac{k_j}{q_{max}}$$

- The third case represents the time period for which the pre-signal red duration would fall partially or entirely within the queue discharge period at the pre-signal. This would imply some loss of discharge capacity at the main signal. The bounds for this case are:

$$R + x_2 \frac{k_j}{q_{max}} < t_m < \frac{q_{max} R}{q_{max} - q_A}$$

- The fourth case represents the time period for which the pre-signal red duration would fall partially or entirely within the free flow period at the intersection, but vehicles stopped at the pre-signal would still be able to discharge within the same cycle. This would imply some capacity loss at the main signal. The bounds for this case are:

$$\frac{q_{max} R}{q_{max} - q_A} < t_m < C - \frac{q_{max}(t_{add} + r_{ps})}{q_{max} - q_A} + r_{ps}$$

[2] It is not crucial to differentiate between the bus and car speeds in the theoretical analysis. Therefore, for the remainder of the section and the next, it will be assumed that buses and cars travel at the same free-flow speed, $v$ km/h. This assumption is especially reasonable for urban environments with speed limits in which buses and cars travel at the same speed.

- The fifth case represents the time period for which the pre-signal red duration would fall partially or entirely within the free flow period at the intersection, but some of the vehicles stopped at the pre-signal would not be able to discharge within the same cycle. This would imply large vehicle delays as some vehicles would have to wait for the next green period to discharge. The bounds for this case are:

$$C - \frac{q_{max}(t_{add} + r_{ps})}{q_{max} - q_A} + r_{ps} < t_m < C + r_{ps}$$

In practice, the red phase at the pre-signal could be triggered whenever a bus reaches the detection location. This operation would only require the detection of the bus; no coordination between the pre-signal and the main signal, or knowledge of the queue length would be needed. However, operating the pre-signal in this naive way will impart delays onto cars with no apparent benefit to some buses. For example, buses with virtual arrival times in cases 4 or 5 could travel through the intersection unimpeded by car queues even if the pre-signal was not in operation. Initiating the pre-signals to provide 'priority' to these buses would not provide them with any travel time savings, but doing so could impart non-trivial delays to cars in both travel directions. Similarly, buses with virtual arrival times in case 1 would still have to queue at the main signal's red period. Initiating the pre-signal during this time would merely provide the bus with a small travel time saving as it is able to gain a slightly more advantageous position in the queue; however, the majority of its delay at the intersection would still occur while queued during the red main signal.

In light of this, three different strategies are considered and compared for the pre-signal operation. The first is the *naive strategy* in which the pre-signal is triggered for all bus arrivals. As discussed, this strategy is expected to have the most detrimental impacts to car traffic. However, this strategy will serve as a baseline as it realistically represents situations in which operation of the pre-signal is not coordinated with the main intersection signal and sufficient detection is not available to identify the length of the car queue. The remaining strategies represent cases in which the pre-signal is coordinated with the main intersection signal and sufficient detection exists. The most extreme case is when the pre-signal is triggered only when it is expected to have the least detrimental impact to car traffic. We refer to operating the pre-signal in this way as the *targeted strategy*. In this case, the bus would trigger a red pre-signal only when there is a large queue that the bus can skip, i.e., starting when the queue reaches $x_2$, and ending when the queue downstream of $x_2$ has dissipated (case 2). This strategy minimizes the negative effects that the pre-signal would have on cars because they would not have to stop multiple times, and it provides priority to buses when the queue lengths are the longest. The final strategy, which we call the *semi-targeted strategy*, tries to find a balance between the two extreme cases presented above. In this case, the bus would trigger the red pre-signal beginning when the queue reaches $x_2$ until the entire queue dissipates (cases 2 and 3). With this strategy, buses could get priority even when the car queues are very small; however, the car flow would be affected for a longer portion of the cycle.

This under-saturated analysis assumes that the impacts of the pre-signal are fully absorbed within at most two cycles. Conservatively, this requires that the following inequality must hold for the arrival demand:

$$q_A < \frac{q_{max} \times (L_C - R)}{L_C + r_{ps}} \tag{3}$$

For most realistic cases, this inequality implies that the analysis described in this section holds for volume to capacity ratios ($V/C^*$) of up to $\sim 0.85$, where $V$ is the demand rate, and $C^*$ is the capacity of the main signal. For $V/C^*$ ratios closer to 1, the intersection signal would become over-saturated when the pre-signal is applied, dramatically increasing the car delays. Therefore, pre-signals are not expected to improve total person delay for $0.85 < V/C^* < 1$. The proposed strategy can still be used to provide bus priority for these demand levels. However, the capacity metric might be more meaningful in this case.

The analysis of car and bus delays using queuing theory for the three operating strategies are discussed in the three following sections.

### 3.1. Naive strategy

Queuing diagrams made up of cumulative curves (Daganzo, 1997) were used to theoretically determine the change in car and bus delays that would be experienced with the pre-signal. Three separate car streams were considered to determine the aggregate impact on cars: (1) those traveling in the same direction as the bus; (2) those traveling straight through the intersection in the opposite direction; and, (3) those turning from the cross-street and traveling in the opposite direction.

Fig. 3 presents illustrative queueing diagrams for determining the additional delays imparted onto cars traveling in the same direction as the bus for a virtual bus arrival time to the main signal of $t_m$. In this figure, the virtual arrival of vehicles to the main signal (i.e., the time they would have arrived to the main signal if there was no queueing) in the subject direction is shown with the thickest line. The departure of vehicles from the main signal if the pre-signal did not exist is shown as the lightest line, and the cumulative departure curve when the bus receives priority at the pre-signal is shown as the medium thickness line. The number of cars that can be stored between the pre-signal and the main signal is denoted as $N_{x_2}$. These

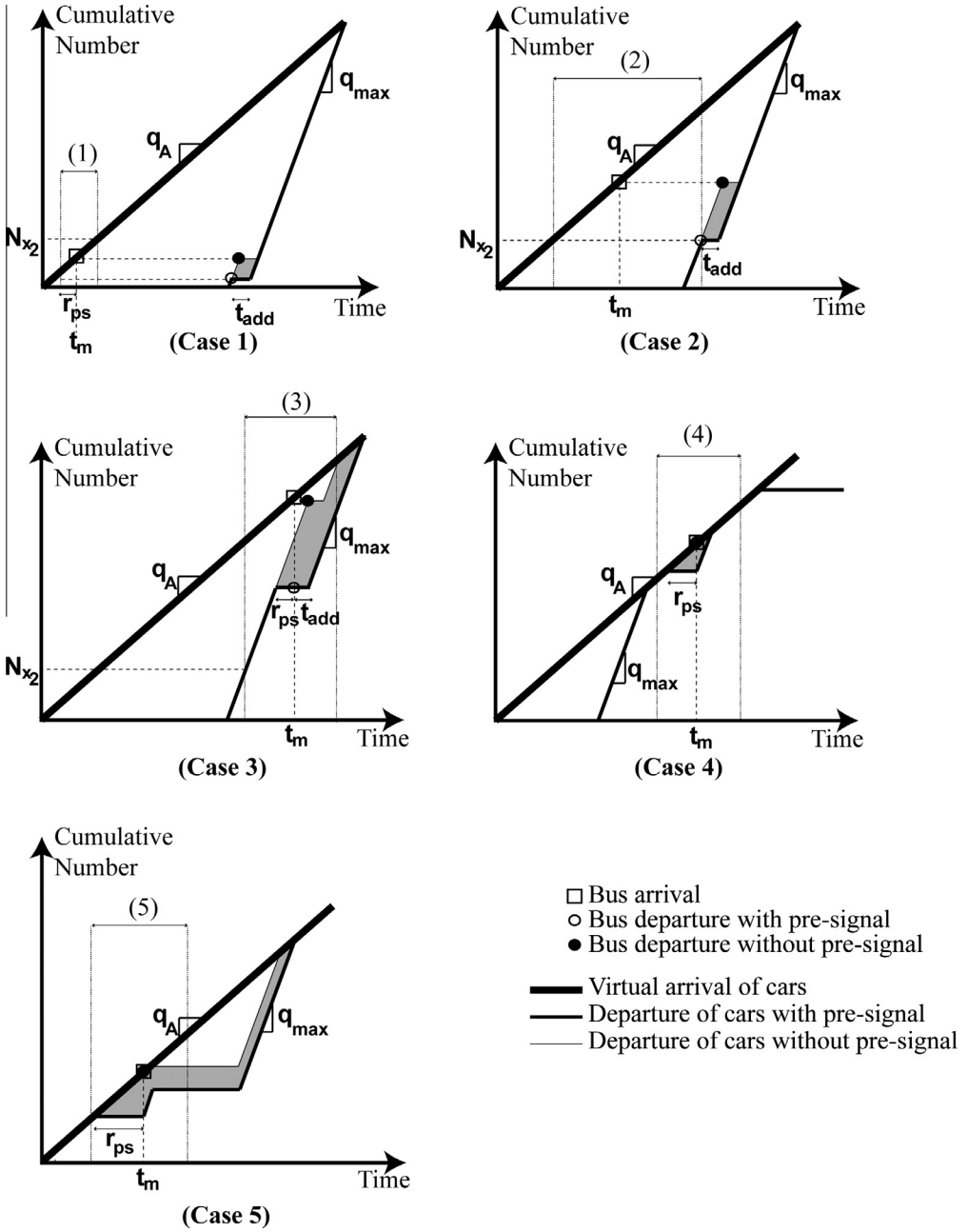


**Fig. 3.** Sample queueing diagrams at the main signal for cars traveling in the subject direction for different bus arrival times.

queuing diagrams also account for the fact that a bus takes some time, $t_{add}$, longer than a car to discharge from a queue. The total additional delay experienced by the cars due to the pre-signal is shown as the shaded area.

In cases 1 and 2, cars are only delayed due to the reordering of vehicles discharging through the intersection; i.e., the cars that would have discharged ahead of the bus without the pre-signal become delayed by an amount $t_{add}$ when the bus is allowed to skip ahead of them. Cases 3 through 5 are different, however, because cars also experience some delay due to the red pre-signal phase during what would be their queue discharge period (case 3) or free flow period (cases 4 and 5) without a pre-signal. In case 3, cars suffer a delay which is equal to the passage time of the bus, plus the additional duration of red experienced at the pre-signal. In case 4, cars experience a delay due to the interruption of their arrival and discharge by the red pre-signal. Case 5 is even more detrimental because some vehicles would have to wait an additional cycle for the next green period before they can discharge. Notice that the delays for the different cases cannot always be described with a single equation since transition periods between cases also exist. The complete delay equations that account for these transition periods are furnished in Appendix A.

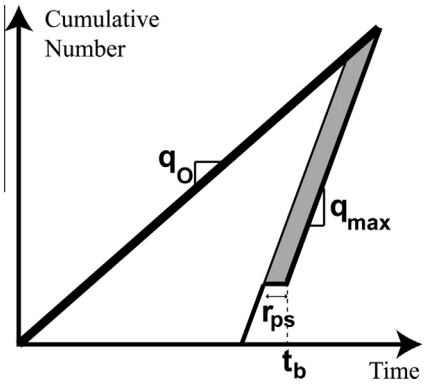


**Fig. 4.** Sample queueing diagram at the main signal for cars traveling in the opposite direction.

Cars traveling in the opposite direction are assumed to have a demand rate of $q_O$ veh/h.[3] Not all of the previous cases apply to these opposite-direction vehicles since they are only affected if the pre-signal turns red during the main signal's green phase. If the pre-signal turns red immediately after the green main signal, all cars discharging from the queue at the main signal experience the same delay (less than or equal to the duration of the red at the pre-signal) since the arrival and discharge rates are the same, and equal to $q_{max}$ (see Fig. 4). However, some cars that would have otherwise discharged at free flow from the main signal also get delayed at the pre-signal and the queue clears later. If the red pre-signal were triggered after the queue at the main signal clears (similar to cases 4 and 5 if $q_O = q_A$), the delay patterns of cars traveling in the opposite directions would be very similar to those observed in Fig. 3. However, the bounds on $t_m$ for the different cases would still be slightly different to account for the time it takes for cars traveling in the opposite direction to reach the pre-signal location.

It is also important to consider the effects of this strategy on cars turning from the cross-street. For this analysis, it is assumed that only a fraction of cars, $p$, turn right towards the pre-signal (assuming no right-turn on red, and no left turning maneuvers although the analysis is qualitatively the same). It is also assumed that the car queue formed at the pre-signal does not spill-back to the intersection. For realistic turning percentages ($p < 25\%$) it was found that this latter assumption would hold for $x_2$ values greater than 7 m.[4] Therefore the results are shown for $x_2 \geqslant 7$ m for the remainder of the paper.

The additional delays for cars turning from the cross-street are conservatively approximated to simplify the analysis. The approximation assumes that only two types of queuing exist: (1) queuing of cars discharging from the cross-street at the maximum rate, which arrive to the pre-signal location at a rate $q_{max}p$, and (2) queuing of cars discharging from the cross-street at their arrival rate, which arrive to the pre-signal location at a rate $q_C p$ (where $q_C$ [veh/h] is the demand rate from the cross-street). There are some queuing patterns that exist where the arrival rate to the pre-signal transitions between these two rates. However, these transition states are ignored due to the marginal benefit obtained from considering them. Instead, it is assumed that the maximum arrival rate holds for these transitional cases, which provides a worst case analysis when considering car delays.

All of the delay components described above are furnished in detail in Appendix A. The expected total additional car delays and expected bus delay savings per cycle can be found by integrating these equations over their respective bounds. To determine the total impact of the pre-signal on the under-saturated intersection, we combine the delay savings to buses and the delays imparted onto cars, weighting each by the average person-occupancy of each vehicle. This provides the total change in person-hours of delay due to the operation of the pre-signal. If this value is less than zero, the strategy not only provides benefits to buses but also to the system as a whole by reducing the total travel time of all people traveling through the intersection.

The bus to car occupancy ratios (which can be thought of as the bus occupancy if car occupancy is $\sim 1$) for which the pre-signal strategy would result in total person delay savings are provided in Fig. 5a as a function of the volume to capacity ratio ($V/C^*$) and $x_2$. As can be seen in this figure, for very low and very high $V/C^*$ more passengers are needed in a bus to obtain total person delay savings. For low $V/C^*$ ratios, car queues are small so buses benefit very little from the pre-signal strategy. For high $V/C^*$ ratios, the pre-signal negatively affects more cars, which increases car delays significantly. Therefore, the pre-signal is best suited for the average of the $V/C^*$ ratios shown. The same basic trade-off exists for the location of the pre-signal. If the pre-signal is placed very close to the main signal, car operations are much worse off so higher bus occupancies are required. However, if the pre-signal is placed too far from the main signal then the bus delay savings diminish.

[3] We assume that the main signal would be under-saturated in the opposite direction as well.

[4] Notice that if this minimum $x_2$ value were not met only the delay calculations for the turning vehicles would require change. The turning vehicles would spillover to the cross street and block cars which desire to travel straight through the intersection. However, for the opposite direction, this minimum value is not required since all vehicles traveling in the opposite direction are included in the delay calculations. In this case, the location of $x_2$ changes only how the queue splits between the upstream and downstream approaches of the intersection.

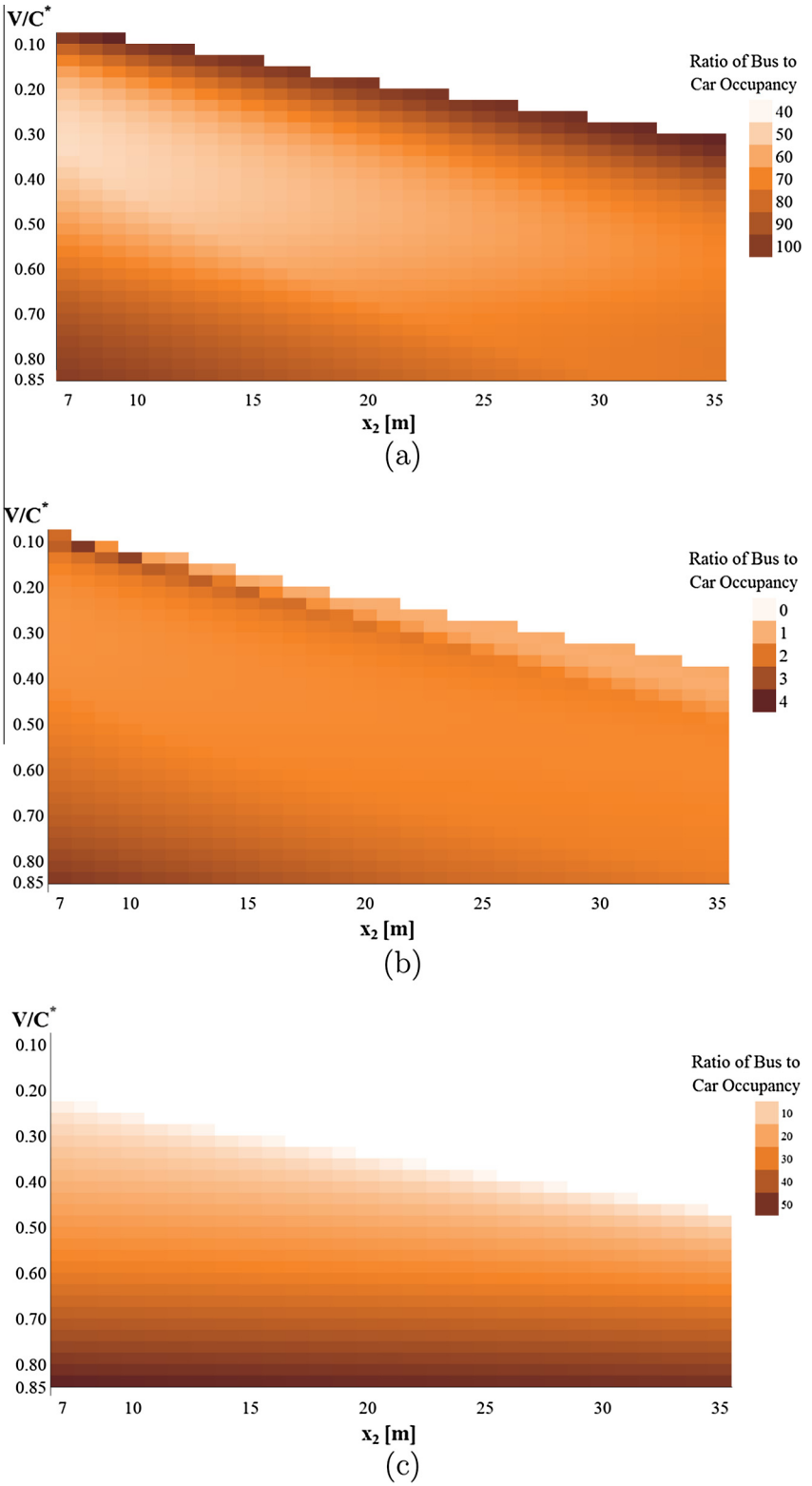


**Fig. 5.** Break-off bus to car occupancy ratios as a function of $V/C^*$ and $x_2$ for: (a) the naive strategy, (b) the targeted strategy, and (c) the semi-targeted strategy as compared to the targeted strategy [$\epsilon = 5$ s, $\delta = 20$ m, $q_{max} = 1800$ veh/h, $k_j = 150$ veh/km, $v = 50$ km/h, $R = 48$ s, $L_C = 90$ s, $p = 0.2$, $q_O = q_C = q_A$].

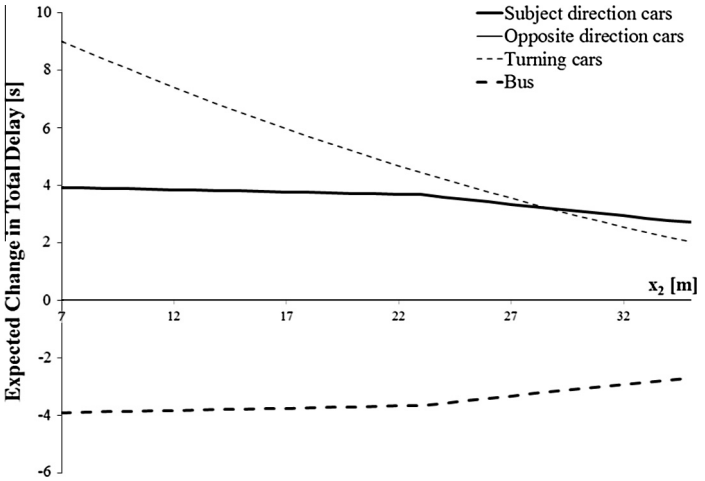


**Fig. 6.** Break-down of change in expected total delay with the targeted strategy for subject direction cars, opposite direction cars, cross-street cars, and buses as a function of $x_2$ [$\epsilon = 5$ s, $\delta = 20$ m, $q_{max} = 1800$ veh/h, $k_j = 150$ veh/km, $v = 50$ km/h, $R = 48$ s, $L_C = 90$ s, $p = 0.2$, $q_O = q_C = q_A$, $V/C^* = 0.84$].

A sensitivity analysis on the two margin of safety parameters—$\delta$ (the margin of safety on $x_1$) and $\epsilon$ (the margin of safety on $r_{ps}$)—is conducted to better understand how sensitivity the car delays are to these values. The changes in delay for $q_a = 700$ veh/h are calculated by individually doubling these values. The results show that car delays are more sensitive to $\epsilon$ than $\delta$. This is true since the red time margin of safety used in the results shown in Fig. 5 was already 45% of the theoretically required value, therefore doubling this value creates a large change in the overall red pre-signal time. Specifically, the additional delay for cars turning from the cross street is the most sensitive to the values of the margin of safety. The delays that the cross street cars experience are much smaller due to the low number of turning vehicles and hence any change in these delays is observed in large percentages. Overall, the total car delay increases by about 20% for the naive strategy (and 10% for the semi-targeted strategy) when $\epsilon$, the red pre-signal buffer time, is doubled and remains relatively constant (for both strategies) when $\delta$, the margin of safety for $x_1$, is doubled. By contrast, bus delays not sensitive to the changes in either $\delta$ or $\epsilon$. This is true since the bus delay savings mostly depend on the length of the skipped queue, which is independent of the value of $x_1$ or $r_{ps}$.

Overall, the pre-signal operated using the naive strategy can provide a net benefit to the system if the ratio of bus to car occupancy is about 50. This level of bus demand would generally only be expected on a few bus lines in a major city, although these values are observed regularly in developing countries. Such high values are required using the naive strategy because the pre-signal operates even when no bus delay savings would be provided. The targeted strategy described next can reduce these unnecessary car delays by operating the pre-signal only when it is expected to have the least detrimental impact to car traffic.

### 3.2. Targeted strategy

The targeted strategy allows for the pre-signal to operate only when there is a queue at the pre-signal location, i.e., only during case 2. By operating the pre-signal in this manner no cars would need to stop twice due to the arrival of a bus. Hence, additional car delays and emissions could be significantly lowered when compared to the naive strategy. While the targeted strategy reduces the total amount of priority provided to buses, buses still receive priority when it is most needed, i.e., when the car queues are the longest. This implies that fewer number of buses would receive priority, however the buses which do receive priority would have large average delay savings.

The bus to car occupancy ratios for which the targeted pre-signal strategy would result in total person delay savings can be seen in Fig. 5b as a function of the volume to capacity ratio ($V/C^*$) and $x_2$. Comparing this figure to Fig. 5a it can be observed that the range of bus to car occupancy ratios for which pre-signals would improve the system become much wider when the targeted strategy is used, as expected. In fact, the targeted pre-signal strategy could improve the overall person delays for bus to car occupancy ratios as low as 2. However, the expected delay savings to all buses are between 50% and 70% lower with the targeted strategy as compared to the naive strategy due to fewer buses receiving priority.

To gain a better understanding of the targeted pre-signal operation, the magnitudes of the expected change in the different delay components (cars traveling in the subject direction, cars traveling in the opposite direction, cars turning from the cross-street, and buses) when a pre-signal is used are plotted in Fig. 6 as a function of the pre-signal location $x_2$. As can be seen, turning cars have the largest delay component since their discharge from the main signal coincides with the red phase of the pre-signal. However, if the pre-signal is placed further away from the main signal this delay component rapidly drops since the pre-signal is operated for a shorter portion of the cycle and the red pre-signal is shorter. The expected delays for cars traveling in the subject direction, however, remain relatively constant as $x_2$ changes since the delay is only experienced

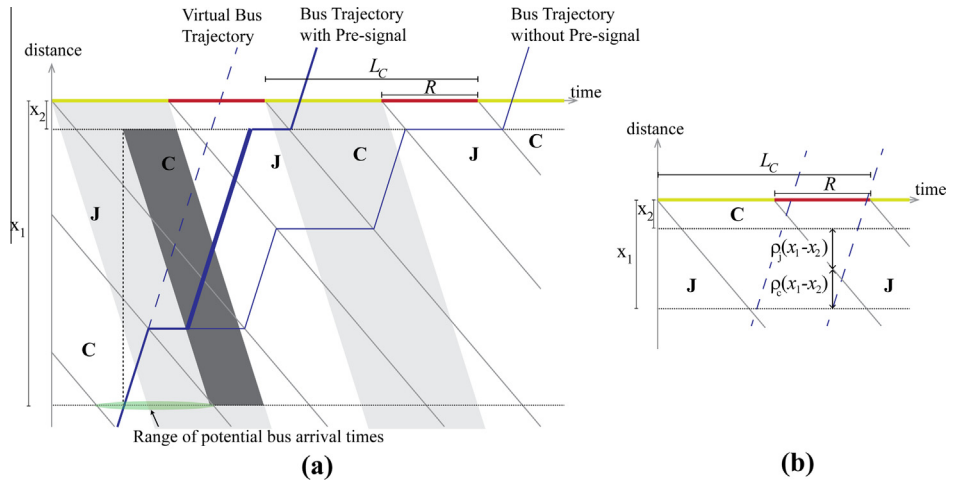


**Fig. 7.** Sample time–space diagrams at an over-saturated intersection.

due to the reordering of approximately the same number of cars. The delays experienced by the cars traveling in the opposite direction are negligible in this figure since the red pre-signal phase overlaps with the green main signal phase only briefly. The bus delay savings also remain similar with increasing $x_2$, since these savings are a direct result of the reordering of cars. The savings shown are the expected values over an entire cycle. Hence, even though the values appear small, some buses can have large delay savings with the use of a pre-signal. Notice too, that these savings have not been weighted by the occupancy of buses, so total person-delay savings should be expected to be much higher.

However, if even more bus delay savings are desired, the semi-targeted strategy (explained next) can be utilized. The semi-targeted strategy tries to find a balance between the naive and targeted strategies by providing bus priority only when the buses would have otherwise experienced delays.

### *3.3. Semi-targeted strategy*

The semi-targeted strategy allows for the pre-signal to operate only during cases 2 and 3 described above. With this operation, the pre-signal only turns red for cars when bus delay savings could be obtained and eliminates all unnecessary additional car delays. With the semi-targeted strategy, the car delays can be reduced between 48% and 61% as compared to the naive strategy. This reduction in car delays corresponds to a relatively small loss in bus delay savings (between 0% and 16%).

The semi-targeted strategy was found to decrease the total person delays if there were more than 20 times more passengers in a bus than in a car (for the same range of $x_2$ and $V/C^*$ values as considered for the targeted strategy). When compared to the targeted strategy, the car delays are higher with the semi-targeted strategy; however, for high bus occupancy levels the semi-targeted strategy could reduce the total person delays more than the targeted strategy. Fig. 5c shows the ratio of bus to car occupancy for which the semi-targeted and targeted strategies would result in the same total person delay. For occupancy ratios above those shown in Fig. 5c, the semi-targeted strategy would result in an even lower total person delay as compared to the targeted strategy. As can be seen from this figure, if there are more than 30–40 times more passengers in a bus than a car (and less than those shown in Fig. 5a), then using the semi-targeted strategy would result in the lowest total person delays as compared to the two other strategies. For lower bus occupancies the targeted strategy is recommended.

## 4. Operation and theoretical analysis of car delays at over-saturated intersections

When the main signal is over-saturated, approaching cars and buses may need to stop multiple times before they are able to discharge through the intersection; see the time–space diagram in Fig. 7a. Queued cars, J, and cars discharging at capacity, C, along with the band of cars traveling in the opposite direction (light gray shaded areas) are shown in this figure. The virtual trajectory of a bus is shown with the dashed line. The trajectory of the same bus without a pre-signal is shown with the thin, solid line. The thick line shows how the bus trajectory would change if a pre-signal were used, and the bolder portion of this line highlights the distance that the bus covers traveling on the bi-directional lane. The dark gray shaded area shows the effect of the pre-signal's red phase on cars traveling in the opposite direction (i.e., capacity loss in the opposite direction). This indicates that an arriving bus may not be able to travel between the detection location and the pre-signal at the maximum speed $v$ as it may encounter car queues before the bi-directional lane segment has time to clear. In these cases, the bus would have to join the car queue and wait until the bi-directional lane segment clears before proceeding to the pre-signal

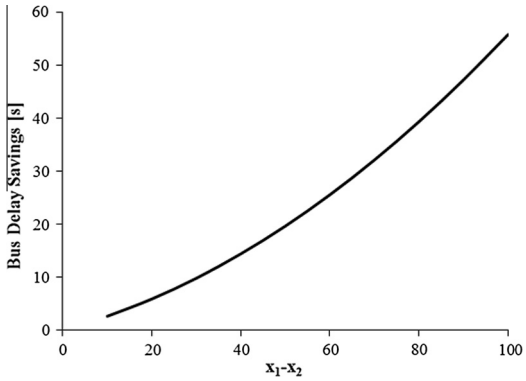


**Fig. 8.** Sample bus delay savings as a function of the distance between the detection location and the pre-signal [$q_{max} = 1800$ veh/h, $k_j = 150$ veh/km, $v = 50$ km/h, $R/L_C = 0.5$].

location unimpeded by car traffic. The red phase at the pre-signal in this case, $r^O_{ps}$, would need to be as long as the sum of the travel times through the bi-directional lane segment of the last car passing the pre-signal in the opposite direction before it turns red and the bus. Again, a safety factor can be added as well such that:

$$r^O_{ps} = \frac{x_1 - x_2}{v} + \frac{x_1 - x_2}{v_b} + \epsilon. \tag{4}$$

The bus delay savings in the over-saturated case are fairly straightforward to determine. The pre-signal would allow the bus to move ahead of the vehicles located between the detection point and the pre-signal location at the time the bus is detected. Therefore, the bus delay savings are simply equal to the time these vehicles would have required to discharge through the intersection. Two sample virtual bus trajectories are illustrated in Fig. 7b. Notice that they differ mainly in their arrival time to the detection location $x_1$. For the first trajectory, there are $(x_1 - x_2) \times k_c$ vehicles between the bus and the pre-signal upon detection. Therefore, the bus will save about $(x_1 - x_2) \times k_c / q_{c^*}$ of travel time on average (where $k_c$ is the density corresponding to $q_{max}$, and $q_{c^*}$ is the capacity of the intersection). For the second trajectory, there are $\rho_c \times (x_1 - x_2) \times k_c + \rho_j \times (x_1 - x_2) \times k_j$ vehicles between the bus and the pre-signal, where $\rho_c + \rho_j = 1$. In this case, the delay savings are $(\rho_c \times (x_1 - x_2) \times k_c + \rho_j \times (x_1 - x_2) \times k_j)/q_{c^*}$ on average.

The total number of cars skipped, and the corresponding delay savings for general cases can be calculated with a simple algorithm based on the bus arrival time to the detection location, $t_d$ (measured from the start of the time the wave separating states J and C arrives to $x_1$). To determine the total number of cars ahead of the bus between $x_1$ and $x_2$, the length of the capacity and jam bands between the detection location and the pre-signal at $t_d$ need to be determined. The length of the band of cars that are traveling at capacity, $\ell_c$, and the length of the queue of stopped cars, $\ell_j$, at the moment of bus detection can be iteratively determined as follows:

$$\begin{aligned} \ell_{c,i} &= \min(x_1 - x_2, w \times (L_C - R - t_d)) && \text{for } i = 1 \\ &= \min(x_1 - x_2 - (\ell_{c,i-1} + \ell_{j,i-1}), w \times (L_C - R)) + \ell_{c,i-1} && \text{for } i > 1 \\ \ell_{j,i} &= \min(x_1 - x_2 - \ell_{c,1}, w \times R) && \text{for } i = 1 \\ &= \min(x_1 - x_2 - (\ell_{c,i} + \ell_{j,i-1}), w \times R) + \ell_{j,i-1} && \text{for } i > 1, \end{aligned} \tag{5}$$

where the loop terminates when $\ell_c$ and $\ell_j$ do not change between iterations.

The expected delay savings of a bus, $d^O_b$, with arrival time $t_d$ to the detection location can then be expressed as:

$$d^O_b = \frac{\ell_c \times k_c + \ell_j \times k_j}{q_{c^*}} \tag{6}$$

For most cases, the average density of the cars between the bus and the pre-signal at the time of bus detection will be closer to $k_c$ than $k_j$; however, as $x_1$ is placed further away from $x_2$ this average density will increase. Therefore, increasing the distance between $x_2$ and $x_1$ is expected to increase the delay saving of buses exponentially. An illustrative example can be seen in Fig. 8 for some realistic values.

The red time required at the pre-signal to facilitate this bus priority will impart delays onto the car traffic in one of two ways. The cars skipped by a bus will each experience an additional delay equal to the discharge time of the bus, $t_{add}$. Furthermore, discharge through the intersection might be interrupted and this will impart some delay upon each car in the queue upstream. If the demand pattern is unknown (i.e., the length of time the intersection is over-saturated is unknown), this latter delay cannot be quantified.[5]

[5] The former delay would be relatively minor when compared to the delay cars experience at an over-saturated intersection.

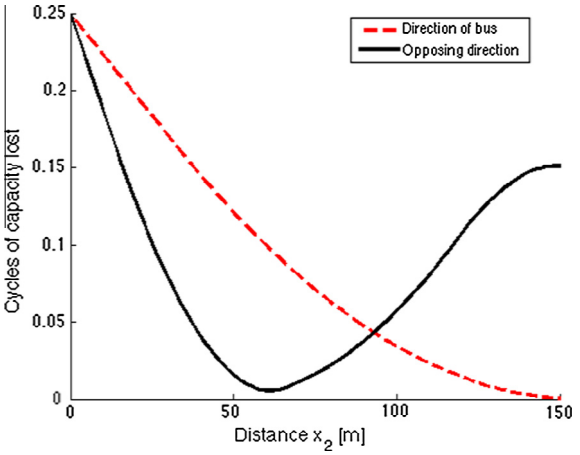


**Fig. 9.** Expected capacity loss at an over-saturated intersection associated with the pre-signal priority strategy for a specific case. [$q_{max} = 1800$ veh/h, $k_j = 150$ veh/km, $v = 50$ km/h, $r_{ps} = 24$ s, $R = 48$ s, $L_C = 90$ s].

Instead, we quantify the negative impacts to cars as the amount of discharge capacity lost at the intersection due to the red pre-signal phase. Variational theory (Daganzo, 2005a,b; Daganzo and Menendez, 2005) is used to calculate this capacity loss. By applying variational theory, the kinematic wave problem is reduced to a simpler shortest path problem (Daganzo and Geroliminis, 2008; Leclercq and Geroliminis, 2013). For our analysis, the zero-cost shortcut created by the red phase of the pre-signal occurs at the time the bus arrives to the detector location and persists for a period of $r_{ps}^{O}$. These arrival times, $t_d$, would be uniformly distributed between the time when the wave separating states J and C arrives to $x_1$ and the time when the wave separating states C and J arrives to $x_1$; see Fig. 7a. Note also that the start of the red pre-signal is not coordinated with the movement of cars in the opposite direction and is entirely based on the location of $x_1$. The red periods of the main signal would also need to be included as shortcuts with zero cost in the shortest path problem. The capacity is then obtained by finding the shortest path starting and ending at the main signal location, and covering a time period that includes the cycle during which the pre-signal was activated. More details on this methodology, as well as analytical formulas that can be used to calculate the capacity loss due to this type of obstruction are provided in Gayah et al. (2014).

Fig. 9 presents the expected capacity reduction at an over-saturated intersection as a function of the pre-signal location while holding the bi-directional lane segment length $x_1 - x_2$ constant at 100 m. Notice that the impacts are non-linear. The capacity loss to the cars traveling in the direction of the bus decreases with the distance between the pre-signal and the intersection; this is a general result due to the coordination of the pre-signal red start time with the backward moving waves emanating from the main signal. However, the impact to the opposite direction is non-monotonic. The reason for this is that the pre-signal activation is coordinated with the direction of bus travel but not with the opposite direction. Overall, the capacity losses are relatively minor: less than 0.25 cycles worth of vehicle discharge are expected to be lost when the pre-signal is located right at the intersection for this particular, but realistic, case. As the pre-signal is moved further back, this amount quickly decreases until negative impacts to cars can be expected to be minimal. This implies that an optimal distance for $x_2$ could be determined if the values of bus delay savings and capacity losses to society were known.

## 5. Simulation results

A related strategy is currently in operation at a signalized intersection in Rapperswil, Switzerland. This location was observed over the course of several days during peak traffic periods to quantify the benefits and drawbacks of the pre-signal on single-lane approaches under more realistic conditions. Unfortunately, car demands at this location were extremely low, even during the peak demand period, such that car queues at the main signal were generally just 1 or 2 vehicles long. Therefore, cars and buses typically only experienced minor delays. As a result, the pre-signal operation was observed only once. In that instance, the bus was able to skip a car queue of approximately 400 m.

To overcome the lack of empirical data, a micro-simulation was created in the AIMSUN software. This simulation mimicked the pre-signal operation described here to verify that this strategy can be used to provide benefits to buses, and to confirm the magnitude of the disbenefits to cars. The simulation included some more realistic features of the pre-signal strategy that would be required for implementation in the field. For example, we relaxed the previous assumption that the distance between the upstream and downstream pre-signal locations ($x_{2u} - x_{2d}$) was insignificant. Instead, the length of this segment was set equal to just over the length of a typical bus (about 12 m) and always kept clear of queued cars in the subject direction so that the bus would have enough room to merge back into traffic after traveling through the bi-directional lane seg-

ment. Bus detection was assumed to be available at both the detection point ($x_1$) and the merge point ($x_{2d}$). The former was used to trigger the red period at the pre-signal while the latter was used to reinstate the green period once the bus had cleared the bi-directional lane segment and merged back onto its original lane. Furthermore, a fictitious signal was placed at the beginning of the bi-directional lane segment that affected only the buses to ensure that these buses did not enter the bi-directional lane segment when a car in the opposite direction was still present. This was included to ensure that no collisions would occur despite the variability in vehicle travel times and acceleration behaviors. In reality, such a signal would not be needed as the bus driver would be able to negotiate this maneuver by simply looking downstream.

Two pre-signal locations considered in the simulation were: $x_{2d} = 15$ m and $x_{2d} = 30$ m. Simulations were performed both without the pre-signal treatment and with the pre-signal treatment for comparison. During these simulations, the main signal had a cycle length of 90 s and 48 s of red time in the subject direction. Simulations were 20 h long with bus headways long enough that the effects of each arriving bus did not overlap. Each simulation contained about 170 individual bus arrivals. The remainder of this section presents the results for under-saturated cases and over-saturated cases, respectively.

### 5.1. Under-saturated simulation results

For the under-saturated case, conservative assumptions were used to provide a worst-case analysis of the level of priority that could potentially be provided to buses, and the cost of providing this priority in terms of the delays imparted onto other vehicles. We implemented the naive operating scheme previously described in Section 3. Therefore, vehicles were stopped at the pre-signal even when the bus was not impeded by car queues at the main signal. Since maneuvers into and out of the bi-directional lane segment required additional time, this resulted in some buses actually experiencing negligible additional delays when the pre-signal strategy was in operation as compared to no pre-signal. Nevertheless, the results paint an optimistic picture for the ability of the pre-signal strategy to provide priority to buses.

For simplicity, car flows were assumed to be the same in the subject direction, opposite direction and on the cross-street. It was assumed that 20% of the vehicles on the cross-street turned on to the opposite lane. Table 1 presents the critical bus to car occupancy ratio that would be required such that the total person-hours of delay would be equal when the pre-signal was operational to when no pre-signal was implemented. These values are on the same order of magnitude as those provided in Fig. 5. In general, the bus to car occupancy ratios decrease with decreasing car flows and increase as the pre-signal is moved further away from the main signal, which again confirm the general trends illustrated in Fig. 5. The only exception appears to be for the car flow of 300 veh/h, which appears to be an exception caused by the stochastic nature of the micro-simulation.

The highest bus to car occupancy ratios shown in Table 1 are not generally expected to be observed in realistic situations, which would seem to be pessimistic for the pre-signal strategy. However, as previously mentioned, these results are for the naive operating strategy in which some buses receive "priority" that is not necessarily required. Table 1 also provides the fraction of individual buses that receive a delay saving of at least 5 s when the pre-signal is implemented. The trends confirm that the pre-signal strategy provides larger benefits to buses as car flows increase (since larger car queues are formed) and smaller benefits to buses as the pre-signal is moved further from the main signal (since the bus can jump less of the car queue). The theoretical cases described in Section 3 can be combined with real-time detection in the field to identify the buses that would benefit the most from priority. The pre-signal could then be implemented only for these cases to provide maximum benefits with minimal negative impacts to cars. As shown by the analysis in Sections 3.2 and 3.3, such targeted implementation could reduce the occupancy ratios for which the pre-signal strategy would provide system-wide benefits by a factor of four or more.

**Table 1**
Results of the simulation for an under-saturated intersection.

| Car flow (veh/h) | $V/C^*$ | Bus to car occupancy ratios | | Bus arrivals with large delay savings | |
|---|---|---|---|---|---|
| | | $x_{2d} = 15$ m | $x_{2d} = 30$ m | $x_{2d} = 15$ m (%) | $x_{2d} = 30$ m (%) |
| 300 | 0.36 | 13.7 | 10.7 | 8.7 | 2.3 |
| 400 | 0.48 | 16.0 | 21.2 | 10.4 | 4.6 |
| 500 | 0.60 | 40.3 | 42.8 | 19.1 | 9.2 |
| 600 | 0.72 | 45.6 | 77.7 | 32.4 | 19.7 |

**Table 2**
Results of the simulation for an over-saturated intersection.

| $x_1 - x_{2d}$ (m) | Bus delay savings (sec/bus) | | Capacity reduction (cycles lost) | | | |
|---|---|---|---|---|---|---|
| | | | Subject direction | | Opposite direction | |
| | $x_{2d} = 15$ m | $x_{2d} = 30$ m | $x_{2d} = 15$ m | $x_{2d} = 30$ m | $x_{2d} = 15$ m | $x_{2d} = 30$ m |
| 80 | 15.3 | 19.8 | 0.54 | 0.58 | 0.13 | 0.23 |
| 100 | 30.8 | 29.1 | 0.58 | 0.61 | 0.15 | 0.25 |

### 5.2. Over-saturated simulation results

For these simulations, both the subject and opposite directions were assumed to be over-saturated to capture the capacity loss in both directions. In addition, the bus delay savings were quantified by comparing average bus delays with and without the pre-signal. The simulations were run assuming multiple pre-signal and detection locations.

The capacity loss and bus delay savings are presented in Table 2. Notice that the magnitudes of the capacity loss are generally higher than those shown in Fig. 9 (which was created for the same signal settings). This is not surprising as the analytical method fails to account for the randomness in vehicle arrivals, driver behavior, and acceleration/deceleration patterns. Nevertheless, the differences are not that great as the results are all within the same general order of magnitude. The bus delay savings increase with the distance between the detection location and pre-signal ($x_1 - x_2$). Furthermore, the bus delay savings do not vary much with the pre-signal location when holding $x_1 - x_2$ constant. The capacity results show the opposite trend: the reduction in capacity in both the subject and opposite direction vary with $x_{2d}$ and are fairly independent of $x_1 - x_2$. The only counter-intuitive trend appears to be that slightly more discharge capacity is lost when the pre-signal is moved further from the intersection. The differences for the subject direction are relatively small and can be attributed to the stochastic nature of the simulation. The differences in the opposite direction are significant in that the capacity loss doubles when the pre-signal is moved further away. However, this difference is attributed to the non-monotonic nature of opposite direction capacity loss with respect to pre-signal location (as illustrated in Fig. 9).

The results are also fairly optimistic with regards to the implementation of pre-signals in practice. Significant bus delay savings can be achieved with the introduction of the pre-signal. The capacity loses are relatively minor (about one half of a cycle worth of vehicle discharge). This is not an insignificant amount, but if bus priority is valued highly then this price might not be too high considering there are few other priority options for buses on intersections with single-lane approaches. Unfortunately, the metrics for bus priority (delay savings) and negative impacts (capacity loss) in the over-saturated case cannot be directly combined to examine total system-wide impacts as in the under-saturated case. Ultimately, each agency will need to decide if the trade-off between car capacity and bus priority is worth it at a given location before implementing the proposed pre-signal strategy.

## 6. Conclusions and discussion

### 6.1. Conclusions

In this paper we proposed using pre-signals to provide priority to buses on intersections with single-lane approaches. The operating strategy creates a bi-directional lane segment on the opposite direction travel lane that buses can use to skip car queues on their original lanes upstream of the intersection. The general details of the strategy were described in Section 2. Three operating strategies for pre-signal were introduced for under-saturated intersections: (1) naive, (2) targeted, and (3) semi-targeted. Note that, the placement of the two pre-signals required for these three strategies is the same. The only difference is in how the pre-signal is timed and when it is activated. Therefore, the strategies can be chosen dynamically, or the pre-signal can be turned off completely. This level of flexibility makes the pre-signals attractive since priority can be provided only if and when necessary.

It was shown that the naive strategy only improves total passenger delay for very large bus occupancy levels (>50–60 times more passengers in a bus than a car) since it operates in a way that might impose delays to cars without any bus delay savings. However, total passenger delays can be further decreased by only implementing the pre-signal when a car queue engulfs the pre-signal location. This strategy could reduce total passenger delays when bus passengers occupancies are only twice that of car passenger occupancies; however, the number of buses which receive priority significantly decreases. As a compromise between these two extreme strategies, the semi-targeted strategy operates only when there is a queue at the main signal that the bus can jump. When compared to the targeted strategy, this strategy could provide the lowest total passenger delays when bus occupancy levels greater than 20–30 (and less than 50–60) times more that of cars.

The analytical results suggest that the pre-signal strategy will provide bus delay savings and/or improved overall total person delays under the following conditions at under-saturated intersections with single-lane approaches:

- $V/C^*$ is less than 0.85.
- The pre-signal is located more than 7 m away from the intersection.
- Low turning ratios from the cross-street are observed (less than 25%).

These bounds are subject to practical constraints that might be considered for field implementation. For example, pre-signals would need to be placed even further than 7 m away from the intersection in practice to keep car queues in the cross direction from spilling over to the intersection location to account for fluctuating demands.

The analytical examination of over-saturated cases suggests that average bus delay savings can be up to 30 s, while the capacity loss can be as much as 25% of a single cycle's worth of discharge. The bus delay savings increase as the length of the bi-directional lane segment increases (larger $x_1 - x_2$); therefore, networks with longer block lengths could benefit more from this strategy. The capacity loss to vehicles in the same direction as the bus decreases rapidly as the pre-signal is moved further away from the intersection. However, the capacity loss in the opposite direction is non-monotonic with respect to the

location of the pre-signal due to the lack of coordination between bus arrivals and the opposite car discharge phases. It is not possible to combine the effects of the bus delay savings and the capacity losses into a single value to determine when the overall system would benefit from this kind of strategy since the relative values of these two components could vary significantly between different agencies.

Micro-simulation tests were also performed under fairly conservative assumptions. The simulations confirm the general trends and patterns obtained from the analytical results. However, the simulations showed that bus delay savings were slightly less, and negative impacts to cars slightly more than what was expected from the theoretical analysis. For example, when the naive strategy was implemented at an under-saturated intersection, the occupancy ratio required to provide system-wide benefit increased to about 80 in one case. However, as the theoretical analysis shows, a less naive strategy can be implemented to reduce this value and make the strategy more viable. The capacity reduction in the over-saturated case also nearly doubled as about 60% of a single cycle's discharge was lost per bus. However, this is a mild loss when compared to removing a lane for car use on a two-lane approach and dedicating it for bus-use only, which reduces capacity by 50% indefinitely. Thus, overall the simulations paint a very optimistic picture about the potential benefits of a pre-signal strategy of this kind in practice.

### 6.2. Discussion

Our analysis finds that the pre-signal strategy can provide positive bus delay savings and reduce total passenger delays at signalized intersections with single-lane approaches. Unfortunately, the magnitude of average bus delay savings might be small when the intersection is undersaturated and this might cause the strategy to appear less beneficial during off-peak periods than peak periods. For example, when the pre-signal is always in operation, the average delay savings when the intersection is undersaturated is only 5–7 s per bus. However, focusing only on this low magnitude of average bus delay savings might mask some of the benefits that can be provided. For one, the average value is skewed by the buses that receive no delay savings. If these buses are ignored (or if the pre-signal is only operated when it will provide buses with larger delay savings), the average delay savings can be as much as 14 s per bus. Secondly, small bus delays savings can result in much larger savings in total passenger delay since buses generally carry more passengers than cars. In many urban areas, buses can carry over 50 people and providing these people with delay savings can have significant impacts in the total passenger delay of the entire transportation system. Thirdly, the higher average delay for the under-saturated intersections examined here is only about 20 s per vehicle. Therefore, the pre-signal strategy can reduce average bus delays by about 25–70% of what would be experienced in the absence of the pre-signal. This is larger than average bus delay savings offered by transit signal priority strategies, which generally ranges from 20% to 25% of the total bus delay (Smith et al., 2005).

Furthermore, these somewhat modest delay savings are experienced at just a single signalized intersection. Since most buses travel through many signalized intersections along urban routes, these delay savings can quickly add up if the proposed strategy is implemented at multiple locations along a route. Even if only implemented at few locations, the small delays savings experienced can also have larger impacts on bus reliability. Previous work has shown that very small disturbances can have result in buses bunching together since bus transit systems are inherently unstable (Daganzo, 2009). The pre-signal strategy can improve reliability by reducing the delays of buses which would have otherwise had the largest delays. This is particularly vital since transit users place a premium on the reliability of transit systems (Paine et al., 1967; Golob et al., 1972).

On the other hand, more dynamic strategies for operating the pre-signals could improve the bounds of application. For example, by relaxing the constraint that the two pre-signals need to turn red simultaneously, car delays and the length of car queues could be further reduced. However, this would require that additional information is known (e.g., length of the current car queue on the subject approach and the current phase of the main signal) at the time of the pre-signal implementation. With this additional information, the pre-signal timing could be optimized (i.e., the red signal initiation and duration at each of the two pre-signal locations) such that the total person delay would always be minimized.

Of course, this pre-signal strategy would require careful signing and regulations to ensure that buses are able to use the opposite lane in a safe and efficient manner. For example, a section of the travel lane would have to be demarcated to let drivers know that queues should not block the portion of roadway needed by buses to merge safely back into its lane. A careful and rigorous enforcement effort would most likely be needed to ensure that drivers heed these instructions when the strategy is first implemented. Promisingly, the existing implementation of such a pre-signal strategy in Switzerland suggests that such efforts can be successful in practice.

Our analysis focused only on cases in which a single bus arrives during a cycle. Thankfully, the strategy can be applied to situations in which multiple buses arrive within the same cycle on a single-lane approach. For example, it is likely that two buses arriving in the same cycle could utilize the same pre-signal red-time to jump the existing car queue since the red time at the pre-signal is long enough to clear a lane of cars. In this case, the change in additional delays imparted onto to cars would be insignificant, while the benefits would be greater as two buses would have received priority (instead of just one). On the other hand, if two buses arrive at very different times within the same cycle, this could result in more restrictions to car movement, which would significantly increase car delays. The results of this paper could easily be used to iden-

tify which subset of buses arriving during the same cycle could be provided with priority while minimizing the negative impact to cars. Buses arriving outside of these times could simply be treated as a car and not receive priority. Of course, further work is needed to explore the interactions that might be created when multiple buses arrive within a cycle to see if it possible to initiate the pre-signal multiple times in a cycle to provide all buses with priority.

Another aspect worthy of future exploration is the operation of this pre-signal strategy on opposing approaches of the same intersection. In this case, care would have to be taken when pre-signals on the opposing approaches triggered at the same time to ensure that car queues from the other approach do not inhibit the buses from using the opposite lane as a temporary priority lane to bypass car queues on its approach lane. For the joint operation of two opposing pre-signals, additional detection would likely be needed to ensure that the pre-signal operates only when the bus will not be interrupted. Initial efforts in this area suggest this type of joint operation is likely to be successful.

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## Appendix A. Delay equations for under-saturated intersections

The equations for bus delay savings, and additional car delays described in Section 3 are furnished in this appendix. All equations are derived using the variables defined in Section 3 and geometric relations. Also, the transitions between the different cases are considered here. For simplicity, the upper-bound of each case, $i$, is denoted as $t_i$ from here on in. Then each case can be further broken down as:

- Case 1 remains the same.
- Case 2 is broken down into two sub-cases:
  - Case 2.1 for which the bounds are:
    $t_1 < t_m < t_1 + r_{ps}$
  - Case 2.2 for which the bounds are:
    $t_1 + r_{ps} < t_m < t_2$
- Case 3 is broken down into two sub-cases:
  - Case 3.1 for which the bounds are:
    $t_2 < t_m < t_2 + r_{ps}$
  - Case 3.2 for which the bounds are:
    $t_2 + r_{ps} < t_m < t_3$
- Case 4 is broken down into two sub-cases:
  - Case 4.1 for which the bounds are:
    $t_3 < t_m < t_3 + r_{ps}$
  - Case 4.2 for which the bounds are:
    $t_3 + r_{ps} < t_m < t_4$
- Case 5 is broken down into two sub-cases:
  - Case 5.1 for which the bounds are:
    $t_4 < t_m < L_C$
  - Case 5.2 for which the bounds are:
    $L_C < t_m < t_5$

The delay equations are provided for the naive strategy. The same equations hold for the semi-targeted and targeted strategies based on when the pre-signal is active.

### A.1. Bus delay savings

The bus delay savings, $d_{b,i}$, for the above described cases, $i$, can be expressed as:

- Case 1

$$d_{b,1} = \frac{q_A r_{ps}}{q_{max}} \tag{A.1}$$

- Case 2
  - Case 2.1

$$d_{b,2.1} = \frac{q_A r_{ps}}{q_{max}} \tag{A.2}$$

  - Case 2.2

$$d_{b,2.2} = \frac{q_A t_m}{q_{max}} + R - t_2 \tag{A.3}$$

- Case 3

$$d_{b,3} = R - t_m\left(1 - \frac{q_A}{q_{max}}\right) \tag{A.4}$$

- Case 4

$$d_{b,4} = 0 \tag{A.5}$$

- Case 5

$$d_{b,5} = 0 \tag{A.6}$$

*A.2. Additional delay for cars traveling in the same direction as the bus*

The additional delay for cars in the same direction, $d_{c,same,i}$, for the above described cases, $i$, can be expressed as:

- Case 1

$$d_{c,same,1} = q_A t_{add} r_{ps} \tag{A.7}$$

- Case 2
  - Case 2.1

$$d_{c,same,2.1} = q_A t_{add} r_{ps} \tag{A.8}$$

  - Case 2.2

$$d_{c,same,2.2} = q_A t_{add}(t_m - t_1) \tag{A.9}$$

- Case 3
  - Case 3.1

$$d_{c,same,3.1} = q_A((t_{add} + t_m - t_2)(t_m - t_1) + \frac{1}{2}\left(\frac{q_{max}(2(R + t_{add}) - (t_m + t_2)) + 2q_A t_m}{q_{max} - q_A}\right)(t_m - t_2)) \tag{A.10}$$

  - Case 3.2

$$d_{c,same,3.2} = (t_{add} + r_{ps})(q_A t_m - q_{max}(t_m - r_{ps} - R)) + q_A r_{ps}\frac{1}{2}\left(\frac{q_{max}(2(R + t_{add} - t_m) + r_{ps}) + 2q_A t_m}{q_{max} - q_A}\right) \tag{A.11}$$

- Case 4
  - Case 4.1

$$d_{c,same,4.1} = q_{max} r_{ps}(t_3 + r_{ps} - t_m) + \frac{1}{2}\frac{q_{max} q_A r_{ps}^2}{q_{max} - q_A} \tag{A.12}$$

  - Case 4.2

$$d_{c,same,4.2} = \frac{1}{2}\frac{q_{max} q_A r_{ps}^2}{q_{max} - q_A} \tag{A.13}$$

- Case 5
  - Case 5.1

$$d_{c,same,5.1} = 2q_{max} R\left(\frac{q_{max} r_{ps}}{q_{max} - q_A} - (L_C + r_{ps}) + t_m\right) + \frac{1}{2}\frac{q_{max} q_A r_{ps}^2}{q_{max} - q_A} \tag{A.14}$$

  - Case 5.2

$$d_{c,same,5.2} = \frac{1}{2}\frac{q_{max} q_A}{q_{max} - q_A}(L_C + r_{ps} - t_m)(2R + L_C - t_m) \tag{A.15}$$

### A.3. Additional delay for cars traveling in the opposite direction to the bus

The additional delay for cars in the opposite direction, $d_{c,opposite,i}$, apply only for when the main signal is green. Therefore, the cases are slightly different for these sets of equations. Hence, below the delay equations, along with their bounds are provided.

- For $R+\frac{2x_2}{v} < t_m < R+\frac{2x_2}{v}+r_{ps}$

$$d_{c,opposite,2} = \frac{1}{2}\frac{q_{max}q_O}{q_{max}-q_O}\left(\left(t_m-\frac{2x_2}{v}\right)^2 - R^2\right) \tag{A.16}$$

- For $R+\frac{2x_2}{v}+r_{ps} < t_m < \frac{q_{max}R}{q_{max}-q_O}+\frac{2x_2}{v}+r_{ps}$

$$d_{c,opposite,3} = \frac{1}{2}\frac{q_{max}q_O}{q_{max}-q_O}\left(\left(R+r_{ps}\right)^2 - R^2\right) - \left(t_m-\frac{2x_2}{v}-r_{ps}-R\right)q_{max}r_{ps} \tag{A.17}$$

- For $\frac{q_{max}R}{q_{max}-q_O}+\frac{2x_2}{v}+r_{ps} < t_m < L_C-\frac{q_{max}r_{ps}}{q_{max}-q_O}+r_{ps}+\frac{2x_2}{v}$

$$d_{c,opposite,4} = \frac{1}{2}\frac{q_{max}q_O r_{ps}^2}{q_{max}-q_O} \tag{A.18}$$

- For $L_C-\frac{q_{max}r_{ps}}{q_{max}-q_O}+r_{ps}+\frac{2x_2}{v} < t_m < L_C+\frac{2x_2}{v}$

$$d_{c,opposite,5.1} = 2Rq_{max}\left(r_{ps}\frac{q_{max}}{q_{max}-q_O} - (L_C+r_{ps}) + t_m - \frac{2x_2}{v}\right) + \frac{1}{2}\frac{q_{max}q_O r_{ps}^2}{q_{max}-q_O} \tag{A.19}$$

- For $L_C+\frac{2x_2}{v} < t_m < L_C+r_{ps}+\frac{2x_2}{v}$

$$d_{c,opposite,5.2} = \frac{1}{2}\frac{q_{max}q_O}{q_{max}-q_O}\left(L_C+r_{ps}-t_m+\frac{2x_2}{v}\right)\left(2R+L_C-t_m+\frac{2x_2}{v}\right) \tag{A.20}$$

### A.4. Additional delay for cars turning from the cross-street

The additional delays for cars turning from the cross direction, $d_{c,cross}$ can be expressed as in the equations below.

- For $\frac{2x_2}{v} < t_m < r_{ps}+\frac{2x_2}{v}$

$$d_{c,cross,1} = \frac{1}{2}q_{max}\left(t_m-\frac{2x_2}{v}\right)^2\frac{p}{1-p} \tag{A.21}$$

- For $r_{ps}+\frac{2x_2}{v} < t_m < \frac{q_{max}(L_C-R)}{q_{max}-q_C}+r_{ps}+\frac{2x_2}{v}$

$$d_{c,cross,2} = \frac{1}{2}q_{max}r_{ps}^2\frac{p}{1-p} \tag{A.22}$$

- For $\frac{q_{max}(L_C-R)}{q_{max}-q_C}+r_{ps}+\frac{2x_2}{v} < t_m < R+\frac{2x_2}{v}$.

$$d_{c,cross,3} = \frac{1}{2}q_{max}r_{ps}^2\frac{q_C p}{(q_{max}-q_C p)(1-p)} \tag{A.23}$$

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