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Instantaneous communication capacities of vehicular ad hoc networks

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ABSTRACT

Wireless communications among vehicles, roadside infrastructures, and traffic management centers can enable the development of next-generation Intelligent Transportation Systems so as to tackle basic traffic problems associated with driving safety, road congestion, and vehicle emissions. This paper analytically investigates the instantaneous communication capacities of vehicular ad hoc networks (VANETs), which measure the upper bounds of the message transmission rates of vehicles. Subject to interference among wireless transmissions, the broadcast capacity is defined by the maximum number of successful receivers, and the unicast capacity by the maximum number of successful senders. With the protocol communication model and uniform vehicular traffic patterns, we derive closed-form formulas for the capacities as functions of transmission range r, interference ratio δ, vehicular density ρ, and channel capacity W. We show that broadcast capacities are approximately $\frac{W}{(2+\delta)r\rho+1}$ for uni-directional communications, and $\frac{2W}{(2+\delta)r\rho+1}$ for bi-directional communications; while unicast capacities are approximately $\frac{W}{(1+\delta)r\rho+1}$ for uni-directional communications, and $\frac{W}{(1+\delta)r\rho+1}$ for bi-directional communications. For general vehicular traffic patterns, an optimization model is proposed to calculate the capacities, and a genetic algorithm integrated with the protocol communication model is developed to solve the optimization problem. Finally, the impacts of different transmission ranges, interference ratios, and shock waves on communication capacities are analyzed.

1. Introduction

With the rapid development of information technologies, connected vehicles equipped with wireless communication devices have become more popular for enabling advanced transportation management system (ATMS) (Aoki and Fujii, 1996) and advanced traveler information system (ATIS) (Yang and Recker, 2005). With traffic information collected by connected vehicles, basic traffic problems associated with driving safety, road congestion, and vehicle emissions can be more easily tackled (Luo and Hubaux, 2006). A number of efforts are underway to investigate connected vehicles as a means of developing "internet on roads", such as Fleetnet (Franz et al., 2001) and CarTALK (Reichardt et al., 2002). Recker et al. (2008) proposed Autonet, which used vehicle-to-vehicle (V2V) and vehicle-to-infrastructure (V2I) communications to leverage cooperative, vehicle-centric pervasive computing as a platform for transportation management; meanwhile, Autonet integrated a bundle of services supporting an arbitrary collection of transportation management applications.

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In a road network, connected vehicles equipped with Dedicated Short Range Communication (DSRC) devices form a mobile ad hoc network (MANET), which is also called vehicular ad hoc network (VANET). VANETs have quite distinct features from other wired or wireless communication networks: (i) they are decentralized autonomous systems; (ii) there is no energy constraint practically, as communication devices can be charged by car engines or batteries; (iii) the communication network topology is highly dynamic due to vehicle movements; (iv) vehicles usually have well-defined driving behaviors related to departure times, routes, lanes, and acceleration rates in a road network, and their positions in a road network are governed by various traffic laws; and (v) DSRC devices will penetrate the market gradually. Therefore, the distribution in VANETs of equipped vehicles, i.e., communication nodes, is random, dynamic, and non-uniform.

Two basic characteristics of VANETs are critical for determining communication delays and reliability (Ma et al., 2011), and for designing message routing algorithms (Li and Wang, 2007):

1. The connectivity of such communication networks defines the coverage as well as the maximum information propagation distance. In the literature, the connectivity of VANETs has been well analyzed. Jin and Recker (2006, 2010) proposed recursive models to calculate the connectivity of VANETs for uniform and general vehicular traffic patterns. They showed that such vehicular traffic patterns as shock waves, market penetration rates (MPRs) of equipped vehicles, and transmission ranges would determine the connectivity of a VANET. Wang (2007) applied a transient Markov process to model information propagation along a traffic stream, where communication nodes follow the homogeneous Poisson distribution, and approximated the information propagation distance with a Gamma distribution; the author also developed closed-form formulas for the expected distance and its variance. Ukkusuri and Du (2008) analytically studied the connectivity of VANETs, in which they considered VANETs as a nominal system with disturbance; they also explored the connectivity at every time step with the consideration of traffic flow feature.
2. Communication throughputs and capacities measure information transmission rates. By the communication throughput, we mean the average transmission rate of vehicles under given conditions of vehicular traffic patterns, MPRs of equipped vehicles, transmission ranges, choices of senders and receivers, and signal interferences. Yang and Jin (2015) derived closed-form solutions for the communication throughputs of VANETs. With a protocol communication model, the study showed that communication throughputs were functions of vehicular traffic patterns, transmission ranges, MPRs of vehicles equipped with wireless communications, percentages of senders, and so on. In the literature, there have been different definitions of communication capacities for general wireless communication networks, which are the maximum per-node transmission rates of vehicles. Gupta and Kumar (2000) derived that with n randomly located nodes, if each node was capable of transmitting messages at W bits per second without interference and using a fixed transmission range, the capacity was achieved as a function of $\frac{W}{n \cdot \log n}$ under a non-interference protocol. If the nodes were optimally placed in a disk within the region A, the capacity was determined by the value of $\sqrt{A/n}$. Extending the static networks, Grossglauser and Tse (2002) studied a mobile ad hoc network and showed that the node mobility increased the communication capacity dramatically. Li et al. (2001) studied the capacity of wireless ad hoc network via NS-2 simulation. Yi et al. (2003) investigated the influence of directional antennas on increasing the capacity of a wireless network. Negi and Rajeswaran (2004) obtained the capacity of wireless ad hoc network with ultra wide band communication model. Moreover, capacities of multicast communications were also estimated with the consideration of message routing algorithms (Wang et al., 2008, 2011). One pioneering study on communication capacities of VANETs was done by Du et al. (2009), where the capacity of the entire network was defined as the maximum number of successful concurrent transmissions. They proposed an integer programming (IP) model to study the broadcast capacity of VANETs.

In this article, we extend the study on communication throughputs of VANETs in Yang and Jin (2015) and propose analytical and optimization models for both broadcast and unicast communication capacities of VANETs. Here, we only consider single-hop communications, where at any time slot a successful transmission occurs directly from one sender to one receiver without any intermediate nodes. Even though equipped vehicles can move as fast as 80 mph in VANETs, their displacements during each packet transmission time are rather small (in the order of 4 meters during 100 ms, which is the time for one DSRC transmission) (Chen et al., 2010). Thus, we are concerned with communication capacities of instantaneous VANETs. In this paper, we first analytically estimate communication capacities under uniform traffic streams, where vehicles are evenly placed with the same spacing. Similar to the throughputs, the capacities are functions of the transmission range r, the interference ratio δ, and vehicular density ρ or vehicle distribution. However, they are not functions of the percentage of senders as the throughputs. Moreover, we extend the IP models in Du et al. (2009) and calculate communication capacities under general traffic streams. To satisfy the design of connected vehicle applications, the IP models are also extended to estimate both broadcast and unicast capacities under uni- and bi-directional communications (Tse and Viswanath, 2005). Furthermore, we apply a genetic algorithm (Barricelli, 1954; Goldberg, 1989) to solve for the capacities numerically. Finally, we compare communication throughputs estimated in Yang and Jin (2015) with the capacities to validate both analytical and numerical solutions proposed in this paper.

This study is highly related to but different from Du et al. (2009). In both studies, we are interested in estimating instantaneous communication capacities. But in this study we apply the protocol communication model based on interference ranges to capture the interferences among transmissions, and the physical model based on signal-to-interference-and-noise ratio (SINR) was used in Du et al. (2009). The physical model is more related to physical layer considerations, but the protocol model is capable of approximately capturing the impacts of signal interferences; more importantly, the protocol model enables us to derive closed-form formulas for communication capacities as well as solve the corresponding IP model more efficiently. In addition, this study is an extension of but different from Yang and Jin (2015). In both studies we consider instantaneous information propagation and apply the same protocol communication model for given vehicular traffic patterns. But in Yang and Jin (2015), the percentage of senders and, therefore, that of receivers are assumed to be given a priori; in contract, in this study we attempt to select a set of senders from all nodes so as to achieve maximum broadcast and unicast capacities efficiently.

The rest of the paper is organized as follows. Section 2 introduces a protocol communication model and defined the communication capacities of VANETs. Section 3 derives closed-form solutions for the communication capacities under uniform traffic streams. Section 4 constructs an optimization model for the capacities and develops a genetic algorithm to search for solutions efficiently. Section 5 applies the optimization model and the genetic algorithm to calculate the communication capacities along an uniform traffic and a shock wave. Finally, Section 6 concludes the work in this paper.

2. Definitions of communication capacities

Suppose that there are n vehicles on the road, and they are all equipped with wireless communication devices, i.e., the MPR of the equipped vehicles is 100% (For the MPR less than 100%, only the equipped vehicles are considered). Their locations are denoted by $x_i,\ i = 1, 2, \ldots, n$, and they are numbered from downstream to upstream. We assume that if $i < j$, then $x_i \geqslant x_j$ (see Fig. 1). Table 1 lists all notations defined in this paper.

2.1. A protocol communication model

Different from wired communications, wireless communications have two fundamental aspects: signal fading and interference (Tse and Viswanath, 2005). In this subsection, a protocol communication model presented in Gupta and Kumar (2000) is introduced to describe the necessary conditions that a transmission is successfully received by a recipient over one hop. If vehicle i transmits a message to vehicle $j,\ i,j = 1, 2, \ldots, n,\ i \neq j$, this message is successfully received by vehicle j if and only if.

1. vehicle j is inside of the transmission range of vehicle i, i.e.,

$$|x_i - x_j| \leqslant r_i, \tag{1}$$

   where r_i is the transmission range of vehicle i;
2. vehicle j should be outside the interference range of every other vehicle k that is simultaneously transmitting messages ($\forall k \in \mathscr{S}, k \neq i, j$); i.e.,

$$|x_k - x_j| > (1 + \delta_k) r_k, \tag{2}$$

   where r_k is the transmission range of vehicle k, δ_k represents the interference ratio $\delta_k \geqslant 0$, and $(1 + \delta_k) r_k$ is the interference range of vehicle k.

In the model, the transmission range is mainly determined by the transmission power and signal fading. The interference ratio is generally determined by the strength of signals and background noise. One common setting of the transmission range and the interference range is 250 meter and 550 meters respectively; here the interference ratio is 1.2 (Li et al., 2001). That is, a transmission between a sender and a receiver is successful only when the receiver is within the transmission range of the sender, but outside the interference ranges of other senders. From (1) and (2), we can see that the success of a transmission is determined by the distance between the sender and the receiver in a traffic stream, the transmission range of the sender, and the interference ratios of all other senders.

In this study, we assume that all vehicles have omni-directional antennas, which allow vehicles to send messages to all directions. To define the communication capacities of VANETs, two communication methods, including uni-directional and bi-directional communications, are introduced to manage transmissions. In uni-directional communications, messages are transmitted from downstream senders to upstream receivers; i.e., upstream vehicles will only receive downstream traffic information. For some applications of VANETs in transportation systems, uni-directional communications are reasonable, since an upstream accident usually will not change the driving behaviors of a downstream vehicle. Then, it is not necessary to transmit upstream incident information to downstream vehicles. In contrast, bi-directional communications, where a sender will send messages to both upstream and downstream vehicles, are also important in reality. The method has benefits on communication reliability, message routing strategies, etc. Also, it is widely applied in various VANET applications, such as vehicle-signal coordination systems (Kamalanathsharma et al., 2015; Yang et al., 2016), which require one vehicle to communicate with both downstream traffic signals and upstream approaching vehicles.

We first define a binary random variable, S_i, to represent senders. If node i is a sender, $S_i = 1$; otherwise, $S_i = 0$. We denote the set of the senders by $\mathscr{S} = \{i : S_i = 1\}$, and the set of the other vehicles, which are automatically receivers, by $\mathscr{R}$. The col-

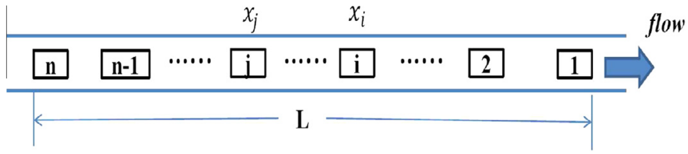

Fig. 1. Distribution of equipped vehicles along a signal-lane freeway segment.

Table 1
Notations.

	Deterministic variables
x_i	Location of vehicle i
r_i	Transmission range of vehicle i
δ_i	Interference ratio of vehicle i
n	Total number of vehicles in the area of study
L	Road length
Π_i	Number of successful receivers of a sender i
Λ	Number of successful senders
W	Wireless channel bandwidth
C_b^1	Broadcast capacity of uni-directional communications
C_b^2	Broadcast capacity of bi-directional communications
C_u^1	Unicast capacity of uni-directional communications
C_u^2	Unicast capacity of bi-directional communications
	Random variables
S_i	1: vehicle i is a sender; 0: otherwise
Y_{ij}	1: a successful communication from vehicle i to j; 0: otherwise
	Regions and sets
$\mathcal{T}_i$	Transmission region of vehicle i
$\mathcal{I}_i$	Interference region of vehicle i
$\mathcal{C}_i$	Successful coverage of vehicle i
$\mathcal{S}$	Sender set
$\mathcal{R}$	Receiver set
$\mathbb{S}$	Sender selection

lection, $\mathbb{S} = \{S_1, S_2, \ldots, S_n\}$, is defined as the sender selection set. The transmission region of vehicle i is denoted by $\mathcal{T}_i$, and its interference region by $\mathcal{I}_i$.

From (1) and (2), the interference region of the sender i in a VANET is defined as

$$\mathcal{I}_i = (x_i - (1+\delta_i)r_i, x_i + (1+\delta_i)r_i). \tag{3}$$

Note that the interference range is always bi-directional. For uni-directional communications, the transmission region of i is

$$\mathcal{T}_i = (x_i, x_i + r_i]. \tag{4a}$$

For bi-directional communications, the transmission region is

$$\mathcal{T}_i = [x_i - r_i, x_i) \cup (x_i, x_i + r_i]. \tag{4b}$$

Then, the successful coverage of the sender i is

$$\mathcal{C}_i = \mathcal{T}_i \cap \left(\cap_{k \neq i, k \in \mathcal{S}} \overline{\mathcal{I}}_k\right), \tag{5}$$

where $\overline{\mathcal{I}}_k$ is the complement region of $\mathcal{I}_k$. And, the successful coverage of all senders is

$$\mathcal{C}(\mathbb{S}) = \cup_{i \in \mathcal{S}} \mathcal{C}_i. \tag{6}$$

We also denote the set of successful receivers of the sender i by $\mathcal{T}_i$,

$$\mathcal{T}_i = \{j | j \in \mathcal{R}, x_j \in \mathcal{C}_i\}. \tag{7}$$

The number of elements in $\mathcal{T}_i$ is denoted by Π_i, which is the number of the successful transmissions of vehicle i, and the number of total successful transmissions by $\Pi(\mathbb{S})$, then

$$\Pi(\mathbb{S}) = \sum_{i \in \mathcal{S}} \Pi_i. \tag{8}$$

For each sender i, when $\Pi_i = 0$; i.e., when it does not have any successful receivers, we say i is an unsuccessful sender. We define $\Lambda(\mathbb{S})$ as the number of successful senders:

$$\Lambda(\mathbb{S}) = \sum_{i \in \mathscr{S}, \Pi_i > 0} 1. \tag{9}$$

Note that, in the problem of searching for the communication capacity, the optimization argument can also be written as the sender selection of variable $\mathbb{S}$. There are totally 2^n possible sender sets. The solution of the optimization problems always exists, and the largest number of transmissions should not be greater than $n-1$. If all nodes are within the transmission range of one node, then $\max_{\mathbb{S}} \Pi(\mathbb{S}) = n-1$.

Lemma 2.1. *For an unsuccessful sender i, we have*

$$\begin{aligned} \Pi(\mathbb{S}) &\leqslant \Pi(\mathbb{S} \setminus \{i\}), \\ \Lambda(\mathbb{S}) &\leqslant \Lambda(\mathbb{S} \setminus \{i\}), \end{aligned} \tag{10}$$

where $\mathbb{S} \setminus \{i\}$ represents the sender selection set which has all elements in $\mathbb{S}$ except S_i. Therefore in the optimal solution, unsuccessful senders shall not exist.

Proof. When $\Pi_i = 0$, for the new sender selection set $\mathbb{S}' = \mathbb{S} \setminus \{i\}$, there is a new set of non-senders $\mathscr{R}' = \mathscr{R} \cup i \supset \mathscr{R}$. For $j \in \mathscr{S}'$, there is $\mathscr{C}_j \subset \mathscr{C}'_j$. Thus, $\mathscr{T}_j \subseteq \mathscr{T}'_j$, and $\Pi_j \leqslant \Pi'_j$. □

Another property is that, in the optimal solution, the successful coverage of all senders should not overlap; i.e.,

$$\mathscr{C}_i \cap \mathscr{C}_j = \varnothing, \quad i \neq j. \tag{11}$$

2.2. Definitions of capacities

In VANETs, information can be disseminated through various methods, and there are two most common modes: broadcast and unicast (Tse and Viswanath, 2005). Broadcast communications allow one sender to transmit messages to all possible nodes in the network; while unicast communications only allow one sender to transmit messages to a single, designated node. Note that a successful receiver must receive a message from one and only one successful sender, but a successful sender can send messages to multiple receivers. In broadcast, all successful transmissions contribute information dissemination, but in unicast only one transmission from one sender is effective. Therefore, the performance of broadcast communications is determined by the number of successful receivers, while that of the unicast communications by the number of successful senders. Moreover, successful transmissions in both modes are governed by the aforementioned protocol communication model.

In the broadcast communication, one sender transmits messages to all other vehicles within its transmission region. Then, the broadcast capacity, the maximum per-node transmission rate (Gupta and Kumar, 2000), is defined as the product of the maximum number of successful concurrent transmissions and the per-node wireless channel bandwidth, $\frac{W}{n}$, i.e,

$$C_b = \max_{\mathbb{S}} \Pi(\mathbb{S}) \cdot \frac{W}{n}. \tag{12}$$

In the unicast communication, one sender only transmits messages to one specific destinations. Generally, message routing algorithms, such as ad hoc on demand distance vector routing (Perkins et al., 2003) and dynamic source routing (Johnson and Maltz, 1996), are associated with unicast communications of VANETs (Li and Wang, 2007; Bernsen and Manivannan, 2009). The unicast capacity is governed by both routing algorithms and optimal assignments of senders and receivers (Wang et al., 2008, 2011). However, introducing routing algorithms with multi-hop communications significantly increases the complexity of the analysis. As a starting point, we investigate the instantaneous unicast capacity under the single-hop communication, where one sender only sends messages to a nearby destination in one hop. Hence, only the optimal assignment of senders and receivers is required to identify the highest message transmission rate of the unicast communication. The unicast capacity is defined as the product of the maximum number of successful concurrent senders and the per-node channel bandwidth, i.e.,

$$C_u = \max_{\mathbb{S}} \Lambda(\mathbb{S}) \cdot \frac{W}{n}. \tag{13}$$

Given the vehicle distribution on a road, i.e, the locations of vehicles $\{x_k\},\ k = 1, 2, \ldots, n$, the transmission range r_k, and the interference ratio δ_k, both broadcast and unicast capacities can be calculated numerically with the model in Section 2.1. Moreover, the studies of capacities also provide the optimal assignment of senders and receivers in a VANET. In the future, with the cooperations of connected and automated vehicles, the optimal assignment can be applied to maximize the communication efficiency.

3. Communication capacities in uniform traffic

In this section, we will analytically study the communication capacities of VANETs in uniform traffic. Considered a straight road with length L and a constant vehicle density ρ, the number of vehicles on the road is $n = \rho L$, and their locations are $x_k = (n-k)/\rho,\ k = 1, 2, \dots, n$. The spacings of all vehicles are the same, $s = 1/\rho$ (see vehicles in Fig. 1 with the same spacing). Moreover, we assume that all vehicles are equipped with wireless communication devices, i.e., they are all potential senders and receivers. To simplify the analysis of capacities, we start with the assumption that all vehicles have the same communication range, r, and interference ratio, δ.

From the protocol model proposed in Section 2, if both vehicle i and k are successful senders, vehicle j is a successful receiver of i, and vehicle l is a successful receiver of k, then they should satisfy the constraints of the protocol model (1) and (2). As shown in Fig. 2, the distance between two closest successful receivers j and l cannot be smaller than δr:

$$|x_j - x_l| \geqslant |x_j - x_k| - |x_l - x_k| \geqslant (1+\delta)r - r = \delta r. \tag{14}$$

3.1. Broadcast capacity

In the broadcast communication, one sender transmits messages to all vehicles within its own transmission region. To achieve the broadcast capacity, a sender shall try to cover as many vehicles within its transmission region as possible, to reduce the number of vehicles interfered, and to increase the number of vehicles receiving messages successfully. With this idea, the uni-directional capacity C_b^1 and the bi-directional capacity C_b^2 are analyzed below.

Without loss of generality, we assume that uni-directional communications only allow message transmissions from downstream to upstream. That is, for a given successful sender, all vehicles within the transmission region have to receive messages from that sender; while only upstream vehicles can receive them successfully (see Fig. 3(a)). The distance between the farthest receiver and all the other senders should be greater than $(1+\delta)r$ (2), which indicates the length of the interference region of one successful sender is

$$LI = |x_f - x_i| + (1+\delta)r, \tag{15}$$

where x_f and x_s is the locations of the farthest receiver and the sender, respectively. (1) indicates that an equipped vehicle can send messages to another one successfully only when their distance is less than the transmission range r. In uniform traffic, the minimum distance between two vehicles is constant and equals the spacing, s. To identify whether there exist successful communications, r and s shall be compared. In the following, formulas for broadcast capacities are derived under different values of r.

1. If $r < s$, it is impossible to find two vehicles satisfying (1); i.e., there is not any successful transmission. Hence, the communication capacity is

$$C_b^1 = 0. \tag{16}$$

2. If $r \geqslant s$, the distance between the farthest receiver and the sender is

$$|x_f - x_s| = \lfloor r\rho \rfloor \cdot s = \left\lfloor \frac{r}{s} \right\rfloor \cdot s,$$

where $\lfloor\cdot\rfloor$ finds the integer floor of a given value. Then the number of vehicles that receive messages successfully from the sender is $n_1 = \lfloor r\rho \rfloor$. In addition, the number of vehicles that are interfered by one successful transmission and fail to receive messages is

$$n_2 = \lceil (1+\delta)r\rho + \epsilon \rceil,$$

where $\lceil\cdot\rceil$ finds the integer ceiling of a given value, ϵ is a small positive value, and $\epsilon \ll 1$.
Hence, the average number of vehicles interfered by the successful transmission is $n_3 = n_1 + n_2$. The broadcast capacity in the uniform traffic is obvious with the consideration of the road boundaries.[1]

$$C_b^1 = \left[\left\lfloor \frac{n}{n_3} \right\rfloor \cdot n_1 + \min\{n_0 - 1, n_1\} \right] \cdot \frac{W}{n}, \tag{17}$$

where $n_0 = mod(n, n_3)$, and $mod(a, b)$ is a modulo operation that finds the remainder of the division of a by b.

In the bi-directional communication, a successful sender transmits messages to all vehicles in both the upstream and the downstream transmission regions. The optimal assignment of senders and receivers is shown in Fig. 3(b). (15) is also

[1] As shown in Fig. 1, traffic bounds are defined as the two ends of the road: $x = 0$ and $x = L$. In this study, we only consider communication capacities among vehicles inside the segment, $0 < x < L$. Hence, we assume that the impact of other vehicles beyond the segment is neglected, and the transmissions to these vehicles are not counted in the capacity estimation.

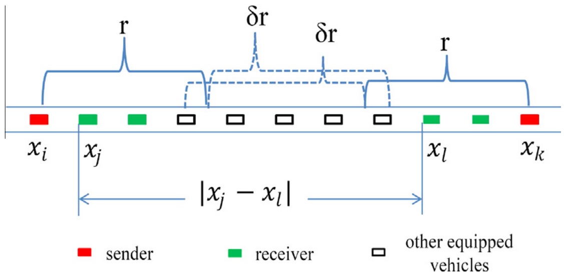

Fig. 2. Distance among two closest successful receivers.

applicable for bi-directional communications. Considering the uniform traffic, we obtain the following results for the broadcast capacity, C_b^2.

1. If $r < s$, it is impossible to find two vehicles satisfying (1); i.e., there is not any successful transmission. Hence, the communication capacity is

$$C_b^2 = 0. \tag{18}$$

2. If $r \geqslant s$, without considering the influence of the traffic boundaries, we derive that for each successful sender, the average number of successful receiver is $2 \cdot n_1$, and the average number of vehicles interfered by one successful transmission is n_3. With the optimal assignment of all senders in the network, we can find the broadcast capacity. If $n < n_3$, the capacity is

$$C_b^2 = [\min\{2n_1, n-1\}] \cdot \frac{W}{n}. \tag{19a}$$

If $n \geqslant n_3$, the broadcast capacity is

$$C_b^2 = \left[\left\lfloor \frac{n}{n_3} \right\rfloor \cdot 2n_1 + n_B\right] \cdot \frac{W}{n}, \tag{19b}$$

where n_B is determined by the road boundaries (see Fig. 4). When $n_0 \leqslant n_1$, the one closest to the road boundary is firstly assigned as the sender as shown in Fig. 4(a), then the number of successful receivers in the n_0 vehicles is $2(n_0 - 1) - n_1$. If the value is smaller than 0, then we remove the sender, and set all the n_0 vehicles near the boundary be unsuccessful receivers. That is, $n_B = \max\{0, 2n_0 - n_1 - 2\}$. If $n_1 < n_0 \leqslant 2n_1 + 1$, the assignment of the senders and receivers near the boundary is shown in Fig. 4(b), where the $(n_0 - n_1)$th vehicle from the boundary is set as the sender then all other vehicles are successful receivers. If $n_0 > 2n_1 + 1$, there are totally $2n_1$ vehicles receiving messages from one sender successfully, and the others are interfered by the sender (see Fig. 4(c)). Hence, the boundary effect, n_B, is determined by the formula below.

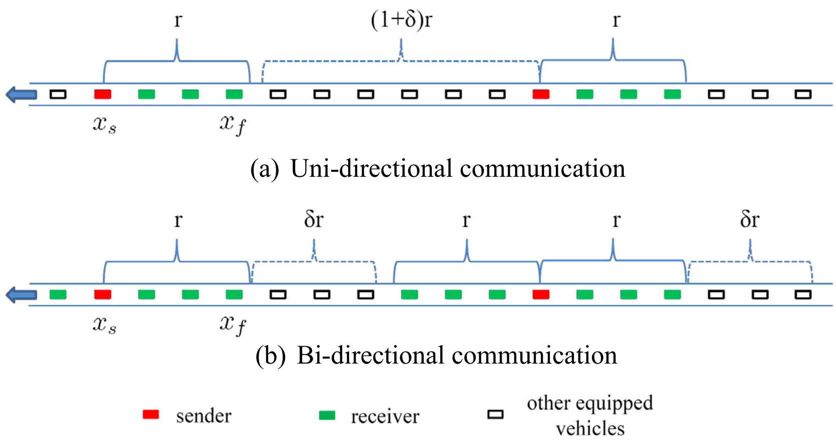

Fig. 3. Optimal node assignments for broadcast capacities in uniform traffic.

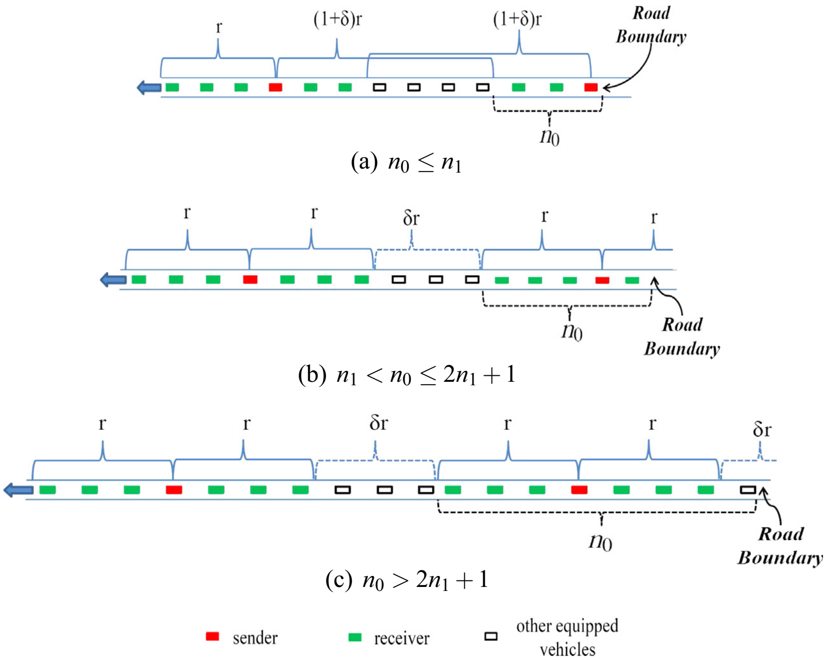

Fig. 4. Boundary effect of bi-directional broadcast communications.

$$n_B = \begin{cases} \max\{0, 2n_0 - n_1 - 2\} & n_0 \leqslant n_1 \\ n_0 - 1 & n_1 < n_0 \leqslant 2n_1 + 1 \\ 2n_1 & n_0 > 2n_1 + 1 \end{cases}.$$

If the road length is sufficiently long; i.e., if the total number of vehicles n is very large, and $n \gg n_0$, then the boundary effect can almost be ignored (see (17) and (19a)). In that sense, the uni-directional broadcast capacity is proportional to $\frac{n_1}{n_3}$, and bi-directional $\frac{2 \cdot n_1}{n_3}$, which are equivalent to $\frac{r\rho}{(2+\delta)r\rho+1}$ and $\frac{2r\rho}{(2+\delta)r\rho+1}$, respectively. Further, when either the transmission range or the vehicle density are large; i.e., when $r\rho \gg 1$, the uni-directional and bi-directional broadcast capacities can be further simplified into $\frac{1}{2+\delta}W$ and $\frac{2}{2+\delta}W$, respectively. Thus the uni-directional capacity is approximately half of the bi-directional capacity. This is intuitively true, as in the bi-directional broadcast communication, a sender can transmit messages to all potential receivers at both upstream and downstream transmission regions. While in the uni-directional communication, a successful sender only transits messages to the receivers in its downstream transmission region, which is just half of that in the bi-directional communication.

3.2. Unicast capacity

In this subsection, we estimate the unicast capacity under an uniform traffic stream. Based on the definition in Section 2.2, the capacity is determined by the maximum number of the successful senders, which is achieved by minimizing the average interference region of each sender. In order to minimize the region, only the closest vehicle of a given sender is chosen as its successful receiver.

In the uni-directional communication, the optimal assignment of senders and receivers are shown in Fig. 5(a), and transmissions only occur from downstream to upstream. With this assignment, the capacity of the unicast communication, C_u^1, is analyzed below.

1. If $r < s$, there is not any successful transmission (1). So, the unicast capacity is

$$C_u^1 = 0. \tag{20}$$

2. If $r \geqslant s$, the length of the interference region of an successful transmission is

$$LI = |x_s - x_c| + (1+\delta)r,$$

where x_c is the closest vehicle in the upstream of the sender. In the uniform traffic, there is

$$|x_s - x_c| = s.$$

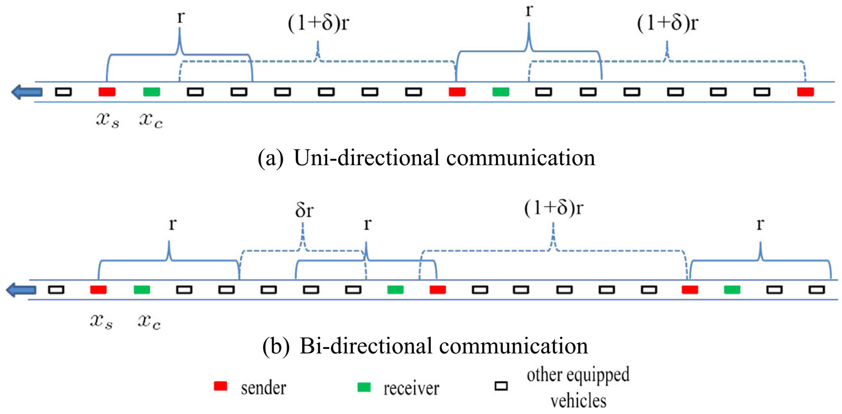

Fig. 5. Optimal node assignments for unicast capacities in uniform traffic.

Hence, the average number of vehicles interfered by one successful transmission is $n_4 = 1 + n_2$. Then the unicast capacity is

$$C_u^1 = \left[\left\lfloor\frac{n}{n_4}\right\rfloor + \min\{mod(n, n_4) - 1, 1\}\right] \cdot \frac{W}{n} \tag{21}$$

In the bi-directional communication, the optimal assignment of senders and receivers are shown in Fig. 5(b). Different from the uni-directional communication, the bi-directional communication allows transmissions from upstream to downstream. The capacity of the unicast communication, C_u^2, can be analyzed in the following steps.

1. If $r < s$, there is not any successful transmissions (1). So, the unicast capacity is

$$C_u^2 = 0. \tag{22}$$

2. If $r \geqslant s$, based on the optimal assignment in Fig. 5(b), the average number of vehicles interfered by two consecutive unicast transmissions is

$$n_5 = 2\left\lceil\frac{(1+\delta)r}{s} + \epsilon\right\rceil.$$

Since $r \geqslant s$, there is $n_5 \geqslant 4$. Hence, the unicast capacity is determined by the formula below with the boundary effect.

$$C_u^2 = \left[2\left\lfloor\frac{n}{n_5}\right\rfloor + n_U\right] \cdot \frac{W}{n}, \tag{23}$$

where n_U represents the number of successful senders at the boundary of the segment. If $n_0' = 1$ (see Fig. 6(a)), where n_0' is the number of vehicles associated with the road boundary and $n_0' = mod(n, n_5)$, the vehicle cannot be a successful sender or receiver, i.e., $n_U = 0$. If $2 \leqslant n_0' \leqslant \frac{n_5}{2} + 1$, the assignment of the senders and receivers near the boundary is shown in Fig. 6(b), and there is $n_U = 1$. Fig. 6(c) shows the assignment of senders and receivers when $\frac{n_5}{2} + 2 \leqslant n_0' < n_5$, and there is $n_U = 2$.[2] Hence, the value of n_U are determined by the formulas below.
If $n_5 = 4$, then

$$n_U = \begin{cases} 0 & n_0' = 1; \\ 1 & 2 \leqslant n_0' \leqslant \frac{n_5}{2} + 1. \end{cases}$$

If $n_5 > 4$, then

$$n_U = \begin{cases} 0 & n_0' = 1; \\ 1 & 2 \leqslant n_0' \leqslant \frac{n_5}{2} + 1; \\ 2 & \frac{n_5}{2} + 2 \leqslant n_0' < n_5 \end{cases}$$

[2] Here, the n_5 shall be greater than 4; otherwise, the condition, $\frac{n_5}{2} + 2 \leqslant n_0' < n_5$, does not exist.

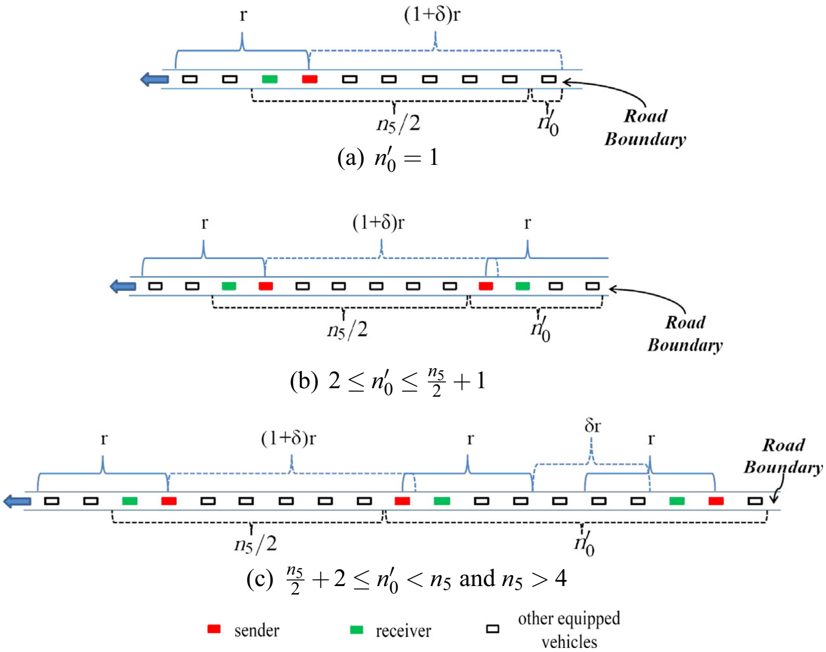

Fig. 6. Boundary effect of bi-directional unicast communications.

Similar to the broadcast communication, if the road length is long enough, i.e., $n \gg n_4$ and $n \gg n_5$, the boundary effect can be ignored (see (21) and (23)). Therefore, the uni- and bi-directional capacities are functions of $\frac{1}{n_4}$ and $\frac{2}{n_5}$, which are equivalent to $\frac{1}{2+(1+\delta)r\rho}$ and $\frac{1}{1+(1+\delta)r\rho}$, respectively. Here, the capacities from the uni- and bi-directional communications are quite similar. The results can be explained by the mechanism of the unicast communication, with which one sender can only send messages to one receiver regardless of uni- or bi-directional communications. Hence, the number of successful senders will be similar in the two types of communications.

Note that the analysis in this section only applies to uniform traffic streams. In reality, due to traffic waves and randomness in human driving behaviors, exactly uniform traffic with evenly distributed vehicles is rarely observed. But, the traffic state near the uniform condition can still be observed once a road reaches a steady state. Hence, the analysis here can be used to estimate communication efficiency of these networks approximately. Moreover, a lot of advanced traffic control systems, such as green driving (Yang and Jin, 2014) and vehicle platooning with cooperative adaptive cruise control (Milanés et al., 2014), require vehicles to travel with a steady speed and maintain a constant spacing. In the future, with the increase of the penetration rate of connected and automated vehicles, it becomes more possible to achieve the optimal steady state and to maintain uniform traffic conditions on the road. In that sense, the analysis of communication capacities under uniform traffic streams will be important on establishing efficient communications and designing reliable applications.

4. An optimization formulation and its numerical solution

In Section 3, we have analytically investigated communication capacities under uniform traffic streams. But, when it goes to general (non-uniform) traffic, where the spacings of vehicles are not the same, analytical capacity estimations will be very difficult due to the unknown distribution of vehicles on the road. Du et al. (2009) constructed integer programming (IP) models to search for uni-directional broadcast capacities numerically. In this section, we extend the models for both uni- and bi-directional broadcast and unicast communications with the protocol communication model, and develop a genetic algorithm to estimate communication capacities under general traffic patterns efficiently.

4.1. An optimization model for communication capacities

In this subsection, we construct an optimization model for communication capacities in general traffic. We first define one Bernoulli random variable, Y_{ij}, to identify successful transmissions. If vehicle i sends packages successfully to vehicle $j, Y_{ij} = 1$; otherwise, $Y_{ij} = 0$, $\forall i,j = 1,2,\ldots,N$. To generalize the applications of the model, we assume that vehicle i $(i = 1,2,\ldots,N)$ has its own transmission range, r_i, and interference ratio, δ_i. From the protocol model, if a vehicle is a successful sender, it has at least one successful receiver. For a successful receiver, it has only one successful sender, and satisfies

(1) and (2). Moreover, the objective of the study is to search for the best assignment of senders to maximize the total number of successful transmissions. Hence, the broadcast capacity can be obtained by the following optimization problem.

$$\max \sum_{i=1}^{n}\sum_{j=1}^{n} Y_{ij} \cdot \frac{W}{n} \tag{24a}$$

s.t.

$$S_k Y_{ij}(|x_k - x_j| - (1+\delta_k)r_k) \geqslant 0, \quad i \neq k,\ i \neq j,\ k \neq j,\ j = 1,\ldots,n; \tag{24b}$$

$$Y_{ij}|x_i - x_j| \leqslant r_i, \quad i \neq j,\ j = 1,\ldots,n; \tag{24c}$$

$$S_j + \sum_{i=1}^{n} Y_{ij} \leqslant 1, \quad j = 1,\ldots,n; \tag{24d}$$

$$\sum_{i=j}^{n} Y_{ij} = 0, \quad j = 1,\ldots,n; \tag{24e}$$

$$S_i = \max_j\{Y_{ij}\}, \quad i = 1,\ldots,n; \tag{24f}$$

$$Y_{ij} = \{0,1\}, \quad i,j = 1,\ldots,n. \tag{24g}$$

The constraints are explained in the following:

1. (24b) gives the constraint of the protocol model for a successful transmission: if both vehicles i and k are successful senders, and j is a successful receiver of node i, then the distance between k and j should be greater than $(1+\delta_k)r_k$.
2. (24c) indicates that, for a successful transmission, the distance between a sender i and its receiver j is smaller than the transmission range r_i. At the same time, this constraint ensures that the capacity is 0; i.e., $Y_{ij} = 0$, when the transmission range r_i is smaller than the spacing of any two vehicles $|x_i - x_j|,\ \forall i \neq j,\ j = 1,2,\ldots,n$.
3. (24d) means that, for a given vehicle j, it could not be a sender or a receiver at the same time.
4. (24e) shows the constraint of the uni-directional communication, with which a successful transmission only happens from downstream vehicles to upstream vehicles, and the reverse transmissions are prohibited. If the bi-directional communication is considered, (24e) should be removed. This revision extends the study in Du et al. (2009) to model the capacities of bi-directional communications.
5. (24f) defines the relationship between a successful sender and its relative successful transmissions. It also indicates that a successful transmission only has one successful sender.
6. (24g) shows the definitions of variable Y_{ij}'s.

(24) can also be applied to estimate unicast capacities, which are estimated by maximizing the number of the successful senders, by replacing (24a) with (25). All the constraints remain the same.

$$\max \sum_{i=1}^{n} S_i \cdot \frac{W}{n} \tag{25}$$

Based on the microscopic traffic flow models, such as car-following models and lane-changing models, vehicle distributions on roads are obtainable. The problem of estimating communication capacities becomes how to find the solutions for (24a) and (25). But, when the number of vehicles on a road is very large, the computational complexity would be very high. In the next subsection, we propose a genetic algorithm to solve this optimization problem efficiently.

4.2. Genetic algorithm

The aforementioned subsection indicates that the communication capacities of a VANET are not only determined by the communication characteristics, including transmission range r_i and influence ratio δ_i, but also related to the distribution of equipped vehicles, $\{x_i,\ i = 1,2,\ldots,n\}$. In a transportation network, even though the locations of equipped vehicles are restricted by the network topology, road traffic signals, and driving behaviors, randomness of vehicular locations due to human driving behaviors still exists and makes it very difficult to find a mathematical solution for capacities under general traffic pattern. In this subsection, we propose a genetic algorithm, which has a distinct advantage on solving optimization problems with random variables (Goldberg, 1989), to find communication capacities (24) and (25). Lemma 2.1 is also applied to improve the computational efficiency of the algorithm.

The genetic algorithm is an optima-searching algorithm to mimic the natural selection and mutation processes of genetics (Goldberg, 1989). This algorithm starts with a set of solutions called initial population. Solutions from one population are taken and used to form a new population. This is motivated by a hope that the new population will be better than the old one. Solutions which are selected to form new solutions are chosen according to their fitness. The more suitable they are, the more chances they have to reproduce.

This study aims at finding a set of senders, $\mathbb{S}$, to maximize the number of successful senders or receivers as well as to estimate capacities. Fig. 7 illustrates the process of the algorithm, and the details of the algorithm are described below.

1. Randomly generate a sender selection set from n vehicles, $\mathbb{S}_k,\ k = 1, 2, \ldots, P$ (P is the population size);
2. Calculate the number of successful transmissions or senders (throughput) of each solution in the population with the communication model described in Section 2.2;
3. Create a new set of solutions by repeating the following steps until the number of solutions in the new set is P;
   (a) Choose the solution with the largest throughput as the first offspring of the new population;
   (b) Randomly choose a pair of solutions from the old population;
   (c) Randomly select a cross point with the probability p_c between $[1, n-1]$, and cross the two selected solutions to create two new solutions;
   (d) Assume that each vehicle has p_m probability to be mutated, i.e., each one has p_m probability to change its states (sender, non-sender);
   (e) Place the new solutions in the new population until there are P solutions there;
4. Replace the old population with the new one as the next generation;
5. If the iteration number is larger than the maximum generation G or the capacity does not change in T generations, stop; otherwise, go to step 2.

In the application of the genetic algorithm, randomly selecting cross point and mutation point is not efficient. Lemma 2.1 indicates that removing unsuccessful sender will increase the value of the objective function. Then, in the algorithm, if one sender is not successful in current generation, we will choose a larger value for p_m to make it a receiver in the next generation. Moreover, (11) shows that the successful coverage of all senders should not overlap each other, so we increase p_m for the uncovered nodes to make them be senders in the next generation.

5. Numerical results

In this section, we simulate several traffic streams to verify the analytical study in Section 3 and the optimization model in Section 4. MATLAB R2009b is applied to model VANETs, including vehicle dynamics and wireless communications with the protocol model.Based on the protocol model, we search for the maximum number of successful senders or receivers under this set. Assume that all selected senders are transmitting messages simultaneously, then the rest of the vehicles can be assigned into three different states: receiver, interfered node and uncovered node. If one vehicle is in the transmission region $\mathscr{T}$ of a sender, it is defined as a receiver. If a node is in the interference region $\mathscr{I}$ of a sender, it is called a interfered node. For all other vehicles, they are set as the uncovered nodes. In our study, we want to maximize the number of the successful senders or receivers, depends on whether we are interested in unicast or broadcast communication capacities. Hence, for one vehicle j, if it is a receiver of vehicle i, and it is not interfered by the other senders, j is a successful receiver of vehicle i. And simultaneously, vehicle i is a successful sender.

5.1. Communication capacities under uniform traffic

We consider a uniform traffic stream at density $\rho = 20$ veh/km on an uninterrupted freeway section. Vehicles are distributed on the road, $[0, L]$ km, with the same spacing $s = 1/\rho = 0.05$ km. We set $L = 5$ km, i.e., the number of vehicles is $n = 100$, and the location of vehicle i as $x_i = (n-i)s,\ i = 1, 2, \ldots, n$. Suppose that all vehicles are equipped with wireless communication devices. We arbitrarily set a various communication ranges $r = \{50, 100, 150, 200, 250\}$ meters and interference ratios $\delta = \{0, 1\}$. [3] In this simulation, we consider a constant wireless channel bandwidth, $W = 6$ Mbps (Jiang et al., 2008).

For the genetic algorithm, the initial population is randomly generated and the population size P is set as 30. The cross rate p_c is 0.6, while the mutation rate is $p_m = 0.0333$. The stopping criteria of this algorithm is that the total number of generations is larger than $G = 500$ or the capacity does not change in $T = 100$ generations. In each generation, if one vehicle is not covered by any transmissions, or it is an unsuccessful sender, the mutation rate is increased to $10 * p_m$.

Fig. 8 illustrates the process of the genetic algorithm on searching for the broadcast communication capacity with $r = 150$ m and $\delta = 1$. The genetic algorithm iterates the population about 270 times to reach the capacity, which indicates the computational complexity for this scenario is approximately $270 \times 30 = 8100$. Compared with the complexity of the optimization problem, 2^{100}, the genetic algorithm is much more efficient on searching for the capacity.

Table 2 shows the uni-directional broadcast and unicast capacities under different communication ranges and interference ratios. The analytical (from the theoretical formulas in Section 3) and the simulated (from the optimization model and the genetic algorithm in Section 4) results are consistent with each other. This serves as a validation for our analysis in Section 3. Table 2 also indicates that, with a larger transmission range, the broadcast capacity gets larger, while the unicast capacity becomes smaller. This is intuitively correct, since in the broadcast communication more vehicles can be covered by

[3] The maximum transmission range of IEEE 802.11p is set as 1000 m (Fisher, 2007), and one common setting of the interference ratio is $\delta = 1.2$ (Li et al., 2001).

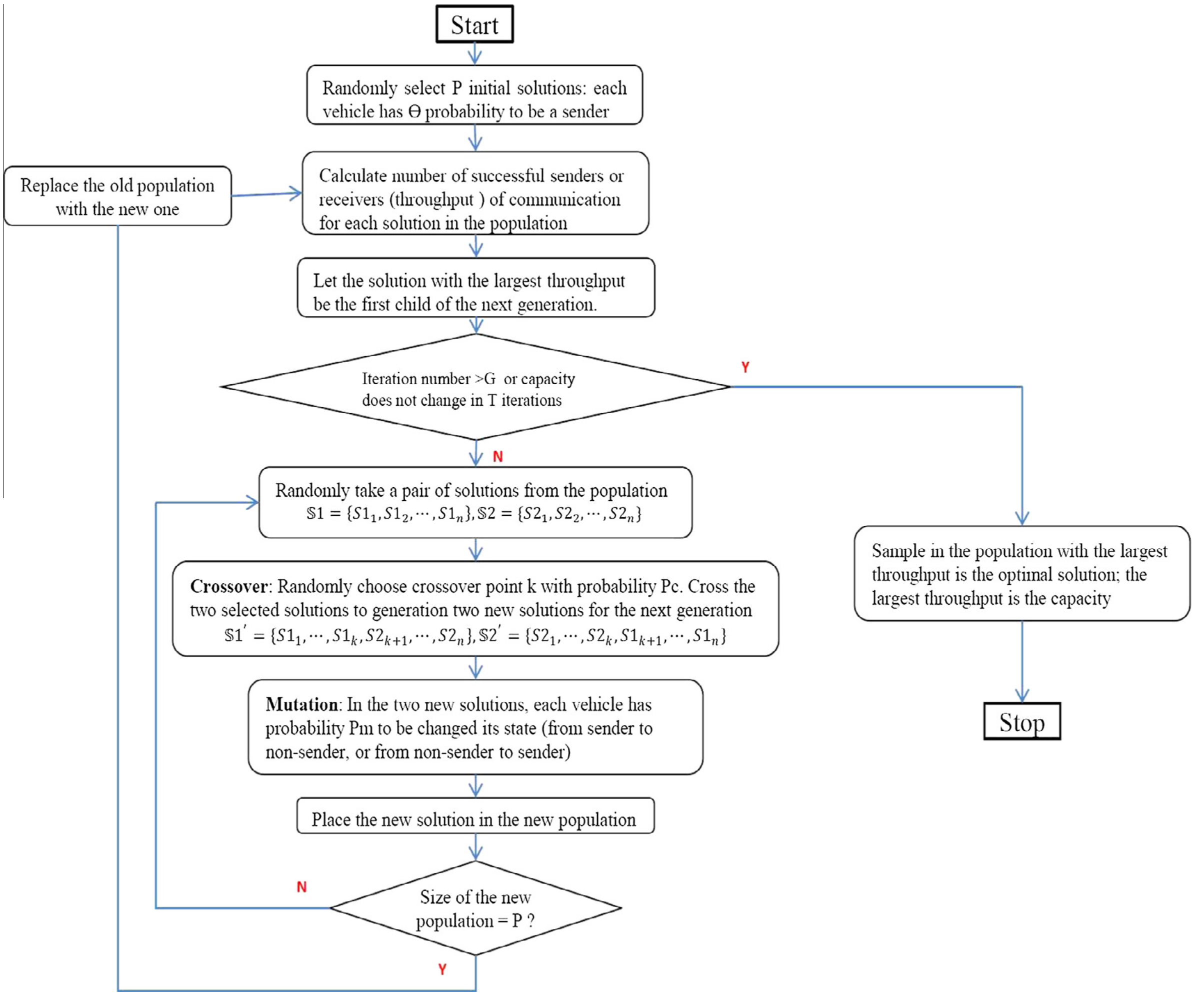

Fig. 7. Flow chart of the genetic algorithm.

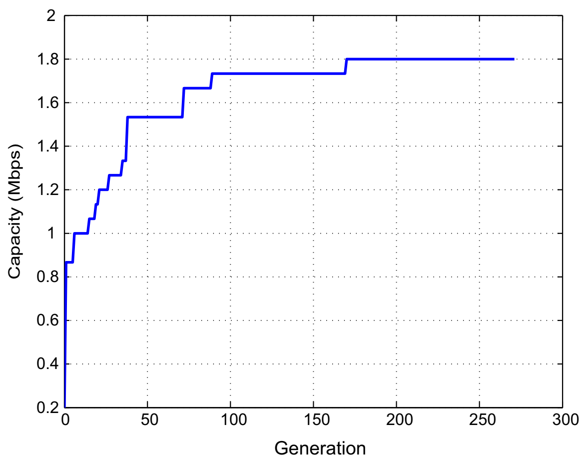

Fig. 8. Broadcast capacity estimation with the genetic algorithm: $r = 150$ m, $\delta = 1$.

Table 2
Comparisons of analytical and simulated uni-directional communication capacities under an uniform traffic.

	Transmission range (m)		50	100	150	200	250
BrO-adcast	$\delta = 0$	Analytical	1.97	2.40	2.57	2.63	2.70
		Simulated	1.93	2.40	2.57	2.63	2.70
	$\delta = 1$	Analytical	1.50	1.73	1.80	1.93	1.97
		Simulated	1.43	1.73	1.80	1.93	1.97
Uni-cast	$\delta = 0$	Analytical	1.97	1.50	1.20	1.03	0.90
		Simulated	1.97	1.50	1.20	1.03	0.90
	$\delta = 1$	Analytical	1.50	1.03	0.77	0.60	0.53
		Simulated	1.43	1.03	0.77	0.60	0.53

senders with a larger transmission range, but in the unicast network, the larger range indicates more vehicles are interfered by a single sender. It is also straightforward to predict that a larger interference ratio reduces both broadcast and unicast capacities.

Table 3 shows the bi-directional broadcast and unicast capacities. As predicted by the analytical results in Section 3, the broadcast capacities with bi-directional communications are almost twice of those with uni-directional communications[4]; the unicast capacities with bi-directional communications are only slightly higher than those with uni-directional communications. Moreover, Table 3 indicates that with larger transmission ranges, the broadcast capacities will be larger, while the unicast capacities become smaller. The trend is similar to the uni-directional communications.

5.2. Communication capacities under a shock wave

In this subsection, we consider the communication capacities on an single-lane road with a stream of traffic subjected to a shock wave. Initially, there are $n = 150$ vehicles located on the section $[0, 4000]$ m, and their locations are

$$x_k(0) = \begin{cases} 4000 - 16.67k & 1 \leqslant k \leqslant 60 \\ 3000 - 33.33(k - 60) & 61 \leqslant k \leqslant 150 \end{cases}.$$

The leading vehicle 1 is traveling with a constant speed, $v_1(t) = 5.11$ m/s. And, Newell's car-following model (26) (Newell, 2002) is applied to describe the dynamics of all vehicles.

$$x_k(t + \tau) = \min\{x_{k-1}(t) - d, x_k(t) + u\tau\}, \tag{26}$$

where d, τ, u represent jam spacing, time gap, and free flow speed, respectively. In this simulation, we set $d = 9$ meters, $\tau = 1.5$ s, and $u = 30$ m/s. With these settings, a shock wave is generated on the road. Similar to the experiment in Section 5.1, we let $W = 6$ Mbps. The traffic stream is simulated for 150 s. Fig. 9 shows the simulated trajectories of all vehicles, where a shock wave is formed and traveling backward with a constant speed.

With the same parameter settings of the communication and the genetic algorithm in Section 5.1, both uni- and bi-directional broadcast and unicast capacities are estimated at $t = 0, 54$ s in Table 4. The results indicate that a shock wave slightly increases the broadcast capacities and reduces the unicast capacities. This phenomena is consistent to the analysis in Section 3, which shows that under the same communication settings, a denser traffic only increase the broadcast capacity slightly due to boundary effect, and decreases the unicast capacity. The conclusion is contradict to common observations that denser traffic generates more transmission conflicts so as to reduce communication efficiency. However, the common observation is based on the assumption that the percentage of simultaneous senders are almost the same. But in the estimation of communication capacities, an optimal percentage of senders is chosen to maximize the transmission rate. With a denser traffic, the percentage of senders is smaller to find the capacity. This phenomenon is also verified in Yang and Jin (2015). Moreover, Fig. 10 shows the broadcast and unicast capacities under uni- and bi-directional communications with $r = 150$ m and $\delta = 0$. We see that the shock wave increases both uni- and bi-directional broadcast capacities, and reduces unicast capacities.

In Yang and Jin (2015), close-form formulas for the mean uni-directional communication throughputs of VANETs are developed. This study is highly related to the capacity analysis, as the capacity is the maximum value of throughput. Here, we compare the average throughput and the capacity when a shock wave is propagating on the road. Using the example above, We calculate both broadcast and unicast capacities and throughputs under different time steps with $r = 150$ m and $\delta = 0$. Results in Fig. 11 indicate that the calculated capacities are larger than the theoretical throughput. Intuitively, this is correct, as the capacity is the maximum value of the throughput. The figures also illustrate that a shock wave increases both the capacity and the throughput under the broadcast communication, while decreases them under the unicast communication. However, the effect of traffic patterns on the communication capacities is not very significant. Moreover, we see throughputs and capacities have the similar trends when they are experiencing a shock wave.

[4] Due to node assignments at the road boundaries, the bi-directional broadcast capacities are not exact twice of uni-directional capacities.

Table 3
Comparisons of analytical and simulated bi-directional communication capacities under an uniform traffic.

Transmission range (m)			50	100	150	200	250
BrO-adcast	$\delta = 0$	Analytical	3.97	4.80	5.03	5.27	5.17
		Simulated	3.73	4.43	4.63	5.03	5.17
	$\delta = 1$	Analytical	3.00	3.37	3.60	3.83	3.67
		Simulated	2.63	3.23	3.30	3.67	3.53
Uni-cast	$\delta = 0$	Analytical	3.00	1.97	1.50	1.20	1.03
		Simulated	2.83	1.80	1.43	1.07	0.97
	$\delta = 1$	Analytical	1.97	1.20	0.90	0.67	0.53
		Simulated	1.87	1.13	0.83	0.67	0.53

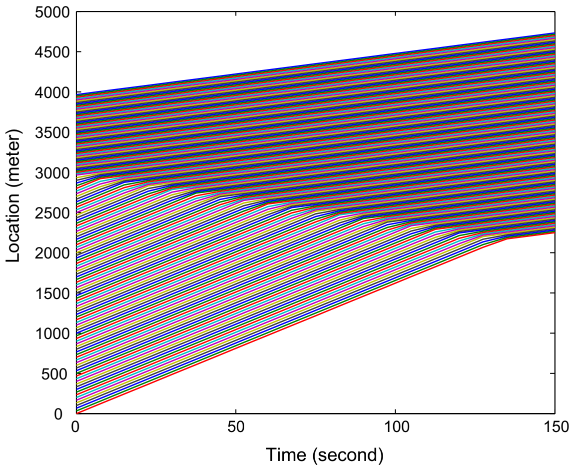

Fig. 9. Vehicular trajectories in a shock wave.

6. Conclusion

In this paper, we investigated the communication capacities to measure wireless message transmission efficiency of VANETs for uniform and general vehicular traffic patterns. The numbers of concurrent successful senders and receivers were applied to derive both broadcast and unicast capacities, respectively, and the optimal assignments of senders and receivers under both uniform and general traffic streams were chosen to estimate the capacities and to maximize message transmission efficiency of a fully connected and automated vehicle system in the near future. In this paper, we first analytically estimated the broadcast and unicast capacities of uni-directional and bi-directional communications under uniform traffic. We derived the capacities as functions of the vehicular density ρ, the communication range r, and the interference ratio δ. The analysis showed that, for uni-directional communications, the broadcast capacity is approximately proportional to $\frac{r\rho}{(2+\delta)r\rho+1}$ (17), while the unicast capacity $\frac{1}{2+(1+\delta)r\rho}$ (21). For bi-directional communications, the broadcast capacity is approximately proportional to $\frac{2r\rho}{(2+\delta)r\rho+1}$ (19a), while the unicast capacity $\frac{1}{1+(1+\delta)r\rho}$ (23). We developed an optimization model and a genetic algorithm integrated with the protocol communication model to search for the capacities. The crossing and mutation probabilities of the genetic algorithm were adjusted based on traffic patterns and communication properties to improve the efficiency of the algorithm. This methodology had potential on obtaining communication capacities in general traffic,

Table 4
Broadcast and unicast communication capacities under a shock wave ($\delta = 0, n = 150$).

Transmission range (m)			50	100	150	200	250
Uni-directional	Broadcast	$t = 0$	1.93	2.33	2.57	2.60	2.73
		$t = 54$	2.00	2.40	2.60	2.63	2.80
	Unicast	$t = 0$	1.47	0.97	0.67	0.53	0.47
		$t = 54$	1.23	0.73	0.53	0.43	0.40
Bi-directional	Broadcast	$t = 0$	3.93	4.57	4.83	5.00	5.03
		$t = 54$	4.13	4.80	4.87	5.03	5.20
	Unicast	$t = 0$	2.03	1.20	0.73	0.63	0.53
		$t = 54$	1.77	0.97	0.67	0.57	0.47

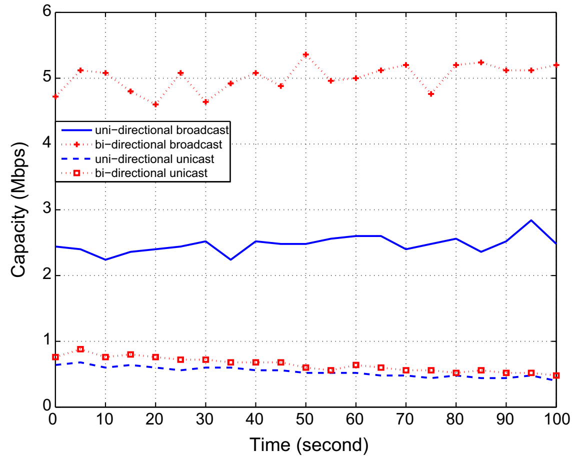

Fig. 10. Communication capacities under a shock wave ($r = 150$ m, $\delta = 0,\ n = 150$).

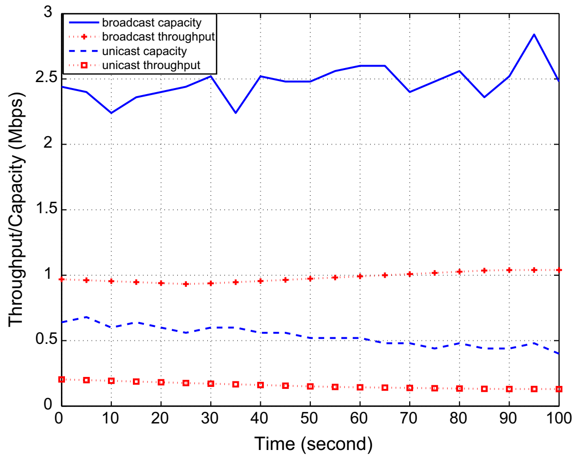

Fig. 11. Uni-directional communication throughputs/capacities under a shock wave ($r = 150$ m, $\delta = 0, n = 150$).

and it was a fundamental work of future capacity studies. Moreover, the influence of traffic dynamics on communication capacities had been investigated. A shock wave in a traffic stream could increase broadcast capacities while decreasing unicast capacities.

In the future we will consider the impact of multi-hop communications and message routing algorithms on both broadcast and unicast capacities. We also plan to apply the analysis of communication capacities to develop efficient message routing algorithms in VANETs. Furthermore, we will analyze more general and realistic traffic and communication scenarios with considerations of market penetration rates of equipped vehicles, stop-and-go waves, SINR communication model, etc. In addition, we will analytically estimate communication capacities under general traffic patterns, and integrate the analysis with advanced traffic control systems, such as green driving and cooperative adaptive cruise, to improve their reliability and efficiency.

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