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Deriving macroscopic fundamental diagrams from probe data: Issues and proposed solutions

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A B S T R A C T

Well-defined relationships between flow and density averaged spatially across urban traffic networks, more commonly known as Macroscopic Fundamental Diagrams (MFDs), have been recently verified to exist in reality. Researchers have proposed using MFDs to monitor the status of urban traffic networks and to inform the design of network-wide traffic control strategies. However, it is also well known that empirical MFDs are not easy to estimate in practice due to difficulties in obtaining the requisite data needed to construct them. Recent works have devised ways to estimate a network's MFD using limited trajectory data that can be obtained from GPS-equipped mobile probe vehicles. These methods assume that the market penetration level of mobile probe vehicles is uniform across the entire set of OD pairs in the network; however, in reality the probe vehicle market penetration rate varies regionally within a network. When this variation is combined with the imbalance of probe trip lengths and travel times, the compound effects will further complicate the estimation of the MFD.

To overcome this deficit, we propose a method to estimate a network's MFD using mobile probe data when the market penetration rates are not necessarily the same across an entire network. This method relies on the determination of appropriate average probe penetration rates, which are weighted harmonic means using individual probe vehicle travel times and distances as the weights. The accuracy of this method is tested using synthetic data generated in the INTEGRATION micro-simulation environment by comparing the estimated MFDs to the ground truth MFD obtained using a 100% market penetration of probe vehicles. The results show that the weighted harmonic mean probe penetration rates outperform simple (arithmetic) average probe penetration rates, as expected. This especially holds true as the imbalance of demand and penetration level increases. Furthermore, as the probe penetration rates are generally not known, an algorithm to estimate the probe penetration rates of regional OD pairs is proposed. This algorithm links count data from sporadic fixed detectors in the network to information from probe vehicles that pass the detectors. The simulation results indicate that the proposed algorithm is very effective. Since the data needed to apply this algorithm are readily available and easy to collect, the proposed algorithm is practically feasible and offers a better approach for the estimation of the MFD using mobile probe data, which are becoming increasingly available in urban environments.

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1. Introduction and background

Network-wide traffic relationships have been the subject of study for at least several decades (Smeed, 1966; Godfrey, 1969; Zahavi, 1972; Herman and Prigogine, 1979; Ardekani and Herman, 1987; Mahmassani et al., 1987, 1984; Olszewski et al., 1995). However, earlier efforts failed to provide a comprehensive dynamic model that was both physically realistic and verified by empirical data. Only recently did researchers finally verify with simulation and empirical data that a well-defined relationship exists between the average flow and density measured across an urban network (Geroliminis and Daganzo, 2008, 2007; Geroliminis and Sun, 2011a; Daganzo, 2007; Daganzo and Geroliminis, 2008). This relationship, known now as the Network or Macroscopic Fundamental Diagram (NFD or MFD), is very helpful for researchers and traffic management agencies to monitor the status of a traffic network, design efficient traffic control strategies, and measure the effectiveness of network efficiency improvement strategies.

Recent research has examined several different applications of the MFD for improved traffic control. One set of examples include gating strategies that carefully limit vehicle inflow into a network to avoid congested states and maximize overall efficiency (Keyvan-Ekbatani et al., 2012, 2013, 2014; Haddad et al., 2013; Geroliminis et al., 2013). Other strategies include congestion pricing strategies that make use of the MFD to determine optimal schemes (Geroliminis and Levinson, 2009; Zheng et al., 2012; Simoni et al., 2015) or routing strategies that used MFD to route across regions in a network (Yildirimoglu et al., 2015; Knoop et al., 2012). In general, these control strategies are sensitive to the functional form of the MFD.

Several studies have examined the factors that influence the attributes of an MFD. Geroliminis and Sun studied the spatial variability of vehicle density and found that it affects the shape, the scatter, and the existence of an MFD (Geroliminis and Sun, 2011b). Fortunately, networks with heterogeneous traffic distributions can be partitioned into more homogeneous regions to obtain more reliable MFDs based on the variance of link densities and spatial compactness of the partitioned regions (Ji and Geroliminis, 2012; Geroliminis et al., 2014). Various other studies have explored how driver routing influences the shape and reliability of MFDs (Saberi et al., 2014; Daganzo et al., 2011; Gayah and Daganzo, 2011; Mahmassani et al., 2013). Studies of arterial road networks controlled by different types of adaptive traffic signal systems reveal that the shape of the MFD depends on the particular signal system used and the level of heterogeneity in the system (Gayah et al., 2014; Zhang et al., 2013).

While it is convenient to use an MFD to describe the traffic status across a network and design traffic control strategies, the data needed to plot the MFD are not always readily available. In theory, the MFD of a network can be easily calculated if the trajectories of all vehicles traveling in the network are known. However, trajectories for all vehicles are generally impossible to acquire. Detector data may be used to estimate MFDs, but detector-based MFDs can have significant errors over the generalized definitions and are dependent on where the detector is placed on each link (Courbon and Leclercq, 2011; Leclercq et al., 2014; Buisson and Ladier, 2009). Fortunately, the availability of GPS technology has made it relatively simple to record the kinematic data from at least a portion of the vehicles traveling in the network. Recent research has shown that the trajectories of a subset of vehicles traveling in the network can be used to satisfactorily estimate current traffic states if the MFD is already known (Gayah and Dixit, 2013) or even to estimate the MFD itself (Nagle and Gayah, 2014) reliably and accurately. Leclercq et al. also explored the combination of detector data for flow estimates and detector data for average speed estimates (Leclercq et al., 2014). Unfortunately, these methods rely on an assumption of uniform probe vehicle distributions across a network, which might be too restrictive for practical use. For example, GPS technologies are likely to be included only in newer vehicles, and certain origin and/or destinations are more likely to have higher penetration rates of these types of vehicles than others. Trajectory-based MFD estimation methods are needed to account for the heterogeneous distribution of probe vehicles. The Leclercq et al. (2014) method also requires that detectors be placed on all links that are included in the region to properly estimate the MFD.

In this study, we test the feasibility of using probe information to estimate MFDs when the probe penetration rate is not uniform and detectors are sporadically placed across the network. The generalized definitions of traffic flow parameters are used to develop proper weighted averages of probe penetration rate across the network that can then be used to estimate average network flow and density. This method assumes that probe penetration rates are known for specific OD pairs or regions. Simulation results using INTEGRATION (Van Aerde and Rakha, 2013a, 2013b; Chamberlayne et al., 2012; Rakha et al., 2012, 2004; Rakha and Zhang, 2004b, 2004a; Van Aerde et al., 1996) verify that these weighted averages of probe penetration rate provide an obvious advantage over simple arithmetic average probe penetration rates in estimating MFDs from a subset of probe vehicle information. Furthermore, an algorithm is proposed to estimate the probe vehicle penetration rate for individual OD pairs/regions, since this information is not likely to be known a priori. This algorithm combines count data from a few fixed detectors located in a network and individual probe vehicle travel times and travel distances. Simulation tests reveal that the OD penetration rate algorithm works well, especially when the probe penetration rate varies greatly from area to area. Given the availability of fixed detector data and probe vehicle data in most networks, this algorithm is practical for advancing the estimates of MFDs using probe data from a simulation environment to the real world.

The rest of this paper is organized as follows. Section 2 derives the methodology to estimate the MFD using an appropriate equivalent average probe penetration rate across the network under the assumption of known probe penetration rates for individual OD pairs. Section 3 provides simulation results that compare the accuracy of MFD estimates using this proposed method with the calculation of a simple arithmetic mean probe penetration rate. Section 4 provides a method to estimate the probe penetration rates for individual (or regional) OD pairs using data from probes and fixed detectors. Section 5 provides some concluding remarks and discussion.

2. MFD estimation using probe vehicles

The MFD provides a relationship between network-wide averages of flow and density, both of which can be used to describe operating conditions within a network. The remainder of this section describes how the MFD can be estimated using probe data when the penetration rate of probe vehicles is consistent across the network and then extends this methodology to the case where probe penetration rates vary across individual regions or OD pairs.

2.1. Estimation of the MFD assuming uniform probe penetration rates

The generalized definitions of Edie (1965) provide precise definitions for the average density (k) and flow (q) in a network, respectively:

$$k = \frac{\sum_1^I t_i}{L_n * T} = \frac{TTT}{L_n * T} \quad \text{and} \tag{1}$$

$$q = \frac{\sum_1^I d_i}{L_n * T} = \frac{TTD}{L_n * T}, \tag{2}$$

where I is the total number of trips recorded in that analysis period (e.g. 15 min); t_i and d_i are the travel time (seconds) and distance (miles), respectively, for trip i during the given analysis period; L_n and T are the network length (miles) and analysis period length (seconds), respectively. Eqs. (1) and (2) only differ in the numerator: the definition of density uses the total time vehicles spend traveling within the network during the analysis period (TTT), while the definition of flow uses the total distance vehicles travel during the analysis period (TTD) as the numerator.

In most real-world applications, the total time and distance traveled will only be known if detailed trajectories of all vehicles are provided. However, if these data are only provided for a subset of vehicles in the network (i.e., those serving as mobile probes), Eqs. (1) and (2) cannot be applied directly. To overcome this limitation, Nagle and Gayah (2014) proposed approximations to estimate density and flow in a network assuming that the fraction of vehicles serving as mobile probes is known (ρ) and same across individual regions or OD pairs. These approximations are:

$$\hat{k} = \frac{\sum_1^{I'} t_{i'}}{\rho L_n * T} \tag{3}$$

$$\hat{q} = \frac{\sum_1^{I'} d_{i'}}{\rho L_n * T} \tag{4}$$

where $t_{i'}$ and $d_{i'}$ are the travel time (seconds) and distance (miles), respectively, for probe vehicle i'; and, I' is the total number of probe vehicles recorded within the analysis period. This method also inherently assumes that the average travel time and travel distance of probe and non-probe vehicles are the same, which should be true if the probe vehicle penetration rate is the same for all OD pairs. The total travel time (distance) is then approximated by scaling the travel time (distance) of probe vehicles by the penetration rate (ρ).[1]

2.2. Estimation of MFD from varied probe penetration rates

Although convenient, the assumption of a constant probe penetration rate across the entire network may be unrealistic in practice. GPS-equipped vehicles that are able to serve as mobile probes are not likely to be distributed uniformly in real traffic networks. More affluent areas are generally expected to have a higher rate of well-equipped vehicles that can collect travel time and speed information that can be used as probe data to inform network estimations. Additionally, the running frequencies of fleet vehicles that are used as probes (such as taxis, freight vehicles, and buses) vary by time over the course of a day. In many cases, the routes used by such vehicles are concentrated and spatially imbalanced across the network. Consequently, the assumption of a single uniform penetration rate is not likely to accurately represent the real distribution of probe vehicles.

Here, we propose a method to estimate the average network density and flow using data available from a few probe vehicles that specifically addresses the fact that probe penetration rates are likely to be non-uniform across the network. We assume that the probe penetration rate for vehicles traveling between a specific origin o and destination d, $\rho_{o,d}$, is fixed for some analysis period and that the penetration rates for all OD pairs are known a priori. The former assumption is

[1] Note that the probe penetration rate may change with time and thus should have a subscript t to reflect this dependence. Instead, all equations are provided here for a single analysis period and thus this subscript is removed to simplify their presentation.

practically realistic if the analysis periods are fairly short (e.g., less than one hour). The latter, however, is especially restrictive as probe penetration rates are not likely to be known for all OD pairs in a network. A method to estimate these values is proposed in Section 4 of this paper. For now, such an assumption is used to simplify the description of the estimation procedure.

Under the assumption that the probe penetration rates for individual OD pairs are known, the arithmetic mean of these individual OD penetration rates would provide a simple estimate of the average probe penetration rate across the entire network. This average probe penetration rate could then be applied in Eqs. (3) and (4) to estimate average network density and flow, respectively. However, this estimate might not accurately capture the probe penetration rate observed across the network. Vehicles traveling between OD pairs further apart are likely to spend more time on the network than vehicles traveling between OD pairs that are closer together, and thus would skew the average mobile probe penetration rate actually observed in the network. This influence needs to be captured in order to generate the appropriate average mobile probe penetration rate to apply within Eqs. (3) and (4) to obtain accurate estimates of flow and density.

Instead, we use the generalized definitions of Edie to determine the appropriate way to calculate the average mobile probe penetration rate in a network with the penetration rates for individual OD pairs that are not equal. Let us first focus on the estimate of average network density. Eq. (1) shows that average density during any period is proportional to the total travel time of all vehicles that traveled in the network within that period. This total travel time can be rewritten as the sum of total travel times for vehicles traveling between every OD pair. If the total travel time of all vehicles traveling between OD pair o, d within a particular analysis interval is defined as $TT_{o,d}$, the total travel time is simply $TTT = \sum_{o,d}^{O,D} TT_{o,d}$, where O and D represent the total number of origins and destinations, respectively. Similarly, define the total travel time of probe vehicles traveling between each OD pair as $TT'_{o,d}$. Under the reasonable assumption that the average travel time for probe and non-probe vehicles between a specific OD is nearly equal, which is realistic if probe and non-probe drivers generally follow the same routes or act in the same way, $TT_{o,d} = TT'_{o,d}/\rho_{o,d}$. Thus, the total travel time for all vehicles can also be written as $TTT = \sum_{o,d}^{O,D}(T'_{o,d}/\rho_{o,d})$.

Examination of Eqs. (1) and (3) reveal that $TTT = \sum_1^{I'} t_{i'}/\rho$ under the assumption of consistent probe penetration rates. We propose now that some equivalent average probe penetration rate exists, ρ_{eq}^k, such that the same equality would hold when probe penetration rates are not consistent across the network; i.e., some ρ_{eq}^k exists such that $TTT = \sum_1^{I'} t_{i'}/\rho_{eq}^k$. This definition of TTT can be combined with the definition from the previous paragraph to estimate this equivalent average penetration rate (for the estimation of network density), which is simply:

$$\rho_{eq}^k = \frac{\sum_1^{I'} t_{i'}}{\sum_{o,d}^{O,D} T'_{o,d}/\rho_{o,d}}, \tag{5}$$

which turns out to be the harmonic mean of individual OD pair probe penetration rates using the probe vehicle travel time between each OD pair as the weight. Substituting this equivalent penetration rate for ρ into Eq. (3) should thus provide a more accurate estimate of the network density when probe penetration rates vary across OD pairs within the network than using a simple arithmetic mean. ρ_{eq}^k can be easily determined using data from the probe vehicles themselves, assuming that the individual OD penetration rate is known.

The same logic can be used to derive the appropriate equivalent average probe penetration rate required to estimate network flow:

$$\rho_{eq}^q = \frac{\sum_1^{I'} d_{i'}}{\sum_{o,d}^{O,D} D'_{o,d}/\rho_{o,d}}. \tag{6}$$

Here $D'_{o,d}$ represents the total distance traveled by probe vehicles between any OD pair od. This equivalent penetration rate can then be substituted into Eq. (4) for ρ to estimate network flow when probe penetration rates vary across ODs within the network.

3. Results from combinations of varied probe penetration rates

The previous section reveals that a weighted harmonic mean probe penetration rate should be used to appropriately estimate average network flow and density from probe data when applying the method suggested in Nagle and Gayah (2014). A simple arithmetic mean is not appropriate as it will not provide an estimate that is consistent with the generalized definitions of Edie. In this section, simulated trajectory data is used to estimate the MFD using the method outlined in Section 2.2 and compared to estimates obtained using a simple arithmetic mean probe penetration rate.

First, the simulation framework and accuracy measures are described. Then, the results for various heterogeneous probe penetration rates under uniform traffic demands are examined. Lastly, the results for various heterogeneous probe penetration rates under heterogeneous traffic demands are examined.

3.1. Network description and accuracy measures for MFD

An idealized 16 $\times$ 16 square grid network with alternating one-way streets is simulated for the tests performed in this study. The network consists of a 544 links, each 400 feet long. For simplicity, origins and destinations are placed at all entry and exit links, respectively; therefore, a total of 64 origin/destination zones are used. For most of tests performed in this subsection, a uniform demand is generated between each origin–destination (OD) pair. A 3-h simulation is used in which the first hour represents a warm-up period, the second hour is the peak demand rate, and the third hour is a recovery period. The demand rate for the first and third hours is 6 vehicles/hour per OD pair, while the peak demand is 17 trips/hour for each OD pair.

To evaluate the accuracy of the estimated MFD, three measures of effectiveness (MOEs) are used (Nagle and Gayah, 2014). These measures of effectiveness represent the root mean square error (RMSE) of the estimates of average network flow, average network density, and combination of average network flow and density simultaneously compared to the ground truth observations. The three measures are defined as follows:

$$RMSE(q) = \sqrt{\sum (q - \hat{q})^2 / N}, \tag{7}$$

$$RMSE(k) = \sqrt{\sum (k - \hat{k})^2 / N}, \quad \text{and} \tag{8}$$

$$RMSE(q,k) = \sqrt{\sum \left[\left(\frac{q - \hat{q}}{q_c} \right)^2 + \left(\frac{k - \hat{k}}{k_j} \right)^2 \right] \Big/ N}. \tag{9}$$

Here, q and k are the ground truth average network flow and density, respectively, calculated using all the trajectory data as per the generalized definitions of Edie. The terms $\hat{q}$ and $\hat{k}$ are the estimated average network flow and density calculated using Eqs. (3) and (4), respectively, where the market penetration rate is estimated using one of the weighted average probe penetration rates from Eqs. (5) or (6). The terms q_c and k_j represent the maximum flow (i.e., capacity) and jam density observed in the network, respectively. N is the number of time intervals within the simulation.

3.2. Varied penetration rates by OD

Six simulation scenarios (A–F) were considered to examine the effects of different overall penetration rates and levels of heterogeneity between penetration rates across OD pairs in different regions on MFD estimation. To simplify the analysis, the network was broken down into the four quadrants shown in Fig. 1. Probe penetration rates for specific OD pairs were defined based on the regional origins and destinations. The specific scenarios considered were:

A. ODs from region **I** to region **II** have a penetration rate of 0.8. All other ODs have a penetration rate of 0.1.
B. ODs from region **I** to region **II** have a penetration rate of 0.5. All the other ODs have a penetration rate of 0.1.
C. ODs within region **I** have a penetration rate of 0.8. All the other ODs have a penetration rate of 0.1.
D. ODs within region **I** have a penetration rate of 0.5. All the other ODs have a penetration rate of 0.1.
E. ODs that have origins in region **I** have a penetration rate of 0.8. All the other ODs have a penetration rate of 0.1.
F. ODs that have origins in region **I** have a penetration rate of 0.5. All the other ODs have a penetration rate of 0.1.

The scenarios were designed to represent idealized probe vehicle distribution patterns that might exist in typical networks. Scenarios A and B are representative of a situation in which a particular corridor has a higher probe penetration rate than other areas. Scenarios C and D are representative of a situation in which a particular region has a higher probe penetration rate than other regions. Scenarios E and F are representative of a situation in which vehicles traveling from a certain region are more likely to be probe vehicles (perhaps because the region is more affluent). The higher probe penetration rates are set to 0.8 or 0.5 and compared to a lower penetration rate of 0.1 to examine the impacts of a difference in probe penetration rates by regions on the estimation results.

The simulation was run once and the trajectory of all vehicles were extracted to obtain the true MFD. Subsets of the vehicle trajectories (representing the probe vehicles) were then randomly extracted based on the scenarios above to provide the data used to estimate the MFD. To eliminate the fluctuations generated by the randomness of sampling, this process was repeated 50 times and the average RMSE is calculated over the 50 samples. Fig. 2 presents the estimated MFD using the proposed methodology and the true MFD using one of the 50 samples for Scenarios A through F. Table 1 presents the MOEs of estimation accuracy for all MFDs obtained for Scenarios A through F using the average results of the 50 samples. These results are provided for two cases: (1) using a simple arithmetic average to compute the mean probe penetration rate; and, (2) using the harmonic mean penetration rate provided by Eqs. (5) and (6). In both cases, the penetration rates for individual OD pairs are assumed to be known a priori. Note that for all MOEs, smaller values indicate better performance.

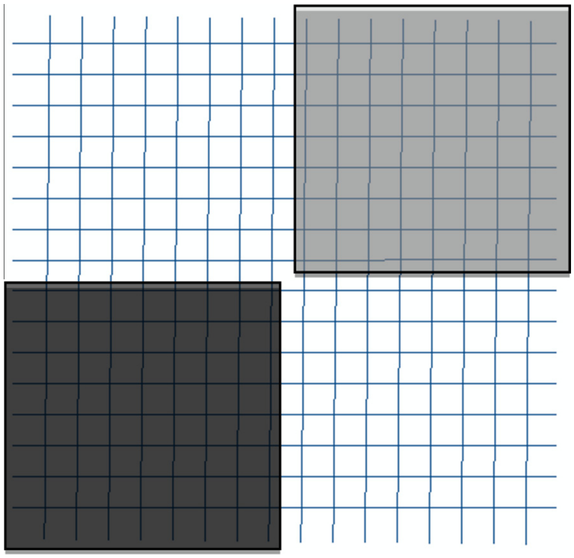

Fig. 1. Idealized grid network with varied penetration rate.

As can be seen, the harmonic mean penetration rate outperforms the arithmetic average penetration rate in almost all cases. The only exceptions appear to be Scenarios E and F, for which the assumption of a uniform probe penetration rate provides better estimations. However, the differences in the RMSEs are rather small for these scenarios. Two-sample T-test results were then performed to compare the RSMEs between the two cases (assuming uniform penetration rates and assuming heterogeneous penetration rates); the results are provided in Table 2. As shown, the differences are not statistically significant in Scenarios E and F; for these scenarios the weighted harmonic mean method is similar to the arithmetic mean method. For the remaining four scenarios, the RMSE generated by the arithmetic average method is significantly larger than the weighted average. A closer examination of these two tables suggests that more accurate estimates are obtained using the harmonic mean method when probe vehicles are more concentrated within a network (Scenarios A through D) than when they are spread out across the network (Scenarios E and F). In the latter case, a simple arithmetic average might be sufficient. This occurs primarily because the harmonic mean method is a weighted average that uses the individual probe travel distances and travel times as weights. When probe vehicles are spread out across the network, these weights would tend to be the same and thus a simple arithmetic mean might be sufficient.

The advantages of using a weighted probe penetration rate also appears to increase with the imbalance of probe penetration rates across individual OD pairs. This can be observed by examining scenario pairs A–B, and C–D. In both cases, the scenario with the larger imbalance of probe penetration rate has the larger difference in RSME between the arithmetic and harmonic methods. To explore this further, a second set of tests is performed in which we fix the total number of probe vehicles and vary the distribution of penetration rate across different regions. The average probe penetration rate across the entire network is kept fixed at 20%, while the probe penetration rate for vehicles traveling within region I is increased from 20% to 90%. Note that the penetration rate for the remaining OD pairs decreases accordingly to maintain the same overall average penetration rate at 20%.

Table 3 provides the resulting RMSEs when the simple arithmetic average method and the proposed weighted harmonic average method is applied. Fig. 3 illustrates these RSME values as well as the benefits of the harmonic method over the arithmetic method as a function of the probe penetration rate imbalance. Note that the orange triangles with the second vertical axis represents the percentage improvement associated with the use of the weighted average rate. As can be seen, more significant advantages are achieved using the harmonic mean penetration rate as the difference between the higher penetration rates versus the lower penetration rates increases. In general, the harmonic mean method provides a consistent RSME value as the penetration rate difference increases. However, the larger differences result in poorer performance using the arithmetic average penetration rate.

3.3. Varied penetration rates by OD with imbalanced congestion levels

The tests performed in Section 3.2 are all based on uniform demand conditions in which each OD pair generates the same number of trips. In reality, network demands are unbalanced (and often highly unbalanced), and some areas of a network become much more congested than others. This congestion can be contained within a specific area in the network, such as the downtown central business district (CBD) or along a certain corridor. How this imbalanced demand will jointly affect the estimated MFD using varied probe penetration rates is explored in this section. For these tests, the original balanced ODs

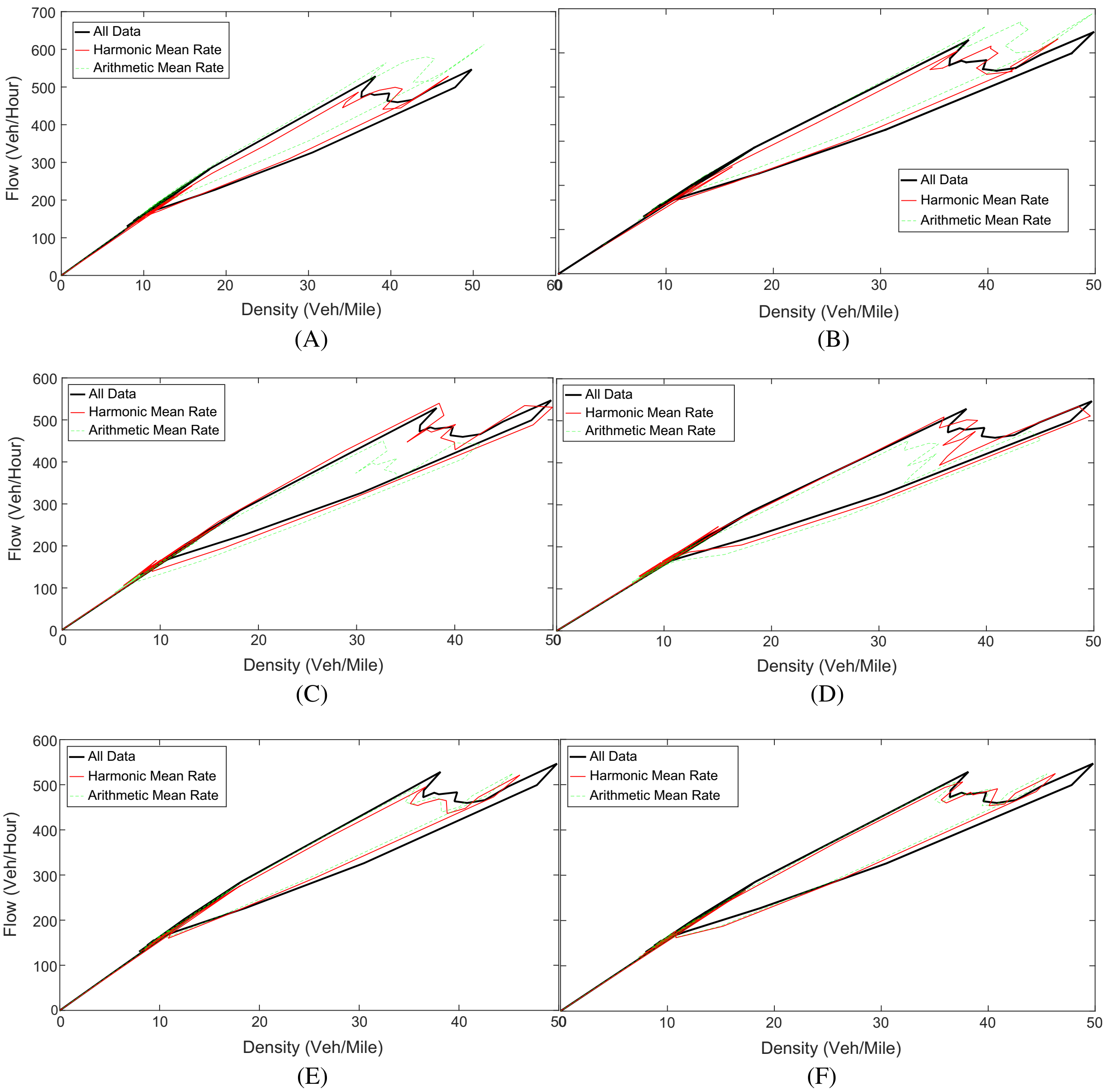

Fig. 2. MFD for Scenarios A through F.

Table 1
RMSE for Scenarios A through F.

RMSE	Scenario A	Scenario B	Scenario C	Scenario D	Scenario E	Scenario F
Arithmetic mean rate						
Average flow (veh/h)	41.9	28.58	66.83	50.73	14.36	14.66
Average density (veh/mi)	2.02	1.66	4.84	3.73	2.07	2.13
Combined flow and density	0.05	0.03	0.08	0.06	0.02	0.02
Harmonic mean rate						
Average flow (veh/h)	22.47	23.72	25.22	25.2	15.59	15.83
Average density (veh/mi)	2.12	2.02	1.82	1.87	1.66	1.72
Combined flow and density	0.03	0.03	0.03	0.03	0.02	0.02

Table 2
Two sample T-test for the difference in RMSEs between Arithmetic Mean Method and Harmonic Mean Method.

	Scenario A	Scenario B	Scenario C	Scenario D	Scenario E	Scenario F
Difference in RMSE	0.0211	0.00477	0.0493	0.0305	−2.00E−05	0.00009
t Value	27.25	6.72	124.39	75.78	−0.19	0.67
Pr > |*t*|	<.0001	<.0001	<.0001	<.0001	0.8532	0.5038

Table 3
Impacts of imbalanced penetration rates.

High rate in region I	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
Total probe vehicles	5961	5943	5937	5947	5925	5896	5934	5949
Arithmetic mean rate								
Average flow (veh/h)	14.24	17.03	19.47	23.07	27.48	33.30	37.49	44.03
Average density (veh/mi)	1.06	1.24	1.40	1.71	1.91	2.34	2.57	3.08
Combined flow and density	0.02	0.02	0.02	0.03	0.03	0.04	0.04	0.05
Harmonic mean rate								
Average flow (veh/h)	14.16	15.85	15.77	16.00	15.07	16.56	15.73	18.01
Average density (veh/mi)	1.04	1.17	1.18	1.17	1.06	1.18	1.13	1.30
Combined flow and density	0.02	0.02	0.02	0.02	0.02	0.02	0.02	0.02

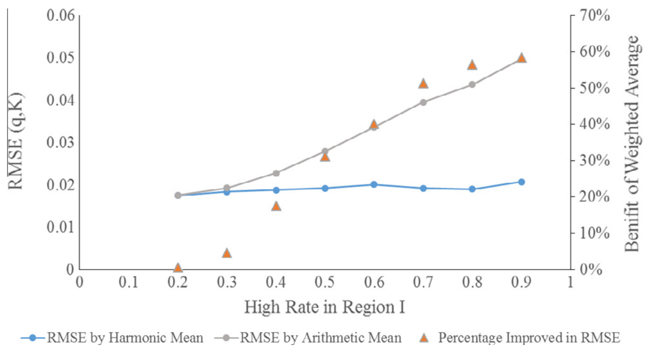

Fig. 3. Benefit of using weighted average for imbalanced penetration rates.

are changed such that demands for ODs within region I are increased by 50% while all other OD demands are decreased to 90% of their original value. Three simulation scenarios (i–iii) with varying OD penetration rates are considered:

i. The penetration rate for ODs within region I is 0.8 while for all the others it is 0.1.
ii. The penetration rate for ODs within region I is 0.5 while for all the others it is 0.1.
iii. The penetration rate for ODs within region I is 0.3 while for all the others it is 0.1.

The results are provided in Table 4. They indicate that when the congestion in the network is contained within a specific area, the harmonic mean method maintains its advantage over using an arithmetic mean. However, this advantage diminishes as the difference in penetration rates decreases between the congested and non-congested areas.

Another set of scenarios are tested where the congestion within the network is limited to a specific corridor. For these tests, the original balanced ODs are changed such demands for trips from region I traveling to region II increase to 150% of their original value, while all demand values for all other OD pairs stay the same. Three simulation scenarios (iv-vi) with varying OD penetration rates are considered:

i. The penetration rate for ODs from region I to region II is 0.8, while for all other OD pairs it is 0.1.
ii. The penetration rate for ODs from region I to region II is 0.5, while for all other OD pairs it is 0.1.
iii. The penetration rate for ODs from region I to region II is 0.3, while for all other OD pairs it is 0.1.

Table 5 provides the results for the corridor congestion case. The results are similar to the congested region case. The benefits of the harmonic mean method shows greater improvement over the use of a simple arithmetic mean probe penetration rate as the probe penetration rate of the congested region increases. This observation is meaningful to the practice of traffic congestion monitoring and control in that usually reproducible congestion occurs within a specific area (such as the

Table 4
RMSE for imbalanced demands and varied penetration rate (congested region).

RMSE	0.8/0.1 Region 1/Others (Scenario i)	0.5/0.1 Region 1/Others (Scenario ii)	0.3/0.1 Region 1/Others (Scenario iii)
Arithmetic mean rate			
Average flow (veh/h)	42.63	37.02	32.3
Average density (veh/mi)	1.94	1.91	1.96
Combined flow and density	0.05	0.04	0.04
Harmonic mean rate			
Average flow (veh/h)	24.9	26.32	27.11
Average density (veh/mi)	1.91	1.88	1.96
Combined flow and density	0.03	0.03	0.03

Table 5
RMSE for imbalanced demands and varied penetration rate in different areas.

RMSE	0.8/0.1 Region I to II/Others (Scenario iv)	0.5/0.1 Region I to II/Others (Scenario v)	0.3/0.1 Region I to II/Others (Scenario vi)
Arithmetic mean rate			
Average flow (veh/h)	104.99	66.31	36.21
Average density (veh/mi)	6.3	3.96	2.24
Combined flow and density	0.12	0.08	0.04
Harmonic mean rate			
Average flow (veh/h)	22.79	23.79	24.92
Average density (veh/mi)	1.92	1.92	1.91
Combined flow and density	0.03	0.03	0.03

downtown area of urban networks) while at the same time the mobile probe penetration rate in such areas might be much higher than the rest of the network. Consequently, the advantage of using a weighted penetration rate will be significant in improving network monitoring in such realistic situations.

4. Probe penetration rate estimation

The simulation results performed in the previous section confirm that the calculation of a harmonic mean penetration rate is needed to accurately estimate average network flows and densities using the methods outlined in Section 2. The benefits over using a simple arithmetic mean penetration rate is especially obvious when probe penetration rates across individual OD pairs and/or the demand is very imbalanced. The method describe and used is based on the assumption that the probe penetration rates for each OD pair are known a priori. However, in practice, these individual OD probe penetration rates are not likely to be known a priori and might change widely over time.

Previous research showed that it is reliable to estimate the penetration rate by sampling probe vehicles at certain fixed traffic detectors in the network (Nagle and Gayah, 2014). The aggregate count of all vehicles, N^d, and probe vehicles, N_p^d, that crossed a detector can be determined and used to estimate the uniform probe penetration rate, $\hat{\rho}$. This method was found to be effective and accurate when the probe penetration rate is uniform across the network. Since the penetration rate is not uniform in this study, as would be the case in a real world application, a method is needed to estimate the individual OD penetration rates to be used in Eqs. (3) and (4) to provide the appropriate equivalent probe penetration rate in a network for traffic state estimation purposes.

We propose a method to estimate the probe penetration rate of individual OD pairs (or regional OD pairs) by combining probe data with data obtained from fixed sensors (e.g. loop detectors) on links. Our method relies on two fundamental assumptions. The first is that detectors located closer to origins (or destinations) with higher probe penetration rates should be more likely to observe higher probe penetration rates. The second is that ODs with higher probe penetration rates should have larger total number of probe trips observed when demands are not extremely imbalanced. The proposed algorithm works by matching the two sets of data (observed penetration rates at individual detectors and observed counts of probe vehicles across individual OD pairs). The k-means clustering analysis method is used to match these two datasets. Using k-means clustering, the two datasets are partitioned into n clusters in such a way that within-cluster differences (measured by the sum of squared differences between the individual observations and the mean value for that cluster) are minimized. These k-mean results are then used to estimate the probe penetration rate of the individual OD pairs as follows:

(1) Use k-means clustering analysis to group the detected penetration rates at individual detectors within the network into n clusters. The mean penetration rate within each of these n clusters are calculated and stored in vector 1.
(2) Aggregate the number of probe vehicles passing each of these detectors by OD pairs.

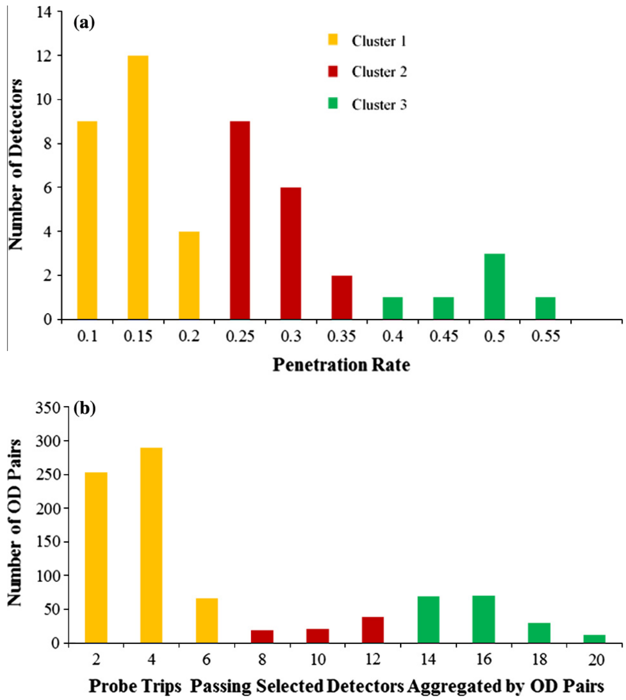

Fig. 4. Histograms of (a) detected penetration rate and (b) observed aggregated probe trips.

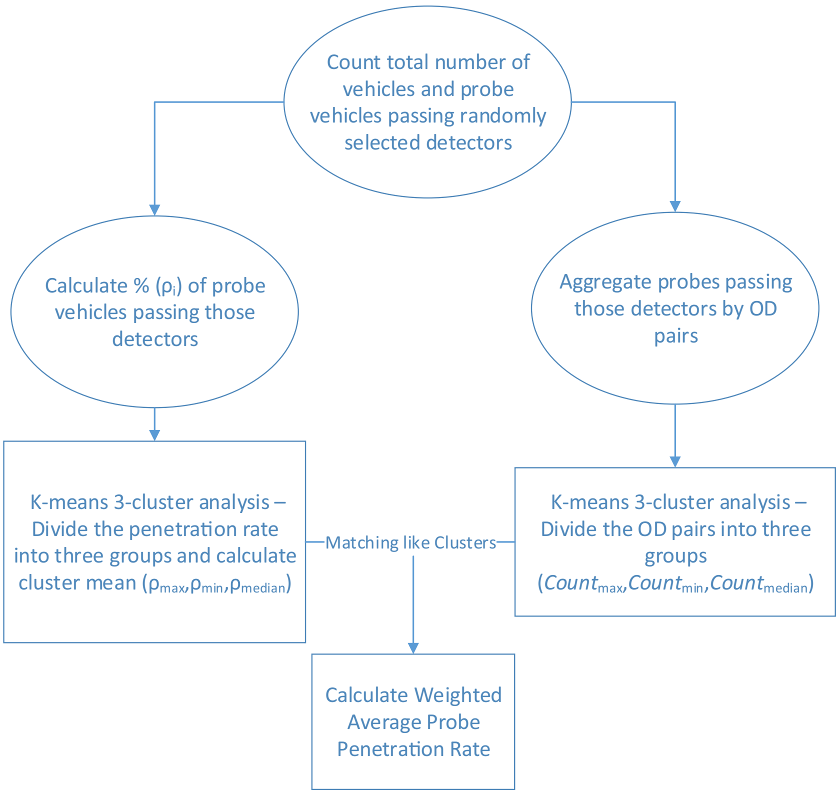

Fig. 5. *K*-mean clustering analysis for estimating weighted average probe penetration rate.

Table 6
RMSE of MFD from estimated weighted average probe penetration rate (0.8/0.1 Upper Right OD/Others).

Arithmetic mean rate					
Average flow (veh/h)	66.83				
Average density (veh/mi)	4.84				
Combined flow and density	0.08				
	2-Clusters	3-Clusters	4-Clusters	5-Clusters	6-Clusters
Harmonic mean rate					
Average flow (veh/h)	199.74	**28.61**	33.12	111.67	276.08
Average density (veh/mi)	16.03	**2.10**	2.43	2.71	3.48
Combined flow and density	0.24	**0.03**	0.04	0.13	0.31

The bold values in the column of "3-Clusters" indicate they are the optimum results.

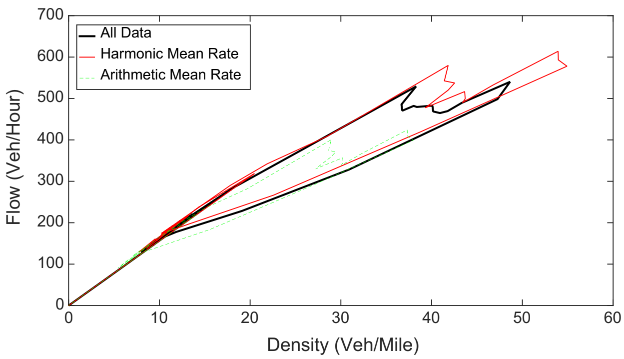

Fig. 6. MFD estimated using cluster method.

(3) Use k-means clustering analysis to group the trip number counts (by OD pairs) into n clusters. The trip count mean within each of these n clusters are calculated and stored in vector 2.
(4) Sort vectors 1 and 2, individually. Pair the two sorted vectors such that each element in vector 1 has a corresponding counterpart at the same rank position in vector 2. Assign each OD pair in vector 2 the corresponding mean penetration rate value at the same ranking position in vector 1.

To illustrate how this algorithm works, we now set the number of clusters to three as an example (as illustrated in Fig. 4). The probe penetration rates are divided into three groups as shown in Fig. 4(a) based on the three clusters. The three mean penetration rates of each of these clusters are defined as cluster 1 (ρ_{min}), cluster 2 (ρ_{median}), and cluster 3 (ρ_{max}) and refer to the min, middle and max values, respectively. Similarly, the vehicle counts by OD pairs are divided into three clusters as shown in Fig. 4(b). The mean values of these counts in each of these groups are defined as cluster 1 ($Count_{min}$), cluster 2 ($Count_{median}$), and cluster 3 ($Count_{max}$) and refer to the max, middle and min values, respectively. These vectors (penetration rates and counts) and then paired together such that the highest values in each vector are matched together, median values are matched together, and minimum values are paired as well. Fig. 5 provides a flowchart for this process for the case when the cluster number is set to 3.

Table 6 presents the RSME of the average network flow and density estimates using the individual OD probe penetration rates using this method for Scenario C (described in Section 3.2). The results illustrate a robust output when the harmonic mean method is applied. Encouragingly, the magnitudes of the RSME values are comparable with the calculations above when the individual OD probe penetration rates were known with certainty.

To examine the impacts that the number of clusters selected may have on the accuracy level of this proposed method, four additional values of n are also tested and considered. The RMSE of each case is also included in Table 6. These results all verify that the optimal number of clusters that should be selected is 3. Fig. 6 provides the MFD resulting from this method. Notice that the matching k-means clustering analysis proves to be very effective in estimating the MFD. Comparing to the results generated when the penetration rate is a known priori (Scenario C from Table 1), the RMSE increases only by a small amount.

5. Summary and discussion

5.1. Summary

In this study, a methodology is proposed to estimate a network's MFD using data from a limited number of probe vehicle trajectories when probe penetration rates are not uniformly distributed across a network. This methodology incorporates the distance and time probe vehicles traverse a network into the calculation of the equivalent probe penetration rates observed in the network using harmonic means. These equivalent penetration rates are then used to estimate network-wide traffic quantities such as average flow and average density using aggregated data obtained from mobile probe vehicles.

Simulation results are used to compare the proposed method with a method using simple arithmetic mean of probe penetration rates previously proposed in the literature under the assumption that individual OD probe penetration rates are known a priori. The results indicate that the proposed harmonic mean method is generally more accurate. This is especially true in situations when the demands in the network are unbalanced and when the penetration rates in the network vary significantly from area to another. In general, the estimates of the MFD vary significantly using the arithmetic average penetration rate especially when the spatial heterogeneity of traffic demand and penetrate rates in the network is large. On the contrary, the methodology proposed in this study generates a more stable result regardless of these imbalances in either demand or penetration rate. This stability of the proposed methodology makes it a more confident estimation tool that is robust to be used in different networks with different congestion patterns.

Because it is not realistic to know the exact probe penetration rate for each OD pair across the network in practical applications, the study proposes an algorithm to estimate the weighted probe penetration rate using fixed sensor data and sample probe data. This methodology involves linking two k-means clustering analyses: one for the detected percentage of probe vehicles passing randomly selected links with detectors in the network and the other for the probe trip counts by OD pairs. The significance of this proposed algorithm is that the two data sources are likely to be easily collected or readily available in practice. Estimating the probe penetration rate by combining loop detector counts with the probe vehicle traveling data, such as origins, destinations, travel times, and distances, is feasible and realistic. The resulting MFD using the estimated weighted average probe penetration rate generates a much smaller RMSE compared to using an average link detected probe penetration rate.

5.2. Discussion

This study provides a significant step towards practical estimations of Macroscopic Fundamental Diagrams using a combination of data that are generally available (detector counts) and data that are becoming increasingly available through a variety of sources (trajectory data from GPS-enabled mobile probe vehicles). This is significant as while MFDs have been proposed as a computationally efficient tool for modeling urban traffic networks and the development of network-wide control strategies, very few empirical MFDs actually exist. The availability of a theoretically valid MFD estimation methodology—such as the one proposed here—is vital towards the growing use of the MFD in urban traffic network monitoring and control.

The proposed methodology has been shown to be consistent with the general definitions of Edie and provides very good results in a simulation environment when probe penetration rates by OD pairs are known a priori. Since this information would never be known, a practical way to estimate these OD probe penetration is also proposed. Although this methodology was developed in a heuristic way, it shows remarkably good performance and results in MFDs that are consistent with those estimated with full a priori knowledge of probe penetration rates across the network. The underlying assumptions that form the basis of this methodology—namely, that observed probe penetration rates should be higher at detectors near OD pairs with high penetration rates and that OD pairs with higher probe penetration rates should have higher total probe trips—are fairly reasonable under a variety of conditions. Of course, these assumptions might not be valid under some conditions. For one, when demands are very imbalanced, trips counts from probe vehicles might not be proportional to probe penetration rates as the total number of trips generated between OD pairs must also be considered. Still, this assumption would hold when traffic demands are fairly uniform, which are when well-defined MFDs are generally to be expected. Vehicle routing might also influence the relationship between penetration rates at fixed detectors and OD pairs. However, again, one would expect penetration rates at detectors to mimic the penetration rates of nearby OD pairs when vehicles route themselves evenly across a network, which again is a reasonable requirement for the existence of MFDs in urban networks.

As with any research effort, further extensions are required. These include:

1. Identifying the impacts of locations of the links where the detectors should be placed to estimate probe penetration rates. In this study, the selection of such links was completely random. However, results might be improved if the detectors were carefully selected based on prevailing traffic patterns. Preliminary suggestions for selecting links are to find locations that are geographically dispersed across the network when congestion is fairly homogeneous, selecting links proportional to the level of congested experienced on the links, or selecting links highly likely to be used by re-routing drivers when the congestion occurs. Furthermore, how varied number of links with detectors will impact the results and how the number links is related to the optimum number of clusters is a topic requiring further examination.
2. Improving the k-mean clustering model by identifying the number of clusters based on features of the network and/or demands in the network. A recent study (Leclercq et al., 2014) concluded that using loop detector data to estimate the MFD is not an optimum solution compared to probe data due to the impacts of spillbacks on detector observations. In the next step of the research, we will vary the penetration rates in the network such that there are more varied penetration rates existing in different areas of the network to identify and quantify the impacts this might have on the proposed methodology.

3. Considering the use of a maximum-likelihood synthetic OD estimator to estimate the OD from the link counts and then computing the OD-specific penetration level using the estimated OD demand. This method will be compared to the OD estimates using the proposed clustering approach that was developed in this paper.
4. Studying the changes and shifting on MFDs generated by network-wide traffic control strategies and better understanding how the MFD can be used as a monitoring and control tool to alleviate traffic congestion.

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