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Solving the User Optimum Privately Owned Automated Vehicles Assignment Problem (UO-POAVAP): A model to explore the impacts of self-driving vehicles on urban mobility

Gonçalo Homem de Almeida Correia[a,*], Bart van Arem[b]

[a] Delft University of Technology, Faculty of Civil Engineering and Geosciences, Department of Transport & Planning, Room 4.22, Stevinweg 1, 2628 CN Delft, The Netherlands
[b] Delft University of Technology, Faculty of Civil Engineering and Geosciences, Department of Transport & Planning, Room 4.13, Stevinweg 1, 2628 CN Delft, The Netherlands

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ABSTRACT

Interest in vehicle automation has been growing in recent years, especially with the very visible Google car project. Although full automation is not yet a reality there has been significant research on the impacts of self-driving vehicles on traffic flows, mainly on interurban roads. However, little attention has been given to what could happen to urban mobility when all vehicles are automated. In this paper we propose a new method to study how replacing privately owned conventional vehicles with automated ones affects traffic delays and parking demand in a city. The model solves what we designate as the User Optimum Privately Owned Automated Vehicles Assignment Problem (UO-POAVAP), which dynamically assigns family trips in their automated vehicles in an urban road network from a user equilibrium perspective where, in equilibrium, households with similar trips should have similar transport costs. Automation allows a vehicle to travel without passengers to satisfy multiple household trips and, if needed, to park itself in any of the network nodes to benefit from lower parking charges. Nonetheless, the empty trips can also represent added congestion in the network. The model was applied to a case study based on the city of Delft, the Netherlands. Several experiments were done, comparing scenarios where parking policies and value of travel time (VTT) are changed. The model shows good equilibrium convergence with a small difference between the general costs of traveling for similar families. We were able to conclude that vehicle automation reduces generalized transport costs, satisfies more trips by car and is associated with increased traffic congestion because empty vehicles have to be relocated. It is possible for a city to charge for all street parking and create free central parking lots that will keep total transport costs the same, or reduce them. However, this will add to congestion as traffic competes to access those central nodes. In a scenario where a lower VTT is experienced by the travelers, because of the added comfort of vehicle automation, the car mode share increases. Nevertheless this may help to reduce traffic congestion because some vehicles will reroute to satisfy trips which previously were not cost efficient to be done by car. Placing the free parking in the outskirts is less attractive due to the extra kilometers but with a lower VTT the same private vehicle demand would be attended with the advantage of freeing space in the city center.

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* Corresponding author. Tel.: +31 152781384.
E-mail addresses: g.correia@tudelft.nl (G.H.d.A. Correia), B.vanArem@tudelft.nl (B. van Arem).

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1. Introduction

A potentially huge disrupter of the mobility system, which once was only dreamt of by futurists, is now rapidly developing: the automated vehicle (AV). Many research groups all over the world are working and competing on vehicle automation development and the first prototypes are starting to roll out from design to reality. One of the most conspicuous models is the Google car (Urmson, 2015).

Although technology can develop under the sheer will to innovate, its innovative technical features may blur our view as to what will really happen in the future. It might fit a purpose at a certain point in time, but its consequences for our daily lives are sometimes difficult to foresee. Automobile technology when it was first developed is a case in point, because hardly any of the car industry visionaries were thinking about the environment when they switched from electric vehicles to combustion motors (Cavadas et al., 2015). Thus, despite the fast technological development in the field of vehicle automation, where pilot testing is showing that we will soon be able to create fully automated vehicles, there is great uncertainty about the subsequent changes to traffic, mobility and cities as we know them (Correia et al., 2015; Fagnant and Kockelman, 2015).

Most of the research done so far on this topic has looked at the impact that several levels of automation and cooperation between vehicles will have on traffic flows on interurban roads. Extensive research has shown the benefits and limitations of several configurations of the fleet composition in a transition time, and the type of traffic element such as a continuous uniform road, a bottleneck or on-ramp on a freeway (Reece and Shafer, 1993; Van Arem et al., 1996; Kesting et al., 2010; Arnaout and Bowling, 2011; Calvert et al., 2011; Hoogendoorn et al., 2014).

With respect to the effects of automation on mobility, which include mode choice, transport costs and parking demand, research has only recently started to look at that topic. Some studies have been published on simulating shared taxi fleets composed of AVs, which the authors foresee will replace privately owned vehicles. It is argued that having fully automated vehicles means that they can be used in practice as a transit system that will be cheaper than taxis (no driver costs) (Martinez et al., 2014). Fagnant and Kockelman (2014) used an agent-based simulation model to study the implications of having a fleet of AVs in a city to serve part of its mobility needs. They concluded that each AV would be able to replace eleven conventional ones but could incur 10% more traveling to reach the next traveler waiting to be picked-up (Fagnant and Kockelman, 2014; Fagnant et al., 2015). The International Transport Forum (ITF) built a model to test the introduction of 100% automated fleets of taxis to satisfy transport demand in a city (Itf, 2015). They modeled a mid-size European city (Lisbon, Portugal) where the only public transport (PT) mode retained besides the automated taxi is the metro system. Results showed that fleet size always decreases: with the metro, each AV could remove 9 out of 10 cars in the city if a maximum 5 min waiting period can be guaranteed, whilst without the metro 5 vehicles would be removed per AV.

Spieser et al. (2014) had the same objective, but they tested the replacement of all vehicles with automated ones for the city of Singapore. They used an analytical mathematical formulation concluding that it would be possible to meet the total personal mobility needs of the entire population with 1/3 of the total number of passenger vehicles currently in operation. Zhang and Pavone (2014) used queuing theory to study the replacement of the taxi demand in Manhattan for a fleet of AVs concluding that 8000 vehicles would be enough to satisfy the existing demand (roughly 60% of the current fleet).

What these recent studies do not discuss is the assumption that people may choose not to own a vehicle, and that they may sometimes be willing to share a private vehicle with strangers in situations where several unrelated people travel in the same vehicle. This has actually been demonstrated to entail a perceived disutility that could be hard to overcome (Correia et al., 2013). While there is a tendency for the motorization rate of Western families to decrease, leveraged by the growth of the shared economy and changes in prioritizing how to spend the available income, owning and using an automobile is still linked to both instrumental and symbolic-affective motives. These will take time to disappear, and one could argue the link will never be entirely broken (Steg et al., 2001). In fact, motorization is still increasing in many countries in Europe as economic conditions improve (European Commission, 2014). Therefore, in this study, we look at the unexplored effects of full-automation in a scenario where vehicles are still mostly privately owned.

An interesting recent study addressed the modeling needs of privately owned AVs by proposing a modified 4-step model. Levin and Boyles (2015) change the classic 4-step model to address the option of empty vehicle relocation after a vehicle drops its driver off. This is compared with the cost of parking at the destination and using public transport for the same trip. Despite recognizing the advantage of an AV being able to move while empty, the method so far ignores the implication that a relocation can lead to higher or lower costs, depending on the next trips due to be served by that vehicle in the same household, because the model is only applied to a peak hour.

We contribute to the research on the impact of AVs by setting two main objectives intended to bridge two important knowledge gaps. The first is to establish a method of assigning privately owned AV trips to a road network, a method that has to go beyond the simulation techniques currently being used and that should be supported by the existing theory on traffic assignment and vehicle routing fields of research. The second is to apply that method to a case study to enable us to disclose some of the potential effects on traffic congestion and parking demand of automating privately owned vehicles.

The method we propose is intended to be used to study the impact of privately owned vehicles in a city where the only two transport options are privately owned fully automated vehicles (level 5) (SAE International, 2014) and PT (e.g. buses). In this context, level 5 automation is defined as: “the full-time performance by an automated driving system of all aspects of the dynamic driving task under all roadway and environmental conditions that can be managed by a human driver”. The model built for that purpose should be able to generate routes and departure times for the vehicle(s) owned by families who live in that city that will satisfy as many trips as it is cost efficient to satisfy, in a scenario with and without vehicle automation (it should be possible to relax the automation property of the vehicles for comparison purposes). This model should measure the likely impact of vehicle automation on the mobility system of a city, in particular the effect on traffic congestion and parking demand, resulting from that transition. For that, a cost-minimization trip assignment problem with respect to mode choice, departure time and route choice is proposed in which the AV costs are determined by considering the driving and parking costs incurred by their use.

The paper is organized as follows: first, the literature on traffic assignment and vehicle routing are reviewed to set up a method for assigning the AV trips to an urban road network. The method is shown and explained in detail in the subsequent section. Then the application to the city of Delft is presented, which is followed by the main results of its use under several scenarios. The paper ends with the main conclusions drawn from this paper and possible directions for future research.

2. Literature review

As far as we know the model that we want to develop and apply is entirely novel. The starting point is research published in two main fields of transportation science which are not often connected: (1) traffic assignment (TA) methods, and (2) vehicle routing problems (VRPs). The first scientific area is mostly developed by transportation engineers and the second by operational researchers, the latter often working on management and industrial engineering problems.

In this section we review these two fields of research to establish the theoretical and methodological components needed to develop what we designate as the Privately Owned Automated Vehicles Assignment Problem (POAVAP) both from a system and a user-optimum perspective.

According to the problem objective that we set out to formulate, it is not enough to determine a route or a set of routes for a relatively small number of vehicles, as is usual with VRPs. The VRP is generally defined as "a problem of designing least-cost delivery routes from a depot to a set of geographically scattered customers, subject to side constraints" (Laporte, 2009). In this paper many vehicles have to be routed in an urban network where congestion effects cannot be ignored. This is the problem addressed in a very significant body of the transportation research literature on TA.

2.1. Traffic assignment

The determination of the flows on each link of a city road network involves a solution of a demand/performance equilibrium problem where the flow on each link is the sum of flows on many paths driven by the vehicles (Sheffi, 1985). The performance of the network can be measured in different ways of which the simplest is to consider a link performance function where the travel time increases with the flow that passes through the link, one of the most classical functions being the BPR (Bureau of Public Roads) function (Ortúzar and Willumsen, 2011). More realistic models consider that the performance of a link is a function of what happens in other links as well, which is the case of heavy traffic in two-way streets, unsignalized intersections or left turn movements. However, this is not considered in this paper for simplification purposes.

In unimodal car traffic networks, equilibrium is said to be reached when no driver can unilaterally change its route and improve his travel time (user-equilibrium condition). The travel times are assumed to be known by the drivers: the principle of perfect information (Wardrop, 1952). In the static approach of the user equilibrium the timing and locations of activities are considered uniform over a period of the day where a so called steady state of traffic flows is hence assumed. The flows are imagined to happen instantaneously at the same time interval in the network (Sheffi, 1985).

The static user equilibrium (SUE) problem can be solved by means of a mathematical program known as the Beckmann's transformation. Mathematical programming is coincidentally one of the methods to solve vehicle routing problems and it is the methodology that in this paper will be used to bridge both research fields in the POAVAP.

The SUE mathematical program is used to bridge as follows:

$$\min z(X) = \sum_a \int_0^{x_a} t_a(\omega) d\omega \tag{1}$$

s.t.

$$\sum_k f_k^{rs} = q_{rs} \forall r, s \tag{2}$$

$$f_k^{rs} \geq 0 \ \forall k, r, s \tag{3}$$

$$x_a = \sum_{r,s,k} f_k^{rs} \times \delta_{a,k}^{rs} \ \forall a \tag{4}$$

where $t_a(\omega)$ is a link performance function, x_a is the flow on arc a, f_k^{rs} is the flow on path k connecting OD pair $r-s$, q_{rs} is the trip rate between origin r and destination s and $\delta_{a,k}^{rs}$ is an indicator binary variable that takes value 1 if link a is on path k between OD pair $r-s$.

The program assumes that vehicle flows are continuous and that the objective function is the sum of the integrals of link performance functions $t_a(\omega)$ that translate the travel time increase with the flow.

This problem can be solved using different methods, one of which is the convex combinations method also known as the Frank-Wolfe algorithm:

Step 0 – Initialization: Perform all-or-nothing assignment based on the free-flow conditions $t_a = t_a(0),\ \forall\ a$. This yields $\{x_a^1\}$. Set counter $n = 1$.
Step 1 – Update: Set $t_a = t_a(x_a^n),\ \forall\ a$.
Step 2 – Direction finding: Perform all-or-nothing assignment based on $\{t_a^1\}$. This yields a set of auxiliary flows (y_a^n).
Step 3 – Find α_n (step) that solves:

$$\min \sum_a \int_0^{x_a^n + \alpha(y_a^n - x_a^n)} t_a(\omega) d\omega$$

Step 4 – Move: Set $x_a^{n+1} = x_a^n + \alpha_n (y_a^n - x_a^n), \forall\ a$
Step 5 – Convergence test: if the travel times do not change significantly from one iteration to the other, stop (the current solution $\{x_a^{n+1}\}$ is the set of equilibrium link flows); otherwise, set $n = n + 1$ and go to step 1.

The computation of an all-or-nothing assignment on Step 0 and Step 2 is very simple and entails assigning all volumes to the shortest paths without considering congestion.

The minimization process in Step 3 is what makes the Frank–Wolfe algorithm more efficient than other methods; however, it entails the need to substitute the step α in the objective function and obtain a direction for the search procedure. In more complex models this may be difficult to do in practice. In the case of the model that we intend to develop, the routing of an AV will depend on a formulation much more complex than model (1)–(4) because it involves the routing of the automated vehicles. An alternative proposed in the literature can be to use a constant step, for instance $\alpha_n = 0.5$, or as indicated by Smock (1962) or Sheffi (1985) $\alpha_n = 1/n$. The critical process in the search for the equilibrium is checking that this equilibrium has indeed been found.

We assume in this paper that every family will try to minimize their travel costs during a day similarly to the individual behavior inherent to the SUE. Nevertheless, in this paper, we consider two transport modes (AVs and PT) in direct competition. Therefore, the POAVAP should assume that there is equilibrium between path choices in the automated vehicular flows as well as equilibrium between two modes: car and PT. In this case, the definition of equilibrium must be extended beyond travel time and it must consider a generalized cost of traveling which may include for instance: out-of-pocket cost, distance and waiting time as examples of impedance factors for choosing a mode of transportation.

In a multimodal network, an equilibrium is said to be reached when "no routing decision or change of mode may improve the generalized cost of transportation" (Florian, 1977). A logit model can be used to distribute the trips between the modes, allowing taking into consideration as many explanatory variables as proven to be significant for the distinction between the modes (Ben-Akiva and Lerman, 1985). To check the convergence of this multimodal system the proportion of shifted demand between the two modes from one iteration to the next can be used (Sheffi, 1985). When little displacement results it means that the equilibrium should have been reached.

The system optimum assignment (Newell, 1980) corresponds to the minimization of the total travel time in the network and can only happen if all drivers agree upon the paths to be chosen or these paths are imposed by an external entity. This is typically used as a performance metric for an urban network (Sheffi, 1985). This type of assignment gains extra relevance in an AV scenario where the driving task may be given to a central computer. The cooperative nature of some of the systems that are being envisioned for the future, focusing by now on traffic performance (Baber et al., 2005; Calvert et al., 2011), may give the primacy of the routing decision to a machine as well. In this paper we assume that the choice of a route stays in the hands of the human/family travelers according to the user-equilibrium, i.e., selfish behavior whereby, each family in this case, aims at minimizing its transport costs, although we recognize that there are methods to influence the drivers to take more socially optimum routes (van den Bosch et al., 2011).

The problem we want to define cannot be considered static over time. It is dynamic by nature, as the trips for each household should be free to be performed at any departure time which by its turn should depend on the expected travel time (once again assuming perfect information). In the dynamic user equilibrium (DUE) assignment problem, traffic is not assumed to be stationary in a time period but it should vary as time develops in a given period. Mathematical programming has been applied for this problem in its system optimum perspective very early with the work by Merchant and Nemhauser (1978). Janson (1991) focused on dynamic user equilibrium proposing a mathematical program dividing the analysis period into several time intervals.

The basic requirements for a dynamic TA can be defined as follows (Heydecker and Addison, 2005; Ortúzar and Willumsen, 2011):

1. Positivity: we are only really interested in non-negative flows on links, paths, trip matrices and costs.
2. Conservation: the model must satisfy flow conservation requirements.
3. FIFO: in real traffic the FIFO (First In, First Out) behavior generally prevails and this must be maintained in the model if proper delays are to be estimated.
4. Minimum travel time: flows do not propagate instantaneously.

5. Finite clearing time: there are no queues left at the end of the modeling period; infinite delays do not occur (as a standard queuing model might suggest).
6. Capacity: there is such a thing as strict capacity constraint in the sense that actual flows cannot exceed it even for a short period of time.
7. Causality: delays are affected by what other vehicles do or have done in the past, not in the future.

A user equilibrium in these conditions is defined as: "Under equilibrium conditions in networks where congestion varies over time and travelers can choose their time of travel, traffic arranges itself so that the total cost associated with travel on those routes that are used by travelers at the time when they are used, are equal and no greater than those on any route at a time when it is not used" (Ortúzar and Willumsen, 2011). In the POAVAP as mentioned before we are interested in capturing a dynamic traffic behavior composed of the several AVs owned by each family. Therefore the method to be proposed has to comply with the previous requirements, although with simplifications as it is not the objective of this paper to provide the best DTA model possible.

2.2. Vehicle routing

One of the major limitations of the previously mentioned methods of TA is that vehicles are generally assumed to be originated in a centroid and heading to another centroid, a decision which translates an OD matrix respecting to a period of time. In the dynamic version there may be a distribution of the vehicles for different time departure intervals but vehicles/people are not allowed to take any decision regarding their succession of trips, or where to stop if parking is required between two trips. These considerations are indeed taken into account in the field of activity based models in which people's activities are modeled in detail with their location and mode of choice depending on land use patterns and existing transportation networks (Salvini and Miller, 2005; Roorda et al., 2008; Bowman and Ben-Akiva, 2001). However, these models are usually too big and data hungry with validation issues. Moreover a car is still seen as a mode and cannot be routed independently of its owners' trips.

From a vehicle routing perspective what we must find for each family is if one or more vehicles should be used for any of the family trips and how these vehicles should be routed and parked in the network to satisfy the transport needs of that household. Typically, as an example, a vehicle may start at home in the morning with all family members on board and drop them off at their destinations complying with schedule constraints. Then it will park itself either in one of those destinations or at a parking lot farther from that area if it is cheaper. Later on the day, in the afternoon, it may return to one of the previous destinations to pick up one of the family members to satisfy another trip (for instance a person going to the gym) later in the evening it will pick everyone up and return them to their house. Eventually if it is cost efficient the car may park itself away from the home location during the night.

The problem of vehicle routing is a classic one in operational research and there are many variations around the definition introduced in the beginning of this section. For an extensive review the reader may consult (Laporte, 2009). The most important variations introduced to the basic definition can have different names and may even overlap and be combined:

- Capacitated VRP (Toth and Vigo, 2002; Ralphs et al., 2003): this is a classic variation of the VRP whereby vehicles have limited capacity on either goods or persons that they can transport in each trip.
- VRP with time windows (Dumas et al., 1991): the time windows establish a mandatory or preferred time window for the pick-up and/or delivery of a good or passenger at a certain node.
- VRP with multiple trips (Brandão and Mercer, 1998): in this modality vehicles can perform several trips starting at the depot.
- Open VRP (Brandão, 2004): "open" means that the vehicle does not need to end its trip at the depot but instead it can park at the last client node.
- Pick-up and delivery problem (Dumas et al., 1991; Savelsbergh and Sol, 1995): vehicles have to transport loads from origins to destinations.
- Dial-a-ride transport (DART) or demand responsive transport (DRT) problem: the pick-up and delivery problem for passengers with the added constraints of restricting the maximum passenger ride time and often with the possibility of the vehicle stopping idle waiting for a next service (Diana and Dessouky, 2004; Jaw et al., 1986; Cortés et al., 2010; Cordeau and Laporte, 2007).
- The time dependent VRP with time windows (TDVRPTW): time-varying travel times in the network with time windows (Figliozzi, 2012).
- Dynamic Vehicle Routing: Routing problem that considers the real-time availability of information for generating a change on the route or schedule of a vehicle (Psaraftis, 1995; Ferrucci and Bock, 2015).

From these definitions we may classify our problem as an open, multiple trip, capacitated VRP with time windows, pick-up and delivery of passengers and time-varying travel times. This is a rather cumbersome definition of the problem, and certainly a new one in the literature. Nevertheless despite the apparent completeness of this definition, two things are still lacking. The first is that VRPs are too much attached to the existence of a so called depot from where vehicles depart and will eventually arrive later. The open VRP relaxes this constraint only apparently because vehicles still depart from a common depot and finish at one of the demand nodes, if there is a return to the depot this is assumed to be done doing the

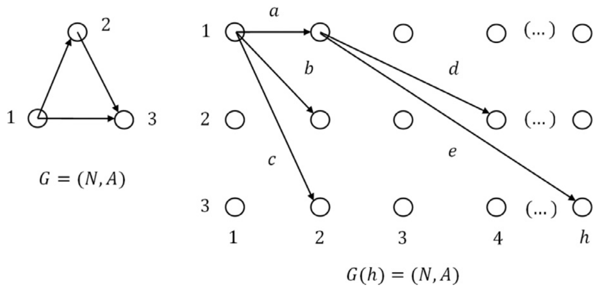

Fig. 1. Time expansion of the road spatial network. Based on Kaufman et al. (1992).

same exact trip in the reverse order. Moreover the formulation of these problems only considers situations where there is no interaction between the routed vehicles and traffic in the road network. The time dependent VRP either with or without time windows intends to acknowledge the significant variations of traffic during a day, however, these travel times are what we may designate as static not changing with any decision taken in the model. The Dynamic Vehicle Routing problem deals with real-time changes in schedules which is not our purpose.

There are almost no examples in the literature of a mathematical programming framework that considers the effect on traffic congestion, nonetheless (Kaufman et al., 1992) have proposed a Mixed Integer Problem (MIP) formulation for the dynamic TA problem which constitutes a good framework to build a connection between the TA problem and the autonomous vehicles' routing. The essential element of the formulation is a time expanded version of a traffic network, $G = (N, A)$, where time is defined for a horizon of h periods, hence $G(h) = (N, A)$, which in practice defines multiple networks depending on the h period. In Fig. 1 it is possible to see how the expansion of a spatial network can be done for a time-space network. On the left a simple spatial network is presented with three nodes and three links. On the right hand side we show the time-space expansion where the vertical axis represents the node and the horizontal axis the time. Each link represents a possible movement except for link a which denotes time spent in the same position. Links b and c represent the possibility of traveling between node 1 and nodes 2 and 3 respectively which take the same time (1 time step) when departing from instant 1. Links d and e represent the travel possibilities between node 1 and nodes 2 and 3 respectively when starting a trip at node 1 at time instant 2. In this case the first trip takes two time steps and the second one three time steps. For simplification purposes other travel possibilities are not represented in the network, for instance traveling possibilities between nodes 2 and 3.

In Kaufman et al. (1992), link travel times are represented by integer variables. Given the values of these variables, the problem is to assign traffic, modeled as a multiperiod multicommodity flow, subject to constraints on capacity implied by the link travel times. The model imposes that: there is no dispersion of platoons within links; vehicles cannot pass each other; there is flow conservation on the time-space network; the travel time experience in the network is dependent on the flow at each time-space link; and a system-optimum objective function is used. The same time-space structure is used for the POAVAP because, as we explain in the next section, this allows to fully specify the AV routes according to the desired automated behavior.

3. The User Optimum Privately Owned Automated Vehicles Assignment Problem (UO-POAVAP)

We present a method to solve what we have designated as the User Optimum Privately Owned Automated Vehicles Assignment Problem (UO-POAVAP) that can assess the impacts on urban mobility resulting from substituting the conventional privately owned family vehicles for AVs, considering that all families act selfishly in choosing their trips, paths and schedules.

3.1. Formulating a System Optimum POAVAP

The model is first developed to be solved from a system optimum perspective (SO-POAVAP) as this formulation is simpler to define in mathematical programming. This means that the objective function will be one that minimizes the total transport cost of all the families in the city. The assumptions of the SO-POAVAP are:

- The trips performed by the household members are either satisfied by the AVs or by PT according to the global generalized transport costs minimization function.
- The generalized transport costs incurred by the household include: vehicle kilometers done by the AVs; PT costs for the trips not satisfied by the AVs; parking costs of the AVs which can vary within the city; and penalties for arriving early or late to each trip destination.
- AVs are allowed to drive empty in the network without any human supervision.
- Each AV routing adds to the traffic flows of the city.
- Each AV has a certain passenger capacity that must be respected.

- The PT trips do not contribute to the traffic flows in the network.
- No external trips to the city are considered in the network.
- The problem is solved to system optimality which means that the total transport costs are minimized without considering any household selfish behavior.

The model is formulated as a MIP problem. Integer variables are used because of the vehicles' routing, meaning that instead of flows being continuous variables, as in model (1)–(4), they are expressed as integer quantities. This has certain implications for the model formulation and convergence process that is discussed later. The proposed MIP formulation is as follows:

Sets

$\boldsymbol{H} = \{1, \ldots, h \ldots H\}$:	set of households in the city, where H is the total number of households.
$\boldsymbol{T} = \{1, \ldots, t \ldots T\}$:	set of time instants in the operation period, where T is the last optimization time instant.
$\boldsymbol{G} = \{1, \ldots, g \ldots T-1\}$:	set of time instants that represent the beginning of a time step between two instants (it does not include the T instant). This also represents the set of time steps which are $T-1$.
$\boldsymbol{I} = \{1, \ldots, i \ldots I\}$:	set of nodes in the network, where I is the number of nodes.
$\boldsymbol{M} = \{1, \ldots, m \ldots M\}$:	set of members of household h, where M is the number of members of household h (for brevity purposes the index of the household is omitted from the set name).
$\boldsymbol{E} = \{1, \ldots, e \ldots E\}$:	set of trips of each member m of household h, where E is the number of trips.
$\boldsymbol{V} = \{1, \ldots, v \ldots V\}$:	set of vehicles of household h where V is the total number of vehicles.
$\boldsymbol{X} = \{1_1, \ \ldots, \ i_{t-1}, i_t, i_{t+1}, \ \ldots, \ I_T\}$:	set of nodes of a time-space network combining the $\boldsymbol{I}$ nodes with the $\boldsymbol{T}$ time instants.
$\boldsymbol{R} = \{\ldots, (\mathrm{i}, \mathrm{j}), \ldots\}$ $i, j \in \boldsymbol{I}, \ i \neq j$:	set of arcs of the road network where vehicles move.
$\boldsymbol{A}_1 = \{\ldots, \mathrm{a}_1(\mathrm{i}_{t_1}, \mathrm{j}_{t_2}), \ldots\}, \ i_T \in \boldsymbol{X}, \ (\mathrm{i}, \mathrm{j}) \in \boldsymbol{R}$:	set of arcs that represent the movement of each vehicle of household h between node i and node j of the road network, between time instant t_1 and $t_2 = t_1 + \delta_{ij}^{t_1}$ where $\delta_{ij}^{t_1}$ is the travel time (in number of time steps) between nodes i and j when the movement starts at time instant t_1. Because travel time changes in function of the vehicles' routing this means that this set of arcs is in constant change and must be a function of the congestion effects on the network.

Data

D_{ij}^{emh}:	has the value 1 if there is an eth trip from member m of household h from node i to node j, $\forall \ i, j \in \boldsymbol{I}, \ e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, \ h \in \boldsymbol{H}.$
Θ_a^{emh}:	desired departure time for the eth trip from member m of household h, $\forall \ e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, \ h \in \boldsymbol{H}.$
a^{emh}:	earliest departure time for the eth trip from member m of household h, $\forall \ e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, \ h \in \boldsymbol{H}.$
Θ_b^{emh}:	desired arrival time for the eth trip from member m of household h, $\forall \ e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, \ h \in \boldsymbol{H}.$
b^{emh}:	latest arrival time for the eth trip from member m of household h, $\forall \ e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, \ h \in \boldsymbol{H}.$
t_{ij}^{max}:	maximum travel time by car in time steps for arc (i, j), $\forall \ (i, j) \in \boldsymbol{R}$.
t_{ij}^{min}:	minimum travel time by car in time steps for arc (i, j), $\forall \ (i, j) \in \boldsymbol{R}$.
t_{ij}^{PT}:	travel time in PT in minutes for trips going from node i to j (in this case we opt for using minutes as unit), $\forall \ i, j \in \boldsymbol{I}$.
Cap^{hv}:	capacity of vehicle v of household h,$\forall \ h \in \boldsymbol{H}, v \in \boldsymbol{V}$.
Lg_{ij}:	Length of arc (i, j) in kilometers,$\forall \ (i, j) \in \boldsymbol{R}$.
μ_h:	expansion coefficient of household h (number of households with the same characteristics in the population), $\forall \ h \in \boldsymbol{H}$.
Q_{ij}:	Capacity of each link (i, j) which is the number of vehicles that go through the link at the highest travel time, $\forall \ (i, j) \in \boldsymbol{R}$.

The parameters for building the generalized transport costs function that should be minimized

ρ:	penalty cost of using PT (specific disutility of the mode).
β:	total travel time cost per minute in PT (includes waiting).
α:	travel time cost by time step in a car.
α_l:	penalty time cost for late arrival at destination by car.
α_e:	penalty time cost for early arrival to destination by car.
ω:	fuel cost per kilometer in a car.
γ:	scale factor between the fuel costs and the parking costs (costs are perceived differently and fuel costs are used as reference).

ς: scale factor between the fuel costs and ticket costs (costs are perceived differently and fuel costs are used as reference).
$Tick$: Ticket cost per PT trip.
Pk_i: Parking cost at location i per time step, $\forall\ i \in \boldsymbol{G}$.

Decision variables

$x_{i_{t_1} j_{t_2}}^{hv}$: binary variable equal to 1 if vehicle v of household h drives on road link (i, j) from time instant t_1 to time instant t_2, $\forall (i_{t_1}, j_{t_2}) \in \boldsymbol{A}_1$, $h \in \boldsymbol{H}$, $v \in \boldsymbol{V}$.
δ_{ij}^t: current travel time by car in time steps for arc (i, j) beginning at time instant t, $\forall\ (i, j) \in \boldsymbol{R}$, $t \in \boldsymbol{T}$.
$w_{i_t}^{hv}$: binary variable equal to 1 if vehicle v of household h parks at node i at time step t, $\forall\ h \in \boldsymbol{H}$, $v \in \boldsymbol{V}$, $i_t \in \boldsymbol{X}$.
Tr_{ij}^{emhv}: binary variable equal to 1 if trip e from node i to node j of member m belonging to household h is done using vehicle v, $\forall\ i, j \in \boldsymbol{I}$, $e \in \boldsymbol{E}$, $m \in \boldsymbol{M}$, $h \in \boldsymbol{H}$, $v \in \boldsymbol{V}$.
P_{ij}^{emhvt}: binary variable equal to 1 if trip e from node i to node j of member m belonging to household h starting at time instant t in vehicle v, $\forall\ i, j \in \boldsymbol{I}$, $e \in \boldsymbol{E}$, $m \in \boldsymbol{M}$, $h \in \boldsymbol{H}$, $v \in \boldsymbol{V}$, $t \in \boldsymbol{T}$.
A_{ij}^{emhvt}: binary variable equal to 1 if trip e from node i to node j of member m belonging to household h finished at time instant t using vehicle v, $\forall\ i, j \in \boldsymbol{I}$, $e \in \boldsymbol{E}$, $m \in \boldsymbol{M}$, $h \in \boldsymbol{H}$, $v \in \boldsymbol{V}$, $t \in \boldsymbol{T}$.
ϕ_{ij}^{emhv}: continuous variable of the difference between the real and desired arrival time of trip e from node i to node j of member m belonging to household h iusing vehicle v, $\forall\ i, j \in \boldsymbol{I}$, $e \in \boldsymbol{E}$, $m \in \boldsymbol{M}$, $h \in \boldsymbol{H}$, $v \in \boldsymbol{V}$.
$l\phi_{ij}^{emhv}$: continuous positive variable of the difference between the real and desired arrival time of trip e from node i to j of member m belonging to household h in vehicle v when the arrival happens after the desired time, $\forall\ i, j \in \boldsymbol{I}$, $e \in \boldsymbol{E}$, $m \in \boldsymbol{M}$, $h \in \boldsymbol{H}$, $v \in \boldsymbol{V}$.
$e\phi_{ij}^{emhv}$: continuous positive variable of the difference between the real and desired arrival time of trip e from node i to j of member m belonging to household h in vehicle v when the arrival happens before the desired time, $\forall\ i, j \in \boldsymbol{I}$, $e \in \boldsymbol{E}$, $m \in \boldsymbol{M}$, $h \in \boldsymbol{H}$, $v \in \boldsymbol{V}$.
L_t^{hv}: discrete variable equal to the number of persons being transported in vehicle v of household h at time step t, $h \in \boldsymbol{H}$, $v \in \boldsymbol{V}$, $t \in \boldsymbol{G}$.
$F_{i_{t_1} j_{t_2}}$: flow of vehicles on arc (i, j) from time instant t_1 to time instant t_2, $\forall (i_{t_1}, j_{t_2}) \in \boldsymbol{A}_1$.

Objective function

$$
\begin{aligned}
Min(C) = & \sum_{\substack{i,j \in I \\ e \in E, m \in M, h \in H}} \left(D_{ij}^{emh} - \sum_{v \in \boldsymbol{V}} Tr_{ij}^{emhv} \right) \times \left(t_{ij}^{PT} \times \beta + Tick \times \varsigma + \rho \right) \times \mu_h \\
& + \sum_{\substack{(i_t, j_{t+\delta_{ij}^t}) \in \boldsymbol{A}_1 \\ h \in H,\ v \in V}} x_{i_t j_{t+\delta_{ij}^t}}^{hv} \times Lg_{ij} \times \omega \times \mu_h \\
& + \left(\sum_{\substack{i,j \in I \\ e \in E, m \in M, h \in H, \\ v \in V,\ t \in T}} \left(A_{ij}^{emhvt} \times t \right) - \sum_{\substack{i,\, j \in I \\ e \in E, m \in M, h \in H, \\ v \in V, t \in T}} \left(P_{ij}^{emhvt} \times t \right) \right) \times \alpha \times \mu_h \\
& + \sum_{\substack{i \in N, t \in G \\ h \in H,\ v \in V}} w_{i_t}^{hv} \times Pk_i \times \gamma \times \mu_h \\
& + \sum_{\substack{i,j \in I \\ e \in E, m \in M, h \in H \\ v \in V}} \left(l\phi_{ij}^{emhv} \times \alpha_l + e\phi_{ij}^{emhv} \times \alpha_e \right) \times \mu_h
\end{aligned}
\tag{5}
$$

This function minimizes the total generalized cost of transportation of all households for one day and has five components (each of which with its own line in Eq. (5)). It considers first the cost of the trips done in PT which includes the value of in-vehicle time, the ticket cost and a penalty cost for opting for PT; then the cost of vehicle fuel which is a function of the kilometers driven; the following component is the value of travel time (VTT) which is a function of the time spent inside the vehicle for all its occupants, hence it cannot be indexed to the x variables as these only represent the time lost by the vehicle itself; the following component regards to the parking costs. Costs of vehicle depreciation are not included in this function as these are regarded as sunk costs not being considered by the traveler in his choice. Early and late arrivals are penalized in the last component of the function in order to speed up the process of finding an optimal solution of a particular routing since there may be many possible routing combinations yielding the same objective function. The assumption $\alpha_e < \alpha < \alpha_l$ is made to avoid cyclical routes which might occur if arriving early is more onerous than traveling (Small, 1982; Levin et al., 2015). Nevertheless we acknowledge that there could be differences in these penalties whether it is a trip to work or a return home for example.

This function is the sum of the costs of two transport options: PT and AV. In the way the model is built, the function translates the utility of the two modes expressed in monetary units. The two underlying generalized cost functions are the following:

$$C(car) = \alpha \times Travel\_Time + Fuel\_cost + \gamma \times Parking\_cost \tag{6}$$

$$C(PT) = \rho + \beta \times Travel\_Time + \varsigma \times Ticket\_cost \tag{7}$$

These functions only have in consideration the mode attributes, whilst it is known and already referred previously that mode choice depends on other factors such as socio-demographic profile of the decision maker as well as other more subjective attributes. The only effect which is not connected to the mode attributes is introduced by a special disutility parameter ρ which intends to represent the average part of the costs which are not being considered in the variables, therefore, denoting a special preference for one of the modes. Most importantly these two functions represent only the deterministic part of the utility of choosing a mode ignoring the random part which would call for using a choice model structure such as a Logit or a Probit. For simplification purposes we assume to ignore in this paper the random part of the utility and its statistical distribution.

By minimizing this objective function, the MIP model is opting for a solution that maximizes the global systematic utility of the mobility of all the households. This means that a cheaper solution for one of the households is balanced with a more expensive one for another.

The objective function is subject to the following constraints

$$Tr_{ij}^{emhv} \leq \sum_{\substack{(i_{t_1} l_{t_2}) \in \boldsymbol{A}_1 \\ t_1 \geq a^{emh} \\ t_2 \leq b^{emh}}} x_{i_{t_1} l_{t_2}}^{hv}, \ \forall i, j \in \boldsymbol{I}, e \in \boldsymbol{E}, m \in \boldsymbol{M}, h \in \boldsymbol{H}, v \in \boldsymbol{V} \tag{8}$$

Assures that a trip e from member m of household h can only be satisfied by vehicle v if that vehicle has passed through node i (trip origin node) after the earliest departure time a^{emv}.

$$Tr_{ij}^{emhv} \leq \sum_{\substack{(l_{t_1}, j_{t_2}) \in \boldsymbol{A}_1 \\ t_1 \geq a^{emh} \\ t_2 \leq b^{emh}}} x_{l_{t_1} j_{t_2}}^{hv}, \ \forall i, j \in \boldsymbol{I}, e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, h \in \boldsymbol{H}, v \in \boldsymbol{V} \tag{9}$$

Assures that a trip e from member m of household h can only be satisfied by vehicle v if that vehicle has passed through node j (trip destination node) before the latest arrival time b^{emv}.

$$\begin{aligned} P_{ij}^{emhvt} &\leq Tr_{ij}^{emhv}, \forall i, j \in \boldsymbol{I}, e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, h \in \boldsymbol{H}, t \in \boldsymbol{T} \\ P_{ij}^{emhvt} &\leq \sum_{\substack{(i_t, l_{t_1}) \in \boldsymbol{A}_1 \\ t \geq a^{emh} \\ t_1 \leq b^{emh}}} x_{i_t l_{t_1}}^{hv}, \forall i, j \in \boldsymbol{I}, e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, h \in \boldsymbol{H}, t \in \boldsymbol{T} \\ P_{ij}^{emhvt} &\geq \sum_{\substack{(i_t, l_{t_1}) \in \boldsymbol{A}_1 \\ t \geq a^{emh} \\ t_1 \leq b^{emh}}} x_{i_t l_{t_1}}^{hv} + Tr_{ij}^{emhv} - 1, \forall i, j \in \boldsymbol{I}, e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, h \in \boldsymbol{H}, t \in \boldsymbol{T} \end{aligned} \tag{10}$$

This set of constraints forces the departure of a trip e to exist at a specific time instant t if the trip is satisfied by a vehicle v and the vehicle passes through the trip departure node at time instant t.

$$\sum_{t \in \boldsymbol{T}} P_{ij}^{emhvt} \leq 1, \forall i, j \in \boldsymbol{I}, e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, h \in \boldsymbol{H}, v \in \boldsymbol{V} \tag{11}$$

Assures that each trip only departs at a specific time instant or that it is not satisfied at all by any vehicle.

$$A_{ij}^{emhvt} \leq Tr_{ij}^{emhv}, \ \forall i, \ j \in \boldsymbol{I}, \ e \in \boldsymbol{E}, \ m \in \boldsymbol{M}, \ h \in \boldsymbol{H}, \ t \in \boldsymbol{T}, \ v \in \boldsymbol{V}$$

$$A_{ij}^{emhvt} \leq \sum_{\substack{(l_t,\ j_{t_1}) \in \boldsymbol{A}_1 \\ t_1 \geq a^{emh} \\ t \leq b^{emh}}} x_{l_{t_1} j_t}^{hv}, \ \forall i, \ j \in \boldsymbol{I}, \ e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \ h \in \boldsymbol{H}, \ t \in \boldsymbol{T}, \ v \in \boldsymbol{V}$$

$$A_{ij}^{emhvt} \geq \sum_{\substack{(l_t,\ j_{t_1}) \in \boldsymbol{A}_1 \\ t_1 \geq a^{emh} \\ t_2 \leq b^{emh}}} x_{l_t j_{t_1}}^{hv} + Tr_{ij}^{emhv} - 1, \ \forall i, \ j \in \boldsymbol{I}, \ e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \ h \in \boldsymbol{H}, \ t \in \boldsymbol{T}, \ v \in \boldsymbol{V} \tag{12}$$

This set of constraints forces the arrival of a trip e to exist at a specific time instant t if the trip is satisfied by vehicle v and there is a vehicle route passing through the trip arrival node at time instant t.

$$\sum_{t \in \boldsymbol{T}} A_{ij}^{emhvt} \leq 1, \ \forall i, \ j \in \boldsymbol{I}, \ e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \ h \in \boldsymbol{H}, \ v \in \boldsymbol{V} \tag{13}$$

Assures that each trip only arrives at a specific time instant or that it is not at all satisfied by any vehicle.

$$\phi_{ij}^{emhv} = \Theta_b^{emh} \times Tr_{ij}^{emhv} - \sum_{t \in \boldsymbol{T}} A_{ij}^{emhvt} \times t, \ \forall i, \ j \in \boldsymbol{I}, \ e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \ h \in \boldsymbol{H}, \ v \in \boldsymbol{V} \tag{14}$$

Computes the difference between the desired and the real arrival time of a trip in time steps. If the trip is not satisfied by a car the variable is zero. This variable is negative if the real arrival time is later than the desired one and positive vice versa.

$$l\phi_{ij}^{emhv} \leq -\phi_{ij}^{emhv}, \ \forall i, \ j \in \boldsymbol{I}, \ e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \ h \in \boldsymbol{H}, \ v \in \boldsymbol{V} \tag{15}$$

Yields the absolute time difference in time steps when the arrival of a trip happens after the desired time.

$$e\phi_{ij}^{emhv} \geq \phi_{ij}^{emhv}, \ \forall i, \ j \in \boldsymbol{I}, \ e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \ h \in \boldsymbol{H}, \ v \in \boldsymbol{V} \tag{16}$$

Yields the absolute time difference in time steps when the arrival of a trip happens before the desired time.

$$\sum_{t \in \boldsymbol{T}} \left(P_{ij}^{emhvt} \times t\right) \leq \sum_{t \in \boldsymbol{T}} \left(A_{ij}^{emhvt} \times t\right), \ \forall i, \ j \in \boldsymbol{I}, \ e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \ h \in \boldsymbol{H} \tag{17}$$

The departure instant of a trip must happen before the arrival instant.

$$\sum_{v \in \boldsymbol{V}} Tr_{ij}^{emhv} \leq 1, \ \forall i, \ j \in \boldsymbol{I}, \ e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \ h \in \boldsymbol{H} \tag{18}$$

A trip is only satisfied by one car and one car alone.

$$L_t^{hv} = \sum_{\substack{i,j \in I \\ e \in E,\ m \in M \\ h \in H \\ t_1 \in T, t_1 \leq t}} P_{ij}^{emhvt_1} - \sum_{\substack{i,j \in I \\ e \in E,\ m \in M \\ h \in H \\ t_1 \in T,\ t_1 \leq t}} A_{ij}^{emhvt_1}, v \in \boldsymbol{V}, t \in \boldsymbol{T} \tag{19}$$

Yield the number of people in each vehicle v of household h at each time instant t.

$$L_t^{hv} \leq Cap^{hv}, v \in \boldsymbol{V}, \ t \in \boldsymbol{T} \tag{20}$$

Assures that the number of persons inside vehicle v of household h is not above the vehicle capacity.

$$\sum_{i \in I} w_{i_t}^{hv} \leq \frac{Cap^{hv} - L_t^{hv}}{Cap^{hv}}, \quad t \in \boldsymbol{G}, \quad v \in \boldsymbol{V}, \ h \in \boldsymbol{H} \tag{21}$$

These constraints impose that when the vehicle is transporting a person, it should not stop idle at any node. It avoids the model producing solutions that may minimize the generalized transport costs but that would not be logical from a practical point of view.

$$\sum_{i \in \boldsymbol{I},\ j_t \in \boldsymbol{X}} x_{i_1 j_t}^{hv} + \sum_{i \in \boldsymbol{I}} w_{i_1}^{hv} = 1, \ v \in \boldsymbol{V}, \quad h \in \boldsymbol{H} \tag{22}$$

Each vehicle v of household h is created and these constraints make sure that the vehicle will only be in one of two possible states in the beginning of the day: stopped or beginning a route. This also means that the variables which characterize the state of a family vehicle will always be created, that is, the model does not have the option of eliminating a vehicle. If the

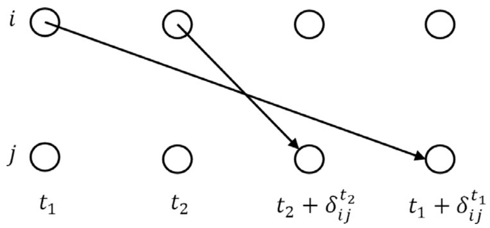

Fig. 2. Example of link travel time inconsistency in the time-space network.

vehicle is not used throughout the day it just stays parked at the same place.

$$\sum_{j_{t_1}\in \boldsymbol{X}} x_{i_t j_{t_1}}^{hv} + w_{i_t}^{hv} = \sum_{j_{t_1}\in \boldsymbol{X}} x_{j_{t_1} i_t}^{hv} + w_{i_{t-1}}^{hv}, \quad \forall i_t \in \boldsymbol{X}, \;\; h \in \boldsymbol{H}, \;\; v \in \boldsymbol{V} \tag{23}$$

These make sure that the vehicles will have continuity of activities in each node throughout the model period.

$$F_{i_{t_1} j_{t_2}} = \left(\sum_{h \,\in\, \boldsymbol{H}, \;\; v \,\in\, \boldsymbol{V}} x_{i_{t_1} j_{t_2}}^{hv} \right) \times \mu_h, \;\; \forall (i_{t_1}, \;\; j_{t_2}) \in \boldsymbol{A}_1, \quad h \in \boldsymbol{H} \tag{24}$$

Flow of vehicles in each road link (i, j) between time instant t_1 and t_2.

$$F_{i_{t_1} j_{t_2}} \le Q_{ij}, \;\; \forall (i_{t_1}, \;\; j_{t_2}) \in A_1 \tag{25}$$

Flow limited by the capacity of each link.

$$\delta_{ij}^{t_1} \ge t_{ij}^{min} + \left(t_{ij}^{max} - t_{ij}^{min}\right) \times \left(\frac{\sum_{t_2 \in \boldsymbol{T}} F_{i_{t_1} j_{t_2}}}{Q_{ij}} \right)^4, \quad \forall (i, j) \in \boldsymbol{R}, \;\; t \in \boldsymbol{T} \tag{26}$$

Non-linear constraint that computes the travel times as a function of the traffic flow. It considers the travel time increase given by the Bureau of Public Roads (Dafermos and Sparrow, 1968): $t = t_0(1 + a \times (\frac{V}{Q})^b)$ where t_0 is the free-flow travel time; V is the volume; Q is the capacity; and a and b are estimation parameters. In this case, an a of $((t_{ij}^{max})/(t_{ij}^{min})) - 1$ and a b of 4 is used for experimental purposes. An inequality is used because in some particular cases link consistency must be imposed (see constraints (28)). In the expression, the summation $\sum_{t_2 \in \boldsymbol{T}} F_{i_{t_1} j_{t_2}}$ does not mean that there can be flows simultaneously for two travel times starting at the same time instant t. Because travel time is a variable, only one of those travel times will exist between the minimum and the maximum. This is guaranteed because of constraints (24) where the flow is computed as a result of summing the $x_{i_{t_1} j_{t_2}}^{hv}$ variables and these will be related to the travel time in the next constraints (27).

$$\delta_{ij}^{t_1} \le (t_2 - t_1) x_{i_{t_1} j_{t_2}}^{hv} + t_{ij}^{max}(1 - x_{i_{t_1} j_{t_2}}^{hv}), \forall (i_{t_1} j_{t_2}) \in \boldsymbol{A}_1, h \in \boldsymbol{H}, v \in \boldsymbol{V}, t_{ij}^{max} \ge (t_2 - t_1) \ge t_{ij}^{min} \tag{27}$$

These two sets of constraints only allow for the existence of routing variables, $x_{i_{t_1} j_{t_2}}^{hv}$, whose time interval (between instants t_1 and t_2) is compatible with the congestion level at link (i, j) defined by constraints (26).

$$t_1 + \delta_{ij}^{t_1} \le t_2 + \delta_{ij}^{t_2}, \;\; \forall \; t_1, \;\; t_2 \in \boldsymbol{T}, \;\; (i, j) \in \boldsymbol{R}, \;\; 0 \le t_1 < t_2 < t_1 + \delta_{ij}^{t_1} \tag{28}$$

These are link consistency constraints that assure that vehicles do not pass one another, i.e., that among two platoons traversing a link, the one which enters later does not leave earlier. This can happen because the problem is defined in a discrete network where low volume starting at a later time instant results in a shorter travel time which should be hindered for consistency purposes (Fig. 2). These constraints have been proposed by Kaufman et al. (1992).

$$x_{i_{t_1} j_{t_2}}^{hv} \in \{1, 0\} \; \forall \; i_{t_1} j_{t_2} \in \boldsymbol{A}_1, \quad h \in \boldsymbol{H}, \;\; v \in \boldsymbol{V} \tag{29}$$

$$w_{i_t i_{t+1}}^{hv} \in \{1, 0\} \; \forall \; i_t i_{t+1} \;\in \boldsymbol{A}_2, \quad h \in \boldsymbol{H}, \;\; v \in \boldsymbol{V} \tag{30}$$

$$Tr_{ij}^{emhv} \in \{1, 0\} \; \forall \; i, \;\; j \in \boldsymbol{I}, \quad e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \;\; h \in \boldsymbol{H}, \;\; v \in \boldsymbol{V} \tag{31}$$

$$L_t^{hv} \in N^0 \; \forall \; t \in \boldsymbol{T}, \;\; h \in \boldsymbol{H}, \;\; v \in \boldsymbol{V} \tag{32}$$

$$F_{i_t j_{t+1}} \in N^0 \ \forall \ i_t, \quad j_{t+1} \in \boldsymbol{X} \tag{33}$$

$$\delta_{ij}^t \in N^0 \ \forall \ (i, \quad j) \in \boldsymbol{R}, \quad t \in \boldsymbol{T} \tag{34}$$

$$\phi_{ij}^{emhv} \in R \ \forall \ i, \quad j \in \boldsymbol{I}, \quad e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \quad h \in \boldsymbol{H}, \quad v \in \boldsymbol{V} \tag{35}$$

$$l\phi_{ij}^{emhv} \in N \ \forall \ i, \quad j \in \boldsymbol{I}, \quad e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \quad h \in \boldsymbol{H}, \quad v \in \boldsymbol{V} \tag{36}$$

$$e\phi_{ij}^{emhv} \in N \ \forall \ i, \quad j \in \boldsymbol{I}, \quad e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \quad h \in \boldsymbol{H}, \quad v \in \boldsymbol{V} \tag{37}$$

Set the domain of the decision variables.

This model follows the 7 rules for a dynamic assignment:

1. Positivity: flows can only be positive by definition of the $F_{i_t j_{t+1}}$ decision variables.
2. Conservation: constraints (23) guarantee continuity of the vehicle routing hence of the flows associated to those vehicles.
3. FIFO: constraints (28) guarantee that vehicles arriving first at the link will leave earlier than those arriving afterwards.
4. Minimum travel time: imposing t_{ij}^{min} guarantees that the travel time can never go lower.
5. Finite clearing time: queuing is not being considered in this simplified model, thus, all traffic will have been cleared by the end of the model run.
6. Capacity: the flow cannot go over the capacity of the link because of constraints (25).
7. Causality: the delay is only dependent on the flow passing at a particular time interval.

The model can be transformed in such a way that it renders the current mobility situation in a city with conventional vehicles. All vehicles are owned by the families, but in this case, they cannot drive without a driver. This can be imposed by the following extra set of constraints:

$$x_{i_t j_{t_1}}^{hv} \leq L_t^{hv} \ \forall \ (i_t, \quad j_{t_1}) \in \boldsymbol{A}_1, \quad h \in \boldsymbol{H}, \quad v \in \boldsymbol{V} \tag{38}$$

These assure that there cannot exist routing variables if the vehicle is empty (L_t^{hv}=0).

3.2. Generating extra cuts to the problem

The search for a routing solution can be accelerated by bounding the problem with extra constraints. These do not change the solution space but tight the bounds on that space by eliminating non-integer solutions of the relaxation process of the traditional branch-and-bound search method. Thus, the gap to find the optimum solution closes faster as certain nodes of the tree are not explored.

$$\sum_{j \in I, \ t_2 \in T} x_{i_{t_1} j_{t_2}}^{hv} \leq 1, \quad \forall \ i \in \boldsymbol{I}, \ h \in \boldsymbol{H}, \ v \in \boldsymbol{V}, \ t_1 \in \boldsymbol{T} \tag{39}$$

These constraints impose that there can only be one routing variable of a particular vehicle v starting at a specific time space node or it does not exist at all. These constraints are already imposed by the interaction of both sets of conservation constraints (22) and (23), where constraints (22) initiate the existence of a vehicle and constraints (23) maintain its existence throughout the model optimization period. However, in the branch-and-bound process it may happen that the variables on the left hand side of constraints (23) sum to the same value of the right hand side, each variable obeying its domain constraints and the sums being above one. Imposing bound constraints (39) will not allow those solution nodes to be explored in the relaxation process.

$$\sum_{i \in I} w_{i_t i_{t+1}}^{hv} \leq 1, \quad \forall h \in \boldsymbol{H}, \ v \in \boldsymbol{V}, \ t \in \boldsymbol{G} \tag{40}$$

Constraints (40) do the same as constraints (39) but for the parking variables.

$$P_{ij}^{emhvt} + A_{ij}^{emhvt} \leq 1, \quad \forall (i, j) \in \boldsymbol{R}, \quad e \in \boldsymbol{E}, \quad m \in \boldsymbol{M}, \ h \in \boldsymbol{H}, \ v \in \boldsymbol{V}, \ t \in \boldsymbol{T} \tag{41}$$

These impose that variables P_{ij}^{emhvt} and A_{ij}^{emhvt} for a specific trip in a specific vehicle cannot take the value 1 at the same time because this would mean that a trip would have the same origin and destination and by definition a trip only happens when there is a movement from one node to another. The previously defined model does not allow this to happen, but imposing extra constraints (41) will allow avoiding exploring nodes where the sum of both these variables is above 1.

3.3. User Optimum POAVAP

The SO-POAVAP is an interesting problem that makes use of the fact that vehicles are automated and, as noted, a centralized transport management system may decide how to transport people in a city with the objective of minimizing total

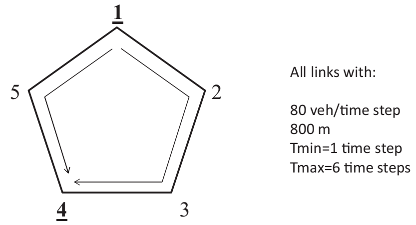

Fig. 3. Simplified network for convergence verification.

transport costs. However, we argue that it is still reasonable for people to maintain some level of selfish behavior by minimizing their own household transport costs. Moreover, the SO-POAVAP is a nonlinear problem which is difficult to solve. Routing all vehicles of all households results in a great number of decision variables and number of constraints, which makes solving it to optimality very difficult.

Nevertheless, we use the SO-POAVAP as the basis to find a user optimum solution for the problem. We do so by assigning the vehicles of each household, one household at a time, to the network in an incremental process. With this method the individual household costs are minimized, not the total costs. The link between those assignments is the travel times in the network, which should depend on all traffic that uses the network, not just the family vehicles being assigned in each increment.

The incremental use of the SO-POAVAP adds the problem of non-equilibrium in the network because the first cars being assigned experience an empty network and the last ones a network already loaded. This is tackled by an iterative process where verifying the equilibrium is an essential part of the model, and this is discussed later in the application.

The UO-POAVAP algorithm has the following steps:

- A list of household members and their trips in a typical day are considered.
- The first family is analyzed and its vehicles routed in the network according to the cost minimization function. Travel times are not updated at this point but capacity in the links is obeyed.
- The vehicles of the next family are routed in the network according to the capacity available left by the previous family.
- After assigning all the households' trips to the network, the updated travel times are computed.
- A new assignment of all the households' trips to the network is done, using the travel times computed in the previous iteration.
- An error is computed between the number of trips satisfied by automobiles in the current iteration and the previous (Sheffi, 1985).

In this algorithm, travel times do not change during the assignment of the full list of households. This has the advantage of allowing the use of an external method to compute shortest paths in the network, which are fed into the assignment process, thereby reducing the number of routing variables. Moreover, the travel time increase given by non-linear constraints (26), can now be eliminated. Whenever a link is used in its full capacity for a specific time instant, this link is taken out of the shortest paths computation.

The following parameters are needed for running the UO-POAVAP:

$\boldsymbol{S} = \{0, \ldots, s..S\}$: number of iterations where S is the maximum (iteration 0 is the initialization of the algorithm).

$\delta_{ij}^{t,s}$: are defined as the travel times in the network in the current iteration s and they are not decision variables as in the SO-POAVAP.

$\delta_{ij}^{t,0} = t_{ij}^{min}, \ \forall (i,j) \in \boldsymbol{R}, \ t \in \boldsymbol{T}$ Initial travel times in each link are defined as the minimum travel times.

error: error between the previous and the current iteration. At this point the number of trips satisfied by the cars will be considered as the reference indicator for that convergence according to what was discussed in the literature review section.

$Trips^s$: are the number of trips satisfied by an automobile in iteration s.

π: limit for the error

$\phi = \frac{1}{s}$: is the coefficient for the equilibrium computation that will balance the contribution of the previous and current assignment for the computation of the volumes and other performance indicators in each iteration.

$Vol^s_{i_{t_1} j_{t_2}}$: are the volumes on link (i, j) from time instant t_1 to t_2 in iteration s.

The following is the pseudo-code of the UO-POAVAP algorithm:

Do until s=S or error <= π
For all households h $\in$ H do {
- Compute Shortest_paths between all nodes of household h using times $\delta_{ij}^{t,s}$;
- All routing variables $x_{i_t j_{t+\delta_{ij}}}^{hv}$ which do not belong to the shortest paths are set to 0;
- Solve the SO-POAVAP for each household h. Model (5)–(41) is run without constraints (26)–(28);
- If the capacity of a link has been reached, take that link off the list of network arcs;

End-for
Update Volumes:
If (s=0) then
$Vol_{i_{t_1} j_{t_2}}^{0} = F_{i_t j_{t_2}}, \forall (i_{t_1}, j_{t_2}) \in \boldsymbol{A}_1$
else
$Vol_{i_{t_1} j_{t_2}}^{s} = (1-\phi) \times Vol_{i_{t_1} j_{t_2}}^{s-1} + \phi \times F_{i_t j_{t_2}}, \forall (i_{t_1}, j_{t_2}) \in \boldsymbol{A}_1$
End-if
Update Travel Times:

$$\delta_{ij}^{t,s+1} = \left\lceil t_{ij}^{min} + (t_{ij}^{max} - t_{ij}^{min}) \times \left(\frac{\sum_{t_1 \in T} Vol_{i_t j_{t_1}}^{s}}{Q_{ij}} \right)^4 \right\rceil, \ \forall \ (i,j) \in \boldsymbol{R}, \ t \ \in \boldsymbol{T} \tag{42}$$

Eq. (42) compute the updated travel times in the network as a result of the iteration assignment according to the BPR function. The result has to be an integer number so that the travel times are compatible with the time space network.
After travel times are computed it is necessary to impose time-space compatibility as it was defined by constraints (28) in the MIP model (5)–(41):
If ($\boldsymbol{\delta_{ij}^{t_1,s+1}}$>2) then
If $(t_2 + \delta_{ij}^{t_2,s+1} < t_1 + \delta_{ij}^{t_1,s+1} \ \forall t_1, t_2 \in \boldsymbol{T}, (i,j) \in \boldsymbol{R}, 0 \le t_1 < t_2 < t_1 + \delta_{ij}^{t_1,s+1})$ *then*
$\delta_{ij}^{t_2,s+1} = t_1 + \delta_{ij}^{t_1,s+1} - t_2$
End-if
End-if
Compute Satisfied trips by car:

$$Trips^0 = \sum_{\substack{ij \in I \\ e \in E, m \in M, h \in H, \ v \in V}} Tr_{ij}^{emhv}$$

Compute Error:
If (s=0) then
$error = Trips^0$
Else
$error = \sqrt{(Trips^s - Trips^{s-1})^2}$
End-if
s=s+1
End-do

3.4. Some numerical experiments in a small network

Before applying the model to the case-study network we apply the method to a small network of 5 arcs and 5 nodes (two ways circulation allowed) (Fig. 3). This is important to understand if the UO-POAVAP is working properly in producing the assignment in the network.

In this simple example, 40 families of one household member living in node 1 have the following two trips: a first trip in the morning at 7 am (a^{emh}) starting at node 1 with destination at node 4 at 7:52:30 am (b^{emh}) minus and plus a slack of 10 min respectively; and a second trip in the end of the day starting at node 4 at 4:30 pm and ending at 5:22:30 pm, with the same slack of minus and plus 10 min. The time step is 2.5 min and we assume that parking is not paid at node 1 which is considered to be the home location. Each family represents 5 real families (total number of families = 200) and the volume delay function is simplified to a square function applied to links that all have the same capacity (80 vehicles per time step of 2.5 min), length (800 m) and travel time ($t_{ij}^{min} = 1$ time steps and $t_{ij}^{max} = 6$ time steps) (Eq. (43)).

$$\delta_{ij}^{t,s} = \left\lceil 1 + (6-1) \times \left(\frac{\sum_{t_1 \in T} Vol_{i_t j_{t_1}}^{s}}{80} \right)^2 \right\rceil, \ \forall \ (i,j) \in \boldsymbol{R}, \ t \ \in \boldsymbol{T}, \ s \ \in \boldsymbol{S} \tag{43}$$

The UO-POAVAP was implemented in the Mosel language and solved using Xpress 7.7, an optimization tool that uses branch-and-bound for solving MIP problems (FICO, 2014). The model is run for 30 iterations (plus initialization) with an objective function that only considers the value of travel time (VTT) ($\alpha = 2$euros/time_step), distance driven by the cars ($\omega = 1$euro/km) and very high PT costs (parking and penalties for early or late arrival are ignored and the high PT costs assure that PT is not used as an option).

The model runs in just a few minutes given the size of the problem. Analyzing the flows and travel times resulting from the trips in the morning only, it was possible to see that in the end of the model run 24 paths of type 1->5->4

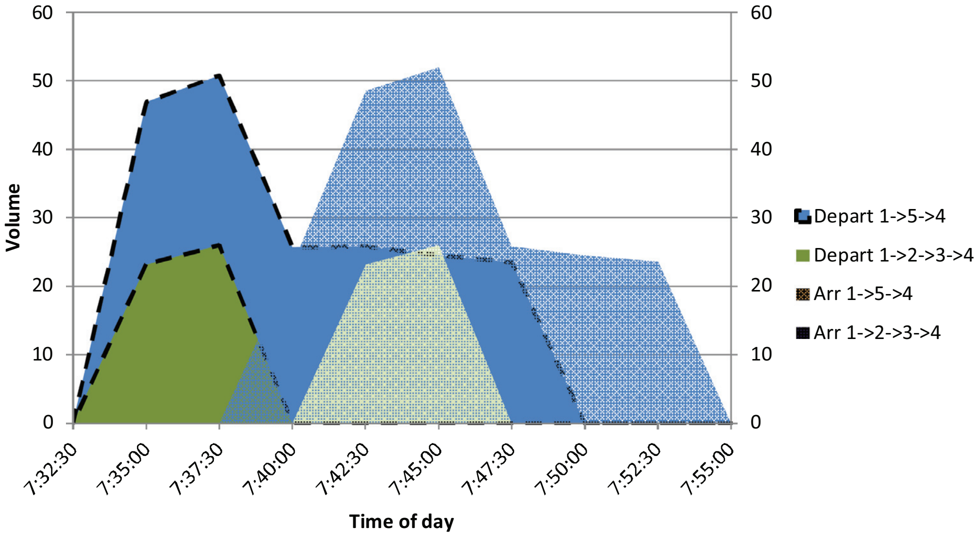

Fig. 4. Departures from node 1 and arrivals at node 4 in the synthetic example.

starting at different periods of the day were used. The 24 paths have an average volume of 8.3 vehicles per time unit (with a small standard deviation of 0.82). The paths all start between 6:50 and 7:50 in 2.5 min intervals. There is no special preference toward any of those hours because there are no penalties for early or late arrival. All of the travel times of the paths have a duration of 2 time units, which is equivalent to 5 min. Thus the solution is pointing for an equilibrium because no traveler may chose a better option by going through nodes 2 and 3. All 400 trips were satisfied by the cars hence there is no variation in the number of trips from one iteration to the next, which in practice yields this criteria not eligible for convergence checking in those situations where the alternative mode is not competitive. The transport costs of the households have an average of 8.7 euros with no deviation. Defining delay as the travel time above the free flow speed, this scenario results in a total delay for all cars of 4.4 h (6.7% of the total driving time).

When adding parking and late/early arrival penalties ($\alpha_l = 3.27$ euros/time_step, $\alpha_e = 0.76$ euros/time_step) the model gives different results. Fig. 4 shows the stacked volume of trips departing and arriving in the morning trip using each of the two paths toward node 4 where it is possible to observe that there is a preference for the path through node 5 because it is the shortest but now there is the use of the longest path given the preference to arrive at a specific time at node 4 (7:52:30). As can be seen all the arrivals happen before the expected arrival time, in which the penalty is lower (7:55:00). The average transport cost of each household is 13.5 euros and its standard deviation in the end of the iterative process is 1.3 euros. The delay is 29.9 h (33% of the total driving time) which is an increase in relation to the previous scenario.

Running the same model with half of the VTT, from 2 to 1 euros/time step, a higher delay of 34 hwas obtained (40.0% of the total travel time) because travelers are now more insensitive to congestion time and try to use the shortest distance more that the shortest time.

In another run, PT costs are considered for each trip, irrespectively of the OD pair, as 6.75 euros. If the two existing trips in each household are satisfied by PT this represents a total cost of 13.5 euros (the average transport costs of the previous case where only cars were used). After the iterative process the car split is 50% of all demand (200 trips) and the average household costs are 13.5 euros which are the PT costs as explained. This result shows that PT is substituting trips which would have a higher cost in case they were satisfied by cars, thus, denoting the necessary equilibrium whereby no family may lower its transport costs by choosing another path by car or by choosing PT.

As a last experiment we consider a changed network where the path 1->2->3->4 no longer exists and there is now a longer alternative through nodes 6 and 7 (Fig. 5). We consider that half of the 200 households have an extra member who has a trip from node 1 to node 7 in the morning and then from node 7 back home to node 1 in the afternoon, with the same schedule as the first member (total number of trips is hence now 600). As can be seen, node 7 is located in a path that can also be an alternative to reach node 4.

We first run the model with the original VTT and a generalized cost for a PT trip of 16 euros, for which we obtain 400 trips satisfied by a car and a delay of 54 h (43.8% of the total travel time). The only trips being satisfied by car are the ones between nodes 1 and 4 because satisfying trips with an extreme in node 7 is too costly given the high VTT. When the model is run with half of the VTT and the same PT price as before 495 trips are satisfied by automobiles and the total delay drops to 46 h corresponding to 29.4% of the total driving time. This is happening because when the VTT is lower, traveling by car gets cheaper and trips which could not be part of the route of the car can now be included in that routing, hence some family cars detour to node 7 and afterwards reach node 4 whilst the other families (families who only have one member) go directly to node 4 because it is the shortest path. This will break up traffic and apparently, in spite of the lower VTT making congestion more attractive, less delay is obtained under such conditions.

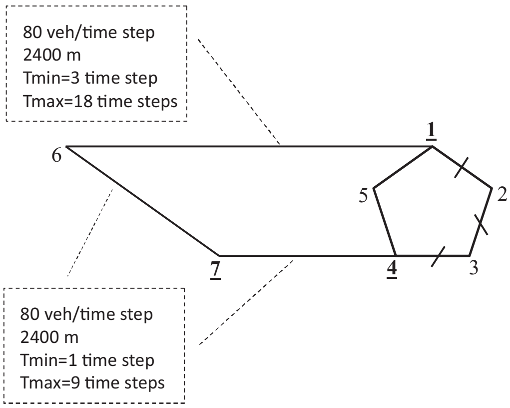

Fig. 5. Simplified network with extra links and extra trips.

Fig. 6. Map of the case study area (topological network on Google Earth satellite view).

4. Application to a quasi-real case study: the city of Delft

4.1. Setting up the case study

The UO-POAVAP is applied to a small city in the Netherlands, Delft, in the province of South Holland. The application is called a quasi-real case-study because not all the data that are used are real. Moreover, despite using real travel data, only the trips of families who travel inside the city during the course of a whole working day in the year 2008 were obtained. This means that traffic flows which are observed in the network cannot be validated in reality. The mode choice model between private vehicle and PT uses the coefficients obtained from a study on multimodal mobility in the Netherlands (Arentze and Molin, 2013). That study provided us with reference values for the generalized cost functions (Eqs. (6) and (7)).

The purpose of the case study is to test and exemplify the model's application and at the same time to get a first look at the type of effects we may expect from the introduction of fully-automated vehicles in urban areas. Fig. 6 shows the simplified road network of Delft superimposed on the satellite view of the region. The city center is marked with an ellipse.

The network has 61 road links and 46 nodes (white lines and black dots). Some of the links have two lanes per way corresponding to the road profile that can be observed in the city. Fig. 6 shows all the nodes and links of the network. The centroids of the TA zones (13 postal code areas) are also indicated (white circles around 13 nodes). All the origins and destinations of the trips are georeferred to the postal code centroids.

The mobility data was obtained from the Dutch mobility dataset (MON 2007/2008). The Dutch government makes this database available for mobility research, in the form of daily information collected on the movements of a sample of individuals. They record the purpose of travel, the origin and destination, transport mode, departure and arrival times. Information about the household is also collected. Details include the composition and size of the household and age, gender and education level of all its members. Until 2008 all trips made by a household were surveyed, which is the information needed to apply the UO-POAVAP.

152 trips were used (29 households sampled), which were made by residents who traveled only within the city of Delft during the surveyed day. Sampling expansion factors for each family were also given for a normal working day, this coefficient varying from 200 to 1300. All modes and motives are included in the sample. With the sampling rate expansion, the 152 trips represent 68,640 trips done by 14,640 households yielding an average sample rate of 0.2%. We could not analyze 14,640 households given the time consuming algorithm, so we used the previously defined expansion coefficient μ_h whereby each analyzed h household represents μ_h number of real households. In this application, a surveyed household that represented, for instance, 480 real households according to the mobility survey was transformed into 24 synthetic households with exactly the same characteristics, which means that an expansion coefficient of $\mu_h = 20$ is used for all $h \in \boldsymbol{H}$. Therefore, for the 14,640 real households, $\frac{14,640}{20} = 732$ synthetic households are analyzed. Therefore the model is therefore capable of greater detail regarding the households characteristics, but our case study was limited by the quality of the original travel survey.

The following simplifications and considerations were used to operationalize the method in its application to Delft (base scenario with automation):

- The time step of the optimization is 2.5 min (time_step).
- The capacity of each link was defined for one hour, which, within the model, is divided uniformly according to the time-step size that is being considered. This means that no rigorous estimation of the link's capacity for each time step of the optimization is computed.
- Only two capacities were considered for the network, one for one lane per direction roads and another for two lanes per direction, which were 1600 and 3200 vehicles (Q_{ij}) respectively. The maximum speeds were assumed to be 50 and 70 km/h respectively for the lower and higher capacity links.
- The departure and arrival times of the surveyed trips in Delft are used as reference to establish the a^{emh} and b^{emh} data vectors with a 15-min slack. For the earliest departure the slack is subtracted from the original departure time, and for the latest arrival the slack is added to the original arrival time.
- The preferred departure and arrival times are assumed to be those recorded in the survey.
- The trips only relate to adults. No extra occupants are allowed for when the trip is to facilitate taking a child to school, hence the representation of the loads in the vehicles is not entirely real. All occupants are assumed to be able to drive an automobile if required.
- All families have at least one vehicle available or the number of vehicles stated in the survey. In practice this allows the model to always consider the possibility of using a car for the family trips. In Delft all families had at least one vehicle.
- All trips within the same zone are assumed to be made by walking, thus, they are not part of the objective function.
- The optimal paths are only computed for a selection of nodes which are called the notable nodes. These include the origins and destinations of the trips made by the household and also any other public parking locations. This definition is useful to reduce the computational time because for instance, when parking charges at the nodes are the same, there is no reason why a node other than an origin or destination of a trip should be chosen for parking (note that there are no parking capacity limitations in the model).

The data needed to run the case study is as follows:

$H = 732$ households
$\mu_h = 20$ families (the same expansion is used for all synthetic households)
$T = \frac{24\ \text{h} \times 60\,\text{min}}{2.5\ \text{min}} + 1 = 577$ time instants
$G = 576$ time steps

t_{ij}^{min}: minimum travel time by car in time steps for each link (i, j) which is obtained from the free flow speed with an absolute minimum of one time step (2.5 min in this application). This introduces limitations because it happens that certain links have lower travel times than the time_step precision. However, this is needed to use the time-space network. It also implies that it is not possible to use big time steps which would accelerate the computation but would make the case-study totally unrealistic. No impedance was considered at the nodes.

t_{ij}^{max}: maximum travel time by car in time steps was computed for a speed of 5 km/h where the curve between the minimum and the maximum travel time is given by the previously referred Bureau of Public Roads curve.

Table 1
Household trips for testing the model.

Member	Trip	Origin (Node)	Destination (Node)	Departure (hour)	Arrival (hour)
1	1	19	41	9.0	9.3
1	2	41	19	11.0	11.3
1	3	19	3	14.0	14.2
1	4	3	19	14.5	15.0
1	5	19	31	15.0	15.2
1	6	31	19	16.0	16.2
2	1	19	18	9.3	9.3
2	2	18	19	9.8	9.8
2	3	19	3	11.8	11.9
2	4	3	31	12.1	12.1
2	5	31	3	12.6	12.6
2	6	3	19	13.3	13.4
2	7	19	31	14.5	14.8
2	8	31	19	16.0	16.3
3	1	19	41	9.0	9.3
3	2	41	19	11.0	11.3
3	3	19	18	14.0	14.0
3	4	18	19	17.0	17.0
4	1	19	3	11.8	11.9
4	2	3	19	12.3	12.4
4	3	19	3	14.5	14.8
4	4	3	19	16.0	16.3
5	1	19	3	11.8	11.9
5	2	3	19	12.3	12.4
5	3	19	31	14.5	14.8
5	4	31	19	16.0	16.3

t_{ij}^{PT}: the travel time in PT was defined for a BUS based PT trip for a commercial speed of 12 km/h. It was computed through the free-float shortest paths in the road network. 5 min were added to all travel times to emulate walking to the BUS stop and waiting for the vehicle.
Cap^{hv}: the capacity of the vehicles was set to 4 passengers per vehicle for all vehicles.
ρ: the generalized cost penalty of using PT is 7.622 euros
β: total travel time cost per minute in PT (includes waiting) is 0.755 euros/min
α: travel time cost by time step in a car is 0.806 $\left(\frac{\text{euros}}{\text{min}}\right) \times \text{time}_{\text{step}}$
α_l: cost for arriving late is 1.306 $\left(\frac{\text{euros}}{\text{min}}\right) \times$ time_step
α_e: cost for arriving early is 0.306 $\left(\frac{\text{euros}}{\text{min}}\right) \times$ time_step
ω: fuel costs by time step in a car are equal to 0.1 $\left(\frac{\text{euros}}{\text{km}}\right)$
γ: scale for the parking costs in relation to the fuel and maintenance costs is 1.81
ς: scale factor of the ticket cost in relation to the fuel and maintenance costs is 2.11
$Tick$: the PT ticket was considered constant and equal to 1.5 euros per trip
Pk_i: the parking costs are 1 euro/h for all locations, which is a compromise between the highest value charged per hour and the hourly rate for staying one day in several parking lots in Delft. By default there are three locations that have free parking: the home location of the household in Delft and two locations outside the city which correspond to nodes 15 and 41 (Fig. 6).

4.2. Running the UO-POAVAP for one household and one iteration in Delft

The model was run for one household and one iteration in the Delft's case study network to see how it performed. The household with most trips (26 trips in total) was chosen. Its traveling information can be seen in Table 1.

As we can see, this household has 5 members who make several trips. It is easy to conclude that the home location of this family is at node 19 because the first trip starts there and the last trip ends there for all household members. Naturally, in this example there are no congestion effects because only one household is being assigned. The expansion coefficient of 20 is ignored, therefore the results relate to one household only.

The problem has 9674 constraints and 9142 variables for the 2.5 min time step defined in the previous section, with a 1-min time step there would have been 38,172 constraints and 40,801 variables, which is a considerable increase, and justifies the use of the 2.5-min time step. In Fig. 7 we can see that finding an optimum solution for the routing of the only family vehicle took 24 seconds.

The solution has a generalized cost of 383.96 euros and 19 of the 26 trips are satisfied by this family's vehicle (the rest by PT). The vehicle was parked for 19.8 h at the home location (node 19) and at node 41 (one of the two free parking lots outside the city).

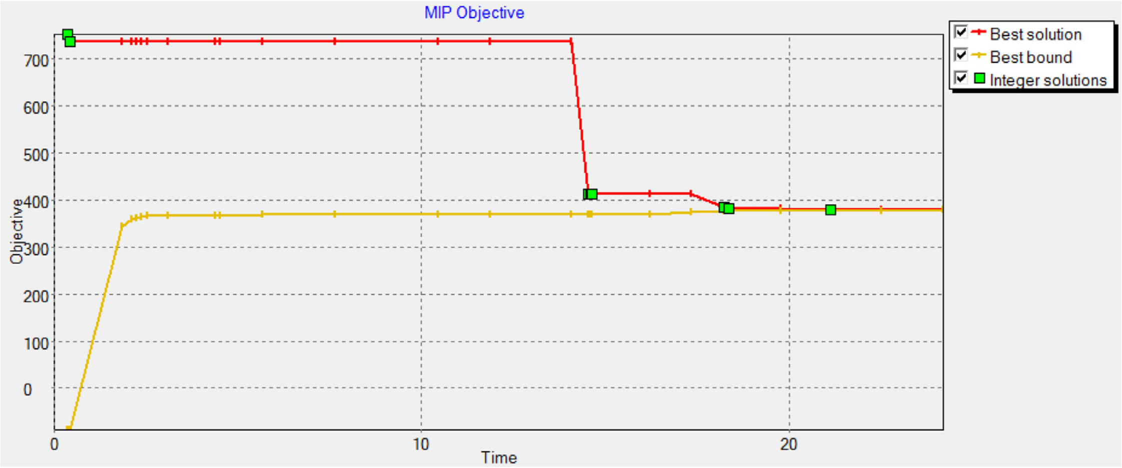

Fig. 7. MIP objective through time (seconds). Output from Xpress-MP.

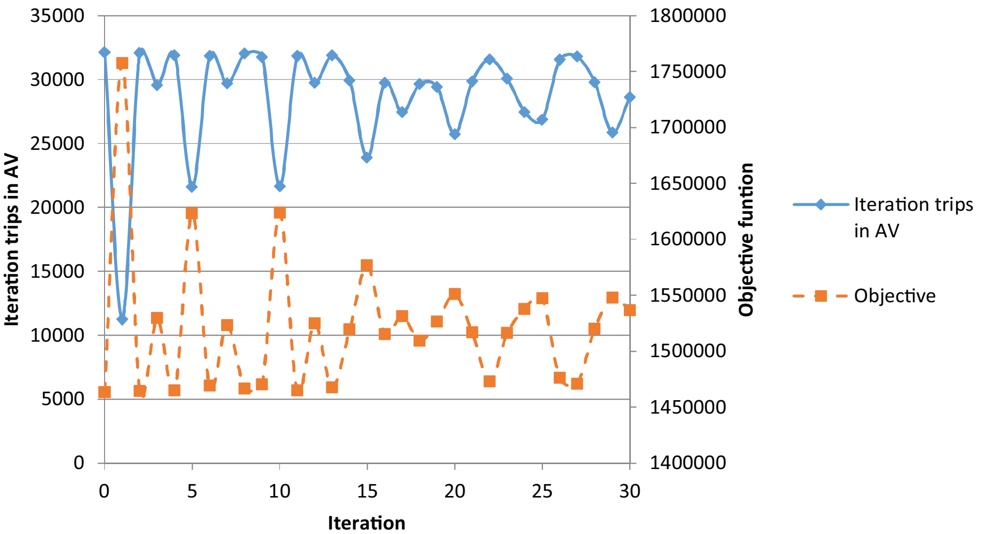

Fig. 8. Performance of the UO-POAVAP for the base scenario with automation.

The car drove 17.5% of the day (about 4.2 h). The total time spent by the family members inside the vehicle was 4.17 h x person. The generalized cost of driving the AVs was 216.5 euros; the generalized cost of the PT trips was 164.9 euros; and the cost of parking was zero since the vehicle only stopped at free parking lots. The remaining part of the generalized cost is related to the schedule penalties.

4.3. Performance of the UO-POAVAP

Before presenting the results of running the model for several scenarios, we first look at the computation performance of the UO-POAVAP when applied to what we call the reference scenario with automation. It is important to check if the convergence of the method is indeed happening, to allow the assumption that a state of equilibrium or at least one close to equilibrium has been reached. The maximum error π is defined as zero, thus, the model will only stops at the maximum number of iterations or when $\pi = 0$.

In Fig. 8 we show two main indicators over the UO-POAVAP running for 30 iterations plus the initialization. The left vertical axis represents the car trips in each iteration resulting from the assignment of the AVs to the network with the current travel times. The right vertical axis displays the objective function values. Together, they show that there is a reduction in the amplitude of the variation of these indicators as time progresses; however, this does not show any tendency for a perfect convergence. Rather, it shows that there will be an oscillatory behavior throughout the algorithm application no matter how many iterations are allowed.

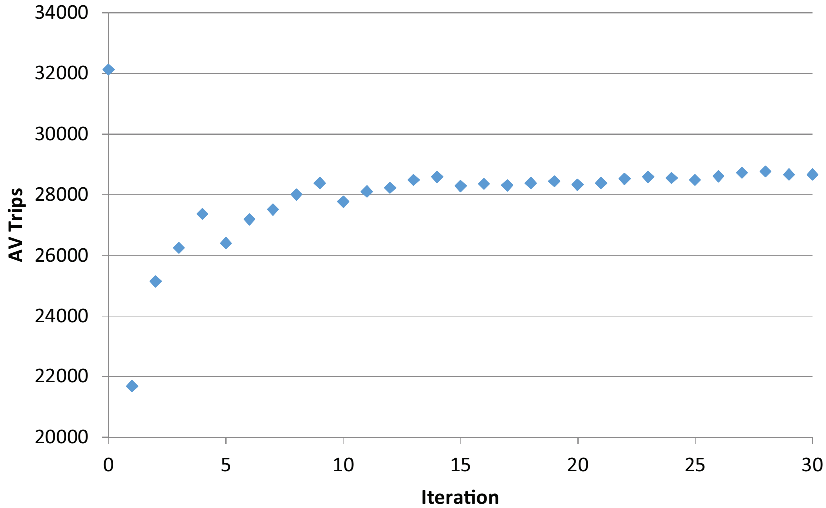

Fig. 9. Number of trips in AVs resulting from each iteration for the base scenario with automation.

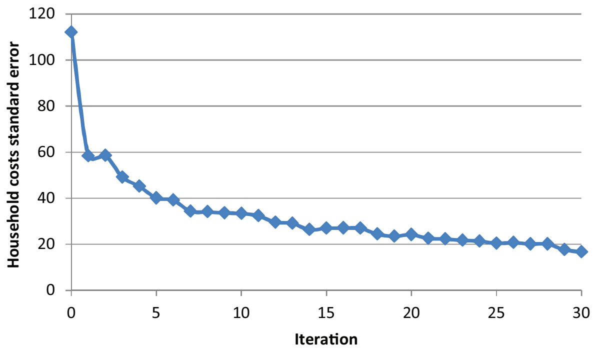

Fig. 10. Household transport costs standard error.

This result is caused by the integer nature of the algorithm that has been proposed for the UO-POAVAP. Both the flows and the travel times are integer. In practice, this means that if in one assignment the number of AV trips is high, the travel time in the links where these vehicles are routed will increase in a double or triple proportion, making them much worst in the next iteration, which will itself lead to fewer trips using AVs. This effect is dissipated as the number of iterations increases, but it will not disappear (as seen in Fig. 8). After running 100 more iterations we confirmed that behavior continues, with the extra observation that the rounded integer travel times obtained by applying the BPR function are highly dependent on decimal volume changes. This makes the converge of these indicators to zero a matter of computer precision.

The final performance indicators produced for each scenario, such as the total driving time, are in fact a combination of the current and previous iterations, using $\phi = \frac{1}{s}$ as in a normal TA. Thus, the chart in Fig. 9 can be produced with the number of AV trips from one iteration to the next. This should not be confused with the "Iteration trips in AV" reported in Fig. 8, which are the result of the current iteration assignment and so are similar to an all-or-nothing assignment, as applied in other traffic equilibrium methods.

A convergence measurement that shows that the algorithm is approaching an equilibrium from one iteration to the other is still lacking, as the number of trips in AV does not allow any conclusions regarding equilibrium of transport costs. Given that what we are proposing is a balance between household transport costs and not a balance between travel times for a period of the day or a peak hour, we use the generalized costs of each household to check for convergence. If the solution is approaching an equilibrium then the costs of the cloned households (several households are repeated in the input data) should be very similar, even if they involve different routes or even different modes, car or PT. This is similar to the approach used in TA, where the travel times for different routes linking the same OD pair are compared.

Fig. 10 shows the standard error between the costs of the same group of cloned households over the iterative process. We can see that this tends to decrease with the number of iterations, which denotes an approximation among the households as the process continues. After 30 iterations the standard error is about 18 euros, which we consider acceptable for running the scenario analysis in the next section.

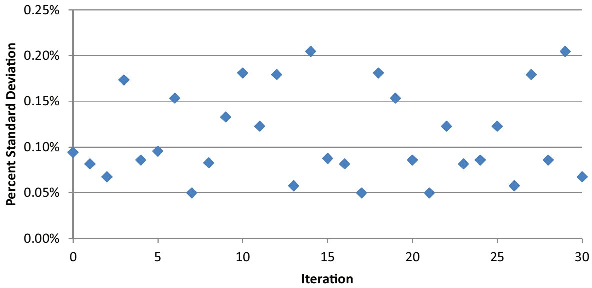

Fig. 11. Standard deviation as a percentage of the mean objective function value of 5 random sets of households over 30 iterations.

Table 2
Scenarios description.

	Scenario	Description
	I) No automation	This is the reference scenario however constraints (38) are imposed as to consider that vehicles are not automated.
With automation	II) Automation - reference	This is the reference scenario as described in the case-study. Vehicles are automated, there is free parking at home and two free external parking lots (nodes 15 and 41).
With automation	III) Paid parking everywhere	In this scenario parking has the same price everywhere, including at the home place of the travelers.
With automation	IV) Free parking everywhere	No parking costs in the entire city.
With automation	V) No free home parking, only at the peripheric nodes (15 and 41)	There is no free parking which means that the only two parking lots where people can park at home for free are located at nodes 15 and 41.
With automation	VI) No free home parking, free central parking at node 23	There is only one parking lot where vehicles are allowed to park without paying, which is located at node 23.
With automation and lower value of travel time	VII) Lower value of travel time	The value of travel time is now considered as half of the original one, this means that α is now $0.403\ (\frac{euros}{\min}) \times time\_step$.
With automation and lower value of travel time	VIII) Lower value of time and no free home parking, only on the peripheral nodes (15 and 41)	Lower value of time and in this case there is no free parking at home and as by default there is only free parking at nodes 15 and 41.
With automation and lower value of travel time	IX) Lower value of time, no free parking at home and free central parking (node 23)	In this scenario there is a combination of lower value of travel time, no free parking at home and the existence of free parking at node 23.

A final analysis is done on the influence of the households' random order on the optimization results. For the same scenario (reference with automation), we ran the model with 5 different randomly ordered sets of households. In Fig. 11 we present the standard deviation of the objective function of the 5 replications as a percentage of the mean value along the iterations. We can see that for this case study of Delft the effect is residual with a maximum of about 0.20% in the 14th iteration. Therefore, we decided to run 30 iterations of one ordered set of households for all the scenarios presented in the next section.

5. Experiments and results

We ran the UO-POAVAP for several policy and future uncertainty scenarios. The key variables for creating these scenarios are the parking charges in the city and the VTT. The consideration of a lower VTT is related to the hypothesis that people will be able to enjoy more the time inside a vehicle, by working, communicating or even hanging out with family and friends in the redesigned interior of the vehicles. At present we can only speculate as this has not been demonstrated by any study. Furthermore, any study being done at the moment cannot be particularly accurate because the fully automated technology is not yet available on the market. Still, the assumption of a reduction in the VTT makes sense given what we know about this important attribute of mode choice.

All scenarios are described in Table 2 and their results are given in Table 3, where scenario II was the one that was used in the previous section to assess the performance of the UO-POAVAP.

Table 3
Scenario testing results.

	Scenario	Generalized Cost Objective Funtion	Generalized Cost per trip	Number of trips in an automobile	Number of trips in PT	Car mode share	Total Active vehicles [1]	Trips per act. veh	Delay (h) [2]	Parking time (h x veh)	Parking time of act. veh (h x veh)	Avg park. time per act. veh (h/veh)	Driving time of act. Veh. (h x veh)	% of congestion	Driving time per act. Veh. (h/veh)	Time inside a veh. aggregate (h x pax)	human travel time per pax (min/pax)	travel time per automobile trip (min)	Total km travelled	% of empty kilometers
	I) No automation	1539100	26.4	26280	34020	43.6%	8844	2.97	110.8*	344631**	205521**	23.2**	6729	1.65%	0.76*	8547	19.51	15.36*	109640	0%*
With automation	II) Automation - Reference	1520000	26.2	28320	31980	47.0%	8296	3.41	142.9	343389	191139	23.0	7971*	1.79%	0.96	9184	19.46	16.89	128602	11.5%
	III) Paid parking everywhere	2262160**	37.7**	12560*	47740**	20.8%*	9025	1.39*	364.4	309334*	174582	19.3*	42026**	0.87%*	4.66**	4003*	19.12*	200.76**	466655	87.4%**
	IV) Free Parking everywhere	1510240	25.7	31000	29300	51.4%	8785	3.53	165.4	342604	202084	23.0	8756	1.89%	1.00	10184	19.71	16.95	132574	10.8%
	V) No free home parking, only at the peripheral nodes (15 and 41)	1563900	26.4	23580	36720	39.1%	7155*	3.30	1023.9	334306	154666*	21.6	17054	2.38%	2.38	7932	20.18	43.39	265974	64.8%
	VI) No free home parking, free central parking at node 23	1509170	25.6	28460	31840	47.2%	8388	3.39	885.5	333792	183744	21.9	17568	5.04%**	2.09	9476	19.98	37.04	240260	53.2%
With automation and lower value of travel time	VII) Lower value of travel time	1290690	23.2	32220	28080	53.4%	8714	3.70**	110.8*	341099	198869	22.8	10261	1.08%	1.18	11896	22.15	19.11	163818	10.3%
	VIII) Lower value of time and no free home parking, only at the peripheral nodes (15 and 41)	1421480	23.7	26360	33940	43.7%	7502	3.51	416.7	331571	160256	21.4	19789	2.10%	2.63	10266	23.37**	45.04	318286	62.28%
	IX) Lower value of time, no free parking at home and free central parking (node 23)	1267430*	21.8*	32720**	27580*	54.3%**	9203**	3.56	993.5**	330511	200015	21.7	20849	4.77%	2.27	12463**	22.85	38.23	291333	50.39%

**maximum *Minimum
[1]vehicles that move either with or without passengers for at least one time step
[2]Delay here is defined as the travel time driven above the free-flow speed

Each scenario was run in an Intel ® Core ™ i7-46000 CPU@2.10 GHz computer with 16GB RAM. For each scenario a maximum number of 30 + 1 iterations were allowed ($S = 30$). The computation time is not indicated on Table 3 but each iteration took about 50 min to be computed. Two results are not presented in the table because they do not change across the scenarios, these are the potential number of trips to be satisfied by an AV, which is always 60,300 (the other 8340 trips of the 68,640 are done in the same zone), and the total fleet, which is always 14,640 (one per family).

Looking first at the objective function value of all scenarios it is possible to observe that the last scenario (scenario IX) is the one that leads to lower generalized transport costs, where there is a lower VTT and there is free central parking at node 23. This shows the considerable importance of the value of human travel time in total generalized costs.

The difference between scenarios VII and IX is that in the latter free parking is offered at node 23, which leads to a significant growth on the delay (1.08–4.77%). This is explained by empty vehicles competing on routes leading to node 23, as can be seen in the high percentage of empty kilometers. Nevertheless, this congestion and its corresponding gas costs seem to be offset by the lower parking costs at node 23. Moreover, we can see that it may be possible to concentrate parking in a few central parking lots that will offset parking charges in residential areas and should allow city centers to be cleared of street parking. When parking is not offered in the city center (only at the peripheral nodes, as in scenario VIII) there is an increase in generalized transport costs and the car mode share falls to 43.7. Nevertheless, this leads to less congestion than compared to locating parking centrally (delay of 2.10% against 4.77%), which may be more desirable from an environmental point of view.

The highest generalized cost of all scenarios is yielded by the “III-Paid parking everywhere” scenario. Probably the most realistic policy in a city is to have different parking rates in different areas; however, we wanted a single scenario to test across-the-board charges. This scenario resulted in a sharp fall in the car mode share from 47% (“II-automation reference”) to 20.8%. The number of active vehicles grows because there are many situations where it is cost-beneficial for the vehicle to move alone without satisfying any trip than to be parked. This can be seen in the huge percentage of empty kilometers: 87.4%. The delay in this scenario is actually not very high, percentage-wise (1.08%), even though the total car driving time is substantial (42,026 h x veh against 7971 h x veh. of the automated reference scenario II), which yields a 200 min average automobile trip duration. Vehicles are moving empty to avoid parking, but they do so locally, without adding to the major traffic flows.

The second highest transport costs are associated with the “V-No free home parking, only at the peripheral nodes (15 and 41)” scenario. Nodes 15 and 41 are on the periphery so the transport costs are higher than the reference (1,563,900 against 1,520,000), whereas when free parking is offered centrally (scenario VI) transport costs actually decrease in relation to the reference scenario, but similarly to what was said before this happens at the cost of added congestion in the city center. Delays are the highest of all scenarios (5.04%), which makes this policy less acceptable under current policy trends of discouraging cars in city centers.

Comparing the reference scenario with and without automation (I and II in Table 3), we see that there is not a large difference for this Delft case study. Nonetheless automation captures about 8% more car trips than the non-automation scenario (a difference of 2040 trips). We must note, however, that this result may be amplified in other case studies by the travel distances, number of trips and number of families in a city. The differences between the scenarios also show a reduction in the overall generalized cost of traveling and greater congestion for the automation scenario. This clearly shows that a vehicle’s independence from its travelers will affect mobility by increasing the number of trips satisfied per vehicle (from 2.97 to 3.41 trips/veh) and eventually worsening congestion because of the extra vehicle travel time (from 0.76 to 0.97 h/trip).

Comparing the reference scenario with automation (scenario II) with the one where the drivers’ VTT drops (scenario VII), the results change considerably: the generalized costs of traveling fall, which can be directly attributed to the lower VTT, and the number of trips done in an automobile rises (from 26,280 to 32,220 trips). What is perhaps more surprising is that congestion actually decreases: despite the fact that there are more vehicles circulating in the city (8714 compared to 8296), each satisfying more trips (on average 3.70 against 3.41), with a slight increase in total travel time per vehicle (1.18 to 0.96 h/veh), the increase in travel time results from longer routes and not necessarily from accumulation in congested links. Those longer routes are apparently being done to facilitate other trips which in the past were not cost efficient to satisfy because of the high VTT, hence the higher passenger travel time (22.15 min/pax against 19.46 min/pax). This change of routes apparently has the positive side effect of generating less competition in the most used links of the network. This confirms what had been concluded in the small example network before.

6. Conclusions and future work

The model described in this paper is the first of its kind reported in the literature. By combining TA and vehicle routing, we have provided a method to solve what we called the User Optimum – Privately Owned Automated Vehicles Assignment Problem (UO-POAVAP). The method takes the approach of a multimodal user equilibrium traffic assignment model, where PT and cars (automatic or not) are the two competing modes. Because fully-automated vehicles are free to move without a human occupant they are able to relocate and park themselves to satisfy as many household trips as is cost efficient for that family.

The method was applied to a small network and to a realistic case study based on the city of Delft in the Netherlands. We found that the number of trips satisfied by the cars in each iteration does not converge to a stable value; it has an oscillatory behavior which can be explained by the integer nature of the routing problem. Combining integer programming with traffic equilibrium methods in a network is not allowing to obtain a well behaved convergence algorithm and that is a noteworthy result from this paper. Results are dependent on the rounding of the vehicle flows and their effect on delays, whereby the optimization tries to reach the best solutions approaching, but never reaching, the threshold where an integer travel time would be higher in each link. Nevertheless, the amplitude of the oscillation of the number of trips decreases with the number of iterations and stabilizes after only a few. Much like in TA, the results are a weighted average between the current and previous iterations, which smoothes the indicators. We checked the user optimum equilibrium in the solutions by means of the generalized transport costs of similar households and concluded that their differences tend to decrease as the number of iterations increases. The random order in which households are analyzed in the method does not bias results, at least for the demand and network of the Delft case-study.

Interesting conclusions can be drawn from the application of the method to the Delft case study. We concluded that the AVs can satisfy more trips than the conventional vehicles and create only a small increase in congestion despite the extra kilometers. It was also possible to conclude that if in the future car users perceive a lower value of travel time in a redesigned vehicle for leisure and work, this could lead to an even greater number of satisfied trips that might actually not come at the cost of more congestion: we have seen that the lower value of travel time can be an advantage by creating an opportunity for satisfying more trips which will reroute the vehicles, thus, having a positive side effect of competing less with other family cars on traffic.

Parking plays an important role in the performance of the system: providing free central parking for all cars in a scenario where everywhere else is paid could apparently lead to a similar or even bigger car trip satisfaction than the one that is achieved when there is free parking at home. This leads to the conclusion that it may be possible to concentrate vehicles in one place. Having free special parking lots on the outskirts of the city is not as attractive as having them in the center because of the extra kilometers, but if the value of travel time is lower we found that the same number of trips satisfied today with conventional vehicles can be served with AVs, but in this case with the added advantage of completely freeing the city center from parked cars.

This paper poses many questions for which we do not have an answer yet. On the methodological side we believe that giving vehicle routing problems some traffic assignment properties and vice versa is useful, however, the methods to do that have to be further investigated in terms of convergence to an equilibrium and in what respects to computation time, which is currently prohibitive for bigger cities. We would also like to find a solution for the system optimum problem and compare it with the user optimum to assess the possible benefits that can accrue from fully controlling the schedule of the trips, however, with the current formulation this may be a very hard task to do given the highly combinatorial nature of routing many vehicles for many trips in the same solution.

On the UO-POAVAP it is still possible to add improvements such as adding other costs, such as pollution, and benefits, such as free space in the city center, in a multi-objective approach involving the perspective of both travelers and society. The realism of the model can also be enhanced by better characterizing the competition of the AVs with the inclusion of a network for PT as well.

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