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A utility-based travel impedance measure for public transit network accessibility

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ABSTRACT

A utility-based travel impedance measure is developed for public transit modes that is capable of capturing the passengers' behaviour and their subjective perceptions of impedance when travelling in the transit networks. The proposed measure is time-dependent and it estimates the realisation of the travel impedance by the community of passengers for travelling between an origin–destination (OD) pair.

The main advantage of the developed measure, as compared to the existing transit impedance measures, relates to its capability in capturing the diversity benefit that the transit systems may offer the society of travellers with different traveling preferences. To clarify the necessity of such capability, we demonstrate the randomness (subjectivity) of travel impedance perceived by transit passengers, through evidence from the observed path choices made in the transit network of the greater Brisbane metropolitan region in Australia.

The proposed impedance measure is basically a nested logit "logsum" composition over a generated set of reasonable path options whose systematic utilities are evaluated based on a discrete choice model previously developed and calibrated for the greater Brisbane transit passengers. As a case study, the proposed impedance measure is calculated for all the origin blocks in the Brisbane area, during the morning commutes to the Central Business District (CBD). The results are presented and discussed, and intuitive and important advantages are demonstrated for the proposed measure.

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1. Introduction

Improving the accessibility of public transport is essential to the sustainability, liveability, and welfare of human societies. An accessible public transport system can encourage people to use and to rely on public transport, leading to a mode shift in favour of public transport (Owen and Levinson, 2015), which in turn decreases the negative effects of automobiles on the environment, and on urban mobility in populated cities. An accessible public transit network can also enhance liveability and welfare of our cities by improving equity, easing work-related commutes, and encouraging the social inclusions of the carless, elderly and disabled (Cervero et al., 1995; Currie and Stanley, 2008).

On the other hand, there are interesting arguments in the literature noting the possible drawbacks of infrastructural development – mainly highways but public transport systems too – and of the increased accessibility of distant suburbs

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to the central business districts of cities. These studies highlight the detrimental effect of such developments from an urban planning and environmental perspective, such as induced long-distance travel demand, urban segregation and separation, energy use, etc., and they emphasize these potential detriments on the sustainability and the quality of our modern cities (Metz, 2008; Banister, 2011; Crozet, 2013). As a result, the analysis of accessibility in general, and the study of transit network accessibility in particular, has become a sensitive topic and important direction of research which has recently attracted a lot of attention among researchers in transport planning, urban geography, and sustainable development. The main goal of the research reported in this paper is to improve estimates of public transit travel impedance, which is an important component in the measurement of accessibility.

During the past few decades, a significant body of literature has contributed to quantifying urban accessibility. A majority of these approaches have agreed on measuring accessibility based on two main components (Hansen, 1959; Pirie, 1979; Handy and Niemeier, 1997; Bhat et al., 2002; Church and Marston, 2003; Kwan et al., 2003; Levinson and Krizek, 2005; Cascetta et al., 2013). These two components are: (1) locations and attractiveness of urban opportunities (benefit side); and, (2) impedances of travelling to these locations from residential areas in the network (cost side). Based on these definitions, more accessible areas are the ones that have lower impedances for travelling to attractive locations.

Transit accessibility has also been defined in a similar fashion, with the only difference that the mode of travel is restricted to public transit (and perhaps walking), and the impedances are calculated based on the public transit network (Hillman and Pool, 1997; Murray et al., 1998; O'Sullivan et al., 2000; Liu and Zhu, 2004; Zhu and Liu, 2004). However, due to the inherent complexities in the transit network that are the result of spatio-temporal constraints in service, accurate calculation of transit costs and travel times in real-sized time-dependent networks has often been compromised. A large body of research in transit accessibility is focused on accessibility to the transit network, as a potential end in itself, instead of serving as a means of transport to other destinations (Murray, 2001; Polzin et al., 2002; Rastogi and Rao, 2002, 2003; Currie, 2010; El-Geneidy et al., 2010). Although an important part of a transit journey is the access from the origin to the public transit service, the spatio-temporal connectivity of the transit network also affects the accessibility of destinations significantly. It has been argued that measuring transit accessibility by only considering the access to the transit network is proper only in high frequency and dense transit networks (Lee, 2005); or, that such a measurement would overestimate the accessibility (Lei and Church, 2010). Moniruzzaman and Páez (2012) investigated accessibility to transit, accessibility by transit, and the mode share, and their case study of the city of Hamilton, Canada, concluded that accessibility by transit is a significant predictor of modal share. Therefore, for creating a positive mode shift to public transport, a closer analysis of transit accessibility and its related component, namely transit impedance, is imperative.

Some researchers have recently acknowledged the importance of accessibility through the transit network and to destinations. With a lack of detailed transit schedule information, they have applied simplified transit cost calculations and made travel time estimates based on average route speeds and route frequencies (O'Sullivan et al., 2000; Liu and Zhu, 2004; Moniruzzaman and Páez, 2012), or based on simplifying assumptions regarding transfer waiting times (O'Sullivan et al., 2000; Lee, 2005; Yigitcanlar et al., 2007; Mavoa et al., 2012; Tribby and Zandbergen, 2012) regardless of the time dependence of the service.

A recent body of research has based accessibility measurement on accurate time-dependent travel times. A majority of these approaches use schedule-based shortest path algorithms that calculate the fastest travel time between the origin–destination (OD) pair in the time-dependent transit network, with walk links for access, egress, and transfer interchanges (Church et al., 2005; Lei and Church, 2010; Lei et al., 2012; Salonen and Toivonen, 2013). This recent research has been successful in calculating the accurate time-dependent travel times in a transit network as a representation of travel cost between OD pairs. However, in this paper we argue that limiting the perceived impedance of transit passengers only to the total travel time might be a strong assumption. And perhaps more fundamentally, by investigating a dataset of actual transit passengers' path choices, we empirically demonstrate that the assumption that the transit impedance is perceived uniformly among the passengers also needs careful revisions.

One inherent feature that differentiates public transit journeys from other transport modes relates to the complexities of public transport, e.g. the spatio-temporal limitations of the service, the importance of transfers, the multimodality of service, and the importance of strategic path choices (Cats, 2011). These complexities, in turn, result in complexities in behaviour, diversity in user preferences, and random variation in perceptions of transit impedance among the users of the system. This research aims to account for these complexities and to rely on a systematic analysis of observed passengers' behaviours in estimating the impedance. To this end, the proposed measure is defined in a utility-based structure as a function of a diverse set of travel attributes calculated for a diverse set of path/mode options in the transit system. Nassir et al. (2015a) estimated a discrete choice model to describe the access behaviour of users based on various service attributes of the transit network between given OD pairs and at given times of service. Their model was estimated using household travel survey (HTS) data collected from the Southeast Queensland (SEQ) region in Australia. For estimating the impedances, we use the utility function estimated in Nassir et al. (2015a), and we apply the proposed set generation algorithm to generate the universal set of reasonable access stop choices and their utility attributes, from which the perceived travel impedance is calculated. A case study is performed on the SEQ transit network to highlight the characteristics of the developed measure.

Section 2 briefly discusses the background of utility-based logsum measures that have been used in the literature of accessibility measurement. In Section 2 we also highlight the methodological contrasts between the proposed method and the existing relevant logsum measures. Section 3 briefly reviews the access choice model in Nassir et al. (2015a). In Section 4, the variability of the observed path choices and the subjectivity of the perceived transit impedances are empirically demonstrated by exploring the path choices recorded in the SEQ travel survey data. This section further highlights the motivation of this research and the necessity of a random utility impedance measurement. Section 5 presents the proposed impedance measure and highlights its advantages when applied to the case study of the SEQ. Section 6 concludes the paper and summarises the findings in this research.

2. The utility-based approach to accessibility

Utility-based accessibility, first developed by Ben-Akiva and Lerman (1979), has been widely accepted and applied in the urban context (Koenig, 1980; Bhat et al., 1998, 1999, 2002; Chen et al., 2007; Gulhan et al., 2013; Nassir et al., 2014). This utility-based definition of accessibility is consistent with the key concept of the total consumer surplus (net benefit) in using the urban opportunities reachable within a given travel impedance (Hansen, 1959; Cascetta et al., 2013). Knowing that the utility of opportunities are subjective (perceived differently among users), the logsum accessibility is an effective technique that estimates the *expected maximum utility* that a user of a system would perceive, given the choice set available to them. The logsum accessibility (A_n) is defined in a general logit choice model as follows.

$$U_{in} = V_{in} + \varepsilon_{in} \tag{1}$$

$$A_n = E\left[\max_{i \in C_n} U_{in}\right] = \frac{1}{\mu} \cdot \ln \sum_{i \in C_n} \exp(\mu \cdot V_{in}) \tag{2}$$

where U_{in} is the actual utility of choice i perceived by person n,[1] V_{in} is the systematic utility that is estimated to describe the choice behaviour of person n, ε_{in} is the random error term, A_n is the perceived accessibility by person n, μ is the scale parameter, and C_n is the set of all alternatives available to person n. In Eq. (2), the perceived accessibility by person n, A_n, is assumed equal to the expected maximum utility of the available choices for a person n, which in turn equals the logsum of all available choices to person n, calculated based on the parameters of the *systematic utility* of the choice model.

Utility-based measures are popular for their ability to capture the random nature of users' preferences, by incorporating: (1) all relevant and measurable attribute variables, and (2) all considered alternatives in modelling user perceptions of their available choice set. An insightful review of utility-based accessibility measures can be found in Geurs and van Wee (2004).

It is important to note that in the existing transit accessibility measures, the random utility has only been applied to the element of destination attractiveness. This is typically calculated using a logsum from a calibrated destination choice model. However in the existing literature, the *travel* impedance element has been modelled simplistically, by measuring travel attributes of a single path—namely the path that is assumed to be optimal for all passengers. There is an exception to this general statement. Bhat et al. (1998, 1999) proposed and developed a "parallel conductance" calculation to represent the composite cost of three alternative modes (auto, transit, and walk) for modelling the destination choice using travel survey data from Boston, MA (USA). Their proposed approach is an alternative logsum technique for measuring the perceived travel impedance considering the combination of available travel modes (Bhat et al., 1998, 1999); however, even in their approach, the travel impedance measure within each mode is represented by only a single path (the assumed optimal path).

In this research, we develop a measure to more accurately estimate the perceived impedance for public transit by identifying the randomness of preferences among the transit passengers and taking into account the availability of diverse alternative path options in a large-scale multimodal transit network. This model not only incorporates a diverse set of attribute variables to describe the systematic utility (or disutility) of the travel to the destination, but also uses a diverse set of path choices, in a logsum form, to account for the randomness of impedance perception. Identifying this gap in the current practice and addressing it properly are the main contributions in this research.

3. Review of access stop choice model

Nassir et al. (2015a) developed a nested logit discrete choice model to predict the choice of transit access stops[2] (i.e. departure stops). They incorporated time-dependent components of the transit service between OD pairs, for modelling the choice of bus stops, railway stations, and ferry terminals, as access points to the public transit network. The model is structured as a nested logit model with two significant nests, train and no-train (bus and ferry). Their explanatory variables are defined in three categories: (1) facility attributes, (2) service attributes, and (3) correction for correlation among alternatives. Facility attributes are variables describing the boarding stop itself, including the mode serving the stop, access walking time from the origin to the stop, and amenity attributes, such as availability of shelter, illumination, boarding slab, etc. Service attributes are regarding the quality of service between the access stop and the destination, which are calculated time-dependently and are either direct or aggregate. The direct service attributes include the measures of time-dependent optimal paths between the alternative stop and the destination (e.g. fastest path, least transfer path, etc.), and the aggregate service attributes include the average measures (travel time, number of transfers, etc.) among all reasonable paths from the stop to the destination.

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[1] Person n is not necessarily one individual person, but a representative of the socio-demographic group that is characterised by the index, n.

[2] The term "stop", in the context of access stop choice modelling in this paper, is reserved for "bus stops", "railway stations", and "ferry terminals"

Nassir et al. (2015a) utilised correction factors to account for the correlation among the error terms of the overlapped alternatives. They proposed three alternative definitions of correction for correlation (*CfC*) attributes to compute and reflect the amount of interdependencies among the stops as a result of their common routes to the destination. Similar to the correction factor in C-logit (Cascetta et al., 1996) and Path Size logit (Ben-Akiva and Bierlaire, 1999), the *CfC* attributes can be interpreted as the fraction of a totally independent alternative that the particular stop represents. Because *CfC* attributes only take negative (and zero) values by definition, a significantly positive coefficient implies effective reduction of the systematic utility for the overlapped alternatives. In this research we use the definition the best model fit (i.e. *CfC3*).

The departure stop choice set generation algorithm is presented in Section 3.1 and the estimated model is presented in Section 3.2.

3.1. Choice set generation algorithm

The choice set generation algorithm used for the model estimation in Nassir et al. (2015a) is utilised in this research to generate the reasonable access stop alternatives (and to compute their attributes) for measuring the time-dependent accessibility of OD pairs in the transit network. For completeness, this algorithm is presented here.

In the terminology adopted in this paper, a transit "path" indicates the recorded trajectory between a given OD pair that may consist of an access walk leg, one or more rides on a bus, a train or a ferry route, one or more transfer walk legs between two consecutive stops, and an egress walk leg to the destination. A transfer is defined as a movement on the pathway (walking) network between an alighting stop and the consecutive boarding stop location, given that this interchange is only taken for the purpose of reaching the final destination.

The set of reasonable access stops for every OD pair is generated specific to the locations of the origin and destination, and to the time and the day (weekday/weekend/holiday) of departure from the origin. The set generation algorithm is a transit schedule-based K-shortest path algorithm that generates the OD paths (including access walks, egress walks, and transfer walks) with the objective to minimize the travel time at each iteration. The time-dependent transit shortest path algorithm used at the core this tool is a modified version of the transit Trip-Based Shortest Path (TBSP) algorithm (Khani et al., 2012; Nassir et al., 2012; Khani, 2013; Khani et al., 2014). The framework of the proposed choice set generation algorithm is shown in Fig. 1.

This procedure is an iterative path finding calculation with a "segment elimination" module that is executed after each path finding. A segment is defined as the combination of a boarding stop, an alighting stop, and a route connecting those two stops. After the TBSP generates a path, in subsequent iterations, all the segments in that path are eliminated; meaning that we eliminate the possibility of that *exact* segment appearing in subsequently generated paths. However, possible variations that overlap that segment could appear in the subsequent paths. For example, if a path includes a segment with a boarding at stop *a* on route *i* and an alighting at stop *b*, the segment *a*-*i*-*b* is eliminated in the following iterations. However, eliminating *a*-*i*-*b* does not prohibit "neighbour" variations of that segment such *a*-*i*-*c*, *a*-*j*-*b*, or *d*-*i*-*b*.

After each path is generated by the TBSP, a reasonability check is performed, with the path considered "reasonable" if the following two criteria hold: (1) the path travel time does not exceed the shortest path travel time plus a threshold called "off-optimality", and (2) the number of transfers does not exceed 3. In addition, three implicit reasonability conditions are

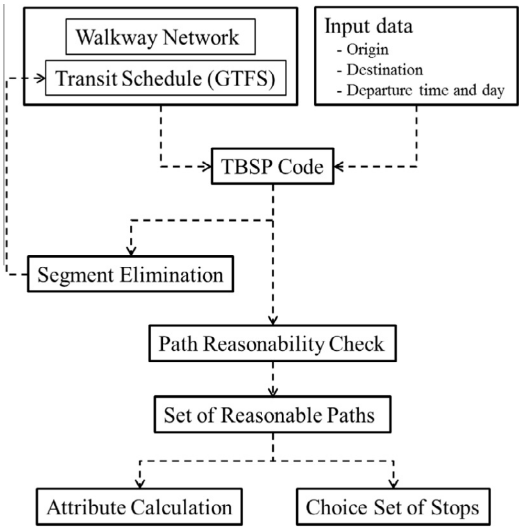

Fig. 1. Choice set generation framework.

embedded in the TBSP shortest path code itself: (1) the transfer walking distance cannot exceed 1 km, (2) the access and egress walks cannot exceed 2 km; and, (3) the waiting time before boarding cannot exceed one hour. Hence, a path that passes the reasonability check will hold these three conditions as well.

The ‘‘off-optimality” threshold in the reasonability check was set to 20 min, which was based on the results from a previous empirical analysis of the smart card data of SEQ passengers (Nassir et al., 2015b). The results in that research indicated that transit passengers are primarily making transit trips with a maximum off-optimality threshold of 20 min. Practically, as soon as a path generated by the K-shortest path code reaches the off-optimality threshold (20 min), the path generation terminates.

The sets of alternative departure stops are implicitly generated by the K-shortest path code. The stop set for each observation is generated specific to the OD and to the time of observation, and is generated by considering all the possible paths that can be taken to travel the OD pair at the time of observation.

The embedded limit of 2 km on the access walks in the path finding procedure guarantees that all the stops are in the walkable range of 2 km. The access walk threshold of 2 km may sound long, but a preliminary analysis of the SEQ household travel survey (HTS) data revealed that the access walk of a large portion (17%) of the observations are longer than 1 km. It has also been reported by other researchers that the observed walking distances for transit access and egress in Brisbane (SEQ) are much greater than the values that are conventionally assumed (Burke and Brown, 2007a, 2007b).

The one hour waiting time threshold is usually not binding for the paths generated in the reasonable set, especially during the daytime and in the urban areas with frequent bus services. This is commonly true because of the 20-min off-optimality threshold eliminating such long waits. The one hour limit is proposed to assure the inclusion of all reasonable paths, particularly rare cases of long waits for transfers when a noticeably faster path does not exist. Such cases may happen during the night time and/or in the suburbs with infrequent service.

3.2. Estimated choice model

The best choice model estimated in Nassir et al. (2015a) is presented with significant attributes defined in Table 1.

Attributes of AccessWalk, FastestTT, MinTransfer, NumRoutes, and Shelter are found to be significantly affecting the passengers' choices in general. It is worth explaining here the reason that FastestTT and MinTransfer are used as attributes of the choice model, instead of ‘‘travel time” or ‘‘number of transfers” that are typically used in transit path choice models. Since the proposed choice model is structured only at the level of access (or departure) stop, and not the entire transit path, the

Table 1
Access stop choice model results (Nassir et al.,2015a).

Number of observations		1237
Initial log-likelihood		−2934.79
Final log-likelihood		−2084.71
Likelihood ratio test		1700.171
ρ^2		0.29
Adjusted ρ^2		0.287
Utility coefficients for significant attributes:	AccessWalk Walk time from origin location to stop (min)	−0.0659 (−7.12)**
	FastestTT Travel time (min) on fastest path to destination from the stop (excluding AccessWalk)	−0.01 (−4.77)**
	MinTransfer Minimum number of transfer among paths from the stop to destination	−0.29 (−5.88)**
	NumRoutes Number of available routes from the stop to destination	0.0223 (4.22)**
	Shelter Binary variable indicating a sheltered stop	0.0529 (2.01)*
	CfC Correction for correlation	0.0555 (2.16)*
Nest coefficients:	Train (train stations)	3.7 (7.3)**
	No-train (bus stops and ferry terminals)	3.55 (4.26)**

Numbers in brackets are *t*-test values.
* Significant at 0.05 level.
** Significant at 0.01 level.

attributes related to ''travel time'' or ''number of transfers'' are aggregated over all the reasonable path alternatives originating from the departure stop to the destination. The aggregate measures tested for this purpose have been the ''mean'' and ''minimum'' over all the path alternatives. For example, in order to capture the travel time impedance of a departure stop to the destination, two alternative measures were included: (1) travel time on the fastest path from the stop to the destination; and (2) mean travel time of all reasonable paths from the stop to the destination. However the first measure (FastestTT) became more significant in explaining the observed behaviours. Therefore, FastestTT is selected as the temporal measure of the impedance between the two stops in the transit network. Same is true about MinTansfer for capturing the directness of service from the stop to the destination.

The estimated coefficients indicate that when an SEQ passenger is choosing an access stop/path, every minute of access walk is equivalent to about 6.6 min of travel time from the stop.

The coefficient of MinTransfer is large in comparison to FastestTT (29 times) and even to AccessWalk (4.4 times). This reveals a generally high disutility considered for the stops that have no direct routes to the destination. People prefer to walk more, and also to spend more time in travel, in order to board at a stop that has a direct connection to their destination. The attribute NumRoutes is also found to be statistically significant; i.e. people consider a positive value for the stops that have multiple route options, as long as the routes can take them to their destination in a reasonable time. Another behavioural observation is that the availability of a shelter is also found significant, indicating that people on average are willing to walk about one more minute to access a sheltered stop.

The mode-specific constants did not become significant but the mode nest coefficients have appeared very significant (significantly different from 1), implying that there is considerable unobserved homogeneity among alternatives in the mode ''Train'' and also among alternatives in the mode ''No-train'', which is effectively captured by the nesting structure and reflected in our accessibility measurement.

An important conclusion from the review of the transit access choice model in Nassir et al. (2015a) is that SEQ passengers' choice behaviour is significantly affected by parameters other than travel time, despite the existing convention in majority of transit accessibility models that calculate the impedance measurement using only total travel time. There are various transit path choice models in the literature that are calibrated for different networks and they similarly report significant impacts of a diverse set of variables on passenger's choices (e.g. Guo and Wilson, 2011; Raveau et al., 2011, 2014). The review of the choice preferences of SEQ passengers concludes that beside the travel time factor, other important attributes that influence SEQ passenger's perception of impedance are AccessWalk, MinTransfer, NumRoutes and Shelter. Inclusion of these variables in the definition of transit utility can highly improve the accuracy of impedance measurement.

Another motive for a random utility approach to measuring transit impedance is ''subjectivity'' or ''randomness'' in the perception of transit service. Even when incorporating all of the measurable service attributes in quantification of impedance, there is a random component in realisation of impedance that is investigated in the following section.

4. Randomness in the perception of impedance

When applying a utility-based approach to transit impedance measurement, the estimated utility of each transit path alternative is a random variable and the extent of this randomness depends on multiple factors, including the explanatory power of measured attributes, heterogeneity of users, measurement errors, etc. In this section, the extent of randomness in the access stop choice model in Table 1 is investigated, by comparing the estimated systematic utility (V_{in}) and the estimated choice probability of the chosen and unchosen alternatives for the 1239 choice observations in the SEQ access choice dataset.

Fig. 2a shows a scatterplot of SEQ choice observations (each data point represents one observation) with respect to the total travel time of the chosen alternative versus total travel time of the fastest alternative in the generated choice set. Out of 1239 observations in the choice dataset, 508 passengers (41.0%) did not choose the fastest stop alternative. On average, SEQ passengers have chosen path alternatives that are 4.7 min (16.0%) longer than the fastest alternative. This observation confirms the previous findings that considering only travel time as a measure of impedance can lead to inaccuracies in the measurement of impedance. However, the focus in this section is on the randomness that exits in the choice model after including all the measurable attributes.

Fig. 2b shows a scatterplot of SEQ choice observations with respect to the estimated systematic utility V_{in} (using the utility parameters in Table 1) of the chosen alternative versus the maximum systematic utility among all alternatives, max (V_{in}). The results indicate that 690 passengers (55.7%) have not chosen the alternative that has the highest estimated utility. On average, SEQ passengers have chosen path alternatives with 18.5% lower estimated utility than the highest utility alternative.

Fig. 2c shows a scatterplot of the SEQ choice observations with respect to the estimated choice probabilities of the chosen versus the highest estimated probability among the alternatives. The results indicate that 631 passengers (50.9%) have not chosen the most probable alternative estimated for them. On average, SEQ passengers have chosen path alternatives with 0.13 unit smaller probability than the most probable alternative (32.8% lower than the maximum probability).

As it is also shown in Fig. 3, about half of the observed passengers have boarded at alternative stops which are ranked between the 2nd and the 20th, when sorted in a decreasing order of estimated choice probabilities. About 40% of passengers have boarded at stops that are ranked between the 2nd and the 6th. This observation indicates that in the estimation of a utility-based impedance measure, even when including various and diverse explanatory variables and using a practically well-fit discrete choice model ($\rho^2 = 0.29$), some unobserved heterogeneity exists among the alternatives and/or among

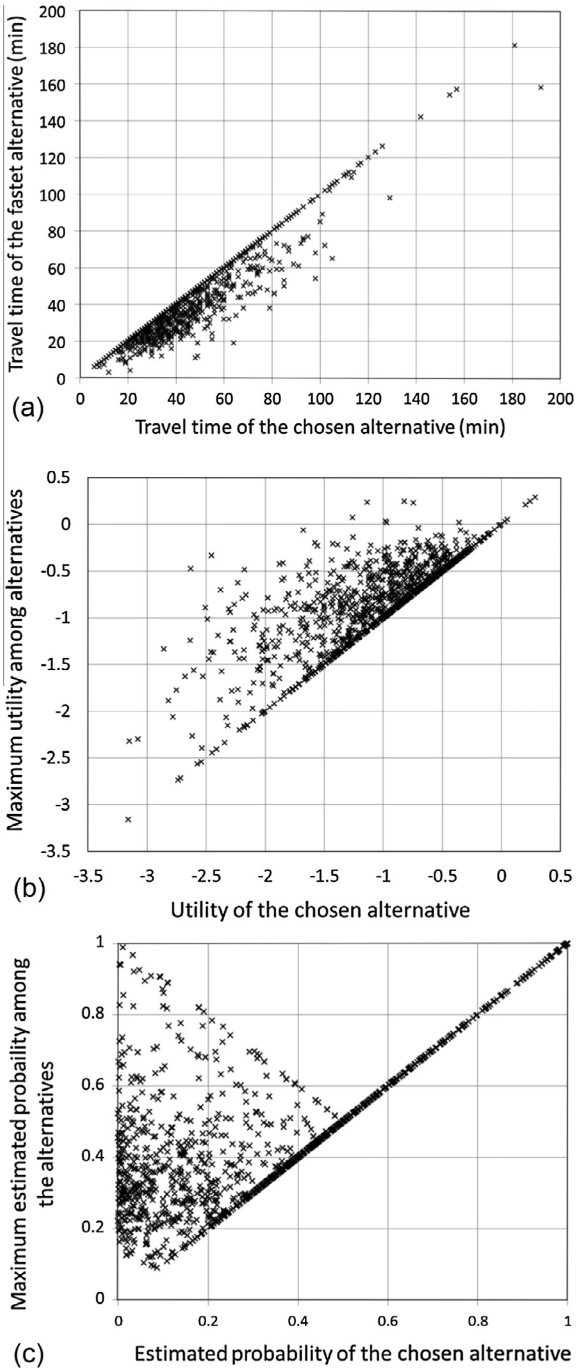

Fig. 2. Scatterplots of chosen vs optimal alternatives, in respect to: (a) total travel time, (b) estimated systematic utility, (c) estimated probability of being chosen.

the passengers' preferences that should be taken into consideration. Therefore, for a utility-based estimation of impedance, in order to: (1) correct for such unobserved heterogeneities and (2) capture the diversity benefit of the transit system, we recommend to incorporate at least a number of highly competing alternatives, instead of a single highest probability alternative.

Mean and standard deviation of the estimated choice probabilities for the top 10 generated alternatives over the 1239 observed choices are shown in Fig. 4. The distribution of estimated probabilities among the generated alternatives also confirm that there is a considerable proportion of the travel demand that is estimated to use alternatives other than the highest

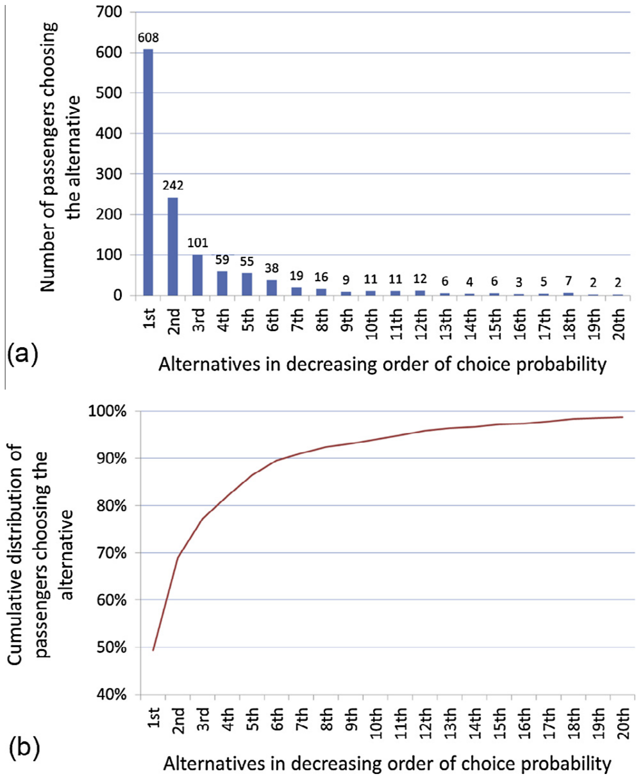

Fig. 3. Distribution of passengers choosing the *n*th highset estimated probability alternatives.

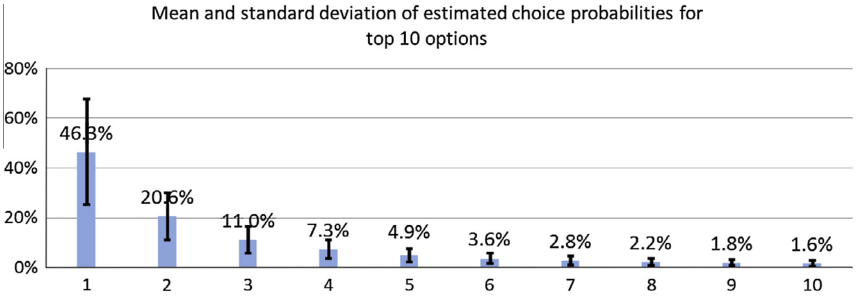

Fig. 4. Distribution of estimated probabilities among top 10 alternatives.

probability alternative. Therefore, despite what is common among the existing approaches to transit accessibility measures (reviewed in Sections 1 and 2 of this paper), measuring the transit impedance through a single representative path may result in certain inaccuracies, especially in dense and multimodal transit networks.

5. Utility-based impedance estimation

The proposed utility-based impedance estimate is based on the utility parameters from the access stop choice model in Table 1. The impedance measure is calculated as the logsum utility of the global set of reasonable access stop alternatives between given OD's and at given times.

Given the nested logit model for the access choice behaviour of transit users, the composite impedance, I_{od}^{τ}, to travel from o to d at time τ is calculated based on nested logsum composition (Koppelman and Bhat, 2006) of all alternatives in the reasonable choice set as shown in Eqs. (3–5).

$$I_{od}^{\tau} = \ln\left[\exp\left(\frac{\omega_{T}^{od,\tau}}{\mu_{T}}\right) + \exp\left(\frac{\omega_{NT}^{od,\tau}}{\mu_{NT}}\right)\right] \tag{3}$$

$$\omega_T^{od,\tau} = \ln \sum_{i \in C_T} \exp(\mu_T \times V_i) \tag{4}$$

$$\omega_{NT}^{od,\tau} = \ln \sum_{i \in C_{NT}} \exp(\mu_{NT} \times V_i) \tag{5}$$

with the following notation:

$\omega_T^{od,\tau}$: the logsum of nest "Train",
$\omega_{NT}^{od,\tau}$: the logsum of nest "No_train",
μ_T: nest parameter for nest "Train" ($1 < \mu_T = 3.7$),
μ_{NT}: nest parameter for nest "No_train" ($1 < \mu_{NT} = 3.55$),
C_T: set of all the generated alternative stops in nest "Train",
C_{NT}: set of all the generated alternative stops in nest "No_train",
V_i: systematic utility of stop i (calculated based on parameters in Table 1).

The impedance measure, I_{od}^{τ}, is calculated from two terms $\omega_T^{od,\tau}$ (logsum of all train stations in the choice set) and $\omega_{NT}^{od,\tau}$ (logsum of all bus stops and ferry terminals in the choice set). The two nest parameters μ_T (=3.7) and μ_{NT} (=3.55) are important in accurate estimation of the perceived impedance. Their values are estimated in the access choice model, and indicate significant correlation among the utility error terms of the alternatives in each nest. This correlation is reflected in the calculation of I_{od}^{τ} and reduces the diversity benefit inside each nest. As a result, the benefit of having two choices in the same nest is less than the benefit of having them in two different nests. This effect can be observed in the case study results, where better values of I_{od}^{τ} are observed in the locations that have access to all transit modes.

Since the proposed impedance measure I_{od}^{τ} is defined as the logsum of the utility of service, the higher amounts of I_{od}^{τ} indicate a better level of service.

The impedance estimation method was tested using the network data from Southeast Queensland, Australia. The transit network data and the schedule regarding the service in May 2009 were provided by the state-wide transit authority, Translink. The transit network with all three modes (bus, train and ferry) included 14,442 stops, 767 routes, and 33,897 timetabled vehicle trips, with different services on weekdays and weekends. The walk network data for SEQ was downloaded from "OpenStreetMap" (http://www.openstreetmap.org/), including local streets, sidewalks, crosswalk connections, walking ramps, footways, and stairways, covering the greater Brisbane area, the Gold Coast and the Sunshine Coast. The walk network included about 250,000 nodes and 340,000 links. Shortest path calculations, geographic matching of the stops, locations, and coordinates were all performed in this network, using ArcGIS (Sandhu and Chandrasekhar, 2006). An average walking speed of 1.2 m/s was used for the estimation of walking times. Moreover, transit facility data for the stops was provided by the Queensland Department of Transport and Main Roads (DTMR). This dataset included information about the convenience amenities, such as shelter, illumination, access walkways, boarding slabs, etc.

Three logsum measures $\omega_T^{od,\tau}$, $\omega_{NT}^{od,\tau}$, and I_{od}^{τ} are calculated for all suburbs in the Brisbane area, SEQ. These measures are calculated for journeys in the morning peak (7 am) of a weekday service, and with the destination located in the Brisbane Central Business District (CBD). Fastest travel time, T_{od}^{τ}, from o to d at the departure time τ is also calculated as a baseline for demonstrating the effectiveness of the proposed impedance measure, I_{od}^{τ}. The calculations were performed at the spatial resolution of a "mesh block" (equivalent of a census block in the USA), which is the smallest geographical area defined by the Australian Bureau of Statistics (2011). The measures were calculated for a total of 24,465 mesh blocks in the Brisbane area.

Fig. 5 shows a scatter plot of the calculated impedance measure I_{od}^{τ} vs. T_{od}^{τ} for all the mesh blocks in the case study. The scatterplot shows a general decline in I_{od}^{τ} with the increase in travel time. This intuitively confirms the general sensitivity of impedance to the travel time (or maybe distance) to the CBD. An important observation that should be made from this graph is that locations that have transit services to the CBD with both train and bus/ferry (black circle data points) show noticeably better travel impedance as compared to locations that only have access with either train mode (red crosses) or bus/ferry mode (grey squares). This confirms the general sensitivity of the impedance to the availability of access from different modes, and highlights the capability of the measure in capturing the diversity benefit that the transit system can offer the community, in a practical case study application.

Another important observation that should be made from Fig. 5 is the vertical pattern of data points in the scatterplot. This implies that there are many locations in the case study area that might have equal travel times to the CBD but the perceived impedance of travel from these locations can vary in a large range (ranging up to three units in some cases). As an example of this, Fig. 6 shows a scaled up map of the suburbs in the West and Southwest of the CBD, colour-coded for different values of the total logsum, I_{od}^{τ}. The mesh blocks with highlighted borders in light blue all have a 22-min travel time to the CBD. The impedance measure I_{od}^{τ} calculated for these mesh blocks, however, ranges from about -0.7 to about +0.5. Among these equidistant mesh blocks, the mesh blocks with smaller values of I_{od}^{τ} are the locations mostly at the bank of Brisbane River (marked by "A") where the access to the transit network is only provided by fewer bus stops from the south. Therefore, it is expected that a portion of the 22 min of travel time from these blocks to the CBD should be the access walk (5–10 min) which is perceived significantly unpleasant among the SEQ passengers. Another possible reason for the lower

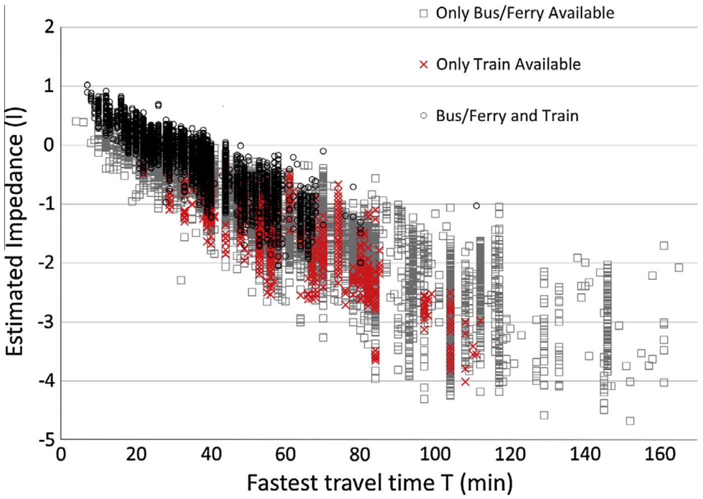

Fig. 5. Case study results, I_{od}^{τ} vs. T_{od}^{τ}.

value of I_{od}^{τ} at these mesh blocks could be related to the diversity of access options that is expected to be limited at the bank of the river (both in-nest and cross-nest).

In the more central mesh blocks (marked by "B") the situation is a little different. These mesh blocks are served more effectively by a larger number of bus stops within a shorter walk, and with multiple bus routes to the CBD. This explains the difference between the estimated impedance of "A" blocks $(-1 < I_{od}^{\tau} < -0.5)$ and "B" blocks $(-0.5 < I_{od}^{\tau} < 0)$; although the travel time for journeys departing at 7 am is equal to 22 min for all of these blocks.

Mesh blocks marked by "C" have a similar estimated impedance as "B", although the density of bus stops are not as high as in "B" blocks. The closer distance of "C" blocks to the train tracks explains the higher I_{od}^{τ}. People in "C" blocks have the option to choose between bus and train, although they probably have to walk a little further to reach any of the modes. The positive effect of having such an option becomes clearer in "D" blocks in which the train station and multiple bus stations are located in a short walk. For the "D" blocks, the I_{od}^{τ} is estimated the highest (about +0.5) among all the mesh blocks that have 22 min travel time to CBD. This example from the case study can demonstrate the sensitivity of the proposed measure I_{od}^{τ} to passengers' utility parameters, and to the diversity of reasonable options.

Fig. 7 shows three maps of the estimated measures $\omega_T^{od,\tau}$, $\omega_{NT}^{od,\tau}$, and I_{od}^{τ} calculated for all suburbs in Brisbane. Fig. 7a is the logsum of the nest "Train" $(\omega_T^{od,\tau})$ associated with only the rail service to the CBD. Fig. 7b is the logsum of the nest "No-train" $(\omega_{NT}^{od,\tau})$ associated with the bus and ferry service to the CBD. Finally, Fig. 7c is the total logsum (I_{od}^{τ}) associated with all public transit modes of service to the CBD. The high level of service that is observed in Fig. 7c as compared to Fig. 7a (only train) and Fig. 7b (only bus and ferry) relates to the diversity of modes available to travel to CBD during the morning peak and reflects how the passenger community perceives it as a positive utility.

There are some interesting observations that can be made from the results in Fig. 7. For one, the logsum utility that the bus/ferry services provide to CBD appears to be greater than that from the train services. This could relate to the high diversity of access options that exists within the bus/ferry nest. However, the effectiveness of the train mode becomes more apparent in Fig. 7c where the combination of all modes comes into play and obviously increases the level of transit service as a whole. The second interesting observation is the general sensitivity of the logsum measures to distance from the CBD. This sensitivity is more strongly observed in bus/ferry service (7b) and in total logsum (7c), as compared to train service (7a). Two possible reasons can be imagined for this: (1) the density of the bus network is much higher in the suburbs closer to the CBD and thus the service is better in the inner suburbs, as compared to more distant locations; (2) the travel speed of trains are higher when compared to bus/ferry, and thus the decline in the train logsum as distances increase is lesser than for bus/ferry.

One theoretical drawback of I_{od}^{τ} as an impedance measure for the transit mode is that it is not directly sensitive to the fare, which is an important determinant of transit impedance. Due to the zone-based structure of transit fares in SEQ, the developed access choice model has not captured the sensitivity of passengers to the fare, because virtually all access options in the choice set share the same fares for the given OD pair. Therefore, passengers' sensitivities to fare are missing in the choice model.

In order to account for transit fares, another impedance measure, $I_{od}'^{\tau}$, is developed that is an adjusted definition of I_{od}^{τ} as follows.

$$I_{od}'^{\tau} = I_{od}^{\tau} + \left(\frac{-0.01}{VOT} \times F_{od}^{\tau}\right) \tag{6}$$

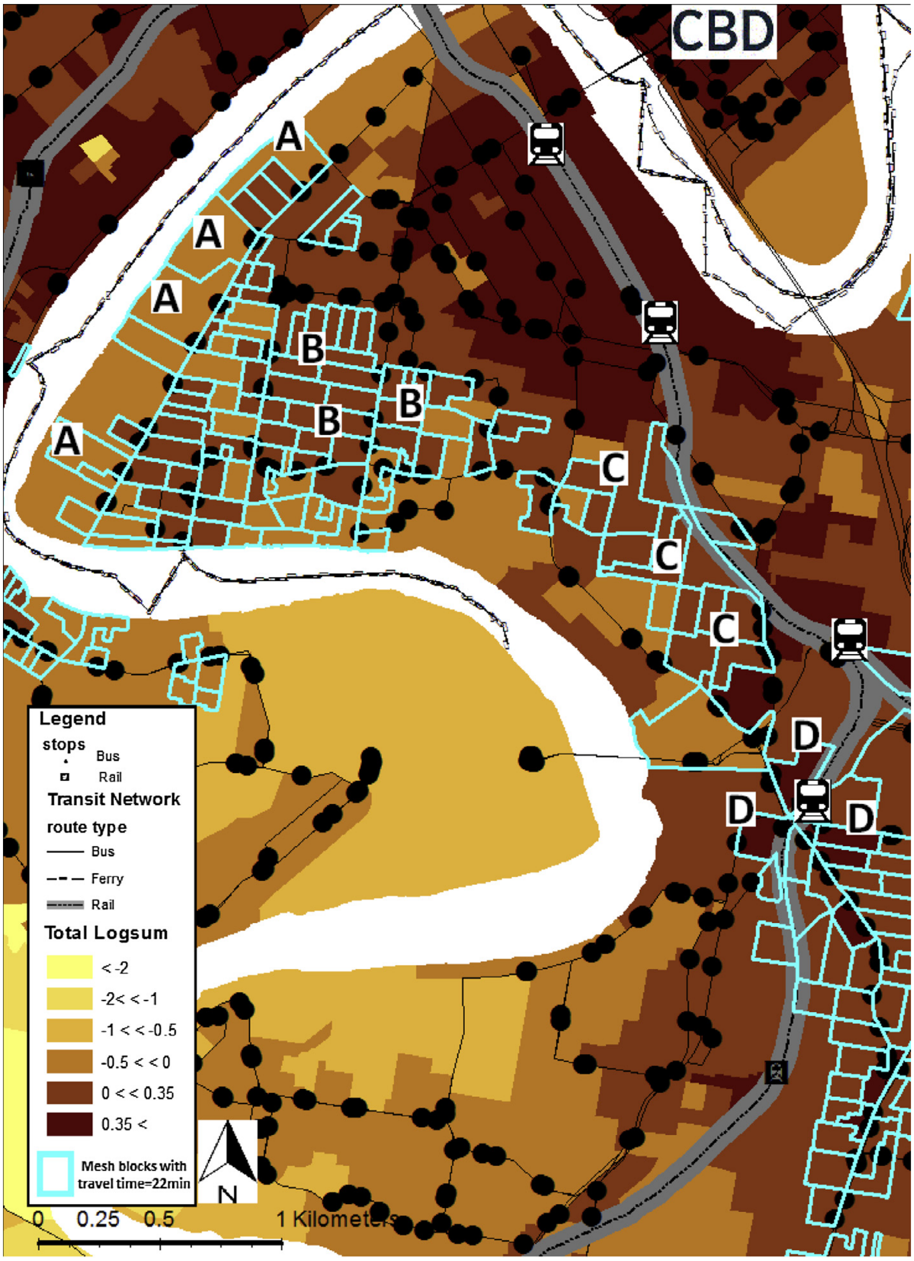

Fig. 6. Example of I_{od}^{τ} in Brisbane.

where F_{od}^{τ} is the transit fare for travelling from o to d at time τ, and the term $\frac{-0.01}{VOT}$ is an estimate of the utility coefficient of fare expenses. This coefficient is calculated by dividing the coefficient of time in the access choice mode (FastestTT, which is -0.01 per minute) divided by the value of time (VOT). The VOT is assumed to be AU$15/hr, and the fares, F_{od}^{τ}, were taken from the SEQ network in 2009.

Fig. 8 shows the two measures I_{od}^{τ} and $I_{od}^{\prime\tau}$ calculated for traveling to the Brisbane CBD at 7 am weekday service from all mesh blocks in Brisbane. As anticipated and as can be visually observed in Fig. 8, $I_{od}^{\prime\tau}$ is slightly smaller than I_{od}^{τ}. Since the transit fares in SEQ have a zone-based structure (ranging from AU$2.40 to AU$18.80), the difference between the two measures I_{od}^{τ} and $I_{od}^{\prime\tau}$ is more evident in the mesh blocks in distant suburbs.

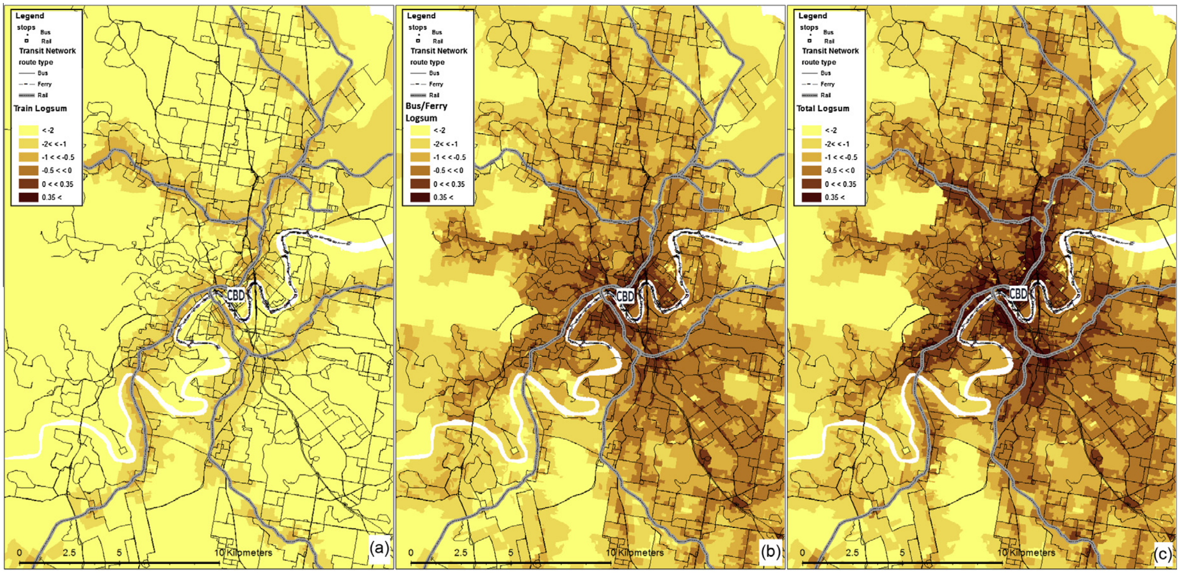

Fig. 7. Estimated measures in Brisbane area: (a) train logsum ($\omega_T^{od,\tau}$), (b) bus/ferry logsum $\omega_{NT}^{od,\tau}$, and (c) total logsum (I_{od}^{τ}).

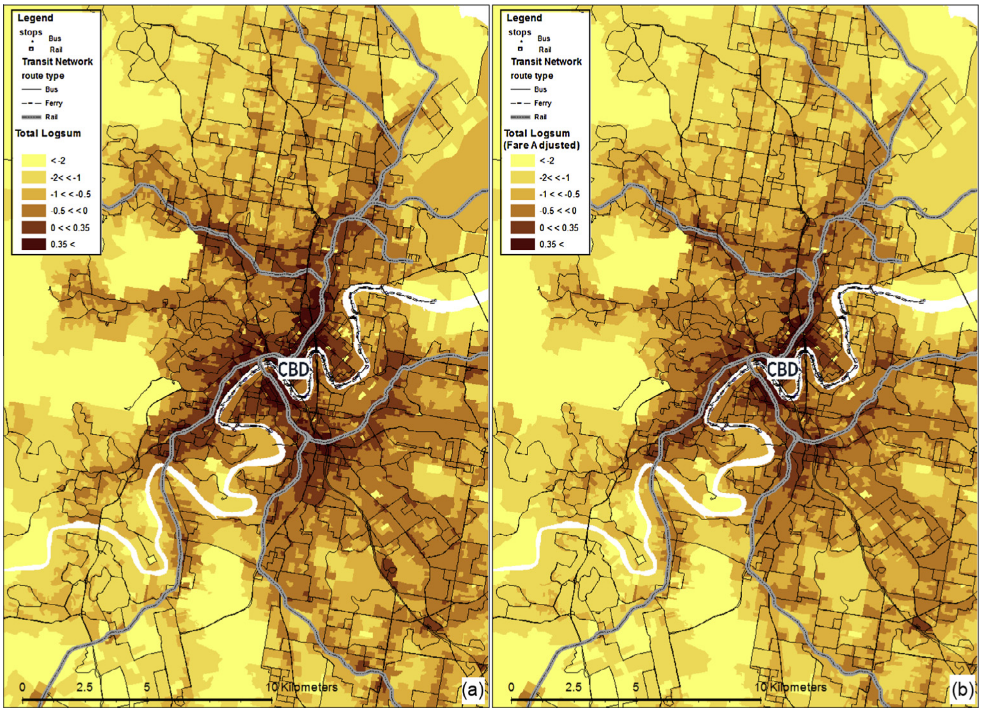

Fig. 8. Estimated measures in Brisbane area: (a) total logsum (I_{od}^{τ}), and (b) fare-adjusted logsum (I'^{τ}_{od}).

6. Conclusions

Transport accessibility measures aim to quantify the total net benefit that residents of a particular geographic area can receive from the proximity or ease of travel to opportunities (or needs) located elsewhere. What makes the accessibility measurement challenging is the subjectivity of these qualities (both on the benefit side and the cost side) and the diversity of perception of these qualities among the residents. This paper contributes to the methodology of this measurement by identifying an important gap in the measurement of transit impedance and by proposing and testing a framework to properly address this gap.

The proposed measure has an important advantage over the existing impedance measures in the literature: it incorporates all reasonable paths and mode alternatives in the measurement, and therefore has the capability to: (1) correct for possible unobserved heterogeneity in the utility model, and (2) to capture the diversity benefit that a transit service can offer to a community of passengers. These advantages become more important in complex multimodal transit networks with an abundance of alternatives and heterogeneous users of the system.

As a case study, the proposed measure is applied to the regional, multimodal, and time-dependent transit network of SEQ, Australia, for a morning peak journey from all of the Brisbane suburbs to the CBD. The impedance measurement is based on the transit access stop choice model that is calibrated using the household travel survey of 2009 in SEQ. This model indicates that SEQ passengers, in general, consider a heavy disutility associated with long access walks and transfers when using the transit service. It is also realised that SEQ passengers consider a significant difference (nested structure) between alternatives from the train mode and other alternatives (bus/ferry). Using the calibrated access stop choice model, these travel behaviour characteristics are reflected into the transit impedance measurement. From the case study it can be demonstrated that the proposed measure effectively captures: (1) passengers' preferences when using the transit service, and (2) availability of diverse alternatives in the transit network.

One potential drawback of the developed accessibility measure is that it does not capture the effect of on-board congestion and the inconvenience associated to it. The reason is that the estimated choice model is calibrated using a regular day household travel survey records, and the public transport system in Brisbane is generally not a crowded systems and is known to perform under the capacity. Therefore, we did not capture the inconvenience associated with potential on-board crowding. However, if the proposed utility-based measure is applied to a crowded network, the disutility of crowding can be captured by the choice model, and can be reflected in the accessibility measurement directly.

One possible direction of future research is to have a closer look at the different socio-demographic group behaviours and reflect the differences among those groups into their perception of impedance for a better understanding of the social equity implications of the results.

Also, in terms of applications to accessibility to work locations, given that the arrival time is usually fixed, one important modification to the set generation algorithm could be the alteration of the TBSP algorithm for preferred (fixed) arrival times to the destination, instead of the fixed departure time from the origin.

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References

Banister, D., 2011. The trilogy of distance, speed and time. J. Transp. Geogr. 19 (4), 950–959.

Ben-Akiva, M., Lerman, S.R., 1979. Disaggregate travel and mobility choice models and measures of accessibility. In: Hensher, D.A., Stopher, P.R. (Eds.), Proceedings of the Third International Conference on Behavioural Travel Modelling, Tanunda, South Australia, Croom Helm, London.

Ben-Akiva, M., Bierlaire, M., 1999. Discrete choice methods and their applications to short term travel decisions. In: Handbook of Transportation Science. Springer, pp. 5–33.

Bhat, C., Govindarajan, A., Pulugurta, V., 1998. Disaggregate attraction-end choice modeling: formulation and empirical analysis. Transport. Res. Rec. J. Transport. Res. Board 1645 (1), 60–68.

Bhat, C., Handy, S., Kockelman, K., Mahmassani, H., Gopal, A., Srour, I., Weston, L., 2002. Development of an Urban Accessibility Index: Formulations, Aggregation, and Application. Center for Transportation Research, The University of Texas at Austin.

Bhat, C.R., Carini, J.P., Misra, R., 1999. Modeling the generation and organization of household activity stops. Transport. Res. Rec. J. Transport. Res. Board 1676 (1), 153–161.

Burke, M., Brown, A., 2007a. Active transport in Brisbane: how much is happening and what are its characteristics. In: Proceedings of State of Australian Cities National Conference. University of South Australia, Adelaide.

Burke, M., Brown, A., 2007b. Distances people walk for transport. Road Transp. Res. J. Aust. New Zeal. Res. Pract. 16 (3), 16.

Cascetta, E., Cartenì, A., Montanino, M., 2013. A new measure of accessibility based on perceived opportunities. Proc. Soc. Behav. Sci. 87, 117–132.

Cascetta, E., Nuzzolo, A., Russo, F., Vitetta, A., 1996. A modified logit route choice model overcoming path overlapping problems: specification and some calibration results for interurban networks. In: Proceedings of the 13th International Symposium on Transportation and Traffic Theory. Pergamon, Lyon, France.

Cats, O., 2011. Dynamic Modelling of Transit Operations and Passenger Decisions, PhD Thesis, Technion-Israel Institute of Technology, Faculty of Civil and Environmental Engineering.

Cervero, R., Rood, T., Appleyard, B., 1995. Job Accessibility as a Performance Indicator: An Analysis of Trends and their Social Policy Implications in the San Francisco Bay Area. University of California Transportation Center.

Chen, A., Yang, C., Kongsomsaksakul, S., Lee, M., 2007. Network-based accessibility measures for vulnerability analysis of degradable transportation networks. Networks Spatial Econ. 7 (3), 241–256.

Church, R.L., Marston, J.R., 2003. Measuring accessibility for people with a disability. Geogr. Anal. 35 (1), 83–96.

Church, R.L., Noronha, V., Lei, T., Corrigan, W., Burbidge, S., Marston, J., 2005. Spatial and Temporal Utility Modeling to Increase Transit Ridership. California Department of Transportation, Sacramento, CA, USA.

Crozet, Y., 2013. The three stages of accessibility: the coming challenge of urban mobility. Sustain. Transp. Chin. Cities 3, 79.

Currie, G., 2010. Quantifying spatial gaps in public transport supply based on social needs. J. Transp. Geogr. 18 (1), 31–41.

Currie, G., Stanley, J., 2008. Investigating links between social capital and public transport. Transp. Rev. 28 (4), 529–547.

El-Geneidy, A.M., Tetreault, P., Surprenant-Legault, J., 2010. Pedestrian access to transit: Identifying redundancies and gaps using a variable service area analysis. In: Transportation Research Board 89th Annual Meeting.

Geurs, K., van Wee, B., 2004. Accessibility evaluation of land-use and transport strategies: review and research directions. J. Transp. Geogr. 12 (2), 127–140.

Gulhan, G., Ceylan, H., Özuysal, M., Ceylan, H., 2013. Impact of utility-based accessibility measures on urban public transportation planning: a case study of Denizli, Turkey. Cities 32, 102–112.

Guo, Z., Wilson, N.H.M., 2011. Assessing the cost of transfer inconvenience in public transport systems: a case study of the London Underground. Transport. Res. Part A Policy Pract. 45 (2), 91–104.

Handy, S.L., Niemeier, D.A., 1997. Measuring accessibility: an exploration of issues and alternatives. Environ. Plan. A 29 (7), 1175–1194.

Hansen, W.G., 1959. How accessibility shapes land use. J. Am. Inst. Planners 25 (2), 73–76.

Hillman, R., Pool, G., 1997. GIS-based innovations for modelling public transport accessibility. Traffic Eng. Control 38 (10), 554–559.

Khani, A., 2013. Models and Solution Algorithms for Transit and Intermodal Passenger Assignment (development of FAST-TrIPs model), PhD Dissertation, University of Arizona, Department of Civil Engineering and Engineering Mechanics.

Khani, A., Hickman, M., Noh, H., 2014. Trip-based path algorithms using the transit network hierarchy. Networks Spatial Econ. 2014, 1–19.

Khani, A., Lee, S., Hickman, M., Noh, H., Nassir, N., 2012. Intermodal path algorithm for time-dependent auto network and scheduled transit service. Transport. Res. Rec. J. Transport. Res. Board 2284 (1), 40–46.

Koenig, J.G., 1980. Indicators of urban accessibility: theory and application. Transportation 9 (2), 145–172.

Koppelman, F.S., Bhat, C., 2006. A Self Instructing Course in Mode Choice Modeling: Multinomial and Nested Logit Models. US Department of Transportation, Federal Transit Administration, 31.

Kwan, M.P., Murray, A.T., O'Kelly, M.E., Tiefelsdorf, M., 2003. Recent advances in accessibility research: representation, methodology and applications. J. Geogr. Syst. 5 (1), 129–138.

Lee, B.H.Y., 2005. Parcel-level Measure of Public Transit Accessibility to Destinations (PhD Dissertation). University of Washington.

Lei, T., Church, R., 2010. Mapping transit-based access: integrating GIS, routes and schedules. Int. J. Geogr. Inform. Sci. 24 (2), 283–304.

Lei, T.L., Chen, Y., Goulias, K.G., 2012. Opportunity-based dynamic transit accessibility in southern California. Transport. Res. Rec. J. Transport. Res. Board 2276 (1), 26–37.

Levinson, D.M., Krizek, K.J., 2005. Access to Destinations. Elsevier, Oxford, United Kingdom.

Liu, S., Zhu, X., 2004. An integrated GIS approach to accessibility analysis. Trans. GIS 8 (1), 45–62.

Mavoa, S., Witten, K., McCreanor, T., O'Sullivan, D., 2012. GIS based destination accessibility via public transit and walking in Auckland, New Zealand. J. Transp. Geogr. 20 (1), 15–22.

Metz, D., 2008. The myth of travel time saving. Transp. Rev. 28 (3), 321–336.

Moniruzzaman, M., Páez, A., 2012. Accessibility to transit, by transit, and mode share: application of a logistic model with spatial filters. J. Transp. Geogr. 24, 198–205.

Murray, A.T., 2001. Strategic analysis of public transport coverage. Socio-Econ. Plan. Sci. 35 (3), 175–188.

Murray, A.T., Davis, R., Stimson, R.J., Ferreira, L., 1998. Public transportation access. Transport. Res. Part D Transp. Environ. 3 (5), 319–328.

Nassir, N., Khani, A., Hickman, M., Noh, H., 2012. Algorithm for intermodal optimal multidestination tour with dynamic travel times. Transport. Res. Rec. J. Transport. Res. Board 2283 (1), 57–66.

Nassir, N., Ziebarth, J., Sall, E., Zorn, L., 2014. A choice set generation algorithm suitable for measuring route choice accessibility. Transport. Res. Rec. J. Transport. Res. Board 2430 (2), 170–181.

Nassir, N., Hickman, M., Malekzadeh, A., Irannezhad, E., 2015a. Modelling transit access stop choices. Transport Res. Rec. J. Transport. Res. Board 2493, 70–77.

Nassir, N., Hickman, M., Ma, Z., 2015b. Activity detection and transfer identification for public transit fare card data. Transportation 42, 683–705. http://dx.doi.org/10.1007/s11116-015-9601-6.

O'Sullivan, D., Morrison, A., Shearer, J., 2000. Using desktop GIS for the investigation of accessibility by public transport: an isochrone approach. Int. J. Geogr. Inform. Sci. 14 (1), 85–104.

Owen, A., Levinson, D.M., 2015. Modeling the commute mode share of transit using continuous accessibility to jobs. Transport. Res. Part A Policy Pract. 74, 110–122.

Pirie, G.H., 1979. Measuring accessibility: a review and proposal. Environ. Plan. A 11 (3), 299–312.

Polzin, S.E., Pendyala, R.M., Navari, S., 2002. Development of time-of-day-based transit accessibility analysis tool. Transport. Res. Rec. J. Transport. Res. Board 1799 (1), 35–41.

Rastogi, R., Rao, K.K., 2002. Survey design for studying transit access behavior in Mumbai City, India. J. Transport. Eng. 128 (1), 68–79.

Rastogi, R., Rao, K.K., 2003. Defining transit accessibility with environmental inputs. Transport. Res. Part D Transp. Environ. 8 (5), 383–396.

Raveau, S., Muñoz, J.C., de Grange, L., 2011. A topological route choice model for metro. Transport. Res. Part A Policy Pract. 45 (2), 138–147.

Raveau, S., Guo, Z., Muñoz, J.C., Wilson, N.H., 2014. A behavioural comparison of route choice on metro networks: time, transfers, crowding, topology and socio-demographics. Transport. Res. Part A Policy Pract. 66, 185–195.

Salonen, M., Toivonen, T., 2013. Modelling travel time in urban networks: comparable measures for private car and public transport. J. Transp. Geogr. 31, 143–153.

Sandhu, J., Chandrasekhar, T., 2006. ArcGIS Network Analyst Tutorial. ESRI.

Tribby, C.P., Zandbergen, P.A., 2012. High-resolution spatio-temporal modeling of public transit accessibility. Appl. Geogr. 34, 345–355.

Yigitcanlar, T., Sipe, N., Evans, R., Pitot, M., 2007. A GIS-based land use and public transport accessibility indexing model. Aust. Planner 44 (3), 30–37.

Zhu, X., Liu, S., 2004. Analysis of the impact of the MRT system on accessibility in Singapore using an integrated GIS tool. J. Transp. Geogr. 12 (2), 89–101.