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Bicycle lane priority: Promoting bicycle as a green mode even in congested urban area

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ABSTRACT

The main obstacles to boosting the bicycle as a mode of transport are safety concerns due to interactions with motorized traffic. One option is to separate cyclists from motorists through exclusive bicycle priority lanes. This practice is easily implemented in uncongested traffic. Enforcing bicycle lanes on congested roads may degenerate the network, making the idea very hard to sell both to the public and the traffic authorities. Inspired by Braess Paradox, we take an unorthodox approach to seeking latent misutilized capacity in the congested networks to be dedicated to exclusive bicycle lanes. The aim of this study is to tailor an efficient and practical method to large size urban networks. Hence, this paper appeals to policy makers in their quest to scientifically convince stakeholder that bicycle is not a secondary mode, rather, it can be greatly accommodated along with other modes even in the heart of the congested cities. In conjunction with the bicycle lane priority, other policy measures such as shared bicycle scheme, electric-bike, integration of public transport and bicycle are also discussed in this article. As for the mathematical methodology, we articulated it as a discrete bilevel mathematical programing. In order to handle the real networks, we developed a phased methodology based on Branch-and-Bound (as a solution algorithm), structured in a less intensive RAM manner. The methodology was tested on real size network of city of Winnipeg, Canada, for which the total of 30 road segments – equivalent to 2.77 km bicycle lanes – in the CBD were found.

1. Introduction

A futurist author, H.G. Wells (1866–1946) stated: “Cycle tracks will abound in utopia” (Stephenson, 2015). The bicycle as a green and sustainable mode of transport is gaining ground (Mesbah et al., 2012; Milne and Melin, 2014; Smith, 2011). In one estimate half of the morning trips in the US is less than 5 miles (Stephenson, 2015), should it be made by 24 min cycling, no job is left for transport engineers. Governments across the world have started to invest in more bicycle facilities (Duthie and Unnikrishnan, 2014; Mesbah et al., 2012; Smith, 2011). A strong correlation has been reported between the usage rate of bicycles and health indices (Milne and Melin, 2014). Fortunately the use of bicycle is on the rise (Brady et al., 2010), so much so some coined the term of “bicycling renaissance” (Pucher et al., 2011). The main obstacles to boosting the bicycle as a

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regular mode of transport are safety concerns due to interactions with motorized traffic (Buehler and Dill, 2015; Habib et al., 2014; Menghini et al., 2010).

Based on GPS dataset, a recent study in the United States suggests cyclists give high value to off-street bike paths and enhanced neighbourhood bikeways with traffic calming features (Broach et al., 2012). A similar observation has also reported for cases in Canada (Su et al., 2010). In other words, "segregation" is the cyclists' most heralded slogan. In the Netherland which is the mecca of the cyclists, the public see the separated bicycle lanes an indispensable part of their transport system (Stephenson, 2015). Retrofitting existing facilities at no (or least) cost to better accommodate cyclists and pedestrians has emerged as an effective tool in the hands of policy makers (Buehler and Dill, 2015) One option is to segregate bicycles and motorized vehicles by providing exclusive lanes for the cyclists (Lin and Liao, 2014; Mesbah et al., 2012; Smith, 2011). The importance of bicycle lane has been correlated to the "bikeability" of the cities (Habib et al., 2014). Empirical analyses strongly suggest that bicycle lanes improve safety of both cyclists and motorists on multi-lane roadways (Brady et al., 2010). Even in narrow streets where space is scarce a simple lane marking in the shared lanes (known as "Sharrows") can greatly contribute to betterment of driving behaviours and hence the safety (Brady et al., 2010; Meng, 2012).

Bicycle lanes come at the expense of restricting the motorists to less space (Alliance for Biking & Walking, 2014), which may lead to a much worse traffic circulation and hence more severe congestion. This genuine fear has precluded the introduction of bicycle lane in many cities. Despite great efforts to analyse network design problem – notably road and transit network design – (Bagloee and Ceder, 2011; Bagloee et al., 2013b, 2015; Farahani et al., 2013; Mesbah et al., 2011a, 2011b; Sarvi et al., 2016) the literature has yet to address the Bicycle Priority Lanes Design (BPLD) problem.

The introduction of bicycle lanes needs to be viewed in the context of the motorized modes at the urban network level. A recent review on the literature sheds light in the shortcomings of the methodologies based on which the importance of network level approaches has been highlighted (Buehler and Dill, 2015).

As such one can divide the task into two categories: uncongested and congested urban networks:

- In uncongested urban networks, bicycle lanes cause no congestion. Under this category, bicycle lanes are relatively unproblematic, and one can follow standard procedures in urban design and planning.
- In congested urban networks, bicycle lanes are more contentious, and debate arises when a portion of the road space of the already congested network is reserved for cyclists.

Mesbah et al. (2012) consider the BPLD problem as a bilevel programing problem and a genetic algorithm was developed as a solution method. In their attempt although the both transport classes of bicycle and motorized vehicle are considered, the interaction between these modes is not considered (the problem is modelled as if they are operating on two separate networks). Furthermore the application of their method to a large size network has yet to be addressed. Duthie and Unnikrishnan (2014) investigate the design of an integrated bicycle network while the impact of congestion is overlooked. Lin and Liao (2014) tackle the BPLD problem with an all-out binary programing framework. Enforcing all the variables as a binary variable makes the solution computationally prohibitive as the size of the problem increases. Regardless the congestion is largely overlooked.

The intent of this study is to address the BPLD problem in the context of a congested city, and we show that even in this context there might be some latent spare capacities that can be released and allocated to the cyclists without worsening the overall congestion. This seemingly unorthodox notion is rooted in the Braess Paradox (Braess et al., 2005) that is; adding road to the network may worsen the traffic circulation. In other words, there might be some roads in an existing network whose closure could improve traffic circulation (Bagloee et al., 2013a).

This study contributes to the literature by addressing the BPLD problem in the congested cities considering three important features: (i) network-wide impact, (ii) congestion, and (iii) scalability to real-size networks. We model the BPLD problem as a bilevel programing problem. In the upper level the total system cost is minimized, while the lower level accounts for the behaviour of the users (motorists and cyclists). Specifically, the lower level models a Multiclass User Equilibrium (MUE) traffic flow. The bilevel programing problems are proven to be NP-hard, a term referring to utmost difficulty in solving the respective problems (Bard, 1998; Jeroslow, 1985).

The necessity of studying mixed modes traffic flow (bicycle with motorized mode) is rooted in the fact that, it is not always possible or feasible to provide a fully-fledged and connected network of exclusive bicycle lanes. In other words, having mixed mode roads in some part of the (bicycle) network is inevitable. In a similar fashion, shared lanes between motorized modes such as heavy trucks and cars are omnipresent in traffic modelling. Hence we articulate the problem as a multiclass traffic flow model using the concept of bias term (Spiess, 1984) that is, both motorized modes and bicycles will experience a common delay term plus an exclusive term (the bias term). Nevertheless, arriving at a proper estimation for the parameters of the roads' delay functions including the bias terms for car and bicycle requires a field survey and model calibration. Using the bias term is computationally efficient and has been consolidated by much empirical evidence (INRO, 2009). Other alternative methods give raise to either microsimulation or asymmetric delay functions via approaches such as Variational Inequality and Complementarity Methods which are computationally expensive.

Given a set of candidate roads where bicycle lanes can be allocated, and a budget to cover the implementation costs (marking, curb raising, etc.), the decision variables are binary variables (1 or 0) associated with the candidate roads. The value 1 indicates a bicycle lane is allocated, and 0 that it isn't. Inspired by the work of Leblanc (1975) we develop a purpose built Branch-and-Bound (BB) as search algorithm. The algorithm is then applied to the large size dataset of the city of Winnipeg, Canada through a phased process which resulted in 3 km bicycle lanes in the Central Business District (CBD).

The rest of the article is organized as follows: In the next section the mathematical formulation of the problem is presented; The solution algorithm is developed in Section 3, followed by numerical results in Section 4; The conclusion is provided in Section 5.

2. Mathematical formulation

Let be:

$\bar{A}$: Set of roads currently with mixed modes (bicycle and motorized modes) but are considered as candidate to be exclusively used by bicycle and the rest of the roads are denoted by A. Note; in order to preserve the connectivity of the network, the roads that are speculated to give away a lane to bicycle must have at least two lanes; should they become nominated to give away one lane as an exclusive bicycle lane, they still would have at least one lane remained (the candidate roads henceforth is simply called "candidate"). For ease of formulation, we adopt the following convention: consider road ℓ (with for instance three lanes), it is replaced with two new links: (i) link $\ell' \in \bar{A}$ with only one lane pending to either remain as a mixed mode road or become an exclusive bicycle lane or road[3] and (ii) road $\ell'' \in A$ with two lanes for mixed mode use.

N: set of nodes.
B: budget available to cover the costs of bicycle lane implementations such as marking, pavement, and curb raising.
y_a: binary decisions variable associated with candidate $a \in \bar{A}$; 1: to be used as an exclusive bicycle lane and 0: to remain mixed use road or lane.
c_a: Implementation cost associated with candidate $a \in \bar{A}$.
$x_a, \bar{x}_a$: motorized and bicycle traffic flow in passenger car equivalent or unit ("pce" or "pcu") on link $a \in A \cup \bar{A}$ respectively (Note; the network available to the motorists and cyclists are A and $A \cup \bar{A}$ respectively, hence $x_a, \bar{x}_a \geqslant 0$ for $a \in A$ and $x_a = 0, \bar{x}_a \geqslant 0$ for $a \in \bar{A}$).
$t_a(x_a + \bar{x}_a)$: general travel time of road $a \in A \cup \bar{A}$, a non-decreasing BPR function of link flow $x_a + \bar{x}_a$ (called delay function) (Sheffi, 1985; Spiess, 1990). The general travel time is a term to describe the disutilities involved in making a trip such as delay, travel time, petrol cost, fare, waiting time, parking fee, toll, pollutions, and safety. For the sake of simplicity henceforth we call it "travel time".
A_n^-, A_n^+: set of links starting and ending at node n respectively; $A_n^-, A_n^+ \subset A \cup \bar{A}$.
$b_a, \bar{b}_a$: additional delay (constant bias) perceived by motorized and bicycle mode of travelling on link $a \in A \cup \bar{A}$.
I: set of origin–destination pairs (OD pairs) $I \subset N \times N$.
$q_i, \bar{q}_i$: motorized and bicycle travel demand in pcu for OD $i \in I$ respectively.
P_i: set of paths between origin–destination $i \in I$.
$h_k, \bar{h}_k$: total motorized and bicycle flows on path $k \in P_i$ respectively.

The bilevel BPLD problem may be written as (all variables and parameters are considered non-negative unless otherwise stated):

$$\min \sum_{a \in A \cup \bar{A}} (x_a + \bar{x}_a) \cdot t_a(x_a + \bar{x}_a) + \sum_{a \in A \cup \bar{A}} \left(x_a \cdot b_a + \bar{x}_a \cdot \bar{b}_a\right) \tag{1}$$

$$\text{S.t. } y_a = 1 \text{ or } 0, \quad a \in \bar{A} \tag{2}$$

$$\sum_{a \in \bar{A}} c_a \cdot y_a \leqslant B \tag{3}$$

$$\min \sum_{a \in A \cup \bar{A}} \int_0^{x_a + \bar{x}_a} t_a(x_a + \bar{x}_a) dx + \sum_{a \in A \cup \bar{A}} (x_a \cdot b_a + \bar{x}_a \cdot \bar{b}_a) \tag{4}$$

$$\text{S.t. } \sum_{k \in P_i} h_k + \bar{h}_k = q_i + \bar{q}_i, \quad \forall i \in I \tag{5}$$

[3] Hence bicycle lanes are separately denoted by one-lane roads, we then alternatively refer to them as bicycle lane or bicycle road.

$$x_a = \sum_{i \in I} \sum_{k \in P_i} h_k \delta_{a,k} \quad \delta_{a,k} = \begin{cases} 1 & a \in k \\ 0 & a \notin k \end{cases}, \quad a \in A \cup \overline{A} \tag{6}$$

$$\bar{x}_a = \sum_{i \in I} \sum_{k \in P_i} \bar{h}_k \delta_{a,k} \quad \delta_{a,k} = \begin{cases} 1 & a \in k \\ 0 & a \notin k \end{cases}, \quad a \in A \cup \overline{A} \tag{7}$$

$$\bar{x}_a \leqslant U \quad a \in \overline{A} \tag{8}$$

$$x_a \leqslant (1 - y_a) \cdot U \quad a \in \overline{A} \tag{9}$$

where in the upper level (Eq. (1)) the total travel time is minimized. Eq. (2) sets out the binary decision variables. Eq. (3) ensures feasibility of the solutions with respect to the implementation costs of the projects versus available budget. In the lower level (Eqs. (4)–(9)), the MUE traffic flow is ensured (INRO, 2009; Spiess, 1984). Although Eq. (8) is redundant, it is intentionally placed in the constraints to emphasize that cyclists can use a candidate road either exclusively (if it turns out to be $y_a = 1$) or mixed with motorized mode (i.e. $y_a = 0$). Note that U is a sufficiently large number which can be considered as $\sum_i q_i$. If it is decided that road a to become an exclusive bicycle lane/road (i.e. $y_a = 1$), then Eq. (9) makes the respective road closed to motorized mode (i.e. $1 - y_a = 0$). The above equations treat the MUE traffic assignment in a simplistic form in which all different classes using link a are subject to the same congestion level (based on the total traffic volume) plus an additional bias term denoted by $b_a, \bar{b}_a$ for motorized and bicycle travel respectively. For instance the total travel time perceived by motorists on road a is $t(x_{a+}\bar{x}_a) + x_a \cdot b_a$, and for cyclists it is $t(x_{a+}\bar{x}_a) + \bar{x}_a \cdot \bar{b}_a$. Obviously, delay functions including the bias terms need to be calibrated based on field survey data.

Nonetheless it is worth noting that a comprehensive inclusion of multiclass into the traffic assignment comes at the cost of facing a non-convex optimization with asymmetric delay functions in which the feasibility and uniqueness of the solutions are not guaranteed. A variety of algorithms based on methods such as Variational Inequality, Complementarity Method, Fixed-Points and Entropy Maximization have been proposed in the literature (Bar-Gera and Boyce, 1999; Chen et al., 2011; Dafermos, 1972; Florian and Morosan, 2014; Nagurney, 2000; Nagurney and Dong, 2002; Zhang and Chen, 2010). Despite all these efforts, there is no consensus in the literature (Boyce, 2014) and how to approach the multiclass traffic assignment problem is still an ongoing debate.

Moreover, some scholars advocate microsimulation methods to better replicate the interaction of the bicycle and cars. Luo et al. (2015) present a recent reviews on the microsimulation-based methods. Nonetheless, the applications of the microsimulation-based methods cannot be extended to the large size networks. Apart from the scalability concern, Li et al. (2015b) have recently shown that the available simulation models first need to be thoroughly and rigorously evaluated. For instance in one case study, the fundamental diagram[4] observed in the field survey was different from that obtained from previous simulation models.

Despite the complexities involved, the above formulation (Eqs. (4)–(9)) is empirically proven to be a reliable method for the MUE traffic assignment (INRO, 2009). In this study, Eqs. (4)–(9) have been coded using Frank–Wolfe algorithm as a module in EMME 3 a leading transport planning application (INRO, 2009) and it is called upon in the algorithm whenever needed.

As for bicycle routing models found in the literature, our findings suggests that, cycling time along with other factors pertaining to the characteristic of the roads such as slope are of highest importance. Some studies provide alternative bicycle routing methodologies. Ehrgott et al. (2012) advocate considering travel time and multiple incommensurable objectives pertaining to the suitability of the routes. Consideration of multi objective functions gives raise to the issue of non-unique solutions. Furthermore the interaction of bicycles and motorized vehicles as well as the impact of congestions have yet to be addressed in their methodologies. Congestion is the most important thing which is overlooked in the literature. Recently consideration of bicycles with motorized vehicles under an integrated multi-class traffic model subject to congestion has been emphasized in the literature. Nonetheless, no real applications has been reported (Li et al., 2015a). In light of the aforementioned knowledge gap we employed the multiclass routing model.

3. Branch-and-Bound algorithm

3.1. Initialization

The BB algorithm is conducted on a tree comprising nodes and branches where each node (if it is not a dead end) is followed by two successor nodes. The bilevel BPLD problem expressed in Eqs. (1)–(9) is a mixed integer programming problem with $|\overline{A}|$ binary decision variables which constitute the solution space. Each node represents either a complete feasible solution or a "partial solution", representing a subarea of the solution space.

[4] It is a famous diagram in transport science that shows the relation between flow rate, density and speed. These three aggregate indices are deemed fundamental, hence, a reliable simulation model is expected to properly replicate them, let alone other detailed indices.

The algorithm starts by first solving the MUE traffic assignment for the do-nothing scenario (no bicycle lanes are allocated) that is $y_a = 0,\ a \in \overline{A}$. For this run and throughout the tree once the MUE is solved for a feasible binary solution, the best value of the objective function (total travel time, Eq. (1)) is kept as the incumbent value. Each node of the tree represents a sub area of the solution space. The tree is built up via an iterative process as follows: a node of the tree is selected at which two branches crop up to make two new nodes. The branching is made at one candidate, one branch represent binary value of 1 and the other one 0.

It is highly desirable to arrive at optimum solution in earlier iterations by expanding the tree at the right candidates. Knowing the right candidates means that having the conundrum solved. We are still able to find some clues or insights based on the following heuristic. A scenario with all the projects – irrespective of the budget – is made and a MUE traffic assignment is computed. The ensuing traffic volume, the capacity and the cost associated with individual candidates are invaluable indications to make an educated guess about the possible best solution. Let us define a merit index as:

$$(vc_a \cdot (\bar{x}_a/x_a) \cdot (x_a + \bar{x}_a))/c_a \tag{10}$$

where vc_a stands for volume per capacity ratio of candidate $a \in \overline{A}$. The rationale behind the proposed merit index is: (i) the first term pick up the congested roads as candidates which is the main subject of this research (ii) the second term ensures that under all things being equal the road that carries more bicycles gains more priority. (iii) The third term guarantees that under all things being equal the road that carries more traffic gets more priority. (iv) At the end, the index is normalized by the implementation costs, to make the task economically more efficient. In the course of branching, the projects are chosen from the top of the sorted merit list until the budget is depleted.

3.2. How does the BB work?

Each node in the tree represents a discrete solution either partial or complete. Consider a "partial solution" (01022) corresponding to five binary variables that represents a scenario in which the first three components are decided to become either bicycle lane represented by "1" or not represented by "0" and the last two (represented by "2") are unspecified (or yet to be decided). The index sets $I_0(z)$, $I_1(z)$, $I_2(z)$ are used to represent the projects decided as no-bicycle-lane, bicycle-lane and yet-to-be-decided respectively at node z. For the above mentioned partial solution we have: $I_0(z) = \{1, 3\}$, $I_1(z) = \{2\}$, $I_2(z) = \{4, 5\}$. The set of descendants (all possible completions) of the partial solution (01022) at node z on the tree is shown by $S(z) = \{y \in R^{|A'|} | y_1 = 0, y_2 = 1, y_3 = 0, y_a = 0/1, a = 4, 5\}$.

The tree iteratively grows up, at each iteration a node z of the tree representing a partial solution is chosen, out of which, an undecided project (represented by value "2") is selected and is assigned two values "1" bicycle-lane and "0" no-bicycle-lane on two newly emerged branches leading at two new nodes. In other words $S(z)$ is partitioned into two parts by the two new nodes. For each new node z of the tree, LB_z, lower bound on the objective function (Eq. (1)) for all the solutions in the descendants set $S(z)$ is evaluated. The lower bound LB_z is compared against the incumbent value denoted by UB^*. In case LB_z rests above UB^* we can argue that descendant solutions represented by node z (i.e. $S(z)$) will never render any better solution than the current incumbent value. Hence the algorithm stops branching at node z that is known as "fathoming". The fathoming action can be mathematically expressed as follows:

$$(UB^* - LB_z)/UB^* \leqslant \varepsilon \tag{11}$$

where ε is a pre-specified relative gap. In case node z does not satisfy fathoming criterion the branching proceeds until the budget is depleted or no more undecided projects are left which leads to a complete solution. The traffic assignment is carried out once a complete solution is found and a new upper bound (UB_z) is computed. The newly computed upper bound replaces the incumbent value if $UB_z < UB^*$. The process carries on until there are no more partial solutions left.

3.3. Node selection and branching rules

As the process proceeds and the tree expands, the algorithm may find a lot of nodes with partial solutions and wondering which one to choose for branching. Once a node is selected, the algorithm still has to pick one undecided project of the corresponding partial solution to conduct branching. To this end there are some methods requiring solving additional problems or retrieving the entire database to finding hopefully best node and branching. As the size of the network increases such methods become computationally intensive.

Alternatively, we consider the order of the projects in the descendingly sorted merit list as priority for the node selection and branching. As for branching, there is only one rule: choose the very next undecided project in the corresponding partial solution. As for node selection the algorithm follows two rules: (i) choose the deepest node of the tree (ii) in case of two nodes at the same level choose the one made of a branch associated with $y_a = 1$.

Some advantages of the proposed depth-first search: (i) finding good solution quickly and thus achieving better fathoming – given the fact that the projects are sorted on merit basis it make more sense to go deep into the tree and selecting the next best project for branching in a greedy manner hoping that the optimum solution lies deep there. (ii) Algorithm needs not to save/retrieve/process the information of the entire tree.

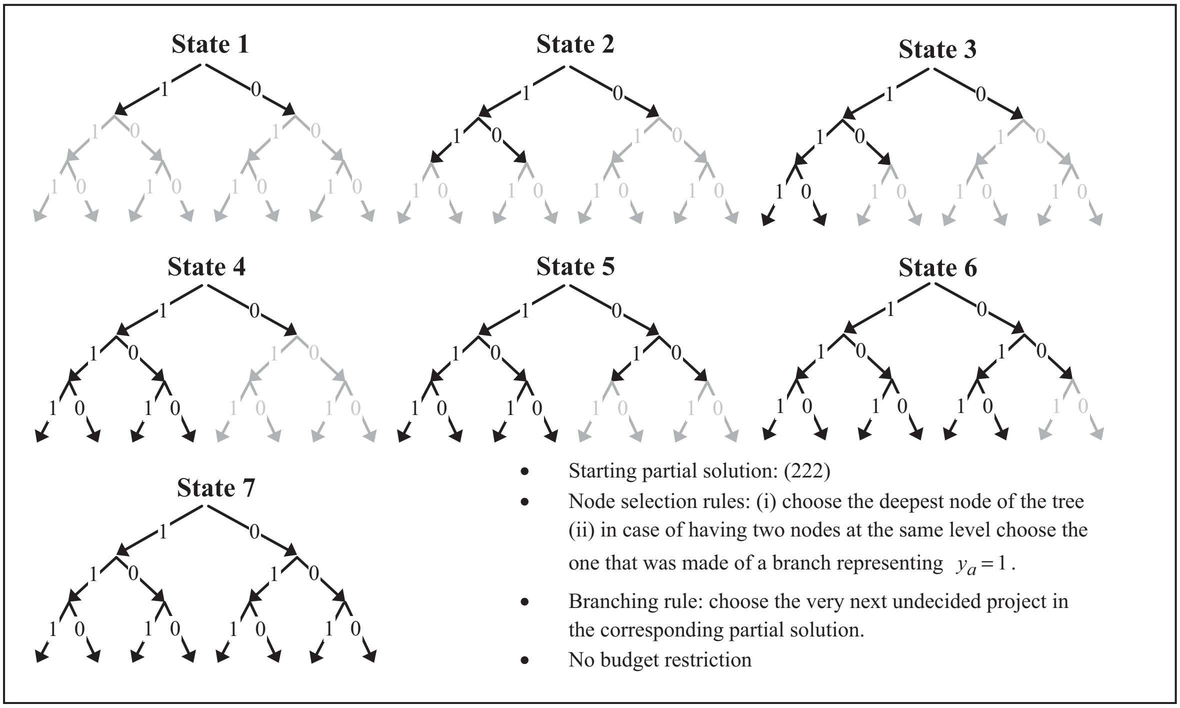

Fig. 1. Proposed node selection and branching in the Branch-and-Bound algorithm.

At each node the algorithm just need to move forward as much as possible on the paths consists of $y_a = 1$ branches. In case there is no space for such moves, the algorithm moves only one node back to the previous node and then move through the $y_a = 0$ and then follow $y_a = 1$ branch (if any). Fig. 1 illustrates gradual built up of the tree based on the above mentioned rules for a case consists of three candidates.

As the algorithm proceeds deep into the BB's tree, it may stop due to fathoming. There are two cases at which fathoming occur: (i) budget depletion and (ii) reaching at inferior lower bound. Should fathoming happens (in either case), there would be no point to proceed on that particular edge or node. Hence the algorithm moves one node back and resumes navigation on the other edge. Navigation over the BB's tree stops when there is no unfathomed node left. In other words, termination of the algorithm is pegged to having the entire BB's tree processed. It is worth noting that budget depletion can occur many times. If an edge/node representing a sub-optimal becomes fathomed due to budget depletion, it does not mean that the entire algorithm terminates. In such cases, the cursor of the algorithm moves to a new node (as described before).

As the tree structure expands the algorithm does not need to remember the already taken paths nor the paths ahead. As shown in Fig. 1, it just needs to know the lower bounds of the nodes on the current path plus the best solution found – so far – which is a string of binary values (0/1) and the corresponding incumbent value. For example if the current node is (11002) the next move is to processing node (11001) followed by the node represented by (11000). For the third move the algorithm moves three nodes back to reach at node (10222).

3.4. Tight lower bound for Branch-and-Bound

Compared to the UE, it is proven that the best traffic pattern (or least cost) is the System Optimal traffic flow (Patriksson, 1994; Sheffi, 1985). Now, given a partial solution, in order to arrive at a valid lower bound, one needs to solve a SO based problem as follows (Leblanc, 1975): (1)...(3),(5)...(9)[5] which is a mixed integer nonlinear programing (MINLP) problem.

Instead of directly solving the MINLP problem – which is highly difficult for large size networks – a valid but loose[6] solution can be found as follows: set all undecided candidates to mixed use (i.e. all "2" in partial solution must turn to "0") and calculate total travel time of the SO flow. Such a conservative estimation is not a healthy lower bound. The ratio of total travel time of the UE flow to the SO flow is called Price of Anarchy and for transportation network it can be as high as 2.15 (Roughgarden and Tardos, 2002). Hence it becomes highly unlikely to cut the solution space due to finding the lower bounds above the incumbent value. If it happens, the algorithm has to process every single solution of the solution space, that is, the algorithm has no superiority against exhaustive enumeration. In fact it is much worse, because, in addition to calculating merely

[5] Note: in the partial solutions some of the binary variables have already been assigned values (0 or 1) which makes the SO problem easier to solve.

[6] Loose means the lower bound value is too low, we rather to get a tighter lower bound.

the upper bounds as required in the enumeration, the algorithm has to calculate the lower bounds too. The following exposition offers an easy way to find a much tighter lower bound value.

The Multiclass SO (MSO) flow can be easily computed using the Frank–Wolfe by replacing $t_a(.)$ the delay function in Eqs. (4)–(9) with $\tilde{t}_a(.)$ as follows:

$$\tilde{t}_a(u_a) = t_a(u_a) + u_a \cdot \frac{\partial t_a(u_a)}{\partial u_a} \quad a \in A \cup \overline{A} \tag{12}$$

where $u_a = x_a + \bar{x}_a$. The deep gap between SO and UE emerges from the second term in the right side of the equation which is the additional externality cost that the commuters must pay. The two functions $t, \tilde{t}$ show benign and similar behaviour as long as the volume is below the capacity. As the volume gets close to (or exceeds) the capacity level the externality cost increases rapidly that results in a much deeper gap between the total travel times in MSO and MUE flows. In order to abate such mathematically-driven deep gap we propose $0 \leqslant \alpha \leqslant 1$ a coefficient to the externality term as follows:

$$\tilde{t}_a(u_a) = t_a(u_a) + \alpha \cdot u_a \cdot \frac{\partial t_a(u_a)}{\partial u_a} \quad a \in \overline{A} \tag{13}$$

As alpha gets close to zero the MSO flow gets close to MUE and the gap vanishes. It is worth noting that the alpha addresses the unfortunate trade-off between computational time and accuracy of the algorithm. The lower alpha ensures a faster and less accurate algorithm. The value of alpha can be identified as per the modeler's discretion depending on the computational technology at the time and how affordable is the computational time.

4. Numerical demonstration

For numerical implementation of the proposed methodology we use the real size transportation data of the city of Winnipeg, Canada, which is a standard benchmark in the literature (Bar-Gera, 2015) (it is also provided by INRO in EMME 3 (INRO, 2009)). The case study comprises of 154 zones, 903 nodes, 2995 directional links. There are 20 different BPR functions considered for the Winnipeg model (INRO, 2009). The BPR functions comply with following format: $t0^*(1 + a^*(x/c)^\wedge b)$ where $t0$, x and c are free flow travel time, traffic volume and capacity respectively. In addition, "a" and "b" are parameters which vary from 0.5423 to 1.1491 and 3.5038 to 6.8677 respectively[7]. The most congested area in the city is the Central Business District (CBD) served by 444 roads for which the bicycle lanes are sought in this study. The average volume capacity ratio in the CBD was calculated above 80% which is regarded as "working at capacity" condition equivalent to level of service E (HCM2010, 2010). The extent of the citywide road network and the CBD are shown in Fig. 2. The total citywide motorized vehicle and bicycle demands for a typical one hour peak are 84,324 and 4107 respectively. Given the widely accepted passenger car unit (pcu) of 0.2 for bicycle (Khan and Maini, 1999; Salter and Hounsell, 1996; Wang et al., 2008) the bicycle demand is equivalent to 822 equivalent vehicle in the peak hour which is mostly concentrated in the CBD. The methodology is coded in a Visual Basic environment linked to MS Access and MS Excel to communicate the data and synchronized with EMME 3 to solve the traffic assignment problems. A desktop PC with a 3.70 GHz CPU and 64 GB of "RAM" is employed.

The maximum speed of bicycle is considered to be 25 km/h (HCM, 2000). Due to higher flexibility of bicycle movement in congested areas, its interaction with motorized classes has yet to be investigated. In terms of travel time, despite relatively low speeds, the bicycle can compete with the motorized modes in the inner-city areas including CBDs, where the speed of traffic is controlled and kept low (Jensen et al., 2010; Sustrans, 2014). Therefore the bias terms in Eq. (1) for both bicycle and motorized classes are assumed identical. Nevertheless arriving at a proper estimation for the parameters of the delay functions including the bias terms for car and bicycle requires field survey data and calibration process. As for the bias term in particular and the delay function in general, recent studies endorse application of delay functions to model cyclist behaviour. A comprehensive survey in the United Sates revealed that cyclists are more concerned with travel time and distance and less sensitive to other characteristics for commuting trips (Broach et al., 2012). In particular, the findings suggest that cyclists are sensitive to the effects of distance, traffic volumes, slope, intersection control (e.g. presence or absence of traffic signals). All these factors can be included in a convex delay function.

As noted before, BPR delay functions are widely used as delay functions. The BPR is a multinomial function of order of 4 which can be calibrated using any statistical soft ware such as SPSS based on the field survey data. As noted above, the survey data must entail information pertaining to the roads individually such as traffic volumes of various classes (car, truck, bicycle, etc.), distances, slope of the roads, and type of intersection control.

First the candidate roads are identified. Table 1 shows the candidate roads to accommodate the bicycle lanes in the CBD identified as per Eq. (10) which accounts for 151 roads of total lengths of 20 km. Second, the BB algorithm is run over the candidate roads so as to identify the roads to yield a network of bicycle lane without deteriorating the total travel time of the system. Given the total length of candidate roads (20 km) the plan is to lay down bicycle lanes up to 10 km in the

[7] As seen, the delay functions are merely function of respective links flow, which is required by Frank–Wolfe (FW) algorithm to solve link based assignment problem. In other words the FW algorithm is not able to consider delay at junctions (nodes). Explicit consideration of junction delay gives raise to asymmetric travel times which in turn calls on different and more complicated methodologies such as variational inequality and complementarity method to solve the traffic assignment problem. Instead, in the undertaken Winnipeg case study (as issued by INRO, 2009) there is a pool of different BPR functions. In fact, delay at junctions has been implicitly considered for the junctions' approaching links in the calibration process.

Fig. 2. Winnipeg case study; the extent of the network undertaken in the analysis and the Central Business District (CBD).

CBD subject to caring for the overall performance of the system including the motorized traffic. As discussed earlier the overall performance of the system is quantified as total disutilities or travel time. The result of the first traffic assignment (on which Table 1 is derived) indicates that the total travel time of the existing network (no-bicycle-lane scenario) is 2,615,545 min per peak hour. In the course of taking road spaces from motorized traffic in favour of bicycles which may

Table 1
The candidate roads to yield a bicycle lane in the CBD, Winnipeg.

No	Inode	Jnode	Len	Vc	Vlmtr	Vlcls	M	No	Inode	Jnode	Len	Vc	Vlmtr	Vlcls	M	No	Inode	Jnode	Len	Vc	Vlmtr	Vlcls	M
1	1050	1047	0.07	2.24	3287	66	2033	51	936	949	0.1	1.21	881	24	298	101	920	919	0.06	0.88	344	9	132
2	1047	1046	0.09	2.43	3565	74	1364	52	1053	1052	0.24	1.71	2531	41	297	102	909	908	0.14	1	1235	18	122
3	1051	1050	0.11	2.24	3287	66	1332	53	1059	1051	0.24	1.38	2542	51	297	103	902	905	0.2	1.07	1311	22	122
4	983	982	0.06	1.69	2484	47	1323	54	936	932	0.1	1.32	969	22	297	104	908	906	0.19	1.19	1467	19	118
5	1010	1009	0.06	1.87	2762	42	1059	55	995	990	0.1	1.29	1425	23	293	105	1038	1039	0.17	0.86	1264	23	118
6	951	950	0.03	1.19	1467	26	988	56	982	981	0.17	1.29	1903	38	292	106	958	959	0.16	1.05	774	18	110
7	1008	1007	0.07	1.76	2609	39	890	57	949	965	0.1	1.19	872	24	289	107	910	909	0.17	1.01	1250	18	110
8	982	1003	0.04	1.5	1104	23	834	58	1039	1040	0.07	0.86	1274	23	285	108	970	944	0.09	0.86	954	11	103
9	1041	1040	0.07	1.63	2412	35	763	59	937	948	0.1	1.05	1541	27	283	109	894	895	0.16	0.92	1131	18	99
10	**986**	**985**	**0.09**	**1.64**	**2422**	**41**	**716**	**60**	**906**	**905**	**0.15**	**1.48**	**1817**	**28**	**282**	**110**	**931**	**930**	**0.1**	**1.13**	**445**	**9**	**91**
11	987	986	0.1	1.7	2515	41	671	61	891	892	0.21	1.47	2719	40	277	111	947	967	0.1	1.06	414	8	91
12	980	979	0.09	1.43	2101	41	671	62	981	980	0.18	1.29	1903	38	273	112	1025	1022	0.15	1.05	1174	13	90
13	1018	1017	0.05	1.13	2084	29	636	63	1061	1060	0.09	2.13	840	11	273	113	1039	1038	0.17	0.92	1357	16	90
14	1046	1045	0.07	1.35	1998	32	604	64	1051	995	0.11	1.3	1440	23	265	114	944	941	0.11	0.86	954	11	87
15	783	1054	0.07	1.06	2601	39	577	65	948	966	0.1	1.01	1484	26	261	115	918	917	0.08	0.86	334	8	85
16	1044	1043	0.1	1.5	2207	38	559	66	931	937	0.1	1.01	1489	25	256	116	901	902	0.27	1.13	1397	20	82
17	1031	1032	0.05	1.48	1651	19	557	67	981	1005	0.1	0.97	1432	26	252	117	1027	1026	0.09	1.29	512	6	81
18	989	988	0.16	1.98	2926	44	551	68	1011	1015	0.1	1.27	1413	20	247	118	1022	1002	0.12	0.94	1043	10	77
19	988	987	0.12	1.64	2419	40	545	69	1061	1041	0.18	1.41	2080	31	245	119	969	968	0.17	1.16	687	11	75
20	**1011**	**1010**	**0.12**	**1.73**	**2554**	**37**	**514**	**70**	**911**	**910**	**0.09**	**1.08**	**1325**	**20**	**243**	**120**	**992**	**991**	**0.21**	**0.95**	**1407**	**16**	**75**
21	1004	1003	0.05	1.55	1145	16	506	71	1053	993	0.13	1.9	1407	16	234	121	970	969	0.18	1.48	585	9	74
22	990	989	0.19	2.04	3013	46	496	72	1036	1031	0.11	1.4	1556	18	232	122	1029	1028	0.17	1.38	544	9	72
23	1012	1011	0.17	1.51	3723	55	478	73	966	981	0.11	0.97	1432	26	229	123	919	918	0.11	0.88	344	9	70
24	1052	1051	0.12	1.48	2186	38	476	74	1038	1028	0.12	1.07	1983	25	209	124	1026	1020	0.1	1.11	438	6	69
25	908	914	0.06	1.23	1515	23	458	75	914	934	0.11	1.08	1334	21	202	125	930	938	0.1	0.9	352	8	69
26	1043	1044	0.1	1.23	1804	37	439	76	1025	1021	0.1	0.87	1281	23	201	126	938	947	0.1	0.91	356	7	68
27	1009	1008	0.17	1.8	2653	41	430	77	1005	1004	0.15	1.23	1821	24	199	127	906	915	0.11	0.94	368	8	65
28	939	929	0.04	0.86	1583	20	425	78	991	990	0.13	1.07	1587	24	196	128	1031	1029	0.18	1.37	541	8	64
29	1020	1019	0.05	1.65	647	13	420	79	932	936	0.1	0.94	686	20	184	129	910	954	0.16	1.18	464	8	64
30	**1015**	**1032**	**0.06**	**1.27**	**1413**	**20**	**419**	**80**	**912**	**911**	**0.12**	**1.08**	**1325**	**20**	**183**	**130**	**1033**	**1031**	**0.24**	**1.62**	**638**	**9**	**63**
31	1045	1046	0.07	0.91	1333	32	405	81	1044	1045	0.16	0.91	1333	32	181	131	941	940	0.19	1.48	585	8	62
32	1041	1042	0.12	1.35	1997	35	396	82	975	970	0.1	1.14	1269	16	178	132	902	901	0.27	1.06	1179	16	58
33	917	931	0.1	1.23	1812	32	394	83	912	958	0.17	1.39	1025	21	171	133	962	986	0.16	1.04	408	9	58
34	1001	1000	0.05	0.97	1432	20	391	84	1042	1043	0.18	1.09	1607	28	169	134	1039	1026	0.12	1.11	438	6	54
35	965	982	0.1	1.43	1048	27	389	85	949	936	0.1	1	734	17	169	135	1059	1052	0.19	1.06	784	10	51
36	1040	1039	0.07	1.2	1774	22	387	86	902	916	0.12	0.98	718	20	166	136	1021	1020	0.22	1.14	446	10	50
37	1037	1038	0.17	1.68	2483	39	386	87	1052	994	0.12	1.55	1148	13	166	137	948	947	0.14	1.02	402	7	48
38	1035	1036	0.22	2.11	3124	40	385	88	950	964	0.15	1.05	1284	23	162	138	954	955	0.16	0.94	367	8	47
39	892	893	0.11	1.24	2282	34	378	89	965	949	0.1	1	732	16	159	139	940	939	0.17	0.99	583	8	47
40	**901**	**917**	**0.1**	**1.21**	**1790**	**31**	**361**	**90**	**1011**	**975**	**0.1**	**1.08**	**1205**	**15**	**157**	**140**	**955**	**962**	**0.17**	**0.99**	**390**	**8**	**45**
41	1045	1044	0.16	1.5	2207	38	345	91	893	894	0.25	1.37	1683	28	153	141	960	959	0.16	1.01	748	7	45
42	782	783	0.12	1.08	2673	38	334	92	994	991	0.1	1.31	968	12	150	142	988	960	0.16	1	745	7	42
43	901	900	0.12	1.02	2516	39	332	93	1028	1018	0.19	1.04	1915	27	148	143	959	958	0.16	0.99	735	7	34
44	1042	1025	0.12	1.31	1939	30	321	94	964	983	0.16	1.03	1270	22	147	144	1044	999	0.18	1.19	472	5	31
45	1043	1042	0.18	1.5	2207	38	317	95	967	980	0.1	1.34	527	11	145	145	949	948	0.21	0.96	379	7	30

Table 1 (*continued*)

No	Inode	Jnode	Len	Vc	Vlmtr	Vlcls	M	No	Inode	Jnode	Len	Vc	Vlmtr	Vlcls	M	No	Inode	Jnode	Len	Vc	Vlmtr	Vlcls	M
46	934	951	0.09	1.18	1450	24	316	96	986	1001	0.08	1.27	501	9	142	146	947	946	0.17	0.88	346	6	29
47	916	932	0.1	1.24	902	25	315	97	1042	1060	0.24	1.45	1067	23	141	147	966	965	0.2	0.97	383	6	26
48	993	992	0.1	1.9	1407	16	313	98	1028	1027	0.08	1.55	612	7	140	148	1060	1042	0.24	0.89	657	7	25
49	1054	1053	0.18	1.41	2601	39	308	99	900	901	0.12	0.88	1300	19	136	149	937	936	0.24	0.92	362	6	23
50	**1036**	**1037**	**0.18**	**1.57**	**2326**	**35**	**299**	**100**	**982**	**965**	**0.1**	**0.91**	**666**	**15**	**133**	**150**	**950**	**949**	**0.29**	**0.95**	**372**	**7**	**21**
																151	1026	1025	0.24	0.87	516	6	0

Len: length in km; vc: volume-per-capacity; vlmtr and vlcls: motorized and bicycle traffic volume in pcu; M: merit index.

Table 2
Numerical results; application of the algorithm in successive phasing stages; CBD, Winnipeg.

Id	Inode	Jnode	Length (km)	Phase 1	Phase 2	Phase 3	Phase 4	Phase 5
1	1050	1047	0.07	1	1	1	1	1
2	1047	1046	0.09	1	1	1	1	1
3	1051	1050	0.11	1	1	1	1	1
4	983	982	0.06	1	1	1	1	1
5	1010	1009	0.06	1	1	1	1	1
6	951	950	0.03	1	1	1	1	1
7	1008	1007	0.07	1	1	1	1	1
8	982	1003	0.04	1	1	1	1	1
9	1041	1040	0.07	1	1	1	1	1
10	986	985	0.09	0	1	1	1	1
11	987	986	0.1	1	1	1	1	1
13	1018	1017	0.05	0	1	1	1	1
14	1046	1045	0.07	0	0	0	1	1
16	1044	1043	0.1	0	1	1	1	1
17	1031	1032	0.05	0	0	1	1	1
19	988	987	0.12	0	1	1	1	1
26	1043	1044	0.1	0	1	1	1	1
28	939	929	0.04	1	1	1	1	1
30	1015	1032	0.06	1	1	1	1	1
31	1045	1046	0.07	0	1	1	1	1
33	917	931	0.1	1	1	1	1	1
36	1040	1039	0.07	0	0	1	1	1
40	901	917	0.1	0	1	1	1	1
56	982	981	0.17	0	0	1	1	1
58	1039	1040	0.07	1	1	1	1	1
67	981	1005	0.1	0	0	0	1	1
124	1026	1020	0.1	0	0	1	1	1
125	930	938	0.1	0	0	1	1	1
136	1021	1020	0.22	0	0	1	1	1
150	950	949	0.29	0	1	1	1	1
Sum			2.77	14	22	28	30	30
Number of additional roads found as bicycle lane				14	8	6	2	0
Computation time (h)				7.67	1.63	0.65	0.36	0.30
Total travel time (minutes per peak hour)				2,641,623	2,640,722	2,641,456	2,640,724	2,640,724

New roads identified as bicycle priority lanes at respective phases are flagged using shaded values.

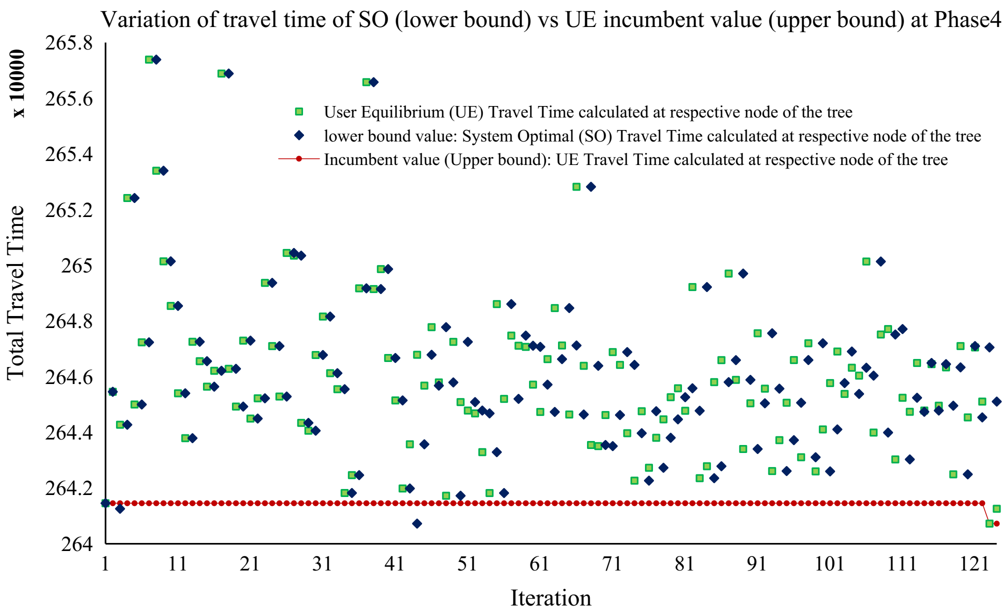

Fig. 3. Phase 4; variation of lower bound and upper bound.

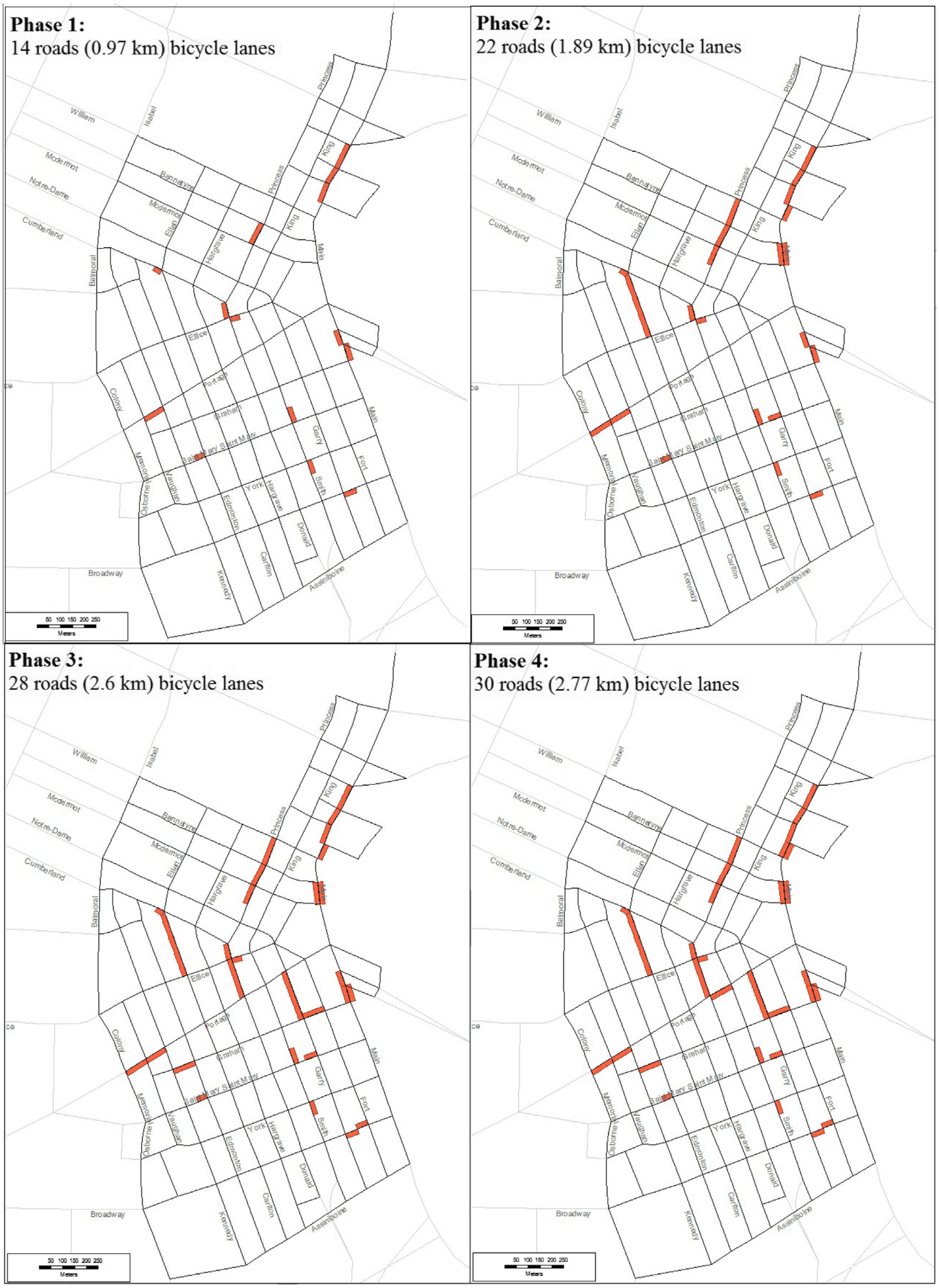

Fig. 4. The roads identified as bicycle lanes in successive phases, CBD, Winnipeg.

adversely deteriorate the overall performance, we consider a ceiling to such adverse impact. With respect to the current total travel time the ceiling is assumed as maximum 1% additional travel time. It is worth noting that achieving 10 km bicycle lanes subject to only maximum 1% additional travel time in the system may not be possible. Hence the 10 km bicycle lanes in the CBD is a target and the algorithms tries its best to lay down as many as possible bicycle lanes.

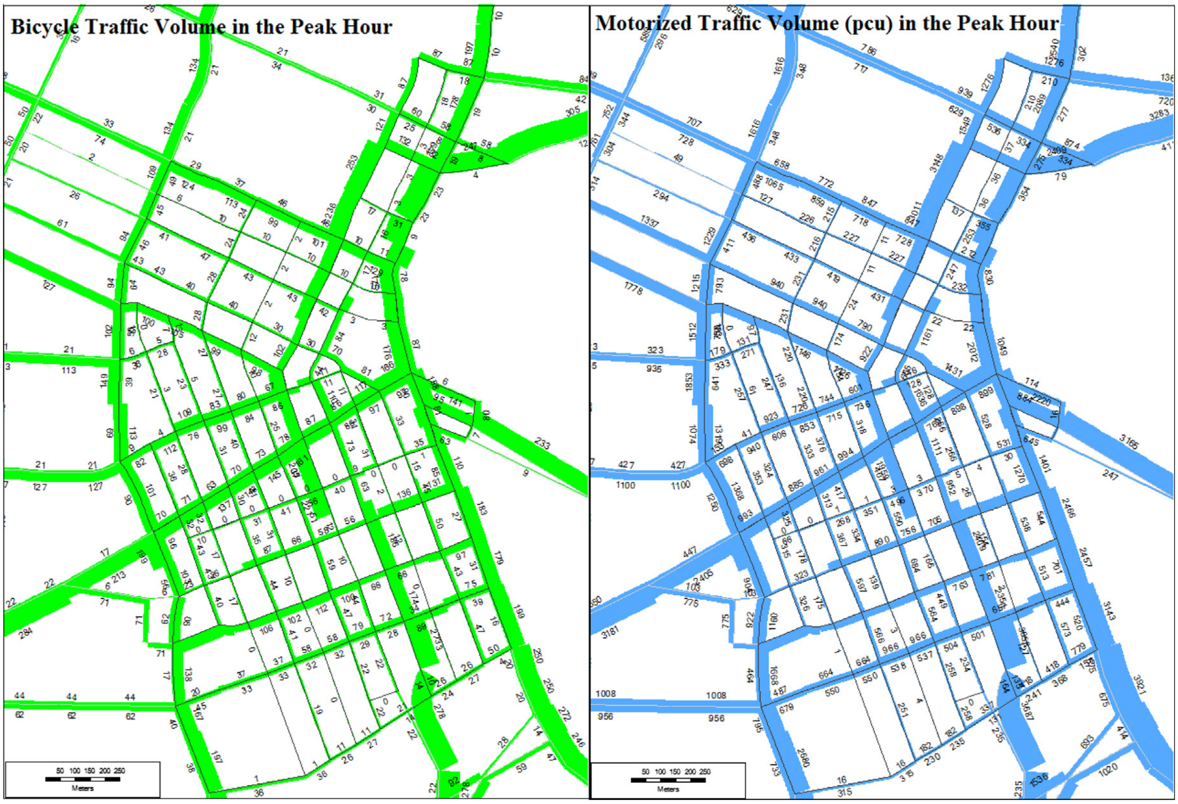

Fig. 5. Ultimate traffic volume in the peak hour after implementing the bicycle lanes, CBD, Winnipeg.

In order to get to a working solution in a timely manner we start with alpha equal to zero in Eq. (12). We will later on show that the computation time is heavily dependent upon the value of alpha.

In order to cope with the intensive size of the problem, given a budget level, the algorithm starts with a very tight budget. In each phase the identified roads for bicycle lanes are carried over to the next phase with additional budget until the budget is consumed or no more bicycle lanes are found. Such a phased approach to the problem is greatly appealing to the industry. Given limited resources, or understandable hesitation and cautiousness with traffic authorities in making new decisions, phasing the projects or conducting the projects through a pilot exercise is a prudent and valid action in practice (Bagloee and Asadi, 2015; Bagloee and Tavana, 2012).

In summary, one first needs to identify a target value of the maximum allowable total travel time. We consider maximum 1% additional travel time of the existing condition. Hence the target total travel time became 2,641,700 (=1.01 $*$ 2,615,545) minutes per peak hour. This value is then set as the incumbent value at the onset of the algorithm. At the end, if the algorithm reaches at a lower incumbent value it gives us motivation to run the algorithm for further phase to find more bicycle lanes. Otherwise (no better incumbent value is found), there would be no more space for a new bicycle lane.

4.1. Phasings

The first phase starts with budget of 1 km. The algorithm was run on 151 candidate roads, as shown in Table 1, for which the computation elapsed time was 7.67 h, and a solution of 14 roads equivalent to 0.97 km with total travel time of 2,641,623 min per peak hour was found. Even if a solution near to the ceiling target is found, it is in general worth continuing until no better solution is found. For the next phase, in our experiment, the do-nothing scenario was updated by incorporating the 14 roads as bicycle lanes and hence the candidate set was updated too. The algorithm was restarted with the same ceiling total travel time and a shrunk candidate set of 137 (=151 − 14) candidate roads. This time the computation lasted 1.63 h and a solution of 8 roads equivalent to 0.92 km with total travel time of 2,640,722 min per peak hour was found.

With regards to the total travel time one may ask why the second solution is superior to the first solution found, in other words, why the algorithm could not find the second solution in the first phase. To answer this question one needs to remember that the undertaken TPLD problem is an NP-hard problem and the alpha (in Eq. (13)) which is supposed to be 1 – to render the global solution – is considered 0 – the minimum possible value – to render a good solution in an efficient and affordable timeframe. We shall show that even for a meagre value of alpha (say half a percent) the computation time tends to infinity).

These newly found 8 roads were also added to the previous 14 roads to update the do-nothing network and the candidate set ($|\bar{A}| = 129$) for the third phase. Application of the algorithm resulted in a solution of 6 roads equivalent to 0.71 km and total travel time of 2,641,456 min per peak hour. The same practice was repeated for the fourth phase which returned a solution of 2 roads equivalent to 0.17 km and the total travel time of 2,640,724 min per peak hour. The algorithm was carried out for the fifth phase and no solution was found. In Table 2 the progressive results in the phases including the 30 roads with total length of 2.77 km selected as bicycle lanes are shown.

As noted before, target value of maximum allowable total travel time based on maximum of 1% additional travel time of the existing condition resulted in an upper bound (UB) or incumbent value at the onset of the algorithm. At the end of each phase, if the algorithm reaches at a lower UB, it means that the entire 1% is not yet depleted, hence it gives us motivation to run the algorithm for further phases to find more bicycle lanes. Otherwise (no better incumbent value is found), there would be no more space for a new bicycle lane. This corresponds to Table 2, Phase 5, where the total travel time is identical to the one of Phase 4. Furthermore there is an interesting observation in the fluctuation of total travel time reported in Table 2: at Phase 2 versus Phase 1 and Phase 4 versus Phase 3, the introduction of more bicycle lanes led to lower total travel time. This means that the bicycle lanes are on Braess infected roads which is the primary design of the proposed methodology.

Fig. 3 illustrates variation of lower bound based on the SO traffic flow and upper bound based on the UE traffic flow pertaining to Phase 4 –as an example – over successive iterations as the algorithm navigates through the BB's tree. As seen, in three occasions (iterations 1, 3 and 44) the lower bound values were found below the incumbent value.

The topographical demonstration of the bicycle lanes laid down on the CBD in the progressive phases is shown in Fig. 4. This figure shows that the latent misutilized road capacities are scattered in the heart of the city at the peak hour where the congestion is the case. The bicycle network can be connected by other means such as exclusive bicycle/parking lane, shared bicycle/parking lane, off road/Shared Path, Copenhagen style (VicRoad, 2000). These results can propel up the arguments made by the bicycle advocates to seek space in the heart of the cities and even in the peak hours of traffic as well as in the heavily congested roads. The ultimate traffic volume in the peak hour (bicycle and motorized) pertaining to the last phase are shown in Fig. 5.

As discussed earlier the value of alpha was first set to zero to find good (not necessarily best) solutions within an affordable time. Hence it was worth trying to seek a solution at this phase by changing the value of alpha. As such a meagre value of 0.005 was set for the alpha and the algorithm was run on a candidate set of the size of ($|\bar{A}| = 121$). After almost 10 days computation no better solution was found and we stopped it.

According to Eq. (13), alpha = 0 means that the SO is lifted to the level of UE. As shown, zero alpha results in practical (although not necessarily global) solution for a real size network. Hence, this does not contradict the scalability of the algorithm, rather it is devised to serve its scalability. In other words the lower alpha compromises the quality of the solution (with respect to the global solution) in order to empower the algorithm to handle real size networks.

As noted before the total travel time of the existing network (with no bicycle lanes) is 2,641,623 min per peak hour. The changes of total travel time over successive phases are presented in Table 2 which varies within 1% above that of the existing network.

5. Conclusion

We started this article with a wishful quote, to which this study aims to contribute. Given many proven advantages of bicycle compared to motorized traffic the studies are still in their infancy. One way to promote the market share of the bicycle mode is to separate bicycles from motorized transport in order to ensuring a safe and streamlined class of mobility. As such, some advocate dedicating road network space to exclusive bicycle lanes at the cost of leaving less space for the motorized traffic. This practice is easily implemented in an uncongested traffic network where the road network is underutilized. However, enforcing bicycle lanes on congested roads may degenerate the network as a whole, making the idea very hard to sell both to the public and the traffic authorities.

With respect to congested networks, we took an unorthodox approach to searching for latent misutilized capacity to be dedicated to exclusive bicycle lanes based upon the Braess Paradox. The problem was formulated as a bilevel mathematical programing with binary elements as decision variables which is widely believed to be an intractable problem to solve. Despite all the complexities involved, the aim of this study was to tailor an efficient and working method to real size networks. Unlike previous attempts, the interaction of bicycles with motorized traffic was considered, and formulated as a multiclass traffic assignment problem in the lower level of the bilevel problem. The objective function in the upper level was to minimize all the disutilities involved in making a trip such as time, cost, parking fee, fare, pollutions, etc., encapsulated as "travel time" for easier reference.

First a set of roads deemed appropriate to accommodate bicycle lanes was identified (set of candidates). Laying out the bicycle lanes comes with some implementation costs and hence a budget is considered in the problem formulation. A successive phasing methodology was developed such that, in each phase a subset of the candidate set is identified, subject to not to degenerating the total travel time of the network above a pre-specified level nor does breaching the budget level. A Branch-and-Bound (BB) algorithm was developed to solve the bilevel problem in each phase. The BB was structured in a less – intensive – RAM manner in order to handle the large scale networks.

The methodology was tested on real size network of city of Winnipeg, Canada, for which the total of 30 roads equivalent to 2.77 km bicycle lanes in the CBD were found.

This article primarily appeals to practitioners, policy makers and traffic authorities in their quest to promote bicycle. The proposed algorithm has been developed as a module in EMME a popular leading planning software. The phasing nature of the methodology makes it a flexible tool in the hands of planners to keep track of the changes in the network and add their engineering judgment to the decision making process. For instance, one can add, remove, or enforce a certain number of candidate roads in the outset of the algorithm based on discretion, vested interest and other concerns. As the title of the article suggests, the main emphasis of this article is to empower policy makers in their uphill battle to fit bicycle infrastructure and lines in tightly – packed roads of the CBDs, without losing the battle to car-obsessed vested interests. In this quest we unearthed the Braess Paradox from the academic literature and used it as a leverage to get the voices of cyclists heard in practice using sound and flawless scientific language.

The methodology proposed can be further improved on several threads:

I. Bicycle transport comes short in competing with motorized transport in long distance trips. One solution can be integrating bicycles into public transport (Flamm et al., 2014; Krizek and Stonebraker, 2011; Wang and Liu, 2013). Some European cities are leading in paving the way for coexistence of environment for both bicycle and public transport. For instance, cyclists can take their bicycles to the metro, or even buses are equipped with special bicycle-racks to accommodate cyclists in their long range commutes. Accordingly a joint problem considering the public transport network needs to be formulated in which the decision variables include which roads are to become bicycle lanes and which roads are to become transit priority. The problem can be extended to a higher level of decision making to adjust the public transport system (in terms of route, fleet, frequency, stop stations. etc.) to accommodate bicycle demand fully.

II. In addition to the exclusive bicycle lane, consideration of other types of bicycle lanes to provide a connected network also deserves to be studied. One possible solution is to tie a range of consecutive road segments together as a single road in the candidate set. Therefore the proposed algorithm will be forced to choose from these tied-up roads which excludes occurrence of secluded short segment bicycle lanes.

III. Given rapid advances in telecommunication and smart phones, there has been a surge of interest in an old concept of bicycle sharing (Angeloudis et al., 2014; Bachand-Marleau et al., 2012; Corcoran et al., 2014; Langford et al., 2013; Pucher et al., 2011; Vogel et al., 2014). How to accommodate fleet size and the required facilities such as bicycle stations as well as the bicycle network remain to be investigated. As such bicycle priority lanes can be tailored to catering to the bicycle sharing schemes. In such schemes, a joint model is needed to identify sharing stations, fleet size of shared bicycles as well as a network of bicycle lanes connecting the stations. Moreover, bicycle sharing schemes by themselves can be extended to a fully-fledged supply chain problem in which a variety of logistic and scheduling issues exists. An example is that of how to move shared bicycles between stations during peak and off peak times and directions.

IV. The relatively new interest in the electric bicycle (E-bike) warrants further investigation. Recent studies have shown a significant changes in the traffic related indices attributed to the presence of the E-bike (Jin et al., 2015). Operational characteristic of the E-bike in terms of coping with long distance commutes, high gradients and speed as well as being dependent on charging stations give a whole new dimension to the problem. First of all given the promising characteristics (long distance, high gradients, and speed)[8], the e-bike must be treated as a new bicycle class distinct from the conventional bicycles which are trickled down to calibrating specific bias terms in the delay functions. Secondly, locating best places for the charging stations should also be included amongst the constraints. Hence the problem of bicycle lane priority is extended to catering to conventional bicycles as well as reaching out to the e-bikes and connecting the charging stations.

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[8] Perils and promises of E-bikes: these promising factors are the main hurdles for the conventional bicycle. Nonetheless being tied to the charging stations (whether at private premises or public) is on the negative side of the e-bikes.

Appendix A

A.1. Efficacy of tree structure in calculation the lower bounds

At each iteration a (parent) node renders two new (offspring) nodes, but it is only required to calculate the lower bound for one of the newly generated nodes. Because the other offspring node inherits the lower bound from the parent node, hence, it makes the computation more efficient. Fig. A-1 depicts this observation graphically: the parent node corresponds to partial solution (1122..22) has lower bound corresponding to string (1100..00) with lower bound value of 85. The branching is made at the third project (the very next project with value of "2") which results in offspring nodes (1112..22) in the left-hand side and (1102..22) in the right-hand side.

For new offspring node (1102..22), the lower bound is computed by replacing the "2"s with "0" which is already calculated for the parent node. Hence it is only imperative to calculate a new lower bound for the other offspring (1112..22) which will not be found better ($88 \nless 85$) (Leblanc, 1975). In fact new lower bound is computed only for the nodes residing on the left wing.

A.2. General assessment of the merit index in Branch and Bound

In this section a general Branch-and-Bound algorithm for solving mixed integer nonlinear programing (MINLP) problem with and without merit index via a simple example is assessed. The example is also a general network design problem: consider network consists of a road (#4) connecting an origin–destination pair with travel demand of $q_{od} = 10$. There are three road construction projects (#1, #2, #3), while the budget can cover maximum two roads ($c_1 = c_2 = c_3 = 1;\ B = 2$). Fig. A-2 shows the network and delay functions associated with the roads and projects.

The objective function is defined as minimizing the total time spent on the network. Hence the MINLP problem can be written as follows:

$$
\begin{aligned}
\min f(x,y) = {} & .125x_1x_1 + .25x_2x_2 + .5x_3x_3 + x_4x_4 \\
\text{s.t.}: {} & x_1 + x_2 + x_3 + x_4 = 10 \\
& x_1 - 10y_1 \leqslant 0 \\
& x_2 - 10y_2 \leqslant 0 \\
& x_3 - 10y_3 \leqslant 0 \\
& y_1 + y_2 + y_3 \leqslant 2 \\
& x_1 \ldots x_4 \geqslant 0;\ y_1 \ldots y_3 \in \{0,1\}
\end{aligned}
$$

From the delay functions of the three projects one can intuitively deduce that the merit order of the candidates is as (y_1, y_2, y_3). As such, the optimal solution is found $(y_1, y_2, y_3) = (1, 1, 0)$, $(x_1, x_2, x_3) = (6.1, 3.1, 0.0, 0.8)$ and $f(x, y) = 7.7$. The computations provided in below were made using GAMS (2014) a leading optimization software.

Details of BB to solve a MINLP problem can be found in any integer programing text book (Floudas, 1995; Li and Sun, 2006). Below is an overview of a generalized BB.

Step 0. Initialize the upper bound as $UB^* = +\infty$. Find a feasible solution for MINLP as the incumbent solution to be represented by the root node of the tree. Set the root node as the current node identified by $c = 1$.

Step 1. Relax the MINLP on the integer variables and obtain corresponding continuous variables $(y_1^c \ldots y_j^c)$ and v^c the value of the objective function.

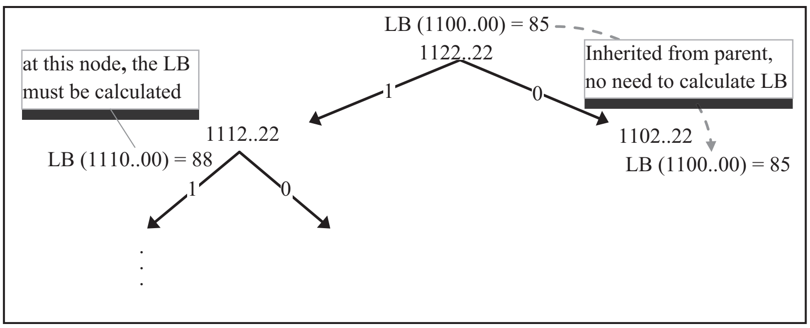

Fig. A-1. Computation of lower bound in the tree structure of Branch-and-Bound.

Step 2. if $v^c \geqslant UB^*$ or the current node represents a feasible solution (all y are integer) then fathom the current node. Consider the current node as incumbent solution if the current node is a feasible solution and it renders a better solution than the incumbent solution (i.e. $UB^* = \min(v^c, UB^*)$.
Step 3. If there is no unfathomed node left, stop, the incumbent solution is optimal. Otherwise select an unfathomed node as the current node. Then choose a y whose value in the current node is not integer $(y \neq [y])$ and split the solution space in two domains one by adding $y \leqslant$ and the other one by $y \geqslant [y] + 1$ in the constraints ($[y]$ returns the first integer value before y). Represent these two subareas by adding two branches at the end of the current node of the tree. Go to Step 1.■

In Step 3, one needs to choose an integer variable to be split for which we proposed the concept of merit index. Projects are first sorted based on their merit indices in a string and it is used for split process. In the following exercise performance of BB with and without merit index is evaluated.

A.3. Branch and Bound; sorted string based on merit index is (y_1, y_2, y_3)

Step 0. Set $UB^* = +\infty$ and the initial feasible solution as the incumbent solution: $(y_1^0, y_2^0, y_3^0) = (0, 0, 0)$, $z^* = 100$ in the root node of the BB's tree. Set the current node $c = 1$.

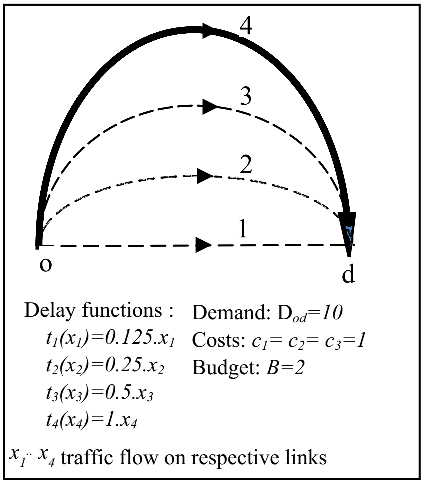

Fig. A-2. Network design example.

i	Y^i	X^i	v^i	ub^*
0	0,0,0	0.0,0.0,0.0,10.0	100.0	100.0
1	0.5,03,0.1	5.3,2.7,1.3,0.7	6.7	100.0
2	1,0.3,0.1	5.3,2.7,1.3,0.7	6.7	100.0
3	1,1,0	6.1,3.1,0.0,0.8	7.7	7.7
4	1,1,0.2	7.3,0.0,1.2,0.9	9.1	7.7
5	0,0.6,0.3	0.0,5.2,2.9,1.4	14.3	7.7

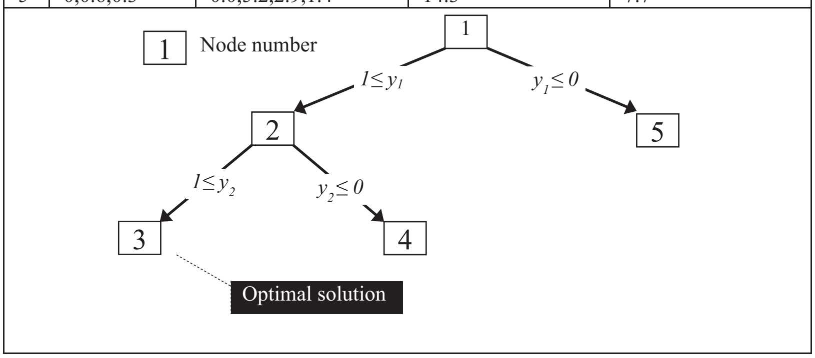

Fig. A-3. Results of Branch and Bound method with merit index.

i	Y^i	X^i	v^i	ub^*
0	0,0,0	0.0,0.0,0.0,10.0	100.0	100.0
1	0.5,03,0.1	5.3,2.7,1.3,0.7	6.7	100.0
2	0.5,0.3,1	5.3,2.7,1.3,0.7	6.7	100.0
3	0,1,1	0,5.7,2.9,1.4	14.3	14.3
4	0.7,0,1	7.3,0,1.8,0.9	9.1	14.3
5	1,0,1	7.3,0,1.8,0.9	9.1	9.1
6	0,0,1	0,0,6.7,3.3	33.3	9.1
7	0.6,0.3,0	6.2,3.1,0,0.8	7.7	9.1
8	0.6,1,0	6.2,3.1,0,0.8	7.7	9.1
9	1,1,0	6.2,3.1,0,0.8	7.7	7.7
10	0,1,0	0,8,0,2	20	7.7
11	0.9,0,0	8.9,0,0,1.1	11.11	7.7

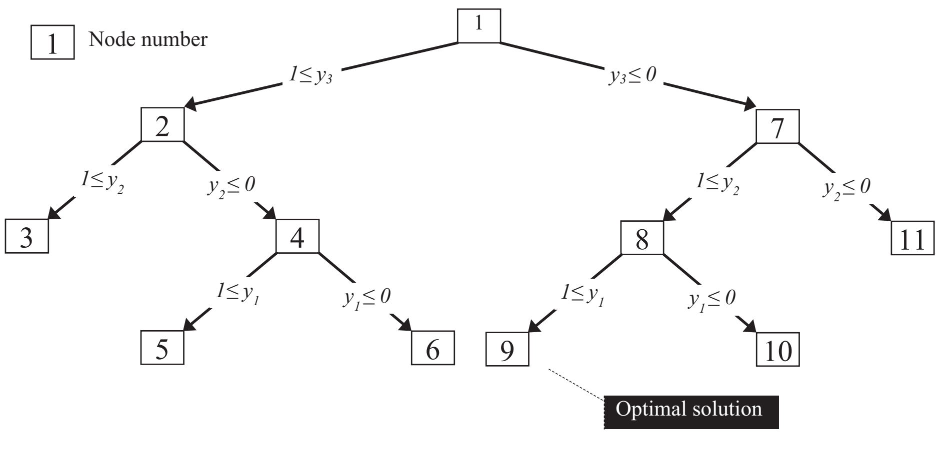

Fig. A-4. Results of Branch and Bound method without merit index.

Step 1. Solve the relaxed (continuous) version of the MINLP which renders optimal value of $UB^1 = v^1 = 6.7$ and solution $(y_1^1, y_2^1, y_3^1) = (0.5, 0.3, 0.1)$.
Step 2. Since $v^1 = 6.7 \ngeqslant UB^* = +\infty$ the current node cannot be fathomed. Update the best upper bound $UB^* = \min(6.7, +\infty)$. Since the current node does not represent a feasible solution it is considered as unfathomed node.
Step 3. if there is no unfathomed node, consider the incumbent solution as the optimal and terminate. Select the current node which is the only unfathomed node (so far) for branching. Then select y_1 which has the maximum value for branching one with additional constraint $y_1 \geqslant 1$ and the other with $y_1 \leqslant 0$. This leads to two new nodes. Select the former as the current node and go to Step 1. The configuration of the tree structure as well as the detail of calculations are shown in Fig. A-3.

A.4. Branch and Bound; Without merit index, a randomly sorted string (y_3, y_2, y_1)

Similar steps need to be taken which is summarized in Fig. A-4. A quick comparison highlights significant and constructive role of the merit index in efficacy of the BB, such that number of attempts to reach at global solution and total computational time increase almost three folds should no merit index is considered.

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