# Mobility Weakens the Distinction Between Multicast and Unicast

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***Abstract*—Comparing with the unicast technology, multiple flows from the same source in multicast scenario can be aggregated even if their destinations are different. This paper evaluates such distinction by the multicast gain on per-node capacity and delay, which are defined as the per-node capacity and delay ratios between multi-unicast and multicast ($m$ destinations for each multicast session). Particularly, the restricted mobility model is proposed, which is a representative mobility model characterizing a class of mobility models with different average moving speeds. The theoretical analysis of this model indicates that the mobility significantly decreases the multicast gain on per-node capacity and delay, though the per-node capacity of both unicast and multicast can be enhanced by mobility. This finding suggests that mobility weakens the distinction between multicast and unicast. Finally, a general framework of multicast study is constituted by analyzing the upper-bound ($\Theta(m)$), the lower-bound ($\Theta(1)$) and the main determinants of the multicast gain on both per-node capacity and delay regardless of mobility model.**

***Index Terms*—Capacity, delay, multicast, restricted mobility, unicast.**

## I. Introduction

NETWORK scaling laws for large-scale wireless ad hoc networks have been extensively studied since the seminal work by P. Gupta and P. R. Kumar [2]. They focus on the unicast scenario of random wireless networks with $n$ static nodes. This work shows that the per-node throughput upper-bound $\Theta\left(\frac{1}{\sqrt{n}}\right)$[1] can be achieved by their scheme. However, there are a large number of wireless mobile devices in the real world. Thus, many researchers focus on the network scaling laws of mobile networks [3]–[10]. With the help of mobility, long distance transmission can be realized, which is not allowed in static networks because of the limitation of transmission power and interference. Hence, in contrast with static networks, node mobility can improve the per-node throughput and delay performance. In [3], D. Shah, *et al.* show that $\Theta(1)$ per-node throughput is obtained in random i.i.d. mobility model for unicast scenario.

Nevertheless, [2] and [3] only analyze two extreme cases of mobility, i.e., lowest node speed (static model) and highest node speed (random i.i.d. mobility model). There are also some studies focusing on other mobility models with different node speeds, e.g., random walk models [4], restricted mobility models [1], [5], [6], heterogeneous mobility models [7]. In [5], the network is divided into squares, where the nodes obey i.i.d. mobility model in each square and follow random walk model when moving among different squares. Moreover, [6] studies the case that each node's home-point can only be located in one of $m$ clusters. This model indicates the fact that people are more likely to be within the region where they live. However, the event of long distance movement happens with low probability in our real world, which is demonstrated in the mobility model in [1]. In this model, each node has its own home-point, and the distance between it and its home-point follows power-law distribution. Although this model cannot well describe the continuity of node's mobility, it is suitable for the fast mobile networks. M. Garetto, *et al.* in [1] adopt a multi-hop scheme for the unicast scenario, which can enhance the per-node throughput and delay by optimizing the configuration of their scheme. For example, when the exponent of the power-law distribution parameter $\alpha$ equals to 2, constant per-node throughput and delay can be obtained simultaneously except for poly-logarithmic factors. Furthermore, some researchers study the heterogeneous mobility model which is feasible for the nodes with different mobility models [7].

The studies above drill down into the impact of mobility on unicast per-node capacity, and their results indicate that the mobility enhances the unicast per-node capacity performance. However, unicast is only a special case of multicast standing for a more general traffic pattern. Therefore, the impact of mobility on multicast becomes a hot topic [8]–[10]. In [8], J.J. Garcia-Luna-Aceves *et al.* focus on the multicast scenario for the static networks. In their study, there are $n$ static nodes in the network, and each node has $m$ ($m < n$) destinations. The results demonstrate that the achievable per-node throughput upper-bound is $\Theta\left(\frac{1}{\sqrt{mn\log n}}\right)$ when $m = o\left(\frac{n}{\log n}\right)$, and the upper-bound is $\Theta\left(\frac{1}{n}\right)$ for the case $m = \Omega\left(\frac{n}{\log n}\right)$. By introducing the mobility into multicast networks, [9] analyzes the multicast per-node capacity under random i.i.d. mobility model, which proves that

Manuscript received July 05, 2014; revised November 04, 2014; accepted February 19, 2015; approved by IEEE/ACM Transactions on Networking Editor E. Ekici. Date of publication April 13, 2015; date of current version June 14, 2016. This work was supported by NSF China (No. 61325012, 61271219, 61221001, 61428205); China Ministry of Education Doctor Program (No. 20130073110025); Shanghai Basic Research Key Project (12JC1405200, 11JC1405100); and Shanghai International Cooperation Project (No. 13510711300).

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Digital Object Identifier 10.1109/TNET.2015.2407872

[1]We use standard asymptotic notations in our paper. Consider two nonnegative function $f(\cdot)$ and $g(\cdot)$: (1) $f(n) = o(g(n))$ means $\lim_{n\to\infty} f(n)/g(n) = 0$. (2) $f(n) = O(g(n))$ means $\lim_{n\to\infty} f(n)/g(n) < \infty$. (3) $f(n) = \omega(g(n))$ means $\lim_{n\to\infty} f(n)/g(n) = \infty$. (4) $f(n) = \Omega(g(n))$ means $\lim_{n\to\infty} f(n)/g(n) > 0$. (5) $f(n) = \Theta(g(n))$ means $f(n) = O(g(n))$ and $g(n) = O(f(n))$. (6) $f(n) = \widetilde{\Theta}(g(n))$ means $f(n) = \Theta(g(n))$ when ignoring poly-logarithmic factor.

mobility can also increase the multicast per-node capacity. Another mobility model with limited node speed is given in [10]. In this paper, the node speed is limited by $R$, and the per-node capacity of the network is a non-decreasing function of $R$. The theoretical results also demonstrate that mobility can enhance the multicast per-node capacity.

In order to further understand the above phenomenon, the distinction between the performance enhancements by mobility is analyzed for unicast and multicast. In particular, this paper focuses on a class of mobility models proposed by M. Garetto, *et al.* [1], i.e., restricted mobility model, which includes mobility models such as static model, random i.i.d. mobility model, etc. Furthermore, the node speed is a decreasing function of $\alpha$, which is the exponent of the power-law distribution in this restricted mobility model. Therefore, the impact of mobility on the network per-node capacity can be investigated by adjusting $\alpha$. In this model, the per-node capacity and delay are analyzed for both unicast and multicast, respectively. The results demonstrate that mobility can increase the per-node capacity for both of them. However, it weakens the distinction between multicast and unicast since the opportunity of flow aggregation is reduced by mobility. Moreover, the delay of unicast and multicast are the same in order sense in the restricted mobility model, and the mobility only reduces the multicast delay gain in constant order.

To support the above conclusion, the main *contributions* of this paper are summarized as follows:

- **Contribution on capacity:** This paper focuses on the multicast capacity gain[2] for the restricted mobility model in two traffic patterns, i.e., unicast and multicast. The bottleneck of network per-node throughput is considered, and the per-node capacity is derived for the two traffic patterns, which shows the enhancement of per-node capacity by increasing the moving speed. Denoting the number of destinations as $m$ and the total number of nodes as $n$, and the multicast gain on per-node capacity under different circumstances satisfies $\left\{\Theta(1), \Theta\left(\frac{\log n}{\log \frac{n}{m}}\right), \Theta\left(m^{\frac{\alpha}{2}-1}\right), \Theta\left(\sqrt{m}\right)\right\}$ for the cases $\{0 \leq \alpha < 2, \alpha = 2, 2 < \alpha \leq 3, \alpha > 3\}$, respectively, where $\alpha$ is the exponential coefficient of restricted mobility model. These results demonstrate that mobility significantly decreases the multicast capacity gain. Moreover, the results also indicate that $m$ enhances the distinction as the secondary determinant. Additionally, this paper further studies the upper-bound and lower-bound of multicast capacity gain regardless of the mobility model, which are $\Theta(m)$ and $\Theta(1)$, respectively. Based on these results, the factors determining the multicast capacity gain are summarized, which form a general framework of the multicast capacity.
- **Contribution on delay:** The impact of mobility on multicast delay gain is studied by adopting the flooding scheme, which is capable of achieving the delay lower-bound of the two traffic patterns. By considering the information expansion speed, it can be derived that the multicast gain of optimal delay with limited transmission range is $\Theta(m)$, and the mobility strength reduces it in constant order. In the further study, this paper investigates the upper-bound ($\Theta(m)$) and lower-bound ($\Theta(m)$) of multicast delay gain in a more general network. According to the theoretical results, the multicast delay gain is mainly determined by the difference of delay for different nodes.
- **Differences from previous work:** This work is the first study on the *optimal* capacity and delay performance for the restricted mobility model in both unicast and multicast scenarios. Moreover, different from previous work, this paper mainly focuses on the performance ratios between the two scenarios in order to investigate the impact of mobility on multicast gain. Additionally, some other essential factors determining the multicast gain are also analyzed.

The rest of this paper is organized as follows. In Section II, the network model and definitions are introduced. The per-node capacity is analyzed for both unicast and multicast scenarios in Section III. In Section IV, the delay performance is studied for the two scenarios. In Section V, the multicast gain on per-node capacity and delay are investigated. Finally, conclusion is summarized in Section VI.

## II. System Model and Definitions

### A. Network Model

There are $n$ nodes in the networks. In order to simplify the performance analysis, the shape of network is assumed to be a torus defined as $\mathcal{O}$ with size $\sqrt{n} \times \sqrt{n}$, and all of the $n$ nodes (users) are moving on its surface. Hence, the node density is 1, and edge effect is ignored in this model.

### B. Mobility Model

In our system, time is divided into slots with equal duration. The fast mobility case is considered, in which the node mobility is of the same time-scale as packet transmission. Under such time division, one node can only transmit one packet to another node in one hop when they are connected.

The position of node $i$ ($i = 1, \ldots, n$) at time slot $t$ ($t = 0, 1, \ldots$) is denoted as $X_i(t)$. In random i.i.d. mobility model, the $X_i(t)$ is randomly, uniformly and independently selected in the network in each time slot. Therefore, the moving speed of nodes in the network is $\Theta(\sqrt{n})$ per-timeslot, which cannot be adjusted. In this paper, the restricted mobility model in [1] is adopted instead. In this model, for any $i$ and $t$, the $X_i(t)$ is independently distributed in $\mathcal{O}$. This assumption is widely introduced in network performance research as in [11]–[14]. However, the mobility of nodes in the network is restricted, i.e., the $X_i(t)$ is not uniformly distributed in the networks, which is different from the random i.i.d. mobility model.

In this model, for each node $i$, there is a corresponding home-point located at $H_i$ which is the mobility center of $i$. Moreover, the home-point is static and uniformly, independently distributed in the network. Although the uniform density may appear to be idealized, it can be a good scenario for the initial study of the impact of mobility due to its mathematical tractability. In this model, three kinds of distance are defined. The distance between nodes (users) $i$ and $j$ at time slot $t$ is defined as $\rho_{i,j}(t) = \|X_i(t) - X_j(t)\|$, where $\|\cdot\|$ is the operation of Euclidean distance. The distance between home-points $H_i$ and $H_j$ is represented as $\rho^H_{i,j} = \|H_i - H_j\|$. Furthermore, the distance between node (user) $i$ and its home-point $H_i$ at time $t$ is $\rho_i(t) =$

[2]The ratio of the capacity of multicast and multi-unicast. See Definition 3.

$\|X_i(t) - H_i\|$. In this model, the distribution of $\rho_i(t)$ is modeled as a non-increasing function $f(\rho)$ to ensure that the node is more likely to be close to its home-point. Many papers focus on the distribution function $f(\rho)$ [1], [6], [15]–[17]. In our paper, according to [1], the $f(\rho)$ is expressed as follows:

$$f(\rho) = \frac{s(\rho)}{\int\int_{\mathcal{O}} s(\rho)} \tag{1}$$

where $s(\rho) = \min\{1, \rho^{-\alpha}\}$ and $\alpha \geq 0$. Consequently, the $f(\rho)$ can be further calculated as

$$f(\rho) = \begin{cases} \Theta\left(s(\rho)n^{\frac{\alpha-2}{2}}\right) & 0 \leq \alpha < 2, \\ \Theta\left(\frac{s(\rho)}{\log n}\right) & \alpha = 2, \\ \Theta(s(\rho)) & \alpha > 2. \end{cases} \tag{2}$$

Although this mobility model is not accurate enough comparing with the practical user movement, it is proved to be appropriate for modeling human and vehicular mobility [1], which is supported by many measurements papers [16], [17]. Moreover, it can be found in (2) that the averaged moving distance in each time slot is determined by $\alpha$. Therefore, the moving speed of nodes in the network can be adjusted by $\alpha$, and hence this is an appropriate model for the study of the impact of mobility on network performance.

### C. Communication Model

In this paper, the protocol model is adopted, which is a simplified version of physical model since it ignores the long distance interference and transmission. Moreover, it is shown in [2] that the physical model can be treated as the protocol model on scaling law when the transmission is allowed for the case that the *Signal to Interference Noise Ratio* (SINR) is greater than a given threshold. In this model, a transmission between node $i$ and $j$ is successful if the following inequality is satisfied

$$\|X_i - X_j\| \leq r(n) \tag{3}$$

where $r(n)$ is the maximum transmission range of each node. Moreover, any other transmitting node $k$ must satisfy the inequality as

$$\|X_k - X_j\| \geq (1+\Delta)r(n) \tag{4}$$

where $\Delta > 0$ is a constant factor that depends on the acceptable SINR of the network. Furthermore, the bandwidth of the network is finite and constant. In this model, the transmission range $r(n)$ is assumed to be $r(n) = \Theta(1)$ [2], and therefore each node can meet another node within its transmission range with constant probability.

Furthermore, the TDMA scheme is employed to guarantee that each transmission is successful. In the $K^2$-TDMA scheme, the network is divided into $\frac{n}{r^2(n)}$ cells with equal size $r^2(n)$, and only one of the $K^2$ cells nearby is allowed to be active, i.e., $K^2$-TDMA scheme allows each $K^2$ adjacent cells to be active with a round-robin fashion. $K$ is a constant and satisfies $(K-1)r(n)/\sqrt{2} \geq (1+\Delta)r(n)$. Hence, $K$ is defined as $K = \lceil 1 + (1+\Delta)\sqrt{2}\rceil$. Without loss of generality, one of the cells is denoted as cell $(1,1)$, i.e., $C_{1,1}$. Then, all the cells in the network can be denoted as $C_{a,b}(1 \leq a, b \leq \sqrt{n})$, respectively.

### D. Traffic Models

Two traffic patterns are studied in this paper, which are *Unicast* and *Multicast*. Firstly, in the unicast scenario, each node randomly selects another node as its destination. The transmission from one source to its destination (maybe through multi-hop) is denoted as one unicast session.

For the multicast scenario, each node $i$ randomly selects $m$ different nodes as its destinations. Node $i$ needs to transmit the same packet to all of its destinations. The multicast session is defined as the transmission from the source to all of its destinations (maybe through multi-hop).

### E. Network Performance Metrics and Some Notations

Some definitions of the performance metrics are listed as follows.

*Definition 1:* (*Per-node Throughput*) For a given scheme, the per-node throughput is defined as the maximum achievable transmission rate as in [2]. In $t$ time slots, there are $M(i,t)$ packets transmitted from node $i$ to its destination(s). Firstly, the *long term per-node throughput* is defined by $\lambda_i(n)$ as

$$\lambda_i(n) = \liminf_{t\to\infty} \frac{1}{t} M(i,t). \tag{5}$$

Afterwards, the *per-node throughput* of this model for a given scheme is defined by the maximum $T(n)$ that satisfies

$$\lim_{n\to\infty} \mathbb{P}(\lambda_i(n) \geq T(n) \text{ for all } i) = 1. \tag{6}$$

This paper studies the random networks instead of arbitrary ones. The transport throughput, which is defined as the rate timing the distance, is not appropriate in our networks since it is just defined for arbitrary networks [2].

*Definition 2:* (*Per-node Capacity*) For a given network, the per-node capacity of it is defined as

$$C(n) = \max_{\sigma\in\Sigma} T_\sigma(n) \tag{7}$$

where $\sigma$ is a scheme for the network, $\Sigma$ is the set of all possible schemes, and $T_\sigma(n)$ is the per-node throughput of scheme $\sigma$.

*Definition 3:* (*Multicast Capacity Gain*) For a given network, the per-node capacity of multicast is assumed to be $C_{multi}(n)$. Moreover, if each node has $m$ destinations, each multicast session can be treated as $m$ unicast sessions (multi-unicast), and the corresponding sum per-node capacity is denoted as $C_{m\_uni}(n)$. Comparing the capacity of multicast and multi-unicast, the multicast capacity gain is defined as

$$\xi(n) = \frac{C_{multi}(n)}{C_{m\_uni}(n)}. \tag{8}$$

The multicast capacity gain $\xi(n)$ indicates the enhancement of per-node capacity by multicast transmission.

*Definition 4:* (*Network Delay*) For a given scheme, assuming that the source sends the packet to the network at time slot $t_s$ and the destination receives the packet at time slot $t_d$, the delay is defined as the average value of $t_d - t_s$, i.e., $\mathbb{E}\{t_d - t_s\}$. It should be noted that the queuing delay at source is not considered here, which is the same as in many important work [1], [4]. Moreover, for wireless networks, the operation time spent in coding/decoding is assumed to be negligible compared to the transmission time.

TABLE I
NOTATIONS AND DEFINITIONS

| Notation | Definition |
|---|---|
| $n$ | The total number of nodes in the network. |
| $m$ | The number of destinations for each source in multicast scenario. |
| $\xi(n)$ | The multicast capacity gain. |
| $\zeta(n)$ | The multicast delay gain. |
| $\alpha$ | The parameter of restricted mobility model. |
| $C(n)$ | The per-node capacity performance. |
| $D(n)$ | The delay performance. |
| $D_{SDT}$ | The delay for the case that data is only delivered by short distance transmission. |
| $D_{LDT}$ | The delay for the case that data is only delivered by long distance transmission. |

*Definition 5:* (*Multicast Delay Gain*) For a given network, the network delay of multicast is assumed to be $D_{multi}(n)$. Moreover, if each node has $m$ destinations, the sum delay of the transmissions from the source to them by unicast is denoted as $D_{m\_uni}(n)$. Comparing the delay of multicast and multi-unicast, the multicast delay gain is defined as

$$\zeta(n) = \frac{D_{m\_uni}(n)}{D_{multi}(n)}. \tag{9}$$

The multicast delay gain $\zeta(n)$ indicates the enhancement of delay performance by multicast transmission.

Finally, some essential notations and definitions are listed in Table I.[3]

## III. CAPACITY ANALYSIS FOR UNICAST AND MULTICAST

In this section, the capacity performance is analyzed for both unicast and multicast scenarios. Moreover, the multicast capacity gain based on the results of this section will be discussed in Section V.

### A. Per-Node Capacity of Unicast Scenario

Different from the random i.i.d. mobility model, the probability that two nodes meet each other is related with the distance between them. Considering two arbitrary nodes $i$ and $k$, the probability that $i$ meets $k$ can be expressed as

$$\begin{aligned} p_{i,k} = \sum_{C_{a,b}} \int_{X_i \in C_{a,b}} f(\|X_i - H_i\|) \\ \cdot \int_{X_k \in C_{a,b}} f(\|X_k - H_k\|) dX_k dX_i. \end{aligned} \tag{10}$$

Without loss of generality, we assume that node $i$'s home-point is in $C_{1,1}$ and node $k$'s home-point is in $C_{a_k,b_k}$. Since this paper focuses on the scaling law of the performance, the order of $p_{i,k}$ satisfies

$$p_{i,k} = \Theta\left( S_0^2 \sum_{X_i, X_k \in C_{a,b}, 1 \le a,b \le \sqrt{n}} f(\|X_i - H_i\|) f(\|X_k - H_k\|) \right) \tag{11}$$

where $S_0$ is the size of one cell and $S_0 = \Theta(1)$. The $p_{i,k}$ can be calculated in the similar way to that in [1], and it is expressed as

$$p(\rho) = \begin{cases} \Theta\left(n^{-1}\right) 4 & 0 \le \alpha \le 1, \\ \Theta\left(\rho^{2-2\alpha} n^{\alpha-2}\right) & 1 < \alpha < 2, \\ \Theta\left(\frac{\rho^{-2}\log\rho}{\log^2 n}\right) & \alpha = 2, \\ \Theta\left(\rho^{-\alpha}\right) & \alpha > 2, \end{cases} \tag{12}$$

where $\rho = \sqrt{a_k^2 + b_k^2}$ is the distance between two home-points. It can be found from (12) that the probability is similar to random i.i.d. mobility model when $0 \le \alpha \le 1$. However, for other cases, the probability becomes different. Based on this mobility model, the per-node capacity of the network is derived in the following theorem.

*Theorem 1:* The per-node capacity for unicast scenario under restricted mobility model is

$$C(n) = \begin{cases} \Theta(1) & 0 \le \alpha < 2, \\ \Theta\left(\frac{1}{\log n}\right) & \alpha = 2, \\ \Theta\left(n^{1-\frac{\alpha}{2}}\right) & 2 < \alpha \le 3, \\ \Theta\left(\frac{1}{\sqrt{n}}\right) & \alpha > 3. \end{cases} \tag{13}$$

*Proof:* The proof of this theorem can be divided into two parts. Firstly, (13) is proved to be the upper-bound of the unicast capacity. Afterwards, the corresponding upper-bound achieving scheme is proposed to demonstrate that (13) is achievable. Finally, according to the definition of capacity, the per-node capacity for unicast scenario under restricted mobility model satisfies (13). The detailed proof can be found in Appendix A, and the capacity achieving scheme is proposed as follows.

*Scheme I:* In this scheme, the transmission range satisfies $r(n) = \Theta(1)$, and there are three kinds of transmissions: source to relay (S-R) transmission, relay to relay transmission (R-R) as well as relay to destination (R-D) transmission. In each time slot, when one cell is active according to TDMA scheme, each kind of transmission will be selected with the same probability. Furthermore, one transmission pair belongs to the selected transmission kind will be chosen to be active equiprobably.

- **If $0 \le \alpha \le 3$**, R-R transmission is not allowed. For any source-destination pair, denoting their home-points distance as $\rho$ and the middle point of their home-points as $\mathcal{C}$, the source is only permitted to transmit packet to one relay with home-point located in the circle centered at $\mathcal{C}$ with radius $\frac{1}{3}\rho$. Afterwards, the relay will send the packet to the destination when they are in the same cell.
- **If $\alpha > 3$**, three kinds of transmissions are allowed. Considering the straight line which connects the home-points of source and destination, the set of the cells it lines across can be defined as *Path Set* of this source-destination pair. The cells in the path set are denoted as $C_i$ which is numbered according to the distance between $C_i$ and source's home-point. In addition, $C_0$ denotes the source's home-point cell. Based on such definition, if the packet is hold by the node with home-point in $C_i$, it will be transmitted to node whose home-point is in cell $C_{i+1}$ and deleted from the previous relay. ■

[3] The detailed discussion of $D_{SDT}$ and $D_{LDT}$ can be found in Section IV.

### B. Per-Node Capacity of Multicast Scenario

In multicast scenario, there are $n$ sources and $m$ destinations for each source, which is different from the unicast scenario. The following lemma is proposed to show the number of corresponding sources for each destination.

*Lemma 1:* In multicast scenario, any node, when acting as a destination, has $m$ sources with probability 1 when $n$ goes to infinity.

*Proof:* The proof can be found in Appendix B. ■

Based on Lemma 1, the per-node capacity for multicast scenario is derived in the following theorem.

*Theorem 2:* The per-node capacity for multicast scenario under restricted mobility model is

$$C(n) = \begin{cases} \Theta\left(\frac{1}{m}\right) & 0 \le \alpha < 2, \\ \Theta\left(\frac{1}{m \log \frac{n}{m}}\right) & \alpha = 2, \\ \Theta\left(n^{1-\frac{\alpha}{2}} m^{\frac{\alpha}{2}-2}\right) & 2 < \alpha \le 3, \\ \Theta\left(\frac{1}{\sqrt{nm}}\right) & \alpha > 3. \end{cases} \tag{14}$$

*Proof:* Similar to Theorem 1, the proof can also be divided into two parts: 1) $C(n)$ in (14) is proved to be the upper-bound of multicast per-node capacity. 2) The following Scheme II is proposed to achieve the capacity upper-bound. The detailed proof can be found in Appendix C.

*Scheme II:* In this scheme, the transmission range satisfies $r(n) = \Theta(1)$, and there are four kinds of transmissions: S-R, R-D, R-R and destination to relay (D-R) transmission. In each time slot, one kind of transmission is selected with the same probability. Afterwards, one transmission pair belonging to it is chosen equiprobably.

- **If $0 \le \alpha < 2$**, R-R and D-R transmissions are not allowed. Traditional 2-hop relay scheme without redundancy [9] is employed here. In particular, the source transmits the packet to each relay within its transmission range. Afterwards, the relay will send the packet to the destinations when it is within the transmission range of them.
- **If $\alpha \ge 2$**, Four kinds of transmissions are allowed. Firstly, a *Euclidean Minimum Spanning Tree* (EMST) of the multicast session is generated among the home-points of source and destinations as in [19]. The packet can be transmitted to all the destinations based on the EMST, and it is sent through each edge of the EMST as unicast by employing Scheme I. ■

## IV. Delay Analysis for Unicast and Multicast

In this section, the optimal delay performance is analyzed for both unicast and multicast scenarios, and the corresponding multicast delay gain will be discussed in Section V.

### A. Optimal Delay of Unicast Scenario

To optimize the delay, it is necessary to utilize all the possible hops for one unicast. Therefore, a multi-hop scheme named flooding scheme is employed, which is shown as follows.

*1) Scheme III: (Flooding Scheme)* Four kinds of transmissions (S-R, R-R, R-D, D-R) are allowed in this scheme and will be selected equiprobably in any active cell. When S-R, R-R or D-R transmission is permitted, randomly choose a source (or a relay) and broadcast the packet to all the nodes in the cell simultaneously. Besides, if R-D transmission is selected, an R-D pair will be randomly chosen to be active.

In [1], the authors propose a multi-hop scheme without redundancy. However, the redundancy is introduced in our paper for flooding scheme which is different from theirs. It is well studied in [4], [18] that redundancy can decrease the delay of networks because packet can be transmitted to the destination through all of the possible paths, and the delay is determined by the shortest one. Moreover, it is obvious that if the transmission range is large enough (e.g., $r(n) = \sqrt{n}$), the delay could be $\Theta(1)$. However, it is not reasonable to assume the transmission range to be related with network scale $n$, and therefore the transmission range is assumed as $r(n) = \Theta(1)$.

As introduced in [11], in order to achieve optimal delay, this paper considers the situation that a single packet is delivered over an empty network, which is analyzed in many work about the optimal delay [4]. Therefore, only one source is allowed to transmit its packet to the network until this packet is received by the destination.

However, the flooding scheme in the restricted networks is quite different from the random i.i.d. mobile networks. In the restricted mobility model, the PDF of packet-holding nodes in the next time slot is determined by the packet-holding nodes in current time slot. Therefore, the transmission process in restricted mobility model can be treated as a Markov chain with $2^{n-1}$ states. Moreover, there are $2^{n-2}$ target states and 1 initial state. Hence, it is too complex to obtain the exact order of delay.

Instead, this paper analyzes the upper-bound of the optimal delay. In particular, the transmissions are divided into two groups, i.e., long distance transmission (LDT) and short distance transmission (SDT). The distance between two nodes' home-points is $\Theta(\sqrt{n})$ for each LDT, and the distance is $o(\sqrt{n})$ for each SDT. In this network, if $\alpha$ is small, the probability of LDT is large, and the probability of SDT is small, and vice versa. In order to reduce the analysis complexity, the packet is assumed to be transmitted to the destination through only LDT or only SDT. Therefore, the cooperation between LDT and SDT is ignored, which causes the derived delay greater than the actual delay, and thus it is the delay upper-bound of flooding scheme.

First, the delay of LDT is calculated in the following lemma.

*Lemma 2:* Considering the transmission from source $i$ to its destination $j$ by flooding scheme, the average delay from source to destination by LDT can be expressed as

$$D_{LDT} = \begin{cases} \Theta(\log n) & 0 \le \alpha < 2, \\ \Theta\left(\log^2 n\right) & \alpha = 2, \\ \Theta\left(n^{\frac{\alpha-2}{2}} \log n\right) & \alpha > 2. \end{cases} \tag{15}$$

*Proof:* The proof can be found in Appendix D. ■

In Lemma 2, it shows that delay is great when $\alpha$ is large. The reason is that there are very few LDTs when $\alpha$ is large, and therefore SDT becomes the main issue. The delay of packet transmitted by SDT for the case $\alpha \ge 2$ is shown in the following lemma. The $D_{SDT}$ for $0 \le \alpha < 2$ is not shown here because $p_{SDT} = o(1)$ for this case, and therefore $D_{SDT} = \omega(\log n)$.

*Lemma 3:* Considering the transmission from source $i$ to its destination $j$ by flooding scheme, the delay from source to destination by SDT can be expressed as

$$D_{SDT} = \begin{cases} O\left(\log^3 n\right) & \alpha = 2, \\ O\left(n^{\frac{\alpha-2}{2}} \log n\right) & 2 < \alpha < 3, \\ O\left(\sqrt{n} \log n\right) & \alpha \geq 3. \end{cases} \tag{16}$$

*Proof:* The proof can be found in Appendix E. ■

The delay of the flooding scheme is determined by $D_{LDT}$ and $D_{SDT}$. Since the cooperation between LDT and SDT is not considered, the minimum value of $D_{LDT}$ and $D_{SDT}$ is the upper-bound of optimal delay, which is indicated in the following theorem.

*Theorem 3:* The upper-bound of optimal delay for unicast scenario follows:

$$D(n) = \begin{cases} \Theta\left(\log n\right) & 0 \leq \alpha < 2, \\ O\left(\log^2 n\right) & \alpha = 2, \\ O\left(n^{\frac{\alpha-2}{2}} \log n\right) & 2 < \alpha < 3, \\ O\left(\sqrt{n} \log n\right) & \alpha \geq 3. \end{cases} \tag{17}$$

### B. Optimal Delay of Multicast Scenario

To obtain the optimal delay for multicast scenario, the flooding scheme is also adopted under the same assumption that the transmission range is constant. In Section IV.A, the upper-bound of optimal delay is derived for the unicast scenario. However, by the Scheme III (flooding scheme), each node in the network will receive a replica of the packet from the source within the time scale of (17). Therefore, for multicast scenario, the optimal delay is of the same scale of unicast scenario, which is also proved in random i.i.d. mobility model in [11].

## V. Multicast Gain

The multicast can be treated as $m$-multi-unicast, and the packets can also be transmitted in the network in the same way as unicast. However, with the help of flow aggregation, multicast may perform better than multi-unicast in some conditions, which is considered to be the advantage of multicast. Fig. 1 shows an example of flow aggregation. In this figure, the source needs to transmit a packet to all of its destinations. It can be found that 11 hops are required for multi-unicast case, and only 5 hops are needed in multicast scenario instead. The flows are aggregated in multicast scenario, which is the difference between multicast and unicast. In order to study such distinction, this paper focuses on the per-node capacity and delay ratio of multicast and $m$-multi-unicast, i.e., multicast gain on capacity and delay, respectively.

### A. The Multicast Capacity Gain

From the theoretical results in Section III, it can be found that $\alpha$ and $m$ jointly enhance the multicast gain on per-node capacity, which shows the essential role of mobility and $m$ in multicast. Furthermore, more general upper-bound and lower-bound of multicast gain on per-node capacity will be derived regardless of mobility model in this subsection. Finally, based on above work, it can be concluded that the multicast capacity gain is mainly determined by three factors.

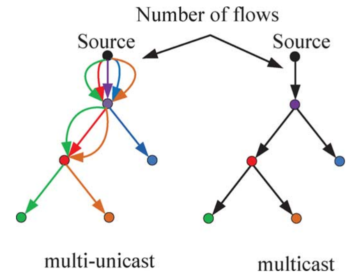


Fig. 1. An example of flow aggregation.

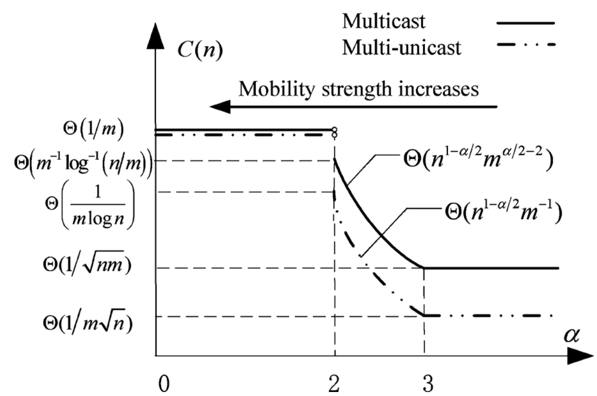


Fig. 2. Per-node capacity comparison between multi-unicast and multicast scenarios.

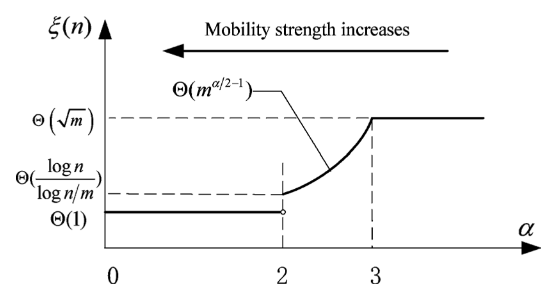


Fig. 3. The impact of $\alpha$ on multicast capacity gain.

If each multicast session is treated as $m$ independent unicast sessions, there are $mn$ unicast sessions in the network. Thus, the per-node capacity of this multi-unicast case is $\frac{1}{m}$ of (13) for the restricted mobility model. This per-node capacity is compared with multicast per-node capacity in (14) in Fig. 2.[4] It can be found that both of the unicast and multicast per-node capacity are non-increasing functions of $\alpha$, which shows the impact of mobility on per-node capacity.

In Fig. 2, it is indicated that the impact of mobility on per-node capacity for unicast and multicast are different. Particularly, since a small $\alpha$ means strong mobility, the per-node capacity enhancement caused by mobility for unicast is greater

[4]There is capacity hopping when $\alpha = 2$ because the convergence of series is changed in this case. The same phenomenon can be found in Fig. 3.

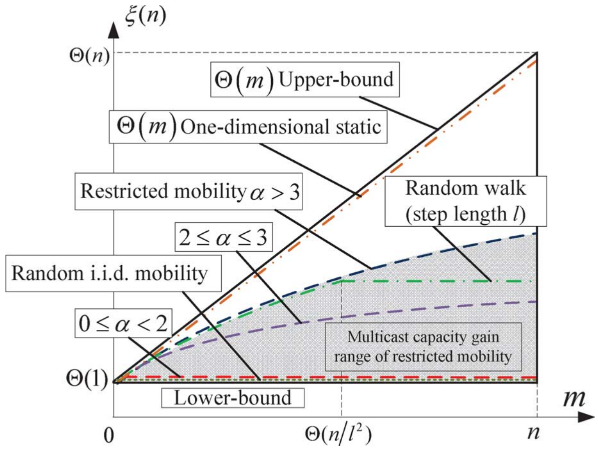


Fig. 4. The impact of $m$ on multicast capacity gain.

than multicast. Therefore, the multicast gain on per-node capacity decreases with mobility strength (increases with $\alpha$)[5]. The per-node capacity gain of multicast is as follows:

$$\xi(n) = \begin{cases} \Theta(1) & 0 \le \alpha < 2, \\ \Theta\left(\frac{\log n}{\log \frac{n}{m}}\right) & \alpha = 2, \\ \Theta\left(m^{\frac{\alpha}{2}-1}\right) & 2 < \alpha \le 3, \\ \Theta\left(\sqrt{m}\right) & \alpha > 3. \end{cases} \tag{18}$$

The impacts of $\alpha$ and $m$ on $\xi(n)$ are illustrated in Figs. 3 and 4, respectively.

In Fig. 3, for a given $m$, the multicast capacity gain increases with $\alpha$ when $2 \le \alpha \le 3$. Otherwise, the multicast capacity gain is not related with $\alpha$ for the case $0 \le \alpha < 2$ and $\alpha > 3$. In Fig. 4, it is shown that $\xi(n)$ is a non-decreasing function of $m$. In particular, there is no multicast capacity gain as long as $0 \le \alpha < 2$, and the gain increases with $m$ when $\alpha \ge 2$. Furthermore, the increasing speed is determined by $\alpha$. The upper-bound, lower-bound and other mobility cases in this figure will be analyzed later.

In fact, the multicast brings about per-node capacity gain due to the cooperation among destinations and relays, i.e., flow aggregation. However, the effect of cooperation is determined by the certainty of node location, namely, nodes will cooperate effectively if their relative locations are certain with high probability in each time slot. For this mobility model, if $\alpha < 2$, the unpredictability of nodes' locations is so strong that the network can be treated as i.i.d. mobility model, and therefore the opportunity of flow aggregation is very low. Furthermore, for the case $2 \le \alpha \le 3$, the stability of nodes' locations becomes stronger with $\alpha$, and the cooperation becomes more effective. Moreover, the number of destinations also impacts the cooperation since it determines the distinction between multicast and unicast. Thus, the per-node capacity gain of multicast is a non-decreasing function of $\alpha$ and $m$. At last, when $\alpha > 3$, the destinations' locations are approximately fixed. Therefore, the multicast capacity gain is independent from $\alpha$, and the cooperation among nodes has a strong impact on the per-node capacity gain. Consequently, this paper considers the mobility to be the first determinant since it determines the form of multicast capacity gain growth, but $m$ only determines the increasing speed.

[5]The mobility strength in this figure can be evaluated by the average moving distance per-timeslot in the restricted mobility model

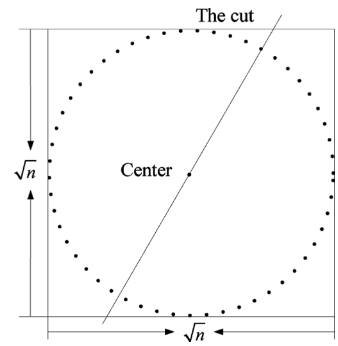


Fig. 5. The network with upper-bound of multicast capacity gain.

The above results are specialized for the restricted mobility model. However, the further study of multicast capacity gain should include more mobility models. Thus, this paper will analyze the upper-bound and lower-bound of multicast capacity gain regardless of mobility model.

For the upper-bound, since each destination is randomly selected, at least one unicast is contained in each multicast. Thus, if the multicast session can be finished during the unicast transmission, the per-node capacity of unicast and multicast will be the same, and therefore the multicast gain on per-node capacity is $\Theta(m)$, which is the upper-bound. A network achieving the upper-bound is proposed as follows. The network size is $\sqrt{n} \times \sqrt{n}$. All of the nodes are static, and their locations are $(\frac{\sqrt{n}}{2}\cos\frac{2\pi i}{n}, \frac{\sqrt{n}}{2}\sin\frac{2\pi i}{n})$, where $i$ is the label of each node. This network is illustrated in Fig. 5, the unicast per-node capacity is derived by analyzing the maximum per-node throughput of the cut in this figure. If the transmission range satisfies $r(n) > \frac{2\pi}{\sqrt{n}}$, it can be easily obtained that only $\Theta(1)$ packets can be transmitted across the cut due to the interference. Therefore, the per-node throughput upper-bound is $\Theta(\frac{1}{n})$, which can be achieved by following scheme: setting $r(n) = \frac{3\pi}{\sqrt{n}}$, for each source destination pair $i$, $j$ (assuming $i < j$), the packet is relayed by $i+1, i+2, \ldots, j$. For the multicast scenario, if the packet is transmitted to the two farthest destinations of both directions, all the other destinations must have already received the packets for the reason that they are relays or within the transmission range of relays. Hence, the multicast can be treated as two unicast between source and the two farthest destinations of both directions, which means the per-node capacity of multicast is also $\Theta(\frac{1}{n})$. Consequently, the multicast capacity gain for this network is $m$, and it is the upper-bound.

For the lower-bound, one multicast session can be treated as $m$ unicast sessions no matter what the mobility model is. Therefore, the per-node capacity of multicast is no less than $m$-multi-unicast, which means the lower-bound of multicast capacity gain is $\Theta(1)$. To achieve the lower-bound, there must be no cooperation among nodes, and the multicast session is equivalent to $m$ unicast sessions. The case $0 \le \alpha < 2$ for the restricted mobility model in this paper is an example of this condition.

The upper-bound ($\Theta(m)$) and lower-bound ($\Theta(1)$) are shown in Fig. 4, and the multicast capacity gain of restricted mobility model is within this range. Moreover, we illustrate the multicast capacity gain of random walk mobility model [3] with step

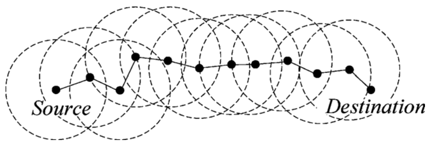


Fig. 6. The transmission path from source to a destination.

length $l$, one-dimensional static model [20][6] and the random i.i.d. mobility model [9] in Fig. 4.

From Fig. 4, it can be found that mobility and $m$ jointly impact the multicast gain on per-node capacity because the opportunity of flow aggregation increases with $m$ but decreases with mobility strength. However, there is a gap between static networks (restricted mobility model $\alpha \to \infty$) and the upper-bound. Hence, there is at least one other factor which impacts the multicast capacity gain. Since the upper-bound is achieved by one-dimensional static model and the multicast capacity gain $\Theta(\sqrt{m})$ can be achieved by two-dimensional static model, it is obvious that the distribution of nodes also impacts the multicast capacity gain. To explain this phenomenon, considering one destination of a multicast session, Fig. 6 shows the transmission path from the source to it, and the corresponding coverage of this transmission (the range of dotted circles). If the other destinations belonging to this multicast session are distributed within the coverage with high probability, the flows are aggregated effectively, and therefore the multicast capacity gain is very high. This example indicated that the distribution of nodes can increase the multicast capacity gain. Consequently, to form a general framework of the multicast capacity gain study, the factors determining the multicast capacity gain are listed as follows:

- The mobility
- The number of destinations
- The node distribution

### B. The Multicast Delay Gain

In order to investigate the multicast delay gain, a general network is considered, and the flooding scheme is employed. In fact, the flooding scheme utilizes all of the possible transmissions to deliver one packet, and therefore the delay of it is optimal for a given transmission range. Defining the flooding scheme delay of the multicast with $m$ destinations as $D_{multi}(n)$ and the flooding scheme delay of each unicast of its corresponding $m$-multi-unicast as $D_{m\_uni,i}(n)$, $(i = 1, \ldots, m)$, the relation between them satisfies

$$D_{multi}(n) = \max_{i=1,\ldots,m} \{D_{m\_uni,i}(n)\}. \tag{19}$$

According to the results in Section IV, the multicast gain on delay for restricted mobility model is $\Theta(m)$ for any $\alpha$, which means the mobility does not affect the multicast delay gain in order sense. However, when $\alpha = 0$, the node's movement satisfies the random i.i.d. mobility model, and therefore the expectations of $D_{m\_uni,i}(n)$ are the same. On the other hand, when $\alpha$ is large, the node is approximately static, and hence the packet arrives at the nodes with the closer home-point earlier, i.e., $\mathbb{E}\{D_{m\_uni,i}(n)\} < D_{multi}(n)$ for some destinations $i$ with home-point close to the source's home-point. Thus, the mobility reduces the multicast delay gain in constant order.

Furthermore, the upper-bound and lower-bound of multicast delay gain is analyzed regardless of mobility model. According to the Definition 5 and (19), the multicast delay gain is upper-bounded by $\Theta(m)$ and lower-bounded by $\Theta(1)$. It is obvious that the upper-bound of multicast delay gain can be achieved by the restricted model in this paper.

Moreover, in order to achieve the lower-bound, the network must satisfy

$$D_{m\_uni,i_{\max}}(n) = \Omega\left(\sum_{i=1,\ldots,i_{\max}-1,i_{\max}+1,\ldots,m} D_{m\_uni,i}(n)\right) \tag{20}$$

where $i_{\max}$ is the corresponding index of the maximum $D_{m\_uni,i}(n)$. For example, considering the network divided into two parts $\mathcal{O}_1$ and $\mathcal{O}_2$ with equal size, there are $n - \frac{cn}{m}$ nodes in $\mathcal{O}_1$ satisfying the random i.i.d. mobility model, and the rest of the nodes are moving in $\mathcal{O}_2$ satisfying the random i.i.d. mobility model, where $c > e$ is constant. Additionally, the distance between two parts is greater than the transmission range. Moreover, with probability $\frac{1}{n^3}$, one node in one part can move to another part in one time slot, and then it moves back to its initial part. Therefore, $D_{m\_uni,i_{\max}}(n) = \Theta(n^3)$ where poly-logarithmic factors are ignored. However, $\sum_{i=1,\ldots,i_{\max}-1,i_{\max}+1,\ldots,m} D_{m\_uni,i}(n) = O(n^3)$, which means (20) is satisfied, and therefore the multicast delay gain is $\Theta(1)$ in this network.

Consequently, the achievable bounds of multicast delay gain are proved, which form a general framework of multicast delay gain. However, the multicast delay gain is not mainly determined by the mobility strength in order sense but by the difference of delay for different nodes in flooding scheme, i.e., the difference between $D_{m\_uni,i}(n)$.

## VI. Conclusion

This paper mainly focuses on the impact of mobility on multicast gain. The analysis of per-node capacity shows that mobility weakens the per-node capacity distinction between multicast and unicast. In particular, the per-node capacity for both unicast and multicast are derived, and two schemes based on restricted relay selection are proposed to achieve the per-node capacity. Afterwards, the multicast capacity gain is calculated, which is utilized to evaluate the distinction between the per-node capacity enhancements for unicast and multicast. Moreover, the essential role of mobility and $m$ in multicast is analyzed. Additionally, the upper-bound and lower-bound of multicast capacity gain are also studied regardless of mobility model, which may guide the multicast study in the future. Three factors which determine the multicast capacity gain are listed in order to form a general framework of multicast capacity. Furthermore, the multicast delay gain is also investigated. The upper-bound and lower-bound of it are derived and proved to be achievable. Moreover, it can be found that the multicast delay gain is not mainly determined by the mobility strength in order sense but by the difference of delay for different nodes in flooding scheme.

[6]The multicast capacity gain for one-dimensional static model is $\Theta(m)$, and it can be obtained in the similar as the network in Fig. 5

## APPENDIX A
## THE PROOF OF THEOREM 1

In order to derive the per-node capacity, a contact graph is considered, in which the nodes are allocated at their home-points respectively. Moreover, an edge can be put between any two-nodes, whose weight is defined as the probability that they happen to be within distance $\Theta(1)$ of each other. Moreover, since this paper focuses on the fast mobility model, the node can only transmit one packet to any other node within distance $\Theta(1)$ of it. Due to the unit density of the network, the contact graph can be treated as a virtual capacitated graph.

Considering a cut dividing the contact graph into two parts with the same size, there are $\Theta(n)$ nodes in each part in average. The two parts are defined as $\mathcal{O}_1$ and $\mathcal{O}_2$, which are illustrated in Fig. 7. Furthermore, the length of the cut is $\Theta(\sqrt{n})$. If the source and destination is within different parts, the packet must be transmitted through the cut. Therefore the sum per-node throughput of these pairs is bounded by the sum weight of the edges across the cut. Since the number of such kind of source-destination pairs is $\Theta(n)$, the per-node throughput upper-bound of the network can also be derived from this bound according to the definition of per-node throughput.

In order to derive the sum probability of the edges across the cut, a node $i$ belonging to $\mathcal{O}_1$ is considered, and its distance from the cut is $\rho_i$. Denoting the sum weight of the edges across the cut with endpoint $i$ as $W_i$, the sum weight of the edges across the cut can be expressed as

$$W = \sum_{i \text{ is in } \mathcal{O}_1} W_i. \tag{21}$$

According to the probability that two nodes happen to be within distance $\Theta(1)$ in (11), $W_i$ can be further expressed as

$$W_i = \sum_{j \text{ is in } \mathcal{O}_2} p(\rho^H_{i,j}). \tag{22}$$

Since the home-points in $\mathcal{O}_2$ are uniformly distributed and the size of the network is $\sqrt{n} \times \sqrt{n}$, the order of $W_i$ in (22) can be computed as

$$\begin{aligned} W_i &= \Theta\left(2\int_{\rho_i}^{\sqrt{n}}\int_0^{\arccos(\rho_i/\rho)} \rho p(\rho) d\theta d\rho\right) \\ &= \Theta\left(2\int_{\rho_i}^{\sqrt{n}} \rho \arccos(\rho_i/\rho) p(\rho) d\rho\right) \\ &= \Theta\left(2\int_{\rho_i}^{\sqrt{n}} \sqrt{\rho^2-\rho_i^2} p(\rho) d\rho\right) \\ &= \Theta\left(\int_{\rho_i}^{2\rho_i} \sqrt{\rho_i}\sqrt{\rho-\rho_i} p(\rho_i) d\rho + \int_{2\rho_i}^{\sqrt{n}} \rho p(\rho) d\rho\right) \\ &= \begin{cases} \Theta(1) & 0 \le \alpha \le 2, \\ \Theta\left(\rho_i^{2-\alpha}\right) & \alpha > 2. \end{cases} \end{aligned} \tag{23}$$

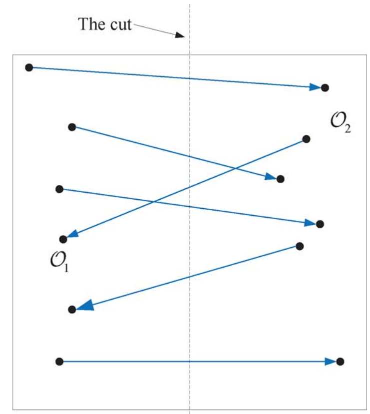


Fig. 7. The packet is transmitted across the cut.

Moreover, since the home-points in $\mathcal{O}_1$ are also uniformly distributed, the order of $W$ in (21) can be computed as

$$\begin{aligned} W &= \sum_{i \text{ is in } \mathcal{O}_1} W_i \\ &= \Theta\left(\sqrt{n}\int_0^{\sqrt{n}} W_i d\rho_i\right) \\ &= \begin{cases} \Theta(n) & 0 \le \alpha \le 2, \\ \Theta\left(n^{2-\frac{\alpha}{2}}\right) & 2 < \alpha \le 3, \\ \Theta(\sqrt{n}) & \alpha > 3, \end{cases} \end{aligned} \tag{24}$$

where the poly-logarithmic factors are ignored for brevity when $\alpha = 3$. Considering that there are $\Theta(n)$ source-destination pairs, the per-node throughput of these pairs can be limited by

$$\Theta\left(\frac{W}{n}\right) = \begin{cases} \Theta(1) & 0 \le \alpha \le 2, \\ \Theta\left(n^{1-\frac{\alpha}{2}}\right) & 2 < \alpha \le 3, \\ \Theta\left(\frac{1}{\sqrt{n}}\right) & \alpha > 3. \end{cases} \tag{25}$$

According to the definition of per-node throughput (i.e., Definition 1), (25) is also the per-node throughput bound of the network. Based on Definition 2, in order to prove that the per-node capacity of unicast scenario equals to (25), it is necessary to testify that the per-node throughput in (25) is achievable. Thus, the Scheme I will be proved to be the capacity achieving scheme in the following part.

Firstly, if $\alpha \le 3$, for the source $i$ and its destination $j$, the probability that source meets a relay node whose home-point is in the circle $\mathcal{O}_{\frac{1}{3}\rho^H_{i,j}}$ centered at $\mathcal{C}$ with radius $\frac{1}{3}\rho^H_{i,j}$ is

$$\begin{aligned} Pr\{i \text{ meets a relay}\} &= \iint_{\mathcal{O}_{\frac{1}{3}\rho^H_{i,j}}} p(\rho) d\mathcal{O} \\ &= \Theta\left(p(\sqrt{n}) \iint_{\mathcal{O}_{\frac{1}{3}\rho^H_{i,j}}} d\mathcal{O}\right) \\ &= \begin{cases} \Theta(1) & 0 \le \alpha < 2, \\ \Theta\left(\frac{1}{\log n}\right) & \alpha = 2, \\ \Theta\left(n^{1-\frac{\alpha}{2}}\right) & 2 < \alpha \le 3. \end{cases} \end{aligned} \tag{26}$$

To calculate the per-node throughput of the network, we assume that $M(i,t)$ packets are transmitted from source $i$ to destination $j$ at time slot $t$. The average delay of the first hop is $Pr^{-1}\{i \text{ meets a relay}\}$. Moreover, if

$t = \omega(\frac{n}{Pr\{i \text{ meets a relay}\}})$, there are $\Theta(n)$ relays hold packets in the circle. Therefore, the destination will meet a relay with probability $Pr\{j \text{ meets a relay}\} = Pr\{i \text{ meets a relay}\}$. Moreover, since there are $\Theta(n)$ relays in the circle and the number of source-destination pairs is $\Theta(n)$, the probability that one source-relay is selected to be active is $\Theta(1)$ when they are in the same cell. Thus, when $t = \omega(\frac{n}{Pr\{i \text{ meets a relay}\}})$, the source will send a packet to relay in $Pr^{-1}\{i \text{ meets a relay}\}$ time slots, and the relay will receive a packet in $Pr^{-1}\{j \text{ meets a relay}\}$ time slots. Hence, the *long term per-node throughput* for this source-destination pair is

$$\lambda_i(n) = \liminf_{t\to\infty} \frac{1}{t} M(i,t) = \Theta(Pr^{-1}\{j \text{ meets a relay}\}). \tag{27}$$

Consequently, the per-node throughput of the network for this scheme is

$$T(n) = \begin{cases} \Theta(1) & 0 \le \alpha < 2, \\ \Theta\left(\frac{1}{\log n}\right) & \alpha = 2, \\ \Theta\left(n^{1-\frac{\alpha}{2}}\right) & 2 < \alpha \le 3 \end{cases} \tag{28}$$

which achieves the upper-bound of per-node throughput in (13) when $0 \le \alpha \le 3$, and there is gap of $\Theta(\log n)$ when $\alpha = 2$.

For the case $\alpha > 3$, the scheme is different. From (12), it is obvious that each node will meet the node whose home-point is $\rho = \Theta(1)$ from its home-point with probability $\Theta(1)$. Therefore, there are $\Theta(n)$ active transmission pairs in each time slot. Furthermore, it takes $\Theta(\sqrt{n})$ hops to transmit one packet from source to destination. Hence, the total throughput of the network is $\Theta(n/\sqrt{n}) = \Theta(\sqrt{n})$ and the per-node throughput $\Theta(\frac{1}{\sqrt{n}})$. Thus, the upper-bound is achieved when $\alpha > 3$.

Consequently, the upper-bound is achievable, which means that this per-node throughput upper-bound is the per-node capacity of the unicast scenario.

## APPENDIX B
## THE PROOF OF LEMMA 1

Since each source randomly selects $m$ destinations, the probability that a given destination $i$ is selected by the given source is $\frac{m}{n}$. Thus, defining the number of node $i$'s sources as $d$, the probability that $d = d_0$ can be expressed as

$$\Pr\{d = d_0\} = \frac{n!}{(n-d_0)!d_0!}\left(\frac{m}{n}\right)^{d_0}\left(1-\frac{m}{n}\right)^{m-d_0}. \tag{29}$$

Therefore, for two constants $0 < c_1 < 1/e$ and $c_2 > e$ ($e$ is the base of the natural logarithm), the probability that $c_1 m < d < c_2 m$ satisfies

$$\begin{aligned}\Pr&\{c_1 m < d < c_2 m\} \\ &= 1 - \sum_{i=1}^{c_1 m} \frac{n!}{(n-i)!i!}\left(\frac{m}{n}\right)^i\left(1-\frac{m}{n}\right)^{n-i} \\ &\quad - \sum_{i=c_2 m}^{n} \frac{n!}{(n-i)!i!}\left(\frac{m}{n}\right)^i\left(1-\frac{m}{n}\right)^{n-i}.\end{aligned} \tag{30}$$

Denote $g(i) = \frac{n!}{i!(n-i)!}(\frac{m}{n})^i(1-\frac{m}{n})^{n-i}$, therefore,

$$\begin{aligned}\frac{g(i)}{g(i+1)} &= \frac{\frac{n!}{i!(n-i)!}(\frac{m}{n})^i(1-\frac{m}{n})^{n-i}}{\frac{n!}{(i+1)!(n-i-1)!}(\frac{m}{n})^{i+1}(1-\frac{m}{n})^{n-i-1}} \\ &= \frac{(i+1)(1-\frac{m}{n})}{(n-i)\frac{m}{n}} \\ &> \frac{i+1}{m}.\end{aligned} \tag{31}$$

According to (31), denoting $I = c_2 m$, the last item of (30) can be bounded as follows:

$$\begin{aligned}&\sum_{i=c_2 m}^{n} \frac{n!}{(n-i)!i!}\left(\frac{m}{n}\right)^i\left(1-\frac{m}{n}\right)^{n-i} \\ &< \sum_{i=I}^{n} \frac{m^{i-I} I!}{i!} g(I) \\ &\overset{(a)}{=} \Theta\left(\sum_{i=I}^{n} \frac{m^{i-I}\left(\frac{I}{e}\right)^I \sqrt{I}}{\left(\frac{i}{e}\right)^i \sqrt{i}} g(I)\right) \\ &= \Theta\left(\sum_{i=I}^{n}\left(\frac{I}{i}\right)^{i+\frac{1}{2}}\left(\frac{e}{c_2}\right)^{i-I} g(I)\right) \\ &= O\left(g(I)\sum_{i=I}^{n}\left(\frac{e}{c_2}\right)^{i-I}\right) \\ &= O(g(I)),\end{aligned} \tag{32}$$

$(a)$ hold due to the fact that $\lim_{n\to\infty}\left(\frac{n}{e}\right)^n \frac{\sqrt{2\pi n}}{n!} = 1$. The $g(I)$ can be further calculated as follows:

$$\begin{aligned}&g(I) \\ &= \frac{n!}{(n-I)!I!}\left(\frac{m}{n}\right)^I\left(1-\frac{m}{n}\right)^{n-I} \\ &= \Theta\left(\frac{\left(\frac{n}{e}\right)^n \sqrt{n}}{\left(\frac{I}{e}\right)^I \sqrt{I}\left(\frac{n-I}{e}\right)^{n-I}\sqrt{n-I}}\left(\frac{m}{n}\right)^I\left(1-\frac{m}{n}\right)^{n-I}\right) \\ &= \Theta\left(\frac{1}{c_2^I}\left[\left(1+\frac{(c_2-1)m}{n-I}\right)^{\frac{n-I}{(c_2-1)m}}\right]^{(c_2-1)m}\sqrt{\frac{1}{I}+\frac{1}{n-I}}\right) \\ &= O\left(\left(\frac{e}{c_2}\right)^I\right).\end{aligned} \tag{33}$$

Therefore, if $m = \omega(1)$, the last item of (30) equals to $o(1)$. Similar results can be found after calculating the second item of (30), i.e., the second item of (30) equals to $o(1)$ when $m = o(n)$. Therefore, $\Pr\{c_1 m < d < c_2 m\} = 1$ when $n$ goes to infinity.

## APPENDIX C
## THE PROOF OF THEOREM 2

In multicast scenario, each node needs to send packets to $m$ nodes which are randomly selected among all the nodes. When $m = \Theta(1)$, the structure of the multicast session is similar to unicast scenario, and therefore this paper mainly focuses on the condition that $m = \omega(1)$. The corresponding per-node capacity can be derived based on the contact graph in the similar way to that in unicast scenario. In particular, considering a circle with radius $\sqrt{\frac{n}{8m}}$ in the contact graph, there are $\Theta\left(\frac{n}{m}\right)$ home-points in this circle with probability 1 due to the uniform

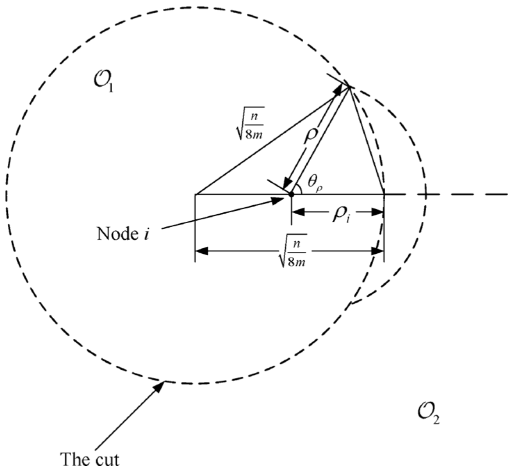


Fig. 8. The relation among $\theta_\rho$, $\rho$, $\rho_i$.

distribution of home-points. Since the nodes with home-points in the circle need to receive packets from the corresponding sources, it is necessary to find the number of sources of these packets, which is denoted as $N_{circle}$. According to Lemma 1, there are $\Theta\left(\frac{nm}{8m}\right) = \Theta(n)$ sources send packets into the circle, i.e., $N_{circle} = \Theta(n)$.

The circle region is denoted as $\mathcal{O}_1$, the region out of the circle is denoted as $\mathcal{O}_2$, which are illustrated in Fig. 8.

The edge of $\mathcal{O}_1$ can be treated as a cut of the contact graph. If node $i$'s home-point is in $\mathcal{O}_2$ and the distance from the edge of $\mathcal{O}_1$ is $\rho_i$, the sum weight of the edges across the cut can also be expressed as in (21), where $W_i$ represents the sum weight of edges across the cut with endpoint $i$. Therefore, the $W_i$ here can be further computed as follows:

$$\begin{aligned} W_i &= \sum_{j \text{ is in } \mathcal{O}_2} p(\rho_{i,j}^H) \\ &= \Theta\left(2\int_{\rho_i}^{\sqrt{n}} \int_0^{\theta_\rho} \rho p(\rho) d\theta d\rho\right) \end{aligned} \tag{34}$$

where $\theta_\rho \in [0, \pi]$ is illustrated in Fig. 8. After some geometric manipulations, the value of $\theta_\rho$ can be computed according to Heron's formula as shown in (35) at the bottom of the page, where $s = \frac{\sqrt{\frac{n}{2m}}+\rho-\rho_i}{2}$. Afterwards, it is necessary to discuss the order of (35). If $\rho \geq \sqrt{\frac{n}{8m}}$, it is obvious that $\theta_\rho > \frac{2\pi}{3}$, and therefore $\theta_\rho = \Theta(1)$. Hence, we only needs to focus on the condition that $\rho < \sqrt{\frac{n}{8m}}$. In this case, after some manipulations, the order of $\theta_\rho$ can be expressed as

$$\theta_\rho = \begin{cases} \Theta\left(\sqrt{1-\left(\frac{\sqrt{\frac{n}{8m}}-\rho}{\sqrt{\frac{n}{8m}}-\rho_i}\right)^2}\right) & \rho_i \geq \frac{1}{2}\sqrt{\frac{n}{8m}}, \\ \Theta\left(\sqrt{1-\left(\frac{\rho_i}{\rho}\right)^2}\right) & \rho_i < \frac{1}{2}\sqrt{\frac{n}{8m}}. \end{cases} \tag{36}$$

Thus, the $W_i$ in (34) can be further derived as

$$\begin{aligned} W_i &= \Theta\left(\int_{\rho_i}^{\sqrt{\frac{n}{8m}}} \int_0^{\theta_\rho} \rho p(\rho) d\theta d\rho + \int_{\sqrt{\frac{n}{8m}}}^{\sqrt{n}} \int_0^{\theta_\rho} \rho p(\rho) d\theta d\rho\right) \\ &= \begin{cases} \Theta(1) & 0 \leq \alpha \leq 2, \\ \Theta\left(\rho_i^{2-\alpha}\right) & \alpha > 2. \end{cases} \end{aligned} \tag{37}$$

Since the nodes are uniformly distributed in the graph, the order of $W$ can be computed as

$$\begin{aligned} W &= \sum_{i \text{ is in } \mathcal{O}_1} W_i \\ &= \Theta\left(\int_1^{\sqrt{\frac{n}{8m}}} \left(\sqrt{\frac{n}{8m}} - \rho_i\right) W_i d\rho_i\right) \\ &= \begin{cases} \Theta\left(\frac{n}{m}\right) & 0 \leq \alpha \leq 2, \\ \Theta\left(n^{2-\frac{\alpha}{2}} m^{\frac{\alpha}{2}-2}\right) & 2 < \alpha \leq 3, \\ \Theta\left(\sqrt{\frac{n}{m}}\right) & \alpha > 3. \end{cases} \end{aligned} \tag{38}$$

It should be noted that the poly-logarithmic factors are ignored in (37) and (38) for brevity. Since there are $\Theta(n)$ sources need to transmit packets into $\mathcal{O}_1$, the per-node throughput of these sources can be limited by

$$\Theta\left(\frac{W}{n}\right) = \begin{cases} \Theta\left(\frac{1}{m}\right) & 0 \leq \alpha \leq 2, \\ \Theta\left(n^{-\frac{\alpha}{2}} m^{\frac{\alpha}{2}-2}\right) & 2 < \alpha \leq 3, \\ \Theta\left(\sqrt{\frac{1}{nm}}\right) & \alpha > 3. \end{cases} \tag{39}$$

According to the Definition 1, (39) is also the per-node throughput bound of the network.

The following part proves that Scheme II can achieve the per-node throughput upper-bound (14). If $0 \leq \alpha < 2$, notice that one node will meet another one with probability $\Theta(\frac{1}{n})$, the mobility model can be treated as the random i.i.d. mobility model. Therefore, the per-node throughput for this case can be derived in the same way as in [9], and the result is shown as follows:

$$T(n) = \Theta\left(\frac{1}{m}\right). \tag{40}$$

For the case $\alpha \geq 2$, the mobility model becomes different from the random i.i.d. mobility model. Considering a time interval $t$ which is large enough, $M(t)$ multicast sessions are finished in $t$. Since each EMST is composed of $m$ edges, and each edge is treated as a unicast session, there are $mM(t)$ unicast sessions. For one unicast session of the EMST, the transmitter meets a relay within $t_{input}(\rho_e)$ time slots, and the receiver meets a relay within $t_{output}(\rho_e)$ time slots if the home-point distance between transmitter and receiver is $\rho_e$. Owing to constant transmission range, a transmission pair will be selected with constant probability when they are within their transmission range. Thus,

$$\theta_\rho = \begin{cases} \arcsin\left(\frac{\sqrt{s\left(s-\sqrt{\frac{n}{8m}}\right)\left(s-\sqrt{\frac{n}{8m}}+\rho_i\right)(s-\rho)}}{2\rho\left(\sqrt{\frac{n}{8m}}-\rho_i\right)}\right) & \rho_i < \rho \leq \sqrt{\frac{n}{8m} - \left(\sqrt{\frac{n}{8m}} - \rho_i\right)^2}, \\ \pi - \arcsin\left(\frac{\sqrt{s\left(s-\sqrt{\frac{n}{8m}}\right)\left(s-\sqrt{\frac{n}{8m}}+\rho_i\right)(s-\rho)}}{2\rho\left(\sqrt{\frac{n}{8m}}-\rho_i\right)}\right) & \sqrt{\frac{n}{8m} - \left(\sqrt{\frac{n}{8m}} - \rho_i\right)^2} < \rho \leq \frac{n}{2m} - \rho_i, \\ \pi & \rho > \sqrt{\frac{n}{2m}} - \rho_i \end{cases} \tag{35}$$

for the case $2 \leq \alpha \leq 3$, the $t_{input}(\rho_e)$ and $t_{output}(\rho_e)$ can be calculated as follows:

$$\begin{aligned} t_{input}(\rho) &= Pr^{-1}\{\text{transmitter meets a relay}\} \\ &= \begin{cases} \Theta(\log \rho_e) & \alpha = 2, \\ \Theta(\rho_e^{\alpha-2}), & 2 < \alpha \leq 3, \end{cases} \\ t_{output}(\rho_e) &= Pr^{-1}\{\text{receiver meets a relay}\} \\ &= \begin{cases} \Theta(\log \rho_e) & \alpha = 2, \\ \Theta(\rho_e^{\alpha-2}), & 2 < \alpha \leq 3. \end{cases} \end{aligned} \tag{41}$$

Since the sum length of all the edges of one EMST is $\Theta(\sqrt{mn})$[19], there must be $\sum_{e=1}^{m} \rho_e = \Theta(\sqrt{mn})$. Recall that there are $M(t)$ multicast sessions, the total transmission time $t$ must satisfy

$$\begin{aligned} t &= \Theta\left(\frac{1}{n}\sum_{e=1}^{mM(t)} \max\{t_{input}(\rho_e), t_{output}(\rho_e)\}\right) \\ &\overset{(a)}{=} O\left(\frac{mM(t)}{n} t_{input}\left(\frac{\sum_{e=1}^{m}\rho_e}{m}\right)\right) \\ &= O\left(\frac{mM(t)}{n} t_{input}\left(\sqrt{\frac{n}{m}}\right)\right). \end{aligned} \tag{42}$$

$(a)$ holds since $t_{input}(\rho)$ is a concave function of $\rho$ and $t_{input}(\rho) = t_{output}(\rho)$. Consequently, based on the definition of per-node throughput, the per-node throughput of Scheme II for $2 \leq \alpha \leq 3$ can be derived as follows:

$$\begin{aligned} T(n) &= \liminf_{t\to\infty} \frac{1}{tn} M(t) \\ &= \Omega\left(\liminf_{t\to\infty} \frac{1}{m t_{input}(\sqrt{\frac{n}{m}})}\right) \\ &= \begin{cases} \Omega\left(\frac{1}{m \log \frac{n}{m}}\right) & \alpha = 2, \\ \Omega\left(n^{1-\frac{\alpha}{2}} m^{\frac{\alpha}{2}-2}\right) & 2 < \alpha \leq 3. \end{cases} \end{aligned} \tag{43}$$

At last, for the case $\alpha > 3$, the network is similar to static one, and the per-node throughput can be obtained in the same way as in [8]. Thus, the result is

$$T(n) = \Theta\left(\frac{1}{\sqrt{mn}}\right). \tag{44}$$

Based on (40), (43) and (44), it can be proved that Scheme II can achieve the per-node throughput upper-bound in (14), and therefore the per-node capacity of multicast is (14) due to its definition.

## APPENDIX D
## THE PROOF OF LEMMA 2

In [9], X. Wang, *et al.* study the delay of flooding scheme for random i.i.d. mobility model. The result shows that the destination will receive the packet in $\Theta(\log n)$ time slots from the beginning. In their model, all the transmissions are LDT with probability 1. Therefore, it is necessary to know the probability of the event of LDT in our model. Based on (12) and $\rho = \Theta(\sqrt{n})$ for LDT, the probability can be obtained as

$$\begin{aligned} p_{LDT} &= \min\{1, \int_{\rho_{i,k}^H = \Theta(\sqrt{n})} p(\rho_{i,k}^H)\rho_{i,k}^H d\rho_{i,k}^H\} \\ &= \begin{cases} \Theta(1) & 0 \leq \alpha < 2, \\ \Theta\left(\frac{1}{\log n}\right) & \alpha = 2, \\ \Theta\left(n^{\frac{2-\alpha}{2}}\right) & \alpha > 2. \end{cases} \end{aligned} \tag{45}$$

Since this lemma just focus on the packets transmitted through LDT, the transmission distance of each packet is the same as that in the random i.i.d. mobility model. The average delay from source to destination by LDT can be expressed as

$$\begin{aligned} D_{LDT} &= \Theta\left(\log n \cdot \frac{1}{p_{LDT}}\right) \\ &= \begin{cases} \Theta(\log n) & 0 \leq \alpha < 2, \\ \Theta(\log^2 n) & \alpha = 2, \\ \Theta\left(n^{\frac{\alpha-2}{2}} \log n\right) & \alpha > 2. \end{cases} \end{aligned} \tag{46}$$

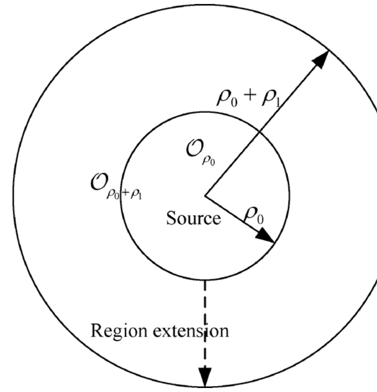


Fig. 9. The region extension from $\mathcal{O}_{\rho_0}$ to $\mathcal{O}_{\rho_0+\rho_1}$.

## APPENDIX E
## THE PROOF OF LEMMA 3

To prove this lemma, we consider the condition that there is an region $\mathcal{O}_{\rho_0}$ of radius $\rho_0$ centered at the home-point of source, and each node with home-point in $\mathcal{O}_{\rho_0}$ holds a packet from $i$ with probability $\Theta(1)$. After $t_{\rho_0\to\rho_1}$ time slots, there is an region $\mathcal{O}_{\rho_0+\rho_1}$ of radius $\rho_0+\rho_1$ centered at the home-point of source, and each node with home-point in $\mathcal{O}_{\rho_0+\rho_1}$ holds a packet from $i$ with probability $\Theta(1)$. This process is called region extension, which is illustrated in Fig. 9, and $t_{\rho_0\to\rho_1}$ is the region extension time from $\mathcal{O}_{\rho_0}$ to $\mathcal{O}_{\rho_0+\rho_1}$. It should be noted that the $\mathcal{O}_{\rho_0+\rho_1}$ is not the ring in Fig. 9 but the circle which covers $\mathcal{O}_{\rho_0}$.

The relation among $t_{\rho_0\to\rho_1}$, $\rho_0$ and $\rho_1$ is analyzed as follows. Considering a node $k$ whose home-point is in the ring $\mathcal{O}_{\rho_0+\rho_1} - \mathcal{O}_{\rho_0}$ and $\Theta(\rho_1)$ from the edge of $\mathcal{O}_{\rho_0}$, the number of cells within $\mathcal{O}_{\rho_0}$, whose distance from $k$'s home-point belongs to $[\rho-1, \rho+1]$, satisfies

$$\begin{aligned} &\Theta\left(\frac{\sqrt{s(s-\rho_0)(s-(\rho_0+\rho_1))(s-\rho)}}{\rho_0+\rho_1}\right) \\ &\quad = \begin{cases} \Theta\left(\sqrt{\rho^2-\rho_1^2}\right) & \rho_1 = O(\rho_0), \\ \Theta\left(\sqrt{\frac{\rho_0}{\rho_1}} \cdot \sqrt{\rho^2-\rho_1^2}\right) & \rho_1 = \omega(\rho_0) \end{cases} \end{aligned} \tag{47}$$

where $s = \frac{\rho_0+(\rho_0+\rho_1)+\rho}{2}$, and (47) holds due to Heron's formula. Afterwards, the probability that $k$ meets nodes whose home-points are within $\mathcal{O}_{\rho_0}$ is calculated. If $\alpha = 2$, the probability is

$$p_{\mathcal{O}_{\rho_0}\to\mathcal{O}_{\rho_0+\rho_1}} = \begin{cases} \Theta\left(\min\left\{1, \frac{\int_{\rho_1}^{\rho_0+\rho_1}\sqrt{\rho^2-\rho_1^2}\rho^{-2}\log\rho d\rho}{\log^2 n}\right\}\right) & \rho_1 = O(\rho_0), \\ \Theta\left(\min\left\{1, \frac{\int_{\rho_1}^{\rho_0+\rho_1}\sqrt{\frac{\rho_0(\rho^2-\rho_1^2)}{\rho_1}}\rho^{-2}\log\rho d\rho}{\log^2 n}\right\}\right) & \rho_1 = \omega(\rho_0). \end{cases} \tag{48}$$

The cells in $\mathcal{O}_{\rho_0+\rho_1} - \mathcal{O}_{\rho_0}$ very close to $\mathcal{O}_{\rho_0}$ are ignored for the reason that such kind of cells are much fewer than others in order sense, and the transmission time is also smaller than others. Hence, the ignorance of these cells does not affect the order of $t_{\rho_0\to\rho_1}$. Thus, the distance between cells in $\mathcal{O}_{\rho_0}$ and $\mathcal{O}_{\rho_0+\rho_1} - \mathcal{O}_{\rho_0}$ is considered to be from $\Theta(\rho_1)$ to $\Theta(\rho_0+\rho_1)$. The region expansion time from $\mathcal{O}_{\rho_0}$ to $\mathcal{O}_{\rho_0+\rho_1}$ can be bounded as

$$t_{\mathcal{O}_{\rho_0}\to\mathcal{O}_{\rho_0+\rho_1}} = O(p^{-1}_{\mathcal{O}_{\rho_0}\to\mathcal{O}_{\rho_0+\rho_1}}\log(\rho_1+\rho_0)). \tag{49}$$

There is a factor $\log(\rho_1+\rho_0)$ in (49) because there are $\Theta(\rho_1(\rho_1+\rho_0))$ nodes in $\mathcal{O}_{\rho_0+\rho_1} - \mathcal{O}_{\rho_0}$. The transmissions out of $\mathcal{O}_{\rho_0+\rho_1}$ and the relay to relay transmissions within the ring $\mathcal{O}_{\rho_0+\rho_1} - \mathcal{O}_{\rho_0}$ during the region extension from $\mathcal{O}_{\rho_0}$ to $\mathcal{O}_{\rho_0+\rho_1}$ are ignored. For a given $\rho_0$, if the $\rho_1$ is too large, it means that too many relay to relay transmissions within the ring $\mathcal{O}_{\rho_0+\rho_1} - \mathcal{O}_{\rho_0}$ are ignored, which will cause the bound of $D_{SDT}$ not tight enough. On the other hand, if the $\rho_1$ is too small, it means that too many transmissions out of $\mathcal{O}_{\rho_0+\rho_1}$ are ignored, which will also cause the bound of $D_{SDT}$ not tight enough. Hence, it is necessary to find the optimal relation between $\rho_1$ and $\rho_0$ to ensure that ignored transmissions are minimized. Thus, it is necessary to discuss $t_{\mathcal{O}_{\rho_0}\to\mathcal{O}_{\rho_0+\rho_1}}$ for different relations between $\rho_0$ and $\rho_1$. Firstly, if $\rho_1 = \omega(\rho_0)$,

$$\begin{aligned} t_{\mathcal{O}_{\rho_0}\to\mathcal{O}_{\rho_0+\rho_1}} &= O\left(\max\left\{1, \frac{\log^2 n}{\sqrt{\rho_0}\rho_1^{-2}\log\rho_1\int_0^{\rho_0}\sqrt{\rho^\dagger}d\rho^\dagger}\right\}\right) \\ &= O\left(\frac{\rho_1^2\log^2 n}{\rho_0^2}\right). \end{aligned} \tag{50}$$

If $\rho_1 = \Theta(\rho_0)$, $t_{\mathcal{O}_{\rho_0}\to\mathcal{O}_{\rho_0+\rho_1}}$ satisfies

$$t_{\mathcal{O}_{\rho_0}\to\mathcal{O}_{\rho_0+\rho_1}} = O\left(\log^2 n\right). \tag{51}$$

Finally, if $\rho_1 = o(\rho_0)$,

$$\begin{aligned} &t_{\mathcal{O}_{\rho_0}\to\mathcal{O}_{\rho_0+\rho_1}} \\ &= O\left(\log\rho_0\max\left\{1, \frac{\log^2 n}{\int_{2\rho_1}^{\rho_0+\rho_1}\frac{\log\rho}{\rho}d\rho + \int_{\rho_1}^{2\rho_1}\frac{\sqrt{\rho-\rho_1}\log\rho}{\rho^{\frac{3}{2}}}d\rho}\right\}\right) \\ &= O\left(\log\rho_0\max\left\{1, \frac{\log^2 n}{\log\rho_0\log\frac{\rho_0}{\rho_1} + \rho_1^{-\frac{3}{2}}\log\rho_1\int_0^{\rho_0}\sqrt{\rho^\dagger}d\rho^\dagger}\right\}\right) \\ &= O\left(\frac{\log^2 n}{\log\frac{\rho_0}{\rho_1}}\right). \end{aligned} \tag{52}$$

Based on the above results, if the $\mathcal{O}_{\rho_0}$ is expanded to $\mathcal{O}_{\rho_0+\rho_1}$ where $\rho_1 = \omega(\rho_0)$, the expansion time is $\Theta\left(\frac{\rho_1^2\log^2 n}{\rho_0^2}\right)$ in (50). However, if the expansion $\mathcal{O}_{\rho_0} \to \mathcal{O}_{\rho_0+\rho_1}$ is treated as exponential growth, i.e., $\mathcal{O}_{\rho_0} \to \mathcal{O}_{2\rho_0} \to \mathcal{O}_{4\rho_0}\cdots \to \mathcal{O}_{\rho_0+\rho_1}$, the extension time can be calculated according to (51). Based on (51), the expansion time is

$$\Theta\left(\sum_{k=1}^{\log\frac{\rho_1}{\rho_0}}\log^2 n\right) = \Theta\left(\log^2 n\log\frac{\rho_1}{\rho_0}\right). \tag{53}$$

For any $\rho_1 = \omega(\rho_0)$, there must be $\log^2 n\log\frac{\rho_1}{\rho_0} = o\left(\frac{\rho_1^2\log^2 n}{\rho_0^2}\right)$. Thus, the ignored transmissions in case $\rho_1 = \Theta(\rho_0)$ is smaller than that in case $\rho_1 = \omega(\rho_0)$. Therefore, $\rho_1 = \Theta(\rho_0)$ performs better than $\rho_1 = \omega(\rho_0)$. Then, $\rho_1 = \Theta(\rho_0)$ and $\rho_1 = o(\rho_0)$ are compared. If the $\mathcal{O}_{\rho_0}$ is expanded to $\mathcal{O}_{\rho_0+\rho_1}$ where $\rho_1 = \Theta(\rho_0)$, it can be treated as the combination of multiple expansions, i.e., $\mathcal{O}_{\rho_0} \to \mathcal{O}_{\rho_0+\widehat{\rho}_1} \to \mathcal{O}_{\rho_0+\widehat{\rho}_1+\widehat{\rho}_2}\cdots \to \mathcal{O}_{\rho_0+\rho_1}$. Based on (52), the expansion time is

$$\sum_{\sum\widehat{\rho}_i=\rho_0}\frac{\log^2 n}{\log\frac{\rho_0^{(i)}}{\widehat{\rho}_1}} = \Omega\left(\sum_{\sum\widehat{\rho}_i=\rho_0}\frac{\widehat{\rho}_i\log^2 n}{\rho_0^{(i)}}\right) = \Omega(\log^2 n) \tag{54}$$

where $\rho_0^{(i)} = \rho_0 + \sum_{j=1}^{i-1}\widehat{\rho}_j$. Thus, the ignored transmissions in case $\rho_1 = \Theta(\rho_0)$ is also smaller than that in case $\rho_1 = o(\rho_0)$. Consequently, $\rho_1 = \Theta(\rho_0)$ is the optimal, and the $D_{SDT}$ for $\alpha = 2$ is bounded by the region expansion as $\rho_1 = \Theta(\rho_0)$ as follows:

$$D_{SDT} = O\left(\sum_{k=1}^{\log n}\log^2 n\right) = O\left(\log^3 n\right). \tag{55}$$

For the condition that $\alpha > 2$, the expansion time can be obtained in the similar way, and the result is shown in (56)

$$t_{\mathcal{O}_{\rho_0}\to\mathcal{O}_{\rho_0+\rho_1}} = \begin{cases} \Theta(\rho_1^\alpha\rho_0^{-2}\log\rho_1) & \rho_1 = \omega(\rho_0), \\ \Theta(\rho_0^{\alpha-2}\log\rho_0) & \rho_1 = \Theta(\rho_0), \\ \Theta(\rho_1^{\alpha-2}\log\rho_0) & \rho_1 = o(\rho_0). \end{cases} \tag{56}$$

Firstly, the relations $\rho_1 = \omega(\rho_0)$ and $\rho_1 = \Theta(\rho_0)$ are compared. If the $\mathcal{O}_{\rho_0}$ is expanded to $\mathcal{O}_{\rho_0+\rho_1}$ where $\rho_1 = \omega(\rho_0)$, it is treated as exponential growth in the same way as $\alpha = 2$, and therefore the expansion time is

$$\begin{aligned} &\sum_{k=1}^{\log\frac{\rho_1}{\rho_0}} 2^{(\alpha-2)k}\rho_0^{\alpha-2}(k+\log\rho_0) \\ &\quad = o\left(\sum_{k=1}^{\log\frac{\rho_1}{\rho_0}} 2^{(\alpha-2)k}\rho_0^{\alpha-2}(\log\frac{\rho_1}{\rho_0}+\log\rho_0)\right) \\ &\quad = o(\rho_1^{\alpha-2}\log\rho_1) \\ &\quad = o\left(\frac{\rho_1^\alpha\log\rho_1}{\rho_0^2}\right). \end{aligned} \tag{57}$$

Thus, $\rho_1 = \Theta(\rho_0)$ performs better than $\rho_1 = \omega(\rho_0)$. Furthermore, to compare $\rho_1 = \Theta(\rho_0)$ and $\rho_1 = o(\rho_0)$, it can be assumed that the $\mathcal{O}_{\rho_0}$ is expanded to $\mathcal{O}_{\rho_0+\rho_1}$ where $\rho_1 = \Theta(\rho_0)$.

If it is treated as the combination of multiple expansions as in the case $\alpha = 2$, the expansion time satisfies

$$\Theta\left(\sum_{\sum \widehat{\rho_i}=\rho_0} \widehat{\rho}_i^{\alpha-2}\log\rho_0\right) = \begin{cases} \omega(\rho_0^{\alpha-2}\log\rho_0) & 2<\alpha<3, \\ O(\rho_0^{\alpha-2}\log\rho_0) & \alpha\geq 3. \end{cases} \tag{58}$$

Hence, $\rho_1 = \Theta(\rho_0)$ is optimal when $2 < \alpha < 3$, and $\rho_1 = o(\rho_0)$ is optimal when $\alpha \geq 3$. Moreover, for the case $\rho_1 = o(\rho_0)$ and $\alpha \geq 3$, it can be easily proved that the optimal $\rho_1$ is $\rho_1 = \Theta(1)$. Therefore, the bound of $D_{STD}$ can be calculated based on the optimal method, and the results are shown in (16).

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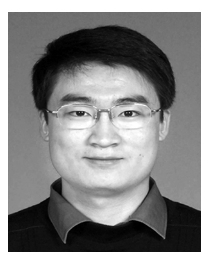

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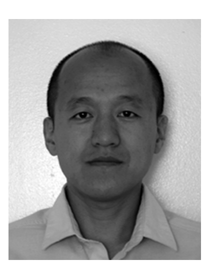

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