Maintenance and Operation of Infrastructure Systems: Review

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Abstract: Infrastructure management plays an important role in sustainable development and is essential for the evaluation of the socioeconomic feasibility of existing projects and the development of new ones. This paper presents a review of key aspects involved in the maintenance and operation of infrastructure under uncertainty. It discusses the main conceptual and theoretical principles and guides the reader through different aspects of the problem by offering a large set of state-of-the-art references. Although the paper does not provide a comprehensive description of all maintenance methods and policies available in the literature, it critically reviews the main approaches. The concepts presented and discussed in this paper provide the basis to build models that can be used for making better decisions for maintaining and operating infrastructure systems. **DOI: 10.1061/(ASCE)ST.1943-541X.0001543.**

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Introduction

Infrastructure systems are an essential component of sustainable development and their optimum operation is central to enable socioeconomic vitality. Optimum operation of structures and infrastructure requires maximizing the availability (i.e., operation over a predefined serviceability threshold) and safety at minimum cost. To achieve this objective, it is essential to define cost-efficient operation policies, among which maintenance plays a significant role.

Maintenance is the combination of the technical and associated administrative actions intended to retain an item or system in, or restore it to, the state in which it can perform its required function [BS3811 (BSI 1984)]. Maintenance of deteriorating systems and replacement problems have been studied extensively in many fields, especially in mechanical, electrical, aerospace, and civil engineering. In addition to several books on operation research, there is a vast amount of papers related to maintenance. In fact, various state-of-the-art reviews on maintenance problems in engineering have been published during the last decades (Pierskalia and Voelker 1979; Sherif and Smith 1981; Sherif 1982; Bosch and Jensen 1983; Valdes-Florez and Feldman 1989; Cho and Parlar 1991; Dekker 1996). There are also many papers available that address the problem of maintenance and operation of specific types of structures (e.g., bridges, pavements, and offshore structures). A comprehensive summary and discussion of existing approaches in civil engineering can be found, for instance, in Frangopol et al. (2004) and Frangopol (2011).

The objective of this paper is to present a review of the main maintenance strategies that contribute to the optimal operation of structures and infrastructure systems under uncertainty. The paper does not provide a description of all methods available in the literature but an overview of the main concepts and strategies. In addition to presenting methods used in civil engineering applications, the paper also describes strategies that have been extensively used in various engineering areas such as aerospace and marine structures (e.g., Melchers 2006), water distribution networks (e.g., Osman and Bainbridge 2011; Grigg 2006), pavement operation, and management (e.g., Archilla 2006).

Initially, the paper presents a set of definitions and a description of the basic concepts involved in a maintenance problem. This includes the identification of alternatives to select performance indicators, a discussion on the structure's performance over time, and the selection of maintenance strategies. In addition, it highlights the importance of end of service life decisions. Then, it presents the basic cost formulation of the maintenance problem, which is the basis for defining optimum operational policies. After describing the maintenance problem, the next part of the paper presents a state-of-the-art description of the main maintenance models. In particular, two useful approaches are discussed in more detail: classic and Markov-based maintenance models. Afterward, a brief discussion on the role of degradation in the definition of a maintenance program is included. This section is followed by a brief summary of the main issues and the approaches used to manage the problem of systems consisting of multiple interconnected components. Finally, the paper presents some key conceptual issues for maintenance management, and it closes with some concluding remarks.

Basic Considerations of the Maintenance Problem

This section presents a review of the aspects relevant to any maintenance program; these are grouped as follows: (1) system's

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performance over time (degradation or deterioration), (2) definition of structural-performance indicators, (3) types and extent of maintenance, (4) inspection problem, and (5) end of service life decisions. These concepts will be used in subsequent sections to formulate maintenance models.

Deterioration Process

Maintenance is required because structures deteriorate with time as a result of various environmental and mechanical stressors (Frangopol 2011). The degradation rate of a particular structure may change dramatically over time due to variations in the demands, or in the internal or external conditions (Sánchez-Silva and Klutke 2016). Thus, understanding deterioration mechanisms, which is key in defining a maintenance program, requires modeling the time-dependent changes of the structural properties and its uncertain nature.

Consider a system that is placed into operation at time $t = 0$ and whose condition decreases with time. The system condition (performance indicator such as strength or resistance) at time t will be defined as $V(t)$. Note that $V(t)$ is a random variable that takes values in the set of positive real numbers. Assume that its value at $t = 0$ is a deterministic quantity that represents the condition when the system is new; i.e., $V(t = 0) = v_0$. Furthermore, if the condition of the structure decreases over time, the accumulated deterioration until time t can be defined as random variable $D(t)$. The value of $D(t)$ depends, among other things, on the structural properties, the demands and the external conditions. Under the assumption that there is no maintenance, the structural condition, $V(t)$, is related with the deterioration, $D(t)$, as follows:

$$V(t) = \max\{v_0 - D(t), 0\} \tag{1}$$

If the history of the system condition is recorded over time, the sets $\{V(t), t \geq 0\}$ and $\{D(t), t \geq 0\}$ constitute stochastic processes.

The failure occurs when the structural condition falls bellow a predefined threshold k^* (with $0 < k^* < v_0$) that represents a serviceability or operation limit for the system (i.e., a safety threshold or limit state) (Fig. 1). Then, an important quantity for maintenance is the system's lifetime, L, which is a random variable defined as

$$L = \inf\{t \geq 0 : V(t) \leq k^*\} = \inf\{t \geq 0 : D(t) \geq v_0 - k^*\} \tag{2}$$

Frequently $k^* = 0$ [as in Eq. (1)], but in some cases it is reasonable to assume $k^* > 0$. Also, the estimation of the structural lifetime L can be interpreted as a first time passage problem; see Melchers (2000) and Streicher et al. (2008).

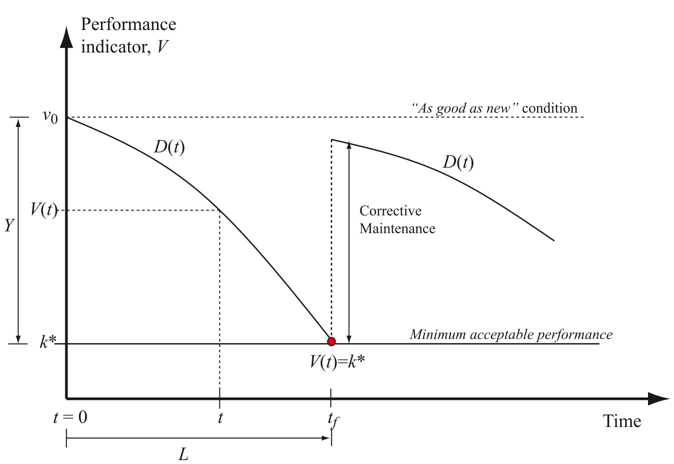

Fig. 1. Description of the system's performance over time

The structural deterioration process, $D(t)$, is frequently divided into *progressive* and *shock-based*. In progressive deterioration, the structure's performance degrades gradually and slowly over time. On the other hand, in shock-based degradation damage accumulates as a result of successive shocks (e.g., effect of earthquakes). There is plenty of literature available describing different degradation models; later in the paper, these models will be presented in more detail.

Structural-Performance Indicators

Maintenance focuses on keeping the structure operating above minimum acceptable performance thresholds. The operation is defined in terms of a structural-performance indicator, $V(t)$, which is a time-dependent function describing the evolution of the condition of the system. Performance indicators may be *qualitative* or *quantitative*. The former are indicative of the overall condition of the structure (e.g., damage index, condition state) and are represented by an arbitrary scale, which frequently uses linguistic terms such as high, moderate, or low. Qualitative performance indicators, presented in the form of a *damage index* or *condition index*, can be defined on a continuous range; e.g., $V(t) \in [0, 1]$, or on a discrete arbitrary scale; e.g., $V(t) \in [0, 1, 2, 3, 4, 5]$ (Liu and Frangopol 2004, 2005a, b, c, d; Petcherdchoo et al. 2008; Frangopol et al. 2009; Ghosn et al. 2016b). The main disadvantage of such qualitative indicators is that the actual structural condition and safety level are not explicitly or adequately accounted for, and that discrete stochastic transitions between condition states may fail to account for actual structural performance (Frangopol and Liu 2007; Frangopol 2011).

On the other hand, quantitative indicators use evaluations defined in terms of some physical characteristic (e.g., displacement, stiffness, strength). In this case, the performance indicator has the units of the physical property evaluated. This distinction is important since the selection of the performance indicator defines to a large extent the relevance of the results and sets limits to the precision of the evaluation. However, given the uncertain nature of the problem, quantitative indicators focus mostly on evaluating the structural performance based on a relevant probabilistic measure, which of course is obtained from an analysis of the physical problem. A common quantitative performance indicator is the time-dependent *reliability index*, $\beta(t)$, which is computed as follows:

$$V(t) = \beta(t) = -\Phi^{-1}[P_f(t)] \tag{3}$$

where P_f = failure probability defined in terms of a limit state function g (e.g., deformation, concrete cracking or loss of nominal resistance); and

$$P_f(t) = \int \cdots \int_{g(\mathbf{p}) \leq 0} f_{\mathbf{p}}(\mathbf{p}, t) d\mathbf{p} \tag{4}$$

with $f_{\mathbf{p}}(\mathbf{p}, t)$ the joint probability density of the vector parameter $\mathbf{p} = \{p_1, p_2, \ldots\}$.

Multiple useful performance measures stem from the definition of the structural lifetime L [Eq. (2)] and are mathematically derived from its probability distribution. The CDF of the lifetime depends on the component characteristics and the failure pattern, and serves as the basis for calculating a number of useful lifetime probability measures such as

- Cumulative probability of failure: cumulative probability of failure $F(t)$ is the probability that the time to failure of a component is less than t; i.e.

$$F(t) = P(L \leq t) = \int_0^t f(u) du \tag{5}$$

- Survivor function: defined as the complement of $F(t)$ and is also referred to as the reliability function; the survivor function gives the probability that the component will not fail before time t and is calculated as

$$R(t) = 1 - F(t) = P(L > t) = \int_t^{\infty} f(u)du \tag{6}$$

An alternative way to write the reliability function is the following:

$$R(t) = P(k^* < V(t) \leq v_0) \tag{7}$$

where k^* = minimum performance threshold and v_0 is the structural condition at time $t = 0$.
- Hazard function: also known as the failure rate, is defined as the conditional probability that a component fails in the time interval $t + dt$ given that it has survived until time t; the hazard function is calculated as (Leemis 1995)

$$h(t) = \frac{f(t)}{R(t)} \tag{8}$$

Due to the simplicity and ease of application relative to other probabilistic approaches, lifetime functions have been also widely used for the maintenance planning of structures (Yang et al. 2006a, b; Okasha and Frangopol 2009, 2010b, c). Also, performance models based on the reliability function and reliability index have been widely implemented for maintenance planning of deteriorating structural systems (Frangopol and Estes 1997; Enright and Frangopol 1999; Estes and Frangopol 2001; Klutke and Yang 2002; Kong and Frangopol 2003b, a, 2004, 2005; Zhu and Frangopol 2012; Saydam and Frangopol 2011; Frangopol and Saydam 2014). Moreover, hybrid models that account for both qualitative measures (e.g., condition-state) and quantitative measures (e.g., reliability index) have been proposed by including the advantages of both approaches (Bucher and Frangopol 2006; Neves et al. 2006a, b). Such models provide a more comprehensive treatment but they may lead to an increased complexity in the problem formulation.

Types and Extent of Maintenance

Any maintenance action requires defining two important aspects: the time at which the intervention is carried out and its extent. Regarding the former, maintenance is frequently classified into *preventive* and *corrective* (Fig. 2). In a preventive maintenance policy, a set of actions (interventions) is defined a priori to keep the structure operating above a certain minimum performance threshold level (not necessarily failure). On the other hand, the actions that occur when the performance threshold is reached or violated are known as corrective maintenances.

In an attempt to be more specific, some authors divide preventive maintenance further into proactive and reactive (Yang et al. 2006a, b). Proactive maintenance refers to any action before the initiation of the failure mechanism (e.g., before crack formation); and reactive maintenance is concerned with any action conducted after the initiation of the failure process (e.g., after crack propagation initiation). Other authors (e.g., Chen and Trived 2005) divide preventive maintenance as follows: (1) *condition-based* and (2) *time-based* preventive maintenance (Legat et al. 1996). In condition-based maintenance, the action taken depends on the state of the system (Scarf 1997b, a; Frangopol 2011). This type of maintenance is only possible if accompanied by an inspection program. On the other hand, in time-based preventive maintenance, maintenance is carried out at predefined time intervals typically defined after a cost-based optimization analysis (Vaurio 1997).

The extent of the intervention plays also an important part in defining an optimum maintenance program. It defines the costs of a maintenance policy and influences the system availability. Interventions carried out to improve the structural condition relative to its original state, can be classified as (Fig. 3):
- Perfect maintenance: restoration of the structure to its *as good as new* condition;
- Imperfect maintenance: restoration of the structure to a condition in between *as good as new* and *as bad as old*;
- Minimal maintenance: restoration of the structure to its *as bad as old* condition; or
- Worse maintenance: intervention that results in (unintentional) reduction in the structural capacity, which may even cause it to fail.

Inspection Problem

In most practical applications, a prespecified long-term maintenance program is hard to implement due to the difficulty of accounting for all factors involved in such decisions (Junca and Sanchez-Silva 2013). Then, sometimes a maintenance policy is defined based on periodic observations of the structural condition; these evaluations are called inspections.

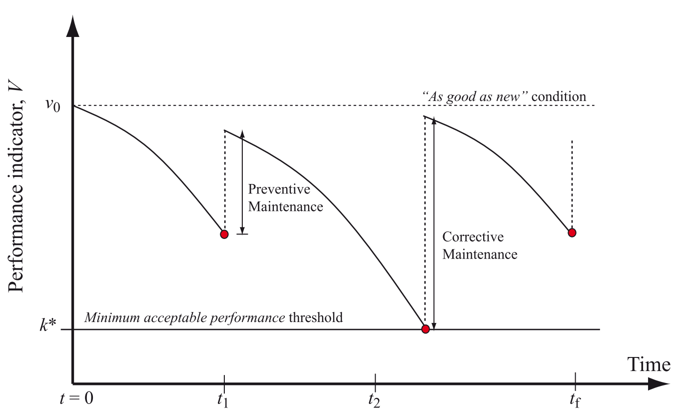

Fig. 2. Preventive and corrective maintenance

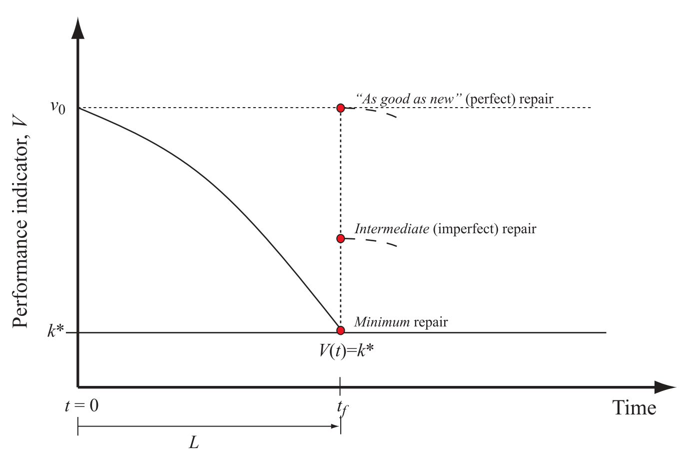

Fig. 3. Extent of maintenance interventions

An inspection is designed to detect the condition of the structure and to define if an intervention is required. Thus, one can define $V(t,\Theta)$ a suitable, monotonically increasing performance indicator with Θ a random vector that takes into account the uncertainties during inspection. Also define S_m as a threshold level, which separates the structure's damage and undamaged conditions. Then, Mori and Ellingwood (2006) proposed a *detectability* function d

$$d[V(t,\Theta)] = \left\{ \begin{matrix} 0 & 0 < V(t,\Theta) \leq S_m \\ 1 & V(t,\Theta) > S_m \end{matrix} \right\} \tag{9}$$

Thus, the case where $d[V(t,\Theta)] = 0$ implies that an intervention is required; and $d[V(t,\Theta)] = 1$, implies that the structure is operating as expected. The function d needs to be characterized probabilistically. Mori and Ellingwood (1994a) argue that the detectability function is not a step function but a monotonically increasing function that has a second-order effect on the limit state probability. Based on Eq. (9), the probability that an intervention is required, $P_R(t)$, may be expressed as

$$P\{d[V(t,\Theta)] = 0\} \tag{10}$$

The uncertain nature of the detectability function leads to the fact that it is not always possible to determine accurately the physical condition of the structure (Valdez-Flores and Feldman 1989; Madanat 1993). Thus, inspections may fail to identify if there is a need for an intervention, or the extent of the required maintenance. There are two approaches to evaluate the accuracy of inspections. The first approach is called *probability of good assessment*, which determines the probability of detecting an event (crack, defect, concentration, etc.) that actually exists. The second approach is called *probability of wrong assessment*, which establishes the probability of detecting an event that does not exist. In the latter, two types of errors can be distinguished:

- Type A: the structure is in a good state (operating above the minimum threshold level) but is judged to be bad and it is repaired.
- Type B: the structure is in a bad state (e.g., failure state) but is judged to be good and it is not repaired.

Further discussions on the accuracy of inspections can be found, for instance, in Pullen and Thomas (1986), Frangopol et al. (1997), Orcesi and Frangopol (2011b, c), Valdez-Flores and Feldman (1989), and Streicher et al. (2008).

Decisions on whether or not to inspect are not only about the detection probability but the associated costs. Then, consider a structure that is inspected at successive time intervals X_k with $k = 1,2,\ldots$ where $X_0 = 0$. It is further assumed that inspection times are negligible, the intervention activities do not change the distribution of the time between failures and the instantaneous failure rate, and that the system cannot fail during inspection. Thus, if C_1 as the inspection cost and C_2 as the cost of leaving an undetected failure per unit time, the total cost per inspection cycle (time between repairs) is given by (Valdez-Flores and Feldman 1989)

$$C(t,\mathbf{Y}) = nC_1 + C_2(Y_n - t) \tag{11}$$

where t = time to failure, $\mathbf{Y} = \{Y_1, Y_2, \ldots\}$ = sequence of inspection times with $Y_{n-1} < t < Y_n$ and $n = 1,2,\ldots$ The optimal inspection policy $\mathbf{Y}^*$ is obtained by minimizing $\mathbb{E}[C(T,\mathbf{Y})]$; where the system failure time, T, is a nonnegative random variable.

A comprehensive description of existing approaches to minimize inspection costs and some variations to the problem formulation can be found in Valdez-Flores and Feldman (1989). Several models for optimizing inspection policies can be found in Nakagawa (2005) and some specific models can be found in Streicher et al. (2008) and Madanat and Ben-Akiva (1994). Several asymptotic solutions have been proposed; for more information see Munford and Shahani (1972, 1973), Keller (1974), and Nakagawa and Yasui (1980).

End of Service Life Considerations

An important part of infrastructure management are the decisions regarding decommissioning. At the end of the structure's service life, L, the owner or stakeholder is presented with various options: the structure may be demolished, or upgraded to extend the service life to $L + \Delta L$. End-of-life impacts that result from these decisions may contribute significantly to the overall lifecycle cost; however, their contribution relative to other lifecycle phases has reportedly varied greatly depending on the system of interest and scope of analysis (Ochoa et al. 2002; Itoh and Kitagawa 2003). Lifecycle analyses that include such end-of-life impacts are often referred to as cradle-to-grave analyses and can support decision making on end-of-life options (ISO 2000; USEPA 2006; Padgett and Tapia 2013). Furthermore, such analyses can support initial design decisions where end-of-life planning is integrated in the initial design, material and system selection.

There are key considerations associated with either *service life extension* or *demolition*. Service life extension may entail rehabilitation of the structure to extend the use for its initial purpose, or may enable an extended structural life with a transition in purpose. In this case, the demands on the structure may be expected to change, but with sufficient capacity enhancement the reliability is still within acceptable limits for an anticipated period ΔT. An example of such service life extension is the common rehabilitation and conversion of industrial or commercial space for residences, typical of modern urban regeneration projects.

As an alternative, if a structure is deconstructed and demolished, this end-of-life stage entails decisions regarding waste generation and management, as well as recovery and recycle or reuse of the structure's contents, components, and material constituents (IISI 2002; Nisbet et al. 2002; Gervasio and da Silva 2008). As an alternative to landfilling of the entire demolished system, some components (e.g., steel beams of a building) may be recovered. The *reuse* of these recovered elements suggests that they are maintained for the same purpose (e.g., steel beams reused in another system in the same form), whereas the *recycling* of the recovered elements often entails material processing and element regeneration or repurposing of the material (e.g., steel beams recycled to produce food cans). The relative benefits and practicality of such end-of-life decisions vary on a case-by-case basis. However, their consideration in a lifecycle analysis is essential for informed decision making.

Maintenance Costs

The financial or economic analysis of the investments in the design and operation of a structure throughout its lifetime is based on the following basic formulation (Rackwitz 2000)

$$\mathbb{E}[Z(\mathbf{p})] = \mathbb{E}[B(t,\mathbf{p}) - C_0(\mathbf{p}) - C_I(t,\mathbf{p})] \tag{12}$$

where $Z(\mathbf{p})$ = cost-benefit relationship (i.e., benefit – costs) and $\mathbb{E}[Z(\mathbf{p})]$ its expected value; $B(t,\mathbf{p})$ = benefit derived from the existence of the structure; $C_0(\mathbf{p})$ = initial investment cost (at $t = 0$); and $C_I(t,\mathbf{p})$ = costs associated with the interventions incurred during the life of the structure. The term $C_I(t,\mathbf{p})$ accounts for both preventive and corrective maintenance as well as structural updates, insurance costs, etc. (Santander and Sánchez-Silva 2008). Also, the characteristics of any maintenance strategy (i.e., time and extension of interventions) are included within the term $C_I(t, \mathbf{p})$. The vector parameter $\mathbf{p}$ takes into consideration all variables that define the performance of the structure. According to statistical decision theory these quantities should be mean values. However, under very specific circumstances this criterion might be modified. For instance Kumar and Gardoni (2014) considered not only the expected value but studied also the effect of the variances. Establishing decision criteria is a subject that has recently attracted the attention of many researchers.

According to Eq. (12), an investment in the construction and operation of the facility makes sense only if $\mathbb{E}[Z(\mathbf{p})] > 0$ for all parties involved; that includes the owner, the builder, the user and society (Rackwitz 2000; Rackwitz and Joanni 2009). Furthermore, the structure is financially optimal for the value of $\mathbf{p}$ that maximizes Eq. (12). A significant amount of research in lifecycle performance of engineering systems has been devoted to find optimum solutions. Some relevant researches on this topic include Rackwitz (2000), Rackwitz and Joanni (2009), Durango and Madanat (2002), Durango-Cohen and Sarutipand (2009), Frangopol (2011), Guillaumot et al. (2003), Madanat et al. (2006), and Junca and Sánchez-Silva (2013b), Biondini and Frangopol (2016). However, it should be stressed that since finding the set of parameters $\mathbf{p}$ that maximizes Eq. (12) is a task conditioned by the maintenance policy (e.g., interventions carried out at fix time intervals), it is not possible to find an optimum for all possible maintenance strategies. Therefore, an important part of the process consists on selecting, for the analysis, those maintenance policies that are reasonably practical.

To solve Eq. (12) it is important to notice that maintenance costs are distributed over the life of the structure, and therefore, they need to be discounted to a reference time; e.g., $t = 0$. The discounting function for small discounting rates γ, such as those used in practice for infrastructure projects, can be formulated as

$$\delta(t) = \exp(-\gamma t) \tag{13}$$

which is equivalent to the standard form $\delta(t) = 1/(1+\gamma)^t$ for small values of γ. Generally, a constant (time-independent) discount rate is assumed. The discount rate is in general difficult to estimate since it depends on many factors and varies throughout the lifetime of the structure. Also, a distinction should be made between financial and economic discounting; however, this is a topic that is beyond the scope of this paper. Typical discount rate values are within the range $0 < \gamma < 7\%$. For instance, Tilly (1997) reports that bridge investments in the United Kingdom should use a discount rate between 4 and 6%. Extensive discussions on the selection of the discount factor can be found in Rackwitz (2000, 2006), Rackwitz et al. (2005), and Streicher et al. (2008).

For a lifecycle cost analysis to be meaningful it is required a well-grounded cost evaluation. However, in practice, the selection of the cost functions required to solve Eq. (12) is not as easy as it may seem. In most case studies, the assessment of the benefits derived from the construction and operation of large infrastructure is difficult to compute accurately; therefore, the benefit term is commonly dropped from Eq. (12) and the analysis focuses on costs only. Regarding the cost of interventions during the structure's lifecycle, C_I, they are frequently divided into direct and indirect costs. *Direct costs* are those directly imposed on the owner; for instance, costs associated with maintenance. On the other hand, *indirect costs* are all costs imposed on the user; i.e., those derived from the loss of functionality of the structure or infrastructure. Further details and a discussion on cost-related issues in lifecycle analysis can be found in Chang and Shinozuka (1996) and Neves et al. (2004, 2006a, b). Finally, in some lifecycle analyses, *costs* may also be evaluated on the basis of other metrics, such as environmental or social impacts. The identification of appropriate discount rates for these metrics remains a pervasive challenge as acknowledged in Padgett and Tapia (2013).

Preventive Maintenance Policies

Basic Concepts

The selection of an optimal maintenance policy should contribute to maximize the cost-benefit analysis as described by Eq. (12). If the focus of the analysis is only on the cost of interventions, an optimal system maintenance policy should be aimed at minimizing the maintenance costs (inspections and interventions) subject to the following two constrains: (1) the structural availability should remain above a prespecified standard, and (2) the safety (reliability) must be within acceptable limits (defined, for instance, in codes of practice).

The limiting structural availability can be defined as (Sánchez-Silva and Klutke 2016)

$$A(\infty) = P(\text{System is operating as } t \to \infty) = \frac{\mathbb{E}[X]}{\mathbb{E}[X] + \mathbb{E}[Y]} \tag{14}$$

where X and Y are random variables describing the length of time the system is operating, and out of service, respectively. Despite the importance of structural availability and its role in defining maintenance programs, in this paper, this issue will not be discussed further; additional information can be found in Nakagawa (2005) and Sánchez-Silva and Klutke (2016). The focus of this section is to outline the basic formulation of the most used approaches to define a maintenance policy. These approaches will be concerned mainly with the trade-off between reliability and maintenance cost.

There are many ways to approach a maintenance program and different models emphasize different aspects of the problem. In this section, maintenance policies are grouped into two general approaches. The first group, *classic maintenance models*, focuses only on the observation of failure times. These models have been used extensively in many industries. The second group includes models developed under the assumption that the system performance over time cannot always be overlooked and, therefore, the maintenance strategy should take into account the condition of the system at the time of the intervention. For the purpose of this paper, *Markov-based models* will be presented as a way to illustrate the point; although many other approaches can be used (Sánchez-Silva and Klutke 2016). As a complement, later in the paper the role of structural degradation in defining maintenance strategies will be described in more detail.

Classic Maintenance Models

In the basic case of maintenance, the actual deterioration mechanism is not taken into consideration and the structural performance is modeled through the failure (maintenance) rate only. A discussion as to how to include the degradation process within a maintenance program is outlined later in the paper.

Basic Maintenance Cost Formulation Problem

The most basic case of a maintenance program can be constructed as follows. Consider a structural system where the times between interventions are independent and identically distributed (*iid*) random variables X (Fig. 4) with PDF $F(x) = P(X < x)$; then, the time to the nth intervention is

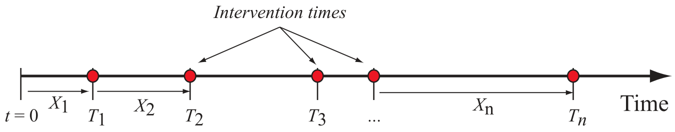

Fig. 4. Illustration of the renewal process

$$T_n = \sum_{i=1}^{n} X_i \tag{15}$$

Then, if $C(T_n)$ describes the cost of the nth intervention (preventive/corrective), the expected total discounted cost of interventions for an infinite time horizon can be computed as

$$\mathbb{E}[C_T] = \mathbb{E}\left[\sum_{n=1}^{\infty} C(T_n)e^{-\gamma T_n}\right] = \mathbb{E}\left[\sum_{n=1}^{\infty} C(T_n)e^{-\gamma \sum_{i=1}^{n} X_i}\right] \tag{16}$$

where γ = discount rate, which is usually assumed to be constant. Based on this basic formulation, three well-known maintenance policies are presented in this section: (1) age replacement, (2) periodic replacement, and (3) minimal repair at failures.

Age Replacement

Consider a structure that is replaced at failure or at a constant time T, whichever occurs first (Fig. 5). The time T is frequently called the planned replacement time. Furthermore, it is also assumed that the intervention time is negligible compared with the time the structure has been in operation. Finally, in every intervention, the structure is taken to its original condition (*as good as new*).

In this policy, failure replacements (i.e., corrective maintenance) occur at random times $X_k (k = 1, 2, \ldots)$ (Fig. 5) with *iid* distributions $F(t) = P(X_k < t)$. On the other hand, preventive maintenance occurs at fixed times. Then, the intervals between replacements caused by either failures or preventive intervention are defined by the random variable $Z_k = \min\{X_k, T\}$ (Fig. 5) with distribution (Nakagawa 2005)

$$P(Z_k \le t) = \begin{cases} F(t) & \text{for } t < T \\ 1 & \text{for } t \ge T \end{cases} \tag{17}$$

Every time there is a failure, the replacement incurs in a cost $C_2 > 0$ (includes both failure and replacement costs); on the other hand, any planned preventive intervention incurs in a cost of $C_1 > 0$, with $C_2 > C_1$. Then, the expected cost in the interval $(0, t]$ is

$$\mathbb{E}[C(t)] = C_1\mathbb{E}[N_1(t)] + C_2\mathbb{E}[N_2(t)] \tag{18}$$

where $N_1(t)$ = number of preventive interventions and $N_2(t)$ = number of failures; both measured in the time interval $(0, t]$. In this formulation the initial cost of the structure is ignored. For an infinite horizon, the target is to find the time T that minimizes the long-run expected cost per unit time; this is

$$C_r(t) = \lim_{t\to\infty} \frac{\mathbb{E}[C(t)]}{t} = \lim_{t\to\infty} \frac{C_1\mathbb{E}[N_1(t)] + C_2\mathbb{E}[N_2(t)]}{t} \tag{19}$$

Since the cycle ends with a preventive replacement at time T, then (Nakagawa 2005)

$$C_r(T) = \frac{C_1\bar{F}(T) + C_2F(T)}{\int_0^T \bar{F}(u)du} \tag{20}$$

$T \to \propto$, describes the case when replacements are made at failures only; in other words, $T = \infty$ means that the structure is never replaced. In this case, the long-run expected cost rate is $C_r(\propto) = C_2/\mu$, where μ is the mean time between failures.

Any policy that implies future investments requires the assessment of the net present value (NPV). Although the expected cost at every cycle is the same, it is still necessary to discount each cycle to $t = 0$. Then, the total expected discounted cost for a finite time span is (Nakagawa 2005)

$$C_r(T) = \frac{C_1e^{-\gamma T}\bar{F}(T) + C_2\int_0^T e^{-\gamma u}dF(u)}{\gamma\int_0^T e^{-\gamma u}\bar{F}(u)du} \tag{21}$$

For the particular case of $T \to \propto$

$$C_r(\infty) = \frac{C_2F^*(\gamma)}{1 - \gamma F^*(\gamma)} \tag{22}$$

where F^* = Laplace-Stieltjes transform of $F(t)$. The case in which failures and interventions are made at discrete times is an extension of previous equations and their expression can be found in (Nakagawa 2005). An expression equivalent to Eq. (22) can be found in Rackwitz (2000) and Rackwitz and Joanni (2009).

Age-replacement policies have been studied extensively with applications in various engineering fields; see for instance Barlow and Proshan (1965), Cox (1962), Cleroux et al. (1979), Sheu and Griffith (2001), Dohi et al. (2000), Kuo and Zuo (2003), Aven and Jensen (1999), and Nakagawa (2005).

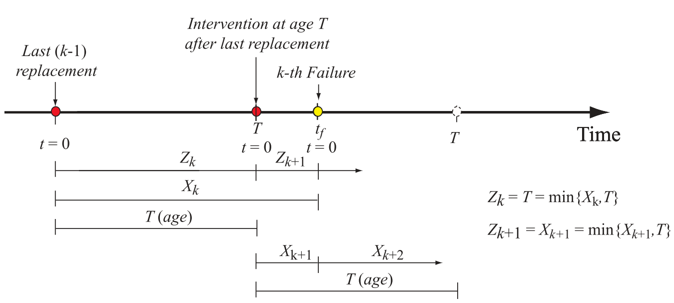

Fig. 5. Description of the age replacement policy

Periodic-Replacement Models

In this maintenance strategy, components are repaired either after failure or all at once at a given time T irrespective of their actual age. Note that in this case, only the failures are observed and components are replaced periodically independently of their condition (Fig. 6). Although this strategy was initially developed for a group of components, it can be applied also to individual elements.

Consider a component that starts operating at time $t = 0$ and it is replaced at periodic times kT with $k = 1, 2, \ldots$ independent of its condition or at failure. When applied to a system, this model assumes that every component has identical failure time distribution $F(t)$. This model also assumes that failures are detected instantaneously, repair times are negligible and interventions take the

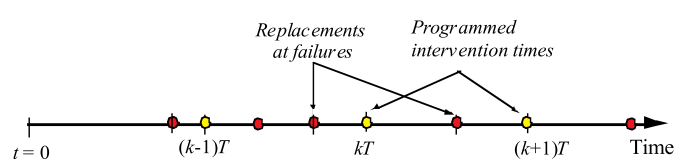

Fig. 6. Periodic replacement model

component to a condition of *as good as new*. Again, the cost of replacement at failure is C_2 while the cost of planned maintenance is C_1. Then, consider a cycle of fixed time T for which the expected cost of maintenance, C, in one cycle is

$$\mathbb{E}[C(T)] = C_1\mathbb{E}[N_1(T)] + C_2\mathbb{E}[N_2(T)] = C_1 + C_2M(T) \quad (23)$$

where $M(T)$ = expected number of failures (failed units) during one cycle (i.e., renewal function; see Barlow and Proshan 1965; Aven and Jensen 1999). A cycle is defined as the time between two programmed interventions. Then, the expected cost rate is

$$C_r(T) = \frac{1}{T}[C_1 + C_2M(T)] \quad (24)$$

If a unit is replaced only at failures (i.e., $C_1 = 0$); and $T = \propto$, then $\mathbb{E}[C_r(\propto)] = C_2M(T)/T = C_2/\mu$. If all costs are discounted with rate γ, the total expected cost for an infinite time span becomes (Nakagawa 2005; Rackwitz and Joanni 2009)

$$C_r(T) = \frac{C_1e^{-\gamma T} + C_2\int_0^T e^{-\gamma u}m(u)du}{1 - e^{-\gamma T}} \quad (25)$$

where $m(t)$ = derivative of the renewal function $M(t)$. The solution to this equation requires solving the renewal function, which is a difficult task and requires usually the use of numerical methods or the definition of bounds as proposed in Barlow and Proshan (1965). In general, there is little difference between age-dependent and periodic repairs unless the failure cost is very large. A comparison between age and block replacements can be found in Barlow and Proshan (1965) and Aven and Jensen (1999).

Minimal Repair at Failures

In this policy, the system is either fully replaced at periodic and predefined times or, in case of failure, subjected to minimal repairs. This strategy is useful in the case of structures where localized damage can occur (e.g., structural damage after an impact or an earthquake). The purpose of minimal repairs is to keep the system operating until the next programmed intervention is conducted. Minimal repairs do not change the structural failure rate between successive programmed replacements.

Assume that failure times have a distribution function $F(t)$ with finite mean and failure rate $h(t) = f(t)/[1 - F(t)]$. The length of a cycle from one replacement to the next is T, which is a fixed and known value. The cost of the minimal repair is C_2 and the cost of the planned replacement is C_1. Therefore, the expected cost of a cycle is

$$\mathbb{E}[C(T)] = C_1\mathbb{E}[N_1(T)] + C_2\mathbb{E}[N_2(T)] = C_1 + C_2H(T) \quad (26)$$

where $\mathbb{E}[N_1(T)]$ and $\mathbb{E}[N_2(T)]$ are the expected number planned interventions or repairs (minimal) that may occur in the time window T; and $H(T)$ is the accumulated failure rate up to time T (Nakagawa 2005). Therefore, the long-run expected cost rate is

$$C_r(T) = \frac{1}{T}[C_1 + C_2H(T)] \quad (27)$$

Similarly to previous cases, the total expected cost for an infinite time span becomes (Nakagawa 2005)

$$C_r(T) = \frac{C_1e^{-\gamma T} + C_2\int_0^T e^{-\gamma u}h(u)du}{1 - e^{-\gamma T}} \quad (28)$$

Modified replacement models can be extended to the discrete case, the replacement of a used component, or the replacement with random, wear-out failures and replacement based on minimum operation thresholds. These models are presented and discussed by (Nakagawa 2005). Several variations of this policy, which is also known as *periodic replacement with minimal repairs*, can be found in many engineering applications; see for instance Morimura (1970), Holland and McLean (1975), Boland and Proschan (1982), Aven (1983), Chen and Feldman (1997), and Pulcini (2003).

Summary of Periodic Replacements

In summary, for the case of periodic replacements, the *expected cost rate* can be evaluated as

$$C_r(T) = \frac{1}{T}\left[C_1 + C_2\int_0^T \varphi(u)du\right] \quad (29)$$

where φ (t) may represent $m(t)$ [Eq. (24)] or $h(t)$ [Eq. (27)] depending on the case considered. Then differentiating $C(T)$ with respect to T and setting it equal to zero

$$\int_0^T u\varphi(u)du = \frac{C_1}{C_2} \quad (30)$$

Then, the value of T that satisfies Eq. (30) corresponds to the optimum preventive maintenance time. For the periodic replacement with discounting

$$C_r(T) = \frac{C_1e^{-\gamma T} + C_2\int_0^T e^{-\gamma u}\varphi(u)du}{1 - e^{-\gamma T}} \quad (31)$$

Finally, other cases that combine replacement models, i.e., age and periodic replacements, as well as those related to imperfect maintenance are discussed in Nakagawa (2005).

Markov-Based Approach

The models presented in the previous section focused mainly on failure and maintenance times. However, the process leading to failure, which is dominated by the degradation mechanisms, is clearly a very important element that needs to be considered in order to define a maintenance policy. This section describes the case in which the changes in the structural condition are discrete and the system performance is described by a Markov process. Markov processes have been studied widely and there is a vast amount of literature available (e.g., Ross 1996; Howard 2007).

Markov Models: Basics

Consider a structure whose state space (i.e., condition) is finite; i.e., $S = \{1, 2, 3, \ldots, n\}$. Then, as the structure degrades, its condition remains the same or changes to a lower condition state at discrete points in time. In this case, the structural performance can be modeled as a *Markov chain*. A Markov chain is a discrete stochastic process $\{X_n, n = 1, 2, \ldots\}$ for which the Markov property holds, i.e., the next state of the structure depends only on its current state and not on its history. In a Markov chain, the transition probability matrix $\mathbf{T}$ is defined by the transition probabilities between all possible condition state pairs. If there are N possible system states, the matrix $\mathbf{T}$ results in a $N \times N$ matrix with $P_{ij} = P(X_{n+1} = j, X_n = i)$, being the probability of moving from state i to state j in one time step. The probability of moving from an initial state i, to any other state j, should be 1

$$\sum_{j=1}^{N} P_{ij} = 1; \quad \text{for } i = 1, 2, \ldots, N \quad (32)$$

In a Markov chain it can be shown that the probability of moving from any state i, to any other state j, in m time steps can be computed as $\mathbf{T}^m$ (Ross 1996). *Semi-Markov* processes represent an extension of the Markov processes and allow for the time between transitions to be random. Semi-Markov processes can be discrete or continuous depending of the distribution of the time between structure's state changes. A special case of semi-Markov processes is the continuous-time Markov process in which the distribution of the time between state changes is exponential,; and therefore the Markov property holds (Ross 1996; Howard 2007). In practice, actual problems are closer to semi-Markov processes than to Markov chains. The use of Markov processes for modeling degradation only requires that the transition matrix, $\mathbf{T}$, is triangular (upper/lower depending of the definition of the problem). However, when modeling both degradation and interventions (e.g., maintenances) there is no restriction on the transition matrix. In this case the model is usually referred to as decision Markov processes, which will be discussed in the next section.

An important drawback of Markov processes lies in the definition of both the transition matrix and the times (random or fixed) at which the system condition changes. Most models define transitions probabilities by making inferences from field data. However, although several attempts have been made to extract these transition probabilities from actual data, obtaining and validating transition probabilities is extremely difficult. Existing approaches to obtain transition probabilities from empirical data include ordered probit models (Baik et al. 2006; Madanat et al. 1995), artificial intelligent techniques such as neural networks (Tran et al. 2010), and the use of expert opinions (Ortiz-Garca et al. 2006). These methods have been applied to many engineering fields, mostly related to infrastructure systems, for example, to the management of wastewater systems (Baik et al. 2006), the prediction of bridge deck systems (Morcous 2006), and for pavement management (Ortiz-Garca et al. 2006; Ben-Akiva and Ramaswamy 1993). In summary, Markov processes are very valuable but the actual quantification of the model input values must be handled with care.

Markov-Decision Processes

A maintenance program based on a Markovian model can be described by the so-called Markov-decision processes. Thus, in a *Markov-decision process* when the system enters in a given state i, the decision of moving to another state j, in the next time interval, depends on both transition probabilities and a finite set of alternative actions $\mathbf{A} = \{a_1, a_2, \ldots, a_k\}$. An alternative, a, is defined by an action (moving from state i to j) and a reward (usually defined as a cost function). The selection of an alternative is a called a decision, and the set of decisions for all states constitutes a policy (Howard 2007). Thus, the objective of Markov-decision processes is to find the best rewarding policy.

Consider a process where for every alternative a, the transition probability can be computed as (Frangopol et al. 2004)

$$P_{ij}(a) = P(X_{n+1} = j | X_n = i, a_k = a) \tag{33}$$

Then, if a stationary policy is selected, this process is known as a *Markov-decision process*; in other words, the decision for an action depends only on the current state of the process. Frangopol et al. (2004) showed that the expected discounted costs over an unbounded time horizon can be obtained from the following recursive equation:

$$W(i) = C(i,a) + \delta(\gamma) \sum_{j=1}^{N} P_{ij}(a) W(j) \tag{34}$$

where $\delta(\gamma)$ = discount function and $W(i)$ are the discounted costs computed over an unbounded time horizon when the system is in state i. The term $C(i,a)$ describes the costs associated with alternative a in state i.

The optimal maintenance intervention policy consists of finding the alternative, a, that minimizes Eq. (34). Frangopol et al. (2004) review some methods to find this solution. One method widely used is the *policy-improvement algorithm*, which successively compares increasingly better policies until no more improvement can be made. A similar iterative algorithm has been proposed by Tijms and van der Duyn Schouten (1984) and Wijnmalen and Hontelez (1992). This problem has been also addressed as a linear programming problem; such as in the Arizona pavement management system; see Golabi et al. (1982), Golabi (1983), Lipkus (1994), Golabi and Shepard (1997), Thompson and Shepard (1994), Thompson et al. (1998), Hawk and Small (1998), and Wang and Zaniewski (1996). Finally, a step-by-step algorithm can be found in Denardo (1982). More details on solution algorithms for Markov decision processes can be found in textbooks by Ross (1970) and Derman (1970). Additionally, extensive work has been carried out by Madanat et al. (1995), Guillaumot et al. (2003), and Madanat et al. (2006).

For the case of semi-Markov decision process, two methods have been used by employing dynamic programming: (1) value iteration and (2) policy iteration (Howard 2007). The value iteration approach can be used for the case of finite time intervals (discrete time semi-Markov decision processes). On the other hand, policy iteration will find a stationary policy that maximizes the gain without discounting and maximizes the expected present value of the future rewards (costs) in the case of discounting (Howard 2007). The particular case of infrastructure inspection and renewal planning is addressed in Kleiner (2001).

According to Frangopol et al. (2004), there are two important issues concerning the use of Markov decision processes for maintenance optimization: (1) the structure's condition state in Markov models is not continuous but discrete and finite, and (2) transition probabilities are difficult to assess and quite subjective. In summary, although there have been several attempts to use Markov decision processes for defining optimum maintenance policies, its implementation in practice still suffers from the same difficulties of Markovian models.

Role of Structural Degradation on Maintenance Models

In the previous section, the maintenance program was defined based on a discrete structural condition state, and the benefits and drawbacks of this approach were outlined. This section presents a description of continuous degradation models and discusses briefly the way in which they can be incorporated in the process of defining a maintenance policy.

Degradation Models

As mentioned before, continuous degradation models can be divided into (1) progressive and (2) shock-based categories. In this section, their basic formulation is presented; also some references, where additional modeling details can be found, are included.

Progressive degradation results in capacity/resistance continuously being removed from the structure at a (usually small) rate that may change (randomly) over time. In reinforced concrete (RC) structures, progressive (gradual) deterioration is caused by phenomena such as creep or chloride ingress, which leads to steel corrosion, loss of effective cross-section of steel reinforcement, concrete cracking, loss of bond, and spalling. The details of these deterioration mechanisms are beyond the scope of the paper but are well described in, for instance, Mori and Ellingwood (1994a, b), Duracrete (2000), Durango and Madanat (2002), Val and Stewart (2005), Biondini et al. (2006, 2008), Biondini and Frangopol (2009), Bastidas et al. (2009), and Kumar et al. (2009). Furthermore, Melchers (1999, 2003, 2005) and Shi and Mahadevan (2001) have studied extensively the effect of corrosion on steel structures. Finally, the combined effect of corrosion and fatigue has also been studied by Zhang and Mahadevan (2001), among others.

Most progressive degradation models available in the literature assume that the type of the degradation process is known, but the parameters are uncertain. The solution to this problem conveys to a parameter estimation problem. Thus, if $V(t)$ is the state of the system at a given time t, which in practice may be expressed in terms of, for example, remaining capacity, reliability, safety, or durability; then, the evolution of the system condition with time may have the following general form:

$$V(t) = \begin{cases} v_0 & 0 \leq t \leq t_e \\ v_0 - h(\mathbf{p}, t - t_e) & t > t_e \end{cases} \tag{35}$$

where v_0 = remaining life of the system at time $t = 0$ and t_e = time before degradation initiation (e.g., time to corrosion initiation). The function h may take a linear or nonlinear form based on the appropriate selection of the vector parameter $\mathbf{p}$, which depends on the problem at hand.

Progressive degradation can be handled also by evaluating variations in the degradation rate, i.e., $\delta(t)$, $t > 0$. Thus, the system state (e.g., remaining capacity measured in physical units per unit time) at time t can be expressed as

$$V(t) = v_0 - \int_0^t \delta(\tau) d\tau \tag{36}$$

where the most common forms of the degradation rate are the following:

- Constant throughout the structure's lifetime, $\delta(t) = \delta$;
- Time dependent (linear or nonlinear), $\delta(t)$; and
- Piece-wise constant (constant over fixed time intervals).

More complex models consider the degradation rate as a stochastic process but their applicability in actual problems is very limited due to their complexity and the frequent lack of data.

Finally, the progressive degradation function can be modeled also as a discrete jump process with random changes at fixed/random time intervals. If the distribution or the time between these changes is selected appropriately, it is possible to find a good approximation to the continuous solution. Examples of this type of model include the geometric process (Lam 2007) and the gamma process (van Noortwijk 2009), which have been used extensively in many applications; for example, see van Noortwijk (2009), Pandey and Yuan (2006), Pandey et al. (2009), and Iervolino et al. (2013).

On the other hand, *shock-based deterioration* is frequently caused by extreme events such as earthquakes, hurricanes, or blasts (including both accidents and terrorists attacks). Shocks can be defined as events that cause a significant change in a system's physical property (e.g., stiffness) in a small time interval. A special case of shock degradation are the so-called sudden (also called catastrophic) failures, cases in which structures fail suddenly and completely at random points in time due to large demands.

Shock-based degradation occurs when a fixed amount of capacity/resistance is removed from the system at discrete points in time (Sanchez-Silva et al. 2011). Then, if Y_i is a random variable describing the degradation caused by shock i, the remaining structural capacity/resistance due to shocks by time t can be computed as

$$V(t) = v_0 - \sum_{i=1}^{N_t} Y_i \tag{37}$$

where N_t represents the number of shocks by time t. In practice, the time between shocks is random; therefore, N_t is also a random variable.

Extensive research has been carried out on mathematical models for shock degradation; for more details see Sánchez-Silva and Klutke (2016), Barlow and Proshan (1965), Aven and Jansen (1999), Nakagawa (2005), and Feldman (1977). Among the first works on this topic in civil engineering were published by Rosenblueth and Mendoza (1971) and Rosenblueth (1976) within the context of earthquake-resistant design optimization. Their ideas were later reconsidered by Rackwitz (2000) to propose a general framework for optimal design and reliability verification. A model for damage accumulation as a result of extreme events has also been proposed by Wortman et al. (1994), Sanchez-Silva et al. (2011, 2012), Riascos-Ochoa et al. (2014), Iervolino et al. (2013), and Sánchez-Silva and Klutke (2016).

Frequently, progressive and shock-based deterioration occur simultaneously (Fig. 7). Thus, for a structural component with initial capacity, v_0, subject to both continuous and sudden damaging events acting independently, the remaining capacity/resistance by time t can be computed as (Sanchez-Silva and Klutke 2016)

$$V(t) = v_0 - \int_0^t \delta(\tau) d\tau - \sum_{i=1}^{N_t} Y_i \tag{38}$$

In some cases, both events are not independent and, therefore, the coupled effect should be taken into consideration. The discussion on modeling deterioration mechanisms is beyond the scope of this paper but further details can be found in Frangopol et al. (2004), Neves et al. (2004), and Sánchez-Silva et al. (2011, 2012). In general, the effects of progressive deterioration on the performance of structures subjected to extreme events has received only limited attention, but studies have shown this combined effect may have an important impact on performance, depending on exposure conditions (Choe et al. 2008; Ghosh and Padgett 2010).

As the structural degradation process becomes more complex, finding an analytical expression for the lifetime distribution becomes more difficult. This complexity may result because, for example, there are different failure mechanisms, or there are not enough data to obtain the parameters of the degradation model. Despite the computational cost, Monte Carlo simulations are always a modeling alternative; however, the main challenge is acquiring information on the statistical descriptors of various random parameters. This difficulty is of particular importance in infrastructure systems given that they operate for long time periods and the degradation processes are very slow.

Maintenance Considerations

In large civil infrastructure systems, understanding and modeling degradation is central to defining optimum maintenance strategies. Then, to define an optimum maintenance intervention program, it is necessary to compute first the lifetime distribution $F_L(t)$ based on the degradation characteristics of the process. This information can be used in any of the models presented at the beginning of this section. Once the lifetime distribution is obtained, the complexities of the degradation process are no longer needed and $F_L(t)$ can be included as an input variable in any cost/benefit relationship function [e.g., Eq. (12)] that leads to an optimum maintenance program. When interventions focus on corrective maintenance (i.e., repair after failure), the lifetime distribution is computed based on a performance threshold k^*. Condition-based preventive maintenance requires defining another threshold a^* with $k^* < a^* < v_0$. Then, the distribution of the time to maintenance is obtained by computing

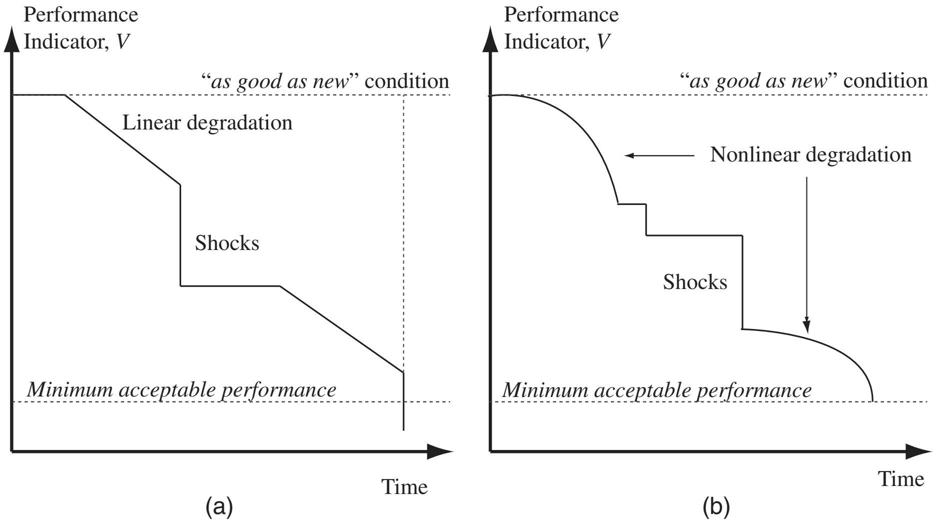

Fig. 7. Description of the combined effect of (a) linear progressive degradation and shocks; (b) nonlinear progressive degradation and shocks

$L_m = \inf\{t \geq 0: V(t) \leq a^*\}$, which is again dominated by the degradation process.

Clearly, when degradation is included in the analysis, the value of gathering information about the evolution of structural condition over time is essential (Saydam et al. 2013). This need has been attracting a lot of attention in recent years. For instance, a growing number of inspection and structural-health monitoring techniques are being used to reduce the uncertainty of the structure's performance evolution and to enhance decisions on maintenance actions (Casas and Cruz 2003; Straub 2011). In summary, combining inspections policies with the value of information and the decision needs (Konakli et al. 2015; Straub 2014) is central to building better inspection and maintenance programs.

Maintenance of Networks and Systems

The strategies presented here have also been used, with the appropriate modifications, for the case of multicomponent systems. The main assumption in these extensions is independence of components' failures. However, for multicomponent systems, an optimal maintenance policy must take into account the dependence between the system components. This dependence may be of three types: economic, structural, and stochastic. The specifics of these aspects will not be discussed here, but further details on interdependent components can be found in Thomas (1986). From the perspective of a system where there is only information about the failure times of components, the expected cost rate of a system consisting on n components that operate independently is (Nakagawa et al. 1984)

$$C_r(T) = \frac{1}{T}\left[C_1 + C_i \int_0^T \varphi_i(u)du\right] \tag{39}$$

where C_i = cost of maintenance of every failed unit and $\varphi_i(t)$ = parameter function $h(t)$ or $m(t)$ of the ith component, as discussed previously. For the case of systems, this approach assumes that all components are replaced together at times $kT(k = 1, 2, \ldots)$. More information on this type of systems can be found in Esary et al. (1973) and Nakagawa et al. (1984). Most maintenance models for multicomponent manufacturing systems can be grouped into the group/block replacement and opportunistic maintenance models (Laggoune et al. 2010; Misra 2008). In the *block/group maintenance* policy, an entire group of components is replaced at periodic intervals defined based on time, cost or both. The main maintenance policies within this group are

- T-age group replacement policy;
- m-failure group policy; and
- m-failure and T-age (m, T) policy.

In the T-age group replacement policy, individual components are all replaced once the system reaches age T (Okumoto and Elsayed 1983). In the m-failure group policy, the system is maintained after m-failures have been reported (Wilson and Benmerzouga 1990; Assaf and Shanthikumar 1987). Finally, the (m, T) maintenance policy calls for a group replacement when the system has reached age T or when m failures have occurred, whichever comes first (Nakagawa 1979; Ritchken and Wilson 1990). The details of these strategies and some additional models are described in Wang (2002).

On the other hand, the so-called opportunistic maintenance comes from the fact that the cost of simultaneous maintenance actions on various components may be less than the sum of the total cost of individual maintenance actions (Laggoune et al. 2010). Therefore, providing the opportunity to carry out preventive maintenance on some components along with the replacement of failed ones, leads to more cost-effective strategy. The so-called warranty models may also be included in this category. Some relevant models of this type can be found in Berg and Epstein (1976, 1978), Zheng and Fard (1991), Dagpunar (1996), Pham and Wang (2000), Wang et al. (2001), and Laggoune et al. (2010).

In civil structural systems, the most common strategy to represent and handle multiple component systems is through block diagrams, which frequently end up in a combination of series and parallel arrangements (Frangopol and Okasha 2009; Okasha and Frangopol 2010c; Saydam and Frangopol 2011; Zhu and Frangopol 2013; Barone et al. 2014). This approach has difficulties in achieving consistency in the selection of performance indicators, the definition of limit states and the collection of data, among others. This leads often to models with large simplifications and too far from reality to be used as a decision tool in practice. Complex system configurations have led to the study of networks, a subject of great importance. In particular, the maintenance and operation of large infrastructure networks have attracted great interest. Most areas of research within this context are associated with the so-called complex network resource allocation problem; for more information see for example Gonzalez et al. (2016), Sánchez-Silva et al. (2009), Gómez et al. (2013), Rokneddin et al. (2013a), and Hernandez-Fajardo and Dueñas Osorio (2013).

Alternatively, for more complex structures where the system configuration is difficult to be modeled using the series-parallel system configuration, nonlinear finite-element analyses can be used to achieve a better estimate of the system reliability. Advanced statistical concepts and tools such as the Latin hypercube sampling and the response surface method allow the efficient integration of the nonlinear finite-element analyses in the probabilistic computation procedure to evaluate the reliability of complex systems such as bridges and ships (Ghosn et al. 2016a; Okasha and Frangopol 2010a, 2012; Padgett and DesRoches 2008; Strauss et al. 2008). Moreover, closed-form solutions to such system level reliability analyses have been pursued and applied to structure and infrastructure systems including matrix-based methods (Song and Kang 2009), recursive combinatorial approaches (Dueñas-Osorio and Rojo 2011), and nonrecursive combinatorial approaches (Dueñas-Osorio and Padgett 2011).

Maintenance Management Considerations

Maintenance management can be defined as the set of actions required to guarantee a prespecified system's performance level. It requires answering questions such as which components to maintain, how to maintain them, and when to carry out maintenance (Lounis and McAllister 2016). The answer to these questions is challenged by the presence of various uncertainties associated with the performance of the structure, the deterioration, and the effect of previous maintenance actions on the structural performance.

Individual Structures

In maintenance management problems of single components, optimum intervention times are selected based on the time-variant performance profiles and the performance index threshold (Figs. 2 and 3). Threshold-based maintenance management approaches have been widely used in literature for structures subjected to, for instance, fatigue, and corrosion (Kwon and Frangopol 2012). However, for systems with multiple deteriorating components and multiple maintenance actions, more elaborate decision strategies have to be used to prioritize interventions and define their extent and execution times. For example, Estes and Frangopol (1999) identified the corrective (i.e., essential) maintenance type to be applied as the one that provides the lowest present cost per year of increase in the service life. Okasha and Frangopol (2010b) proposed an algorithm based on event-tree analysis, which calculates and compares the cost of all available corrective maintenance scenarios to reach the prespecified service life of the structure.

Maintenance management is a complex multiobjective optimization problem (Furuta et al. 2006). Competing objectives include maximizing the lifecycle performance and safety of the structure on one hand, and minimizing the total maintenance or lifecycle cost on the other hand (van Noortwijk and Frangopol 2004; Frangopol et al. 2002; Frangopol and Neves 2008; Frangopol and Messervey 2009; Frangopol and Kim 2011). Multicriteria optimization problems have been formulated for various types of structures at the levels of components, entire structural systems, and networks. For example, Neves et al. (2006a) and Petcherdchoo et al. (2008) proposed an approach for optimizing preventive maintenance actions for deteriorating bridges. The approach considers only one type of maintenance and uses two design variables for the optimization problem—one for the first maintenance application time, and the second for the time interval between design variables. This approach was further enhanced in Neves et al. (2006b) to include multiple preventive maintenance types. Another example is the approach proposed by Okasha and Frangopol (2010b), which considers both corrective and preventive maintenance and uses continuous design variables for the time of application of preventive maintenance and an integer variable to for the optimum number of application of each maintenance type. The approach considers one type of preventive maintenance for bridge girders and also one type for the bridge deck. However, it can select the best corrective maintenance type among a set of predefined options. An optimization approach recently proposed by Kim et al. (2013) distinguishes among two maintenance types (defined by the extent of the maintenance action) in which the optimization provides the critical damage level for each maintenance type that if found during an inspection, the corresponding maintenance type has to be performed. The optimization in Kim et al. (2013) was performed to simultaneously maximize the expected service life and minimize the expected lifecycle cost, which includes the inspection and maintenance costs.

A new important direction of research in this area is the implementation of control-system theory, which is a field widely developed in other engineering areas. An example of the use of control systems for defining optimum maintenance policies can be found in Junca and Sánchez-Silva (2013a). Structural-health monitoring (SHM) may be used with advantage as an indication of the actual structural performance (Frangopol and Messervey 2008; Orcesi et al. 2010; Orcesi and Frangopol 2011a; Okasha et al. 2012). As shown in Fig. 8, information collected during SHM or inspection actions can be effectively used to update and calibrate the parameters of the adopted deterioration models and establish improved intervention plans (Frangopol 2011; Soliman and Frangopol 2014). The integration of such information significantly improves the quality of management decisions by reducing the epistemic uncertainties associated with the performance prediction as well as providing the ability to detect the damage based on the recorded structural responses.

Groups of Structures

In addition to individual structures, maintenance management can be performed for groups or networks of damaged structures. The literature provides many examples for such models, especially for bridge networks subjected to extreme events such as earthquakes or gradual deterioration effects (Augusti et al. 1998; Frangopol et al. 2001; Liu and Frangopol 2005a, b; Frangopol and Liu 2007). For these problems, the relevant questions include which structures should be maintained and the extent of the intervention in order

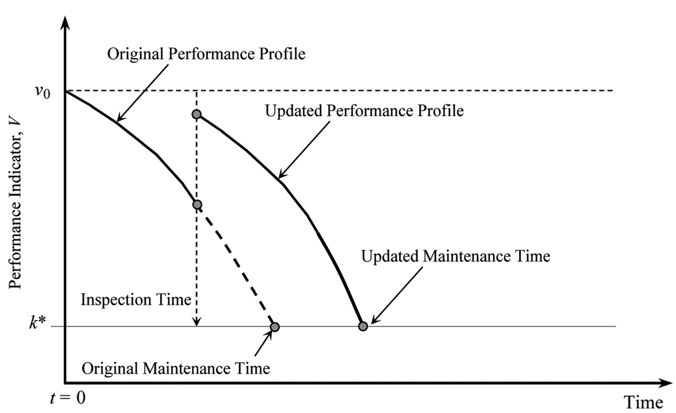

Fig. 8. Updating performance profile and maintenance time based on inspection outcome

to achieve a specified functionality or connectivity through the road network (Bocchini and Frangopol 2011, 2013; Rokneddin et al. 2013a). A key component in this type of study is the network configuration. Liu and Frangopol (2005d) presented an approach for quantifying the bridge network reliability and the bridge reliability importance factors defined as the sensitivity of the bridge network reliability to changes in individual bridge system reliability. In their approach, the network is composed of nodes, points and links in which the reliability is measured in terms of the connectivity between the origin and destination. Ghosh et al. (2013) and Rokneddin et al. (2013b) also propose and apply a bridge reliability assessment in networks (BRAN) methodology to assess network reliability including aging bridge instrumentation data and correlated bridge failures. Connectivity reliability is also considered as the performance indicator in their studies. In Liu and Frangopol (2005a), an approach for establishing the optimum maintenance types and times within the bridge network was proposed. The approach uses event-tree analyses to find an expression for the network connectivity (i.e., reliability), which is subsequently used to find optimal maintenance plan in order to maximize the network connectivity and minimize the total maintenance cost. Finally, recent network-modeling approaches focus on network decomposition and hierarchical system representations (Gomez et al. 2013). In this approach the main focus of the analysis is on integrating the relevance and precision of the model with the decision needs. However, regardless of the adopted performance indicator or the optimization scheme, further extensive research is still required to reduce the gap between theory and practice in this field and to promote the real-world application of these maintenance management methodologies.

Concluding Remarks

Frequently, maintenance policy managers and engineers make decisions heuristically based on their experience and common sense due to the difficulties in finding models that suit the practical realistic issues of actual projects. Within this context, this paper attempts to provide an overview that can be used to support the development of infrastructure management strategies (i.e., maintenance and operation). Recently, the problem of defining optimum maintenance policies has attracted a lot of interest in different sectors—e.g., engineers, politicians, environmental activists, and investment banks—and a large number of technical papers and books have been published in the field of maintenance of deteriorating systems. This paper presents a comprehensive overview of the different aspects involved in managing infrastructure. It also describes different deterioration mechanisms and presents a discussion about existing structural-performance indicators. Moreover, it describes the criteria, variables, and models used to define optimum maintenance policies from both a conceptual and theoretical standpoint. Finally, it presents a discussion on the management of multicomponent systems. The paper provides a large number of references that can be used to support the development of better maintenance models.

Overall, the paper emphasizes the challenges in infrastructure management, which include the definition and quantification of uncertainty, especially for predicting the system performance over time. An essential element associated with this task is finding cost-effective reliable monitoring systems for data acquisition. Also, an important aspect of maintenance programs is the need for better decision-making strategies that can capture the dynamics of system performance as well as the role of all individuals involved in the process (such as owners, users, and operators) and their interaction. The nature of maintenance and operation of large infrastructure systems spans beyond the boundaries of engineering and requires new and more comprehensive approaches.

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