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Numerical and monitoring based Markov Chain approaches for the fatigue life prediction of concrete structures

A. Strauss

Institute of Structural Engineering, Department of Civil Engineering and Natural Hazards, University of Natural Resources and Life Sciences, 1190 Vienna, Austria

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ABSTRACT

The design of special concrete structures, such as foundations for offshore wind turbines, has to be carried out against complex requirements and under challenging environmental conditions. Among these are for example dynamic and cyclic loadings caused by wind and sea waves. Concrete fatigue processes are dominant degradation processes (a) that determine the technical lifetime of such foundations, and (b) that are difficult to determine in occurrence, severity and with regard to the redistribution characteristics in the cross section or structure using classical inspection or monitoring programs. Semi-Markov Chain approaches, which are based on process sojourn time considerations, combined with smart monitoring sensor systems and advanced nonlinear Finite Element simulation methods, present very promising approaches for the realistic description of concrete fatigue processes and their redistribution characteristics and in consequence for the prediction of the remaining lifetime. The objectives of the current work are (a) to present a Semi-Markov Chain approach that is based on smart monitoring information and on advanced nonlinear Finite Element simulations for the realistic assessment and the prediction of concrete fatigue processes, and (b) to demonstrate the Semi-Markov Chain approach on an offshore wind energy tower concrete foundation in Cuxhaven. This work presents a further step toward developing a software tool that can be used to perform reliability assessments of aging concrete structures and to update their reliability with inspection and monitoring data.

1. Introduction

1.1. Motivation

Offshore wind energy (OWE) is among the emerging sustainable energy sources. Engineering design can contribute significantly to its extensive implementation. Various activities have contributed to the knowledge base that is needed for the design and technology development [28]. In Europe, offshore wind turbines are present predominantly in Denmark, Sweden, England, Holland and Norway (among others). These are close to the coast with shallow water depths of up to 10 m. New wind farms are planned in Germany at distances of up to 40–50 km [19] from the shore and with a foundation depth of 30 m. The design of the foundations for offshore wind turbines (OWEA) has to take into account the complex requirements posed by the dynamic and cyclic loading of the construction and of the subsoil caused by wind and waves [20]. In contrast to onshore wind turbines, offshore wind turbines are characterized by: (a) additional loading from waves and tides, (b) increased building dimensions due to foundation construction in deep shore areas, and (c) hydrodynamic force effects on the foundation, which can be up to ten times greater than the aerodynamic loads and hydrodynamic moments on the foundation [16]. The foundations have to fulfill their function under the demanding environmental conditions on the open sea without extensive inspection and maintenance work for 20–25 years [15]. These special conditions require an optimization of fabrication, transportation and installation of the facilities associated with a possible high level of serial prefabrication [19].

1.2. Safety of fatigue endangered concrete structures

The various design provisions are not necessarily compatible with regard to the concept of safety. API [1] follows the “working design stress” which corresponds to the global safety concept. Germanischer Lloyd (GL) [12] permits the partial safety concept as well as the global safety concept, while DIN 1054 [10] exclusively relies on the partial safety concept and its predictions based on the cross section forces. The classification of the safety level is therefore as difficult as the definition of the safety factors. Accordingly, the partial safety factors, as for instance those of DIN 1054 [10], are not readily transmissible on offshore, since the loading

E-mail address: Alfred.strauss@boku.ac.at

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and resistance properties of offshore systems are more uncertain than those of more general concrete structures. These uncertainties with regard to the required safety level and in keeping with the objective to keep concrete structures in operation beyond the defined technical lifetime t_L formed the basis for the development of an integrated monitoring and assessment framework (IMAF). This framework combines monitoring, modeling and prediction methods for the fatigue assessment of endangered concrete structures. A realistic assessment of the possible remaining lifetime t_R of fatigue endangered concrete structures can be assured by a balanced combination of classical and advanced monitoring systems together with numerical modeling predictions [26].

1.3. Performance related monitoring

In the IMAF of this research project, an S-monitoring (e.g., associated with a number of applied cyclic stress ranges) and an R-monitoring concept (e.g., associated with changes in the microstructure and in the strain tensor) were applied on an offshore foundation and on cylindrical laboratory concrete samples [25]. This performance related monitoring concept (see also [29]) enables a continuous and adaptive detection and assessment of not accurately known fatigue processes during and after the planned technical lifetime t_L. It also allows the review of more or less well developed predictive models that are common in the Life-Cycle Engineering (LCE). The performance-related monitoring concept also includes an accompanying numerical modeling module which enables the interpretation of the overall system performance (e.g. performance of the whole foundation tower) and which allows supervision and redundant checking of the results of the prediction models. Nevertheless, according to Gokce et al. [13], the uncertainties in the data collected, determined by means of intermittent testing or monitoring, the limitations of the models, and the non-stationary nature of structural behavior need to be considered. These uncertainties can be incorporated by using special statistical techniques [13].

1.4. Structural identification

Performance-related monitoring increases in significance, as previously mentioned, when supplemented by a numerical model. This numerical model in a first step needs to be adapted to the mechanical properties of the existing structure by means of Structural Identification methods (St-Id). Even at this stage, well designed monitoring systems provide valuable information on input variables for the St-Id and for the associated model calibration variables. Structural Identification (St-Id) can be described according to Catbas et al. [8] and Strauss et al. [23] simply as estimating the properties of a structural system based on a correlation of inputs and outputs for decision-making, or according to a more detailed formulation of Catbas et al. [8]: *A complete* St-Id *process, establishing the decision making needs, developing analytical and numerical models, and conducting field measurements, along with parameter identification using the experimental data for model calibration. The associated results are expected to provide a set of solutions for the performance of a structure for optimal decision making.*

The preparatory or input phase of Structural Identification (St-Id) processes can be structured as: (a) the definition of the relevant performance characteristics, (b) the preparation of a numerical Finite Element Model (FEM) on the basis of existing information on the material and on the structure which has for example been gathered during the planning phase, execution phases and/or inspection phases, (c) the definition of significant loads or load combinations, and (d) the definition of theoretically and experimentally derived degradation processes of the considered performance characteristics. These steps are important for updating concrete fatigue degradation process models such as required for the considered wind energy offshore foundations, as shown in Fig. 1(a) and (b). As mentioned already, the used innovative monitoring systems (see Fig. 1(d)) in combination with Semi-Markov Chain approaches, which will be discussed in the following, have turned out to be essential elements for the St-Id, the condition assessment, and the lifetime t_L prediction (see Urban et al. [25]). The final results of a successful St-Id should include: (a) a representative linear or nonlinear FEM, (b) interesting performance indicators that enable a comparison of monitored with modeled performance indicators for a first verification of model uncertainties, (c) identification of deterioration mechanisms and their level and rate through the lifetime t_L based on temporal comparisons of modeled with monitored performance indicators, (d) robustness quantities, taking into account the system performance or system redistribution processes using information from FEMs.

Please note that the fatigue specific upper stress σ_o and lower stress σ_u, as well as the degradation degrees D, are associated with the Young's modulus according to [21]. Therefore, the Young's modulus has been treated as most important updating parameter.

For example, for the case study discussed in Section 3 and shown in Fig. 1(a) and (b), the following St-Id steps were carried out:

In a first step, a Nonlinear Finite Element (NLF) model and a cyclic load model of the offshore test foundation in Cuxhaven were developed. The Young's modulus distribution of the reinforced concrete E_C along the circular cross section, divided into 36 Young's modulus fields $\mathbf{E}$, was defined as an indicator for the St-Id process. The consecutive step analyses of the cyclic loading on the NLF element model and the associated stepwise adjustment of the *Young's modulus* fields $\mathbf{E_{c,NLF}}$, using the findings of Petkovic [21], enabled the *NLF model based* definition of the *Young's modulus fields* $\mathbf{E_c}$ along the circumference of the cross section for the St-Id at predefined time horizons. The Young's modulus adjustment processes essentially depend on the upper and lower concrete stress levels $S_{c,o}$ and $S_{c,u}$, wherein changes in the Young's modulus E_c cause effects on fatigue characteristics in the cross section fields as defined in fib Model Code 2010 (2013b) and in consequence on their related remaining life time t_R. The concrete stresses in the cross section fields are redistributed during transformation processes to those regions with a higher E_c, where an increase in stress is possible.

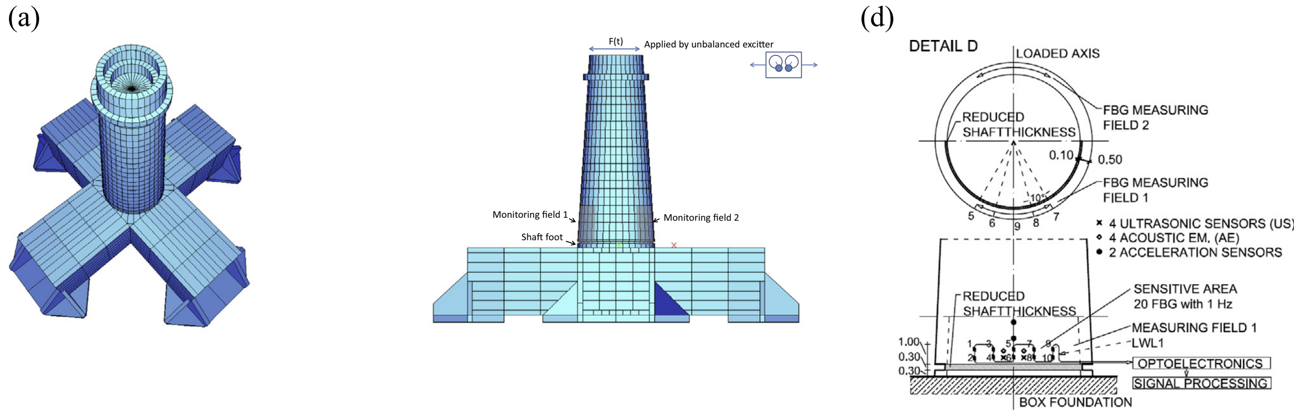
In a second step, the fatigue-associated R-monitoring and S-monitoring information from fibre optical sensor (FOS), ultra sound sensor (US), and acoustic emission (AE) sensor systems were analyzed with respect to the concrete fatigue characteristics. The comparison of the R-monitoring and S-monitoring information with the monitoring results of the laboratory small scale tests and the load depending fatigue findings of Petkovic [21] enabled the *monitoring based* characterized *Young's modulus fields* $\mathbf{E_{c,Mo}}$ distribution *along* the circumference of the cross section.

In a third step, a Bayesian updating approach was used to incorporate the *monitoring based* characterized *Young's modulus fields* $\mathbf{E_{c,Mo}}$ as short term information in the NLF based $\mathbf{E_{c,NLF}}$ so as to allow an adaptation of the NLF model to the existing structural fatigue conditions. Fig. 1(e) shows the Bayesian updated *Young's modulus fields* $E_{cm}/E_{cm,initial}$ up to applied load cycles $4.2 \cdot 10^6$ in blue.[1]

In a fourth step, the NLF models obtained by means of previously described St-Id processes for selected times of loading served as a basis (a) for a deterministic or probabilistic performance assessment, and (b) the prediction of the lifetime t_L or remaining lifetime t_R. Two strategies were considered for the performance

[1] For interpretation of color in Fig. 1, the reader is referred to the web version of this article.

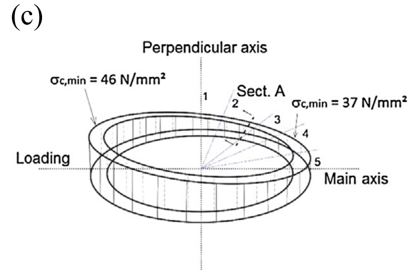
(a)



(b)

Component Description	
Total height of the trial foundation	32.15 m
Shaft height	22.85 m
Pedestal height without upspring slab	8.00 m
Shaft diameter, bottom	8.50 m
Shaft diameter, top	5.80 m
Wall thickness	0.60 m
Pedestal total length	40.00 m
Pedestal width	8.50 m
Ground cover plate	0.60 m
Shaft total weight	7687 kN

(c)



(e)

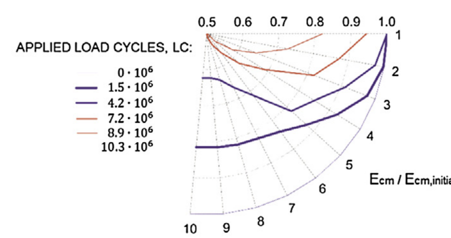


Fig. 1. The discretized finite element model of the offshore wind energy tower concrete foundation in Cuxhaven, with (a) 36 elements in the cross section, (b) main dimensions of the foundation, (c) main axes of the considered cross section, and (d) a detail of the reduced cross section next to the foundation boxes and the associated monitoring layout, (e) normalized Young modulus vs. applied load circles [26].

prediction: (I) the adjusted NLF models were simulated for the cyclic loading up to the life time t_L and until the failure of the entire structure, and (II) a semi Markov Chain approach was used for the prediction of the *Young's modulus fields* $\mathbf{E_c}$ up to the life time t_L. This second strategy, which is based on scattering values, will be discussed in more detail in the following. Uncertainties of modeling, of material properties, of mechanical properties and uncertainties in monitoring were treated for the *St-Id* and the performance assessment according to Ang and Tang [3], Ang and De Leon [2], and Strauss et al. [23] by specifying significant variables, such as probability density functions (PDFs), and using Bayesian methods for the inclusion of short term findings. In particular, this approach is also very appropriate for the considered Semi-Markov Chain prediction procedure, as (a) it enables a continuous incorporation of short term monitoring information, and (b) it provides a more realistic probabilistic-based decision-making and prediction basis for the considered concrete fatigue processes.

The first three St-Id steps are essential for the identification process and can be integrated on different levels with respect to available information. The analyses and monitoring efforts are based on mean value considerations in analogy to the Nonlinear Modeling techniques. The integration of uncertainties into modeling and monitoring systems will be the next step in the current research activity.

1.5. Performance and performance prediction

As mentioned above, a stepwise Finite Element analysis procedure, as shown in Fig. 2(a), was developed for (a) the interpretation of the already recorded and expected results of the installed monitoring systems (see Fig. 2(b)) and the fatigue associated transformation processes, and (b) an efficient model update. The efficient update comprises (i) the adjustment of changes in the Young's modulus E_{cm} in the cross sections A_i at the predefined load steps, (ii) the comparison of the computed strains with the monitored strains, and (iii) the Bayesian updating of the adjusted Young's modulus E_{cm}'' by using the monitored strains, as previously mentioned, as short term information.

As an example, the developed stepwise NLF procedure started with the loading of the undamaged shaft sub cross sections (concrete Young's modulus $E_{cm} = 48.5$ GPa in all sub cross sections) with the body weight and the prestressing of $p_0 = 19 \cdot 1580$ kN. Subsequently, for each $\Delta n = 1000$ applied dynamic load cycles on the head of the tower, the concrete Young's modulus E_{cm} was adjusted in each sub cross section A_i. Specifically, the adjustment was performed according to the experimentally derived functional relationship between E_c^{fat} and N_i/N_f according to Grünberg and Göhlmann (2006). The necessary input N_i/N_f, for determining E_{cm} and E_c^{fat} were obtained in the consecutive computational steps from S–N curves with respect to the stress ranges ΔS of each sub cross section, where the degradation to E_c^{fat} is related as follows:

$$E_c^{fat} = (1 - D) \cdot E_{cm,0} \quad (0)$$

with E_c^{fat} = fatigue reduced concrete Young's modulus, D = damage reduction factor for cyclical loaded concrete and $E_{cm,0}$ = initial concrete Young's modulus. Further findings are documented in Urban et al. [25,26].

1.6. Prediction procedures

To decouple the prediction procedures from the previously mentioned Finite Element analysis and to include historical information from monitoring and numerical analyses, a Markov Chain approach was chosen. The Markov Chain approach, as outlined by Wang et al. [27], was applied in this research to determine the transition probabilities for *do nothing actions* between condition states of the previously mentioned sections of the fatigue endangered circular cross section of the offshore foundation,

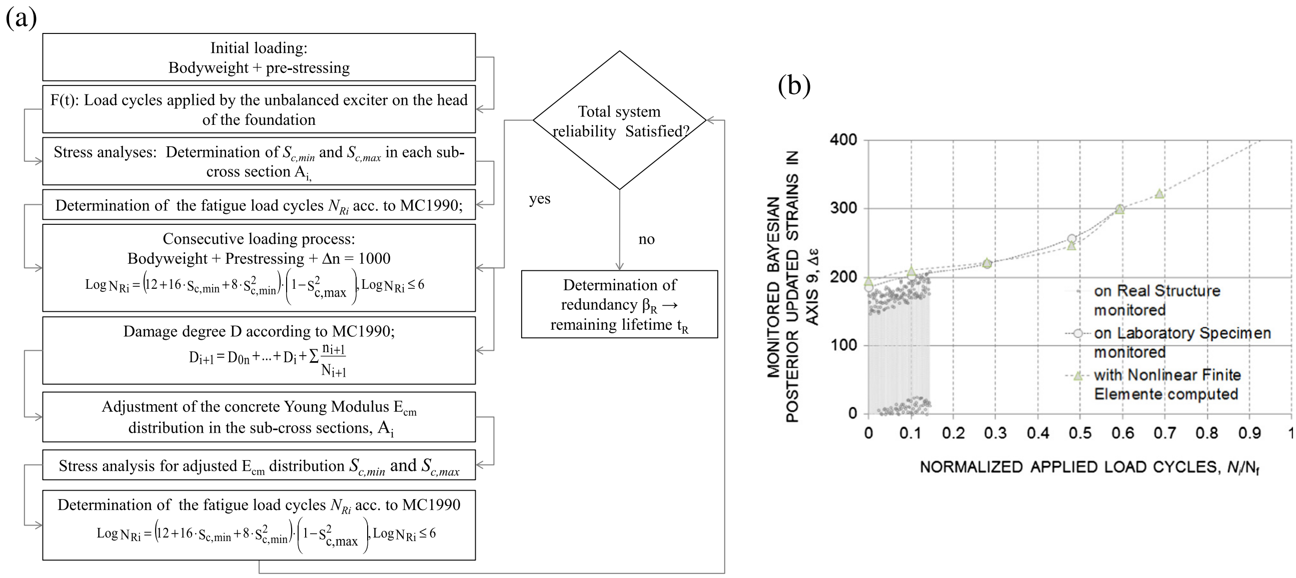


Fig. 2. Framework of the stiffness transformation processes in the cross-sections A_i; (a) Stepwise Finite Element analysis process based on the concrete fatigue formulations of ModelCode 1990 for the adjustment of the Concrete Young's modulus E_c transformation, and (b) the associated fatigue active strain process of axis 9 – numerical derived and monitored on the structure and on laboratory tested specimens.

where *do nothing action* means that there was no intervention in the stiffness deterioration process. It is assumed that once an asset falls out of a condition state, that condition state is not visited again. In the current paper, a Semi-Markov model for the modeling of fatigue processes in concrete structures, where the sojourn time in each condition state is assumed to follow a Weibull distribution, is presented, rendering the method more flexible than the traditional Markov Chain model. The use of such statistical tools as Semi-Markov modeling, which takes into consideration the age of a fatigue process, monitoring as well as numerical information, can provide insights into the fatigue performance of concrete structures at the network level.

2. Markov Chain based prediction models for fatigue endangered concrete structures

2.1. Markov Chain model

A Markov Chain model is a stochastic process that is characterized according to Ross [22] by the following properties. At the moment n, the process $X_n = i$ is in state i. The transition probability $P_{i,j}$ which expresses the transition into state j may be assigned to this process. This probability is expressed mathematically as follows:

$$P\{X_{n+1} = j | X_n = i, X_{n-1} = i_{n-1}, \ldots, X_0 = i_0\} = P_{i,j} \tag{1}$$

for states $i_0, i_1, \ldots, i_{n-1}, i, j$ and where $n \geqslant 0$. The transition probability may, according to Wang et al. [27], be determined on the basis of the following formulation:

$$p_{i,j}(a_k) = \frac{m_{i,j}(a_k)}{m_i(a_k)} \tag{2}$$

for i, j = 1, 0.9, 0.8, 0.7, 0.6, 0.5 and 0.4 and where k = action with impact on the process. The Markov Chain associated parameters i, j indicate the degradation degrees as follows: $D = 1 - i$ or $D = 1 - j$. $p_{i,j}(a_k)$ = the transition probability of a group transitioning e.g. from state i to state j whenever action a_k occurs; $m_{i,j}(a_k)$ = the total number of groups (e.g. sections of a fatigue endangered circular cross section of an offshore foundation) for which state i prevails prior to action k and state j prevails after action k has occurred; $m_i(a_k)$ = the total number of groups for which state i was the case prior to action k. For the current study on the assessment of fatigue induced failure risk in concrete structures using Markov Chain models, a *Matlab* software was developed to interpret Eq. (2). In the basic version of the software program, it is assumed that action a_k has no influence on the process.

The time variable transition probabilities for the properties of interest were determined for the previously indicated 36 cross section segments and for 15 load cyclic levels, i.e. for a total of 540 sets. The transition probabilities were calculated through the results of an artificial numerical simulation that focused on the redistribution processes and was considered as monitoring information. The properties of interest were (a) the concrete Young's modulus E_c of elasticity, (b) the fatigue specific upper stress σ_o, (c) the fatigue specific lower stress σ_u and (d) and the degradation degrees D.

2.2. Semi-Markov model

The basis for developing a Semi-Markov model is a stochastic process with the possible states $0, 1, 2, \ldots$. The Semi-Markov model is characterized by the facts that a stochastic process which enters state i, $i > 0$, (1) transitions into the next state j with a probability of $p_{i,j}$, i, $j > 0$, and (2) the sojourn time between states i and j corresponds with the distribution $h_{i,j}$. A generally applicable method for determining the sojourn time is a Weibull distribution with a probability function that is defined according to Tobias and Trindade [24] as follows

$$h(t) = \frac{\gamma}{\alpha}\left(\frac{t}{\alpha}\right)^{\gamma-1} \cdot e^{-\left(\frac{t}{\alpha}\right)^{\gamma}} \tag{3}$$

with the scale parameter α and the shape parameter γ and where t equals the number of years it will take the considered group (e.g. sections of a fatigue endangered circular cross section) to transition from one state into the next state. The scale parameter α and the shape parameter γ can be determined using the Maximum Likelihood Estimation (MLE) method. In the current study, the numerically determined fatigue performance (which changes due to the impact of the loading cycles) of the cross sectional areas of a foundation for an offshore wind turbine is drawn upon as stochastic process data. In particular, the sojourn times of the groups = cross sectional areas in the state categories,[2] determined via a Weibull distribution, are of interest. The MLE method for estimating the scaling parameter α and the shape parameter γ is quite popular in the field of statistics [6]. Details can be found in a great number of statistical publications, among them also the publication by Birolini [4]. The formulation of single stage probability $Q_{i,j}(t)$ according to Ibe [18], see Eq. (4), serves as the basis for the development of a Semi-Markov model where the sojourn time of a process in a defined state prior to its transition into the next state is known:

$$Q_{i,j}(t) = P[X_{n+1} = j, T_n \leqslant t | X_n = i], \quad t \geqslant 0, \tag{4}$$

where $Q_{i,j}(t)$ is the transition probability that the process will in its next step transition to state j if it is currently in state i. Since the sojourn time T_n in the current state i is smaller than t, it is also possible to formulate:

$$Q_{i,j}(t) = p_{i,j} \cdot H_{i,j}(t), \tag{5}$$

where $p_{i,j}$ equals the transition probability of the Markov Chain. The accumulated probability of the sojourn time is defined as:

$$H_{i,j}(t) = P[G_n \leqslant t | X_n = i, X_{n+1} = j], \tag{6}$$

where G_n equals the sojourn time of the process in state i prior to its transition into state j.

2.3. Semi-Markov processes

Howard [17] developed the following formulation for determining the probability that a temporally continuous Markov process that will at a certain moment n be in state j, which at moment 0 had transitioned into state i:

$$\varnothing_{ij}(n) = \delta_{ij} > w_i(n) + \sum_{k=1}^{N} p_{ik} \int_0^n h_{ik}(m) \varnothing_{kj}(n-m) \tag{7}$$

$$i,j = 1,2,\ldots,N; \quad n = 0,1,2,\ldots; \quad \delta_{ij} = \begin{cases} 1 & i = j \\ 0 & i \neq j \end{cases} \tag{8}$$

$$\Phi^{0,m}(m) = \begin{bmatrix} 1 - \sum_{j=4}^{9} p_{10,j} \cdot H_{10,j}(m) & p_{10,9} \cdot H_{10,9}(m) & p_{10,8} \cdot H_{10,8}(m) & 0 & 0 \\ & 1 - \sum_{j=4}^{8} p_{9,j} \cdot H_{9,j}(m) & p_{9,8} \cdot H_{9,8}(m) & \cdots & 0 \\ \vdots & & 1 - \sum_{j=4}^{8} p_{8,j} \cdot H_{8,j}(m) & \ddots & \vdots \\ & & & & p_{6,4} \cdot H_{6,4}(m) \\ & & & & p_{5,4} \cdot H_{5,4}(m) \\ & & & \cdots & 1 \end{bmatrix} \tag{12}$$

where $\varnothing_{ij}(n)$ = transition probability of the temporally continuous Semi-Markov process for interval $(0,n)$, with state i at moment $n = 0$ and state j at moment n; $w_i(n)$ = probability that the process will shift from its original state i at a moment larger than n. The second term in Eq. (7) describes the probability of the sequences in terms of events where the process transitions for the first time from state i into state k at a moment m, and subsequently throughout the remaining time span $n - m$ proceeds from state k to state j. Via summation of the probabilities of the first transition over all states k and all moments m (between 1 and n), all relevant scenarios are captured comprehensively; p_{ik} = probability of transition from state i to state k; and $h_{ik}(m)$ = probability distribution of the sojourn time from i to k at moment m. Eq. (7) can be written in matrix form as follows:

$$\Phi(n) = W(n) + \int_0^n [P \cdot H(m)] \cdot \Phi(n-m) \quad n = 0,1,2,\ldots \tag{9}$$

Notation $(\cdot)$ implies that the ijth element of matrix P is multiplied with the ijth element of matrix H

$$C(m) = P \cdot H(m) \tag{10}$$

and $C(m)$ is defined as the core matrix according to Howard [17]. The elements of $C(m)$ are $c_{ij}(m) = p_{ij} \cdot h_{ij}(m)$, where p_{ij} is the transition probability of the embedded Markov Chain; and $h_{ij}(m)$ is the probability distribution of the sojourn time in state i prior to the transition into state j at moment m. It is also assumed that the sojourn time in state i prior to the transition into state j equals the time interval between the time when the considered group first enters into state i and its first transition into state j.

Instead of calculating the transition probabilities for interval $(0, n)$, the yearly intervals $(m-1, m)$ can be determined for $m = 1,2,\ldots,n$ and multiplied with each other. The result is an approximation of the transition probabilities for interval $(0,n)$. Assuming that no more than one single transition occurs within e.g. the course of one year, the following simplification is permissible

$$\Phi^{0,n}(n) = \Phi^{0,1} \cdot \Phi^{1,2}, \ldots, \Phi^{n-1,n} \tag{11}$$

where $\Phi^{m-1,m}$ = single transition probability matrix for the time from $m-1$ to m (e.g. mth interval), $m = 1,2,\ldots,n$. For a state transition of a maximum of two category steps in one time interval, the transition probability matrix e.g. for the first year $m = 1$ can be written e.g. for ten assessment categories as:

Please note: As an example, the parameter $j = 4$ in the matrix belongs to $j = 0.4$ and indicates the degradation degree $D = 1 - j = 0.6$. In general, the analysis of fatigue risks (what is of interest in this contribution) only in rare cases exhibited a reduction of more than two category steps for a time interval of one year [26].

The reduction of two or more category steps can be controlled according to the adopted time unit, as there is no "immediate" reduction of more than two categories due to a natural process

[2] The groups are subdivided into five state categories according to their fatigue risk.

such as fatigue. It therefore falls outside the scope of the current study. The transition probability matrix of intervals $m > 1$ can be determined via the following alternative formulation. According to Cleves et al. [9] and Castillo et al. [7], the sojourn time distributions are truncated on the left side at the beginning of each interval. Accordingly, the accumulated distribution of the sojourn time for interval $(m-1, m)$ results in

$$H_{i,jT|T>n-1}(t) = \frac{H_{i,jT}(t) - H_{i,jT}(m-1)}{1 - H_{i,jT}(m-1)}; \quad m-1 < t \leqslant m \tag{13}$$

or for $t = m$ as the accumulated distribution of the sojourn time respectively, it can be written as

$$H_{i,jT|T>n-1}(m) = \frac{H_{i,jT}(m) - H_{i,jT}(m-1)}{1 - H_{i,jT}(m-1)} \tag{14}$$

and the transition probability matrix for interval $(m-1,\ m)$ takes the form of:

$$\Phi^{m-1,m}(m) = \begin{bmatrix} 1 - \sum_{j=4}^{9} p_{10,j} \cdot H_{10,jT|T>m-1}(m) & p_{10,9} \cdot H_{10,9T|T>m-1}(m) & p_{10,8} \cdot H_{10,8T|T>m-1}(m) & 0 & 0 \\ & 1 - \sum_{j=4}^{8} p_{9,j} \cdot H_{9,jT|T>m-1}(m) & \cdots & \cdots & 0 \\ \vdots & & \ddots & \cdots & \vdots \\ & & & \cdots & \cdots \\ & & & \ddots & \cdots \\ & & & \cdots & 1 \end{bmatrix} \tag{15}$$

Using Eq. (14), the single stage probability distributions of the state categories [5] are determined. For the embedded Markov Chain, the total transition probability is assumed to be 1. Consequently, Eq. (15) includes the probability of the sojourn time at the end of the period, provided that the sojourn time at the beginning of the period is available.

3. Case study on an offshore foundation of a wind turbine

Fig. 1(a) presents the NLF element model of the offshore wind energy tower concrete foundation in Cuxhaven, (a) top and side view of the discretized FEM with 36 elements in the cross section, (b) main dimensions of the foundation, (c) main axes of the considered cross section, and (d) detail of the reduced cross section next to the foundation boxes and the associated monitoring layout. The NLF element model of the entire concrete foundation consists of brick volume elements [25] and was generated using the software package SOFISTIK [14]. The brick elements make it possible to monitor the development of the stress distribution across the circular cross section. A concrete strength class C80/95 was chosen for the design layout of the circular shaft and the central section of the foundation, whereas the concrete strength class of the hollow boxes beneath the shaft is of C50/60 in accordance with EN1992. The mean of the concrete Young's modulus of the circular shaft was $E_{cm} = 48.5$ GPa, and was based on $n = 27$ samples taken from the foundation shaft. This high value in E_{cm} is a consequence of the aggregate properties of anorthosite and granite. Fig. 1(b) portrays the main dimensions of the foundation of the considered offshore wind energy plant. In an early phase, the full scale tests on the wind power plant with the unbalanced exciter and in consequence the monitoring on site were stopped due to energy policy decisions in Germany. The monitoring process was simulated numerically, whereas monitoring findings of laboratory fatigue tested reinforced concrete components were used for the NLF element analysis of the fatigue process in the real structure. With respect to the loading regime, the sea state demand was indirectly implemented in the generalized load spectrum as predefined by the contractor, see also [25]. The analyses and evaluations are based on mean value considerations in analogy to the nonlinear modeling.

On the basis of numerically and monitored data, as presented in Fig. 3(b), the parameters of the sojourn time distribution are derived from the Young's modulus condition state of the observed fatigue endangered cylindrical cross section. This fatigue endangered cross section was specified 0.30 m above the boxes of the foundation, as shown in Fig. 1(a) and (d). A horizontal cyclical load was applied with the first frequency, as it was planned for the full scale experiment in Cuxhaven [25], on the top of the foundation tower along the center line of one box. The concrete Young's modulus E_c was assumed to be initially constant across the entire circular cross section, and was then reduced step by step, in accordance with the increasing number of applied cyclic stress ranges $\Delta\sigma$, as illustrated in Fig. 1(a), in each of the 36 discretized cross sections.

The fatigue specific upper stress $S_{c,o} = \sigma_{max}$, and the fatigue specific lower stress $S_{c,u} = \sigma_{min}$ were determined by the dead weight of the structure g_1, the vertical prestressing p_0 of 15 tendons, and the cyclical load on the top of the foundation tower [25]. In consequence, the cyclic stress ranges $\Delta\sigma$ are computed as the difference between the time variable specific upper stress $S_{c,o} = \sigma_{max}(t)$ and the fatigue specific lower stress $S_{c,u} = \sigma_{min}(t)$.

In the early phase, the full scale tests on the wind power plant with the unbalanced exciter were stopped due to energy policy decisions in Germany. Therefore, the monitoring process was completed based on the initial monitoring results from the real structure using the NLF element analysis. The cyclic stress and stiffness performance of the investigated circular cross section were extracted initially from the monitoring system applied on the real structure, and in consequence from the described NLF element analysis which was based on laboratory concrete fatigue tests on structural components and on cylindrical concrete specimens. Fig. 2(b) presents the available monitored and reconstructed cyclic strain data for element 9 of the considered cross section.

As shown in Fig. 2, the fatigue specific upper and lower stress $S_{c,o} = \sigma_{max}$ and $S_{c,u} = \sigma_{min}$ and in consequence the cyclic stress ranges $\Delta\sigma$, which are the basic inputs for the N–S fatigue assessment approaches, changes with the increasing number of applied load cycles, due to the Young's modulus degradation in the cross section.

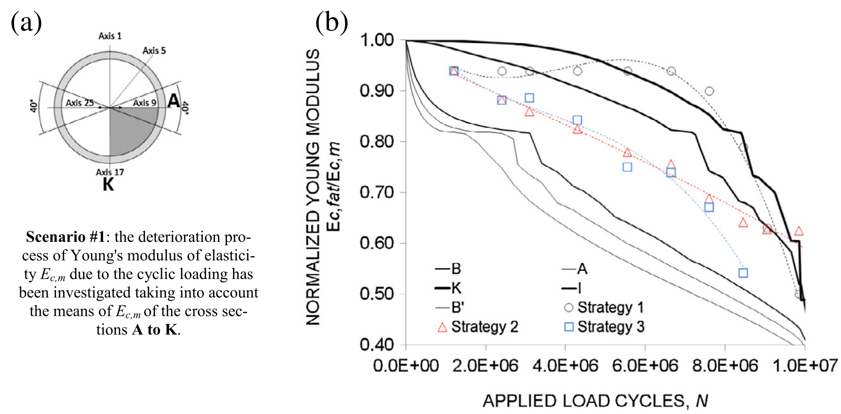


Fig. 3. Numerically determined prediction of the changes in the normalized Concrete Young's modulus E_c in the segments A to K of the circular cross-section of the offshore foundation and the Semi Markov Chain based predictions (for strategies 1–3 according to Eqs. (16)–(18)) on the basis of the transition probabilities of the segment group A to K.

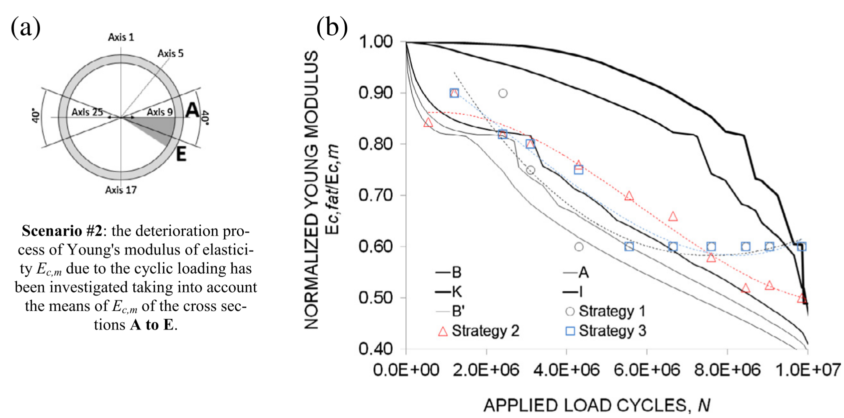


Fig. 4. Numerically determined prediction of the changes in the normalized Concrete Young's modulus E_c in the segments A to K of the circular cross-section of the offshore foundation and the Semi Markov Chain based predictions (for strategies 1–3 according to Eqs. (16)–(18)) on the basis of the transition probabilities of the segment group A to E.

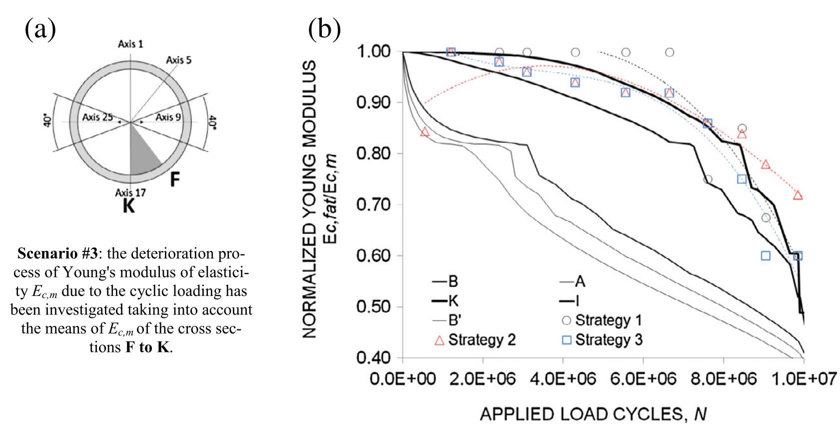


Fig. 5. Numerically determined prediction of the changes in the normalized Concrete Young's modulus E_c in the segments A to K of the circular cross-section of the offshore foundation and the Semi Markov Chain based predictions (for strategies 1–3 according to Eqs. (16)–(18)) on the basis of the transition probabilities of the segment group F to K.

In consequence, the investigated 5 sections (A, B, . . ., E – as shown in Figs 3–5 on top left) next to the Principal Load Direction PLD in axis A experiences an increase, and the perpendicular sections (F, G, . . ., K – as shown in Figs. 3–5 on top left) to the PLD exhibit a reduction in the remaining lifetime t_R. The process is caused by the decreasing stresses $S_{c,o} = \sigma_{max}$ and $S_{c,u} = \sigma_{min}$ in A–E and the increasing stresses in F–K. In other words, the remaining lifetime increases in the PLD since there is a reduction in the stress range of the associated S–N diagram, which is caused by the reduction in the Young's modulus and hence an increase in the applicable stress cycles. The stresses are redistributed to the sections perpendicular to the PLD with the minor reduced Young's modulus regions. The increase in the stress ranges causes, according to S–N diagrams, a reduction in the applicable stress cycles and hence a reduction in the remaining lifetime perpendicular to the PLD.

The numerical analyses served for the prediction of the general fatigue performance of the system, the prediction of the fatigue damage degree D_{fat} according to the S–N approach of fib [11], and the fatigue reliability assessment for each discretized section and the whole cross section.

Figs. 3–5 portray the simulated process of the normalized Young's modulus with regard to the mean value of the initial Young's modulus of elasticity $E_{c,m}$, where the degradation degree D in class 6 corresponds with a reduction of $E_{c,fat}/E_{c,m} = 0.5$. Fig. 3 shows, as is to be expected, a rapid initial drop of the normalized Young's modulus in the segments A, B and B′ and a delayed decrease in segments I and K.

These numerical findings combined with monitored data were the basis for the Markov Chain approach as described in Section 2.3. The Semi-Markov Chain approach was investigated for three scenarios, with regard to the computation of the condition states in the segments by using transition probability matrices. For these investigations, $2 \cdot 15$ transfer matrices $\Phi^{m-1,m}$ and $\Phi^{1,m}$ according to Eq. (15) were generated for predefined constant loading cycles $\Delta N = N_m - N_k$ (with $k = 1,\ldots,14$, $m = k + 1$, $\Delta N = 150{,}000$, and with $k = 1$, $m = 2,\ldots,15$, $\Delta N = m \cdot 150{,}000$) from numerical and monitoring findings. The maximum value $k = 14$ gives the critical load cycles of $N = k \cdot \Delta N = 14 \cdot 150{,}000 = 2 \cdot 10^6$ as defined by the contractor. In particular, the $2 \cdot 15$ transition probability matrices $\Phi^{m-1,m}$ and $\Phi^{1,m}$ were generated for Scenario #1, Scenario #2, and Scenario #3.

In Scenario #1, the deterioration process of Young's modulus of elasticity $E_{c,m}$ due to the cyclic loading was investigated, taking into account the means of $E_{c,m}$ of the cross sections A to K. In Scenario #2 the means of $E_{c,m}$ of the cross sections A to E, and in Scenario #3 the means of $E_{c,m}$ of the cross sections F to K have been taken into account for the investigation (see Figs. 3–5). It was of interest to see how the cross section grouping affects Semi-Markov prediction procedures.

It should be mentioned that there are more non-zero elements in the transition probability matrices $\Phi^{m-1,m}$ for Scenario #1 than for Scenarios #2 and #3, since the fatigue processes and the class change in this narrow range A to E run more concentrated.

In consequence, the prediction of the fatigue conditions or degradation degrees (*class* 1 = no degradation,…,*class* 6 = complete degradation) in the segments of the fatigue endangered cylindrical cross section were characterized according to the transmission procedures as shown in Eqs. (16)–(18):

$$\theta^{m(1)} = \Phi^{m-1,m} \cdot \theta^{m-1} \tag{16}$$

$$\theta^{m(2)} = \Phi^{1,m} \cdot \theta^{1} \tag{17}$$

$$\theta^{m(3)} = \Phi^{1,m} \cdot \theta^{m-1} \tag{18}$$

For the initial assumed fatigue condition θ^1, as presented in Table 1, the prediction strategies as depicted in the above equations, result in the fatigue conditions $\theta^{m(1)}$ of Procedure 1, $\theta^{m(2)}$ of Procedure 2 and $\theta^{m(3)}$ of Procedure 3, as shown in Figs. 3–5. The prediction process was based on the Semi-Markov approach that includes the sojourn time of the historical processes, as described under Clause 2.3. Strategy 1 shows a very reactive course of the prediction on the change in the assessment base, which is the numerically simulated normalized $E_{c,fat}/E_{c,m}$ up to time $m - 1$. Strategy 2 is more of a muted course and reflects the larger span of the transition probability matrices connecting the initial fatigue condition $\theta^{1(2)}$ with the fatigue condition $\theta^{m(2)}$ at prediction time m. Strategy 3 is similar to Strategy 2 in that it is more of a muted

Table 1
Probabilities of the initial fatigue state Φ^1.

Degradation class	1	2	3	4	5	6
θ^1	0.2	0.3	0.2	0.1	0.1	0
$E_{c,fat}/E_{c,m}$	1	0.9	0.8	0.7	0.6	0.5

course due to the large span characteristic of the transition probability. Nevertheless, the results are artificial, since $\theta^{(3)}{}_j$ at the assessment time j are used as updated initial fatigue condition $\theta_1^{(3)}$. These three strategies are all applied on the previously sketched Scenarios #1 to #3. As can be seen from Figs. 3–5, all of the three strategies cover Scenarios #1 to #3 very well. The Semi-Markov prediction processes for (a) Scenario #1 are located, as expected, between the simulated fatigue prediction of segment A and segment K, (b) Scenario #2 are located next to the simulated fatigue prediction of segments A and B, and (c) Scenario #3 are located next to segments K and I.

It can be concluded that the Semi-Markov Chain approach provided for all three strategies and scenarios a robust prediction of the fatigue process in the circular cross section of the considered fatigue endangered offshore foundation. These studies are based on numerical considerations. Nevertheless, the studies also demonstrate that the Semi-Markov Chain is appropriate to be based on continuous and discrete monitoring information. Furthermore, a Semi-Markov Chain prediction which is based on numerical considerations can be improved by including monitoring information in the form of Bayesian updating procedures, which will be the next steps in this research.

4. Summary

A Markov Chain prediction model was developed for the efficient fatigue evaluation of reinforced concrete structures. The model was applied in particular to a fatigue endangered circular cross section of an offshore foundation. The fatigue processes in the circular cross sections of the offshore foundation are characterized significantly by redistribution processes of the concrete Young's modulus E_c. As a consequence, any changes in the transmission probability matrices need to be taken into account and adapted accordingly, particularly where relevant Markov Chain predictions are applied. In order to determine the adaptation requirements of the transmission probability matrices, the entire fatigue process was analyzed using a linear step by step Finite Element fatigue evaluation method.

The concrete Young's modulus E_c was continuously adapted in the 36 segments of the circular cross section of the offshore foundation with respect to the number of applied load cycles, and the time variable transition probabilities $\Phi^{m-1,m}$ and $\Phi^{1,m}$ were determined for $36 \cdot 15 = 540$ groups of (a) concrete Young's modulus E_c of elasticity, (b) fatigue specific upper stress σ_o, (c) fatigue specific lower stress σ_u, and (d) degradation degrees D.

The Semi-Markov Chain approach was investigated for three scenarios, with regard to the number of segments used for the characterization of the transition probability matrices.

The studies show that the Semi-Markov Chain approach investigated with respect to different prediction strategies and scenarios can provide a robust prediction of fatigue processes in particular for circular cross sections e.g. as used for offshore foundations. The studies are based on numerical considerations, and it could be demonstrated that the Semi-Markov Chain is appropriate to be based on continuous and discrete monitoring information. In addition, a Semi-Markov Chain prediction which is based on numerical considerations should include monitoring information e.g. by Bayesian updating procedures.

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