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ORIGINAL PAPER

Improved PID controller tuning rules for performance degradation/robustness increase trade-off

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Abstract Definitely, robustness is an important feature that any control system must take into account, especially considering that the design is usually based on low-order linear models that represent the whole controlled process. The problem is that to include such characteristic implies a degradation in the system's performance. With regard to the previous statement, this paper is concerned with the design of the closed-loop control system, to take into account the system performance to load-disturbance and to set-point changes and its robustness to variation of the controlled process characteristics. The aim is to achieve a good balance between the multiple trade-offs. Here, a PID control design is provided that looks for a robustness increase, allowing some degradation in the system's combined performance. The proposed approach is complementary to the work presented by Arrieta and Vilanova (Simple PID tuning rules with guaranteed M_s robustness achievement, in 18th IFAC world congress, 2011; Ind Eng Chem Res 51(6):2666–2674, 2012. doi:10.1021/ie201655c); Arrieta et al. (Performance Degradation Driven PID controller design, in PID12, IFAC conference on advances in PID control, 2012).

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1 Introduction

Since their introduction in 1940 [16,17] commercial *Proportional-Integrative-Derivative* (PID) controllers have no doubt been the most extensive option that can be found on industrial control applications [13]. Their success is mainly due to their simple structure and the physical meaning of the corresponding three parameters (therefore, making manual tuning possible). This fact makes PID control easier to understand by the control engineers than other most advanced control techniques. In addition, the PID controller provides satisfactory performance in a wide range of practical situations.

With regard to the design and tuning of PID controllers, there are many methods that can be found in the literature over the last 70 years. Special attention is given to the contributions on the *IFAC Workshop PID'00—Past, Present and Future of PID Control*, held at Terrassa, Spain, on April 2000 and more recently on the *IFAC Conference on Advances in PID Control PID'12* that took place at Brescia, Italy, on March 2012, where a glimpse of the state of the art on PID control was provided. Moreover, because of the widespread use of PID controllers, it is interesting to have simple, but efficient methods for tuning the controller.

Since the seminal work of Ziegler and Nichols [37], an intensive research has been done, developing autotuning methods to determine the PID controller parameters

[2,14,25,31]. It can be seen that most of them are concerned with feedback controllers which are tuned either with a view to the rejection of disturbances [19,26] or for a well-damped fast response to a step change in the controller set point [27,29,30]. O'Dwyer [28] presents a complete collection of tuning rules for PID controllers, which show their abundance.

Moreover, in some cases the methods considered only the system performance [2,22], or its robustness [10,20,21]. However, the most interesting cases are the ones that combine performance and robustness, because they face all system's requirements [22,23,34,35].

Taking into account that in industrial process control applications, a good load-disturbance rejection (usually known as *regulatory control*) as well as a good transient response to set-point changes (known as *servo-control* operation) is required, the controller design should consider both possibilities of operation.

Despite the above, the servo and regulation demands cannot be optimally satisfied simultaneously with a one-degree-of-freedom (1-DoF) controller, because the resulting dynamic for each operation mode is different and it is possible to choose just one for an optimal solution.

Considering the previous statement, most of the existing studies have focused only on fulfilling one of the two requirements, providing tuning methods that are optimal to servo-control or to regulation control. However, it is well known that if we optimize the closed-loop transfer function for a step-response specification, the performance with respect to load-disturbance attenuation can be very poor and vice versa [3]. Therefore, it is desirable to get a compromise design, between servo/regulation when using the 1-DoF controller.

The proposed method considers 1-DoF PID controllers as an alternative when *explicit* two-degree-of-freedom (2-DoF) PID controllers are not available. Therefore, it could be stated that the proposed tuning can be used when both operation modes happen and it could be seen as an *implicit* 2-DoF approach (because the design takes into account both objectives, servo and regulation modes) [8,9].

Moreover, it is important that every control system provides a certain degree of robustness, to preserve the closed-loop dynamics, to possible variations in the process. Therefore, the robustness issue should be included within the multiple *trade-offs* presented in the control design and must be solved on a balanced way.

The previous cited methods study the performance and robustness jointly in the control design. However, no one treats specifically the performance/robustness *trade-off* problem, or consider in the formulation the servo/regulation *trade-off* or the interaction between all of these variables. In this sense, an initial stage has been performed by [4,5], providing a simple PID tuning that guarantees a certain level or value for the robustness characteristic taking into account at the same time the balance between the servo and regulation combined performance.

The approach presented in this paper is different, but complementary to the work in [4–6]. We provide a PID design based on the optimality degree of the system's performance. The tuning looks for a robustness increase, choosing an allowed degradation value in the combined performance. Therefore, it can be stated as the main contribution presented in this paper.

The paper is organized as follows. Section 2 introduces the control system configuration, the general framework, as well as some related concepts and methods. In Sect. 3, the performance optimality index is presented, whereas in Sect. 4 the proposed PID tuning is presented . Some examples are shown in Sect. 5 and the paper ends in Sect. 6 with some conclusions.

2 Materials and methods

2.1 Control system configuration

We consider the feedback control system shown in Fig. 1, where $P(s)$ is the controlled process, $C(s)$ is the controller, $r(s)$ is the set point, $u(s)$ is the controller output signal, $d(s)$ is the load disturbance and $y(s)$ is the system output.

The controlled process $P(s)$ will be represented by a First-Order-Plus-Dead-Time (FOPDT) model given by the transfer function of the form

$$P(s) = \frac{K}{1+Ts}\mathrm{e}^{-Ls}, \tag{1}$$

where K is the process gain, T is the time constant and L is the dead time. The normalized dead time is $\tau = L/T$.

This model is commonly used in process control, because it is simple and describes the dynamics of many industrial processes approximately [15]. The availability of the FOPDT models in the process industry is a well-known fact. The generation of such a model needs a very simple step-test experiment to be applied to the process. From this point of view, to maintain the need for plant experimentation to a minimum is a key point when considering the industrial application of a technique.

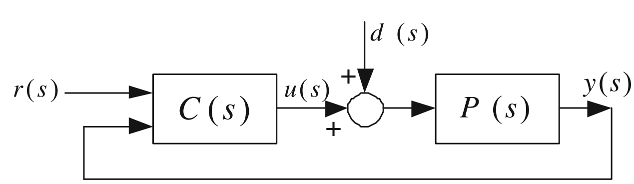

Fig. 1 Closed-loop control system

The output of the ISA-PID controller [15] is given by

$$u(s) = K_p\left(1 + \frac{1}{T_i s}\right)e(s) - K_p\left(\frac{T_d s}{1 + (T_d/N)s}\right)y(s), \tag{2}$$

where $e(s) = r(s) - y(s)$ is the control error, K_p the controller static gain, T_i the integral time constant, and T_d the derivative time constant; the derivative filter constant N is taken as $N = 10$, as it is a usual practice in industrial controllers.

As shown in (2), the derivative mode is applied only to the feedback signal, to avoid extreme instantaneous changes in the controller output signal when a set-point step change occurs [15].

2.2 Performance

One way to evaluate the performance of a control systems is by calculating a cost function based on the error, i.e., the difference between the desired value (set point) and the actual value of the controlled variable (system's output). Of course, as larger and longer time is the error, the system's performance will be worse.

In this sense, a common reference for the evaluation of the controller performance is a functional based on the integral of the error such as integral square error (ISE) or integral absolute error (IAE).

Some approaches have used the ISE criterion, because its definition allows an analytical calculation for the index [36]. However, nowadays it can be found in the literature that IAE is the most useful and suitable index to quantify the performance of the system [15,18,25,31,32]. It can be used explicitly in the design stage or just as an evaluation measure.

The formulation of the criterion is stated as

$$\mathrm{IAE} \doteq \int_0^{\infty} |e(t)|\,\mathrm{d}t = \int_0^{\infty} |r(t) - y(t)|\,\mathrm{d}t, \tag{3}$$

where the index can be a measure for changes in the set point or in the load disturbance.

2.3 Robustness

Robustness is an important attribute for control systems, because the design procedures are usually based on the use of low-order linear models identified at the closed-loop operation point. Due to the non-linearity found in most of the industrial process, it is necessary to consider the expected changes in the process characteristics assuming certain relative stability margins, or robustness requirements, for the control system.

As an indication of the system *robustness* (relative stability), the sensitivity function peak value will be used. The control system maximum sensitivity is defined as

$$M_s \doteq \max_{\omega} |S(j\omega)| = \max_{\omega} \frac{1}{|1 + C(j\omega)P(j\omega)|} \tag{4}$$

and recommended values for M_s are typically within the range 1.4–2.0 [15].

The use of the maximum sensitivity as a robustness measure has the advantage that lower bounds to the gain, A_m, and phase, ϕ_m, margins [15] can be assured according to

$$A_m > \frac{M_s}{M_s - 1}; \quad \phi_m > 2\sin^{-1}\left(\frac{1}{2M_s}\right).$$

Therefore, ensuring $M_s = 2.0$ provides what is commonly considered minimum robustness requirement (that translates to $A_m > 2$ and $\phi_m > 29^o$, for $M_s = 1.4$), we have $A_m > 3.5$ and $\phi_m > 41^o$.

In many cases, robustness is specified as a target value of M_s; however the accomplishment of the resulting value is never checked.

2.4 Simple PID tuning rules for arbitrary M_s-based robustness achievement

In [4], a joint criterion that faces the *trade-off* between the performance for servo and regulation operation and also that takes into account the accomplishment of a robustness level is presented.

A cost objective function is formulated, where J_x^z represents criteria (3) taking into account the operation mode x, for a tuning mode z. The index is stated to get the resulting point (J_r^{rd}, J_d^{rd}) as much closer as possible to the "ideal" one, (J_r^o, J_d^o). Therefore,

$$J_{rd} = \sqrt{\left(J_r^{rd} - J_r^o\right)^2 + \left(J_d^{rd} - J_d^o\right)^2}, \tag{5}$$

where J_r^o and J_d^o are the optimal values for servo and regulation control, respectively, and J_r^{rd}, J_d^{rd} are the performance indexes for the *intermediate* tuning considering both operation modes.

The index (5) is minimized with the aim of achieving a balanced performance for both operation modes of the control system. Also, using (4) as a robustness measure, the optimization is subject to a constraint of the form

$$|M_s - M_s^d| = 0, \tag{6}$$

where M_s and M_s^d are the maximum sensitivity and the desired maximum sensitivity functions, respectively. This

Table 1 PID settings for servo/regulation tuning with robustness consideration

Constant	M_s^d free	$M_s^d = 2.0$	$M_s^d = 1.8$	$M_s^d = 1.6$	$M_s^d = 1.4$
a_1	1.1410	0.7699	0.6825	0.5678	0.4306
b_1	−0.9664	−1.0270	−1.0240	−1.0250	−1.0190
c_1	0.1468	0.3490	0.3026	0.2601	0.1926
a_2	1.0860	0.7402	0.7821	0.8323	0.7894
b_2	0.4896	0.7309	0.6490	0.5382	0.4286
c_2	0.2775	0.5307	0.4511	0.3507	0.2557
a_3	0.3726	0.2750	0.2938	0.3111	0.3599
b_3	0.7098	0.9478	0.7956	0.8894	0.9592
c_3	−0.0409	0.0034	−0.0188	−0.0118	−0.0127

constraint tries to guarantee the selected robustness value for the control system. See [4] for more details.

The results are expressed just in terms of the FOPDT process model parameters (1), in a tuning methodology for PID parameters, with the corresponding desired level robustness as

$$\begin{aligned} K_p K &= a_1 \tau^{b_1} + c_1 \\ \frac{T_i}{T} &= a_2 \tau^{b_2} + c_2 \\ \frac{T_d}{T} &= a_3 \tau^{b_3} + c_3, \end{aligned} \tag{7}$$

where the constants a_i, b_i and c_i are given in Table 1, according to the desired robustness level for the control system.

With the aim of giving more completeness to the previous tuning method, an extension of the approach was proposed, allowing to determine the PID controller for any arbitrary value M_s^d in the range [1.4, 2.0] [5]. Thus, tuning expressions (7) can be rewritten as

$$\begin{aligned} K_p K &= a_1(M_s^d)\tau^{b_1(M_s^d)} + c_1(M_s^d) \\ \frac{T_i}{T} &= a_2(M_s^d)\tau^{b_2(M_s^d)} + c_2(M_s^d) \\ \frac{T_d}{T} &= a_3(M_s^d)\tau^{b_3(M_s^d)} + c_3(M_s^d), \end{aligned} \tag{8}$$

where the constants are expressed as functions of M_s^d. Therefore, from Table 1, each constant a_i, b_i and c_i was generated from a generic second-order M_s^d-dependent polynomial as

$$\begin{aligned} a_1 &= -0.3112(M_s^d)^2 + 1.6250(M_s^d) - 1.2340 \\ b_1 &= 0.0188(M_s^d)^2 - 0.0753(M_s^d) - 0.9509 \\ c_1 &= -0.1319(M_s^d)^2 + 0.7042(M_s^d) - 0.5334 \\ a_2 &= -0.5300(M_s^d)^2 + 1.7030(M_s^d) - 0.5511 \\ b_2 &= -0.1731(M_s^d)^2 + 1.0970(M_s^d) - 0.7700 \\ c_2 &= -0.0963(M_s^d)^2 + 0.7899(M_s^d) - 0.6629 \\ a_3 &= 0.1875(M_s^d)^2 - 0.7735(M_s^d) + 1.0740 \\ b_3 &= 1.3870(M_s^d)^2 - 4.7810(M_s^d) + 4.9470 \\ c_3 &= 0.1331(M_s^d)^2 - 0.4733(M_s^d) + 0.4032. \end{aligned} \tag{9}$$

It is important to note that the tuning just depends on the system's model information and the design parameter M_s^d. Moreover, it is worth noting that each one of the parameters (7) and (8) are generated according to a relation of the form $p_i = a_i \tau^{b_i} + c_i$.

3 Performance optimality index

The analysis exposed here shows the interaction between performance and robustness in control systems. It is possible to say that an increase of robustness implies an optimality loss in the performance (i.e., a degradation), with respect to the one that can be achieved without any robustness constraint.

It is possible to define the degree of optimality of the constrained case, with respect to the unconstrained one (that is the optimum and corresponds to the M_s^d-free case in Table 1).

To quantify the degree of optimality, the following index is proposed

$$I_{\text{Perf}} \doteq \frac{J_{rd}^o}{J_{rd}^{rdM_s}}, \tag{10}$$

where J_{rd}^o is the optimal index value (5), using the tuning (7) for no constraint of M_s (first column of Table 1), which means that the best one can be achieved. Then, $J_{rd}^{rdM_s}$ is the value of index (5) for the cases where the tuning has a robustness constraint.

Note that (10) is normalized in the [0, 1] range, where $I_{\text{Perf}} = 1$ means a perfect optimality and, as much as the robustness is increased, the index $J_{rd}^{rdM_s}$ will increase and, consequently, $I_{\text{Perf}} < 1$, meaning an optimality reduction.

The degree of optimality that each control system achieves, when a desirable value of M_s is stated, can be evaluated taking advantage of the generic tuning rule (8). For each value of $M_s^d \in [1.4–2.0]$, the optimality degree (10) can be obtained.

For each τ, we take advantage of the possibilities of the tuning (8) and (9), to get the PID parameters for any value of $M_s^d \in [1.4–2.0]$ and then compute the degree of optimality

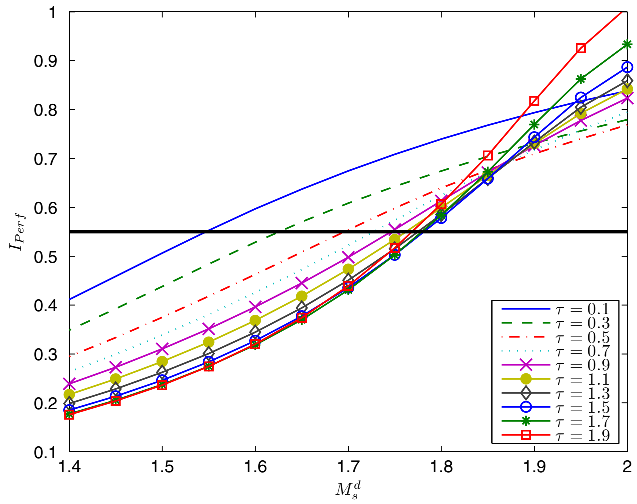

Fig. 2 Variation of the index I_{Perf}

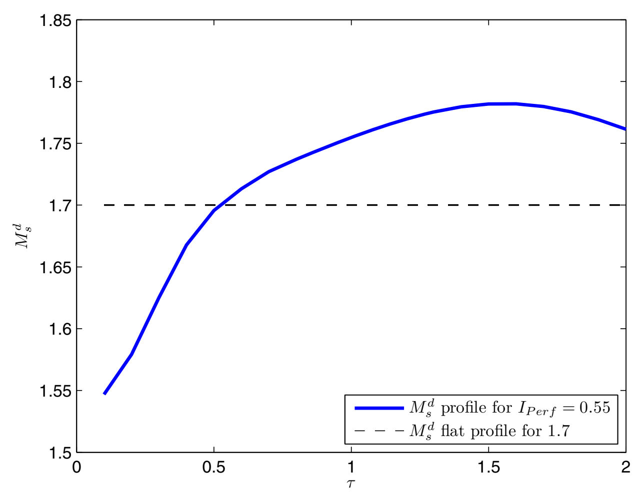

Fig. 3 M_s^d profile corresponding to a 55 % optimality degree

using (10). Figure 2 shows the I_{Perf} variation, as a function of M_s^d, for some values of the normalized dead time, τ.

Note that, as an example, the horizontal line indicates when the degree of optimality is 55 %. With the intersection points of this line and the curves corresponding to the I_{Perf} variation for each τ, it is possible to determine a set of desired robustness that is related with this degree of optimality ($I_{\text{Perf}} = 0.55$). This set of $M_s^d(\tau)$ can be seen as a robustness profile that the tuning should attain to accomplish such a degree of optimality. Figure 3 shows the details where also, just to clarify, there is the case of $M_s^d = 1.7$ that can be considered as a flat profile.

4 PID tunings with performance optimality degree

As it has been shown above, there is a relation (*trade-off*) between the degree of optimality and the increase in the system's robustness. It is possible therefore to find the corresponding M_s^d value for any specific optimality, as a point (M_s^d, I_{Perf}).

Following the above idea for all plants in the range $\tau \in [0.1, 2.0]$, fixing a certain degree of optimality, we can look for the corresponding set of M_s^d values. So, using the robustness profile in tuning (8) and (9), the controller's parameters $[K_p, T_i, T_d]$ can be obtained.

Here, with the aim of facilitating the understanding of the general idea and taking into account that it could be easier to specify a certain degradation, than a degree of optimality (i.e., an optimality loss), we redefine (10) as

$$Deg \doteq 1 - I_{\text{Perf}}. \tag{11}$$

In this sense, a desirable optimality degree of 60 % can be interpreted as a 40 % of degradation. The general concept is exactly the same, but just the way of interpretation is changed.

Using a similar idea to the one exposed in Sect. 2.4, we look for a tuning methodology that uses the degradation as a parameter design, to increase the robustness of the system.

4.1 PID tuning for fixed performance degradation levels

The previously exposed procedure tries to achieve that; by allowing a degradation in the performance, the system's robustness can be increased [6].

We will define a broad classification, to fix the levels according to the information provided in Fig. 2. In this sense, the degradation of the system performance will be used, Deg, as a design parameter.

So, the aim is to obtain profiles of M_s^d for the range of $\tau \in [0.1, 2.0]$. Therefore, the selected optimality degree level must intersect each one of the curves corresponding to each one of the plants. To get a degree of optimality higher than 75 %, the range of considered robustness should be extended to values greater than $M_s^d = 2.0$, but this value is the minimum acceptable robustness. On the other side, to have an optimality degree lower than 45 % the robustness values must be lower than $M_s^d = 1.4$, which is considered as a high robustness level; therefore, decreasing the degree of optimality to less than 45 % (meaning a degradation of more than 55 %) is not justified.

Then, the range of application was established as $Deg \in [0.25, 0.55]$ and therefore the classification as

– Low degradation—$Deg = 0.25$
– Medium–low degradation—$Deg = 0.35$
– Medium–high degradation—$Deg = 0.45$
– High degradation—$Deg = 0.55$.

As stated above, for each stated degradation level and each τ, the corresponding M_s^d value is found. Then, the set of

Table 2 PID tuning settings for the allowed performance degradation

Constant	$Deg^a = 0$	$Deg^a = 0.25$	$Deg^a = 0.35$	$Deg^a = 0.45$	$Deg^a = 0.55$
a_1	1.1410	0.8787	0.7490	0.6292	0.5252
b_1	−0.9664	−0.9280	−0.9348	−0.9444	−0.9492
c_1	0.1468	0.2033	0.2669	0.3195	0.3494
a_2	1.0860	0.8154	0.8664	0.8871	0.8755
b_2	0.4896	0.6431	0.6033	0.5847	0.5830
c_2	0.2775	0.4502	0.3874	0.3466	0.3275
a_3	0.3726	0.2794	0.2757	0.2804	0.2949
b_3	0.7098	0.8765	0.8698	0.8471	0.8123
c_3	−0.0409	−0.0149	−0.0070	−0.0037	−0.0055

robustness values determines the M_s^d profile that is used in the proposed generic tuning (8) and (9) to determine the sets for each parameter of the PID controller. Therefore, with all the parameter sets, the tuning rule can be formulated.

Once again, following a similar idea to that described for the M_s^d case, the aim is to provide a simple tuning and for that we take advantage of the good fitting that equations (7) provide. So, the sets for each PID parameter and for each degradation level are approximated to fit the corresponding equations' form.

The tuning rule remains expressed, according to the form in (7), but the a_i, b_i and c_i constants are given in Table 2, according to the *allowed degradation* level in the system's performance, Deg^a.

Table 2 shows, in the first column, the case for $Deg^a = 0$ that is exactly similar to the one in Table 1 for M_s^d free (without any constraint), but it is included here to give completeness to the approach. Note also that keeping the same tuning expressions (7) provides even more uniformity and simplicity to the proposed approach.

The evaluation of the above proposed tuning rule has to be done taking into account both performance and robustness issues. To study the system's performance, in Fig. 4 there are the indexes J_{rd}, for each case of Deg^a.

Once more, it is important to see how the changes in the performance (due to the imposed degradation) affects the achieved robustness for the system. Figure 5 shows this evaluation, where the optimality decreases (i.e., degradation increases), and the robustness of the system grows up. This is an important aspect because these M_s values represent the profile that should be accomplished to achieve a fixed degradation (meaning a certain degree of optimality).

It can be seen that all results are in agreement with regard to the well-known performance/robustness ratio.

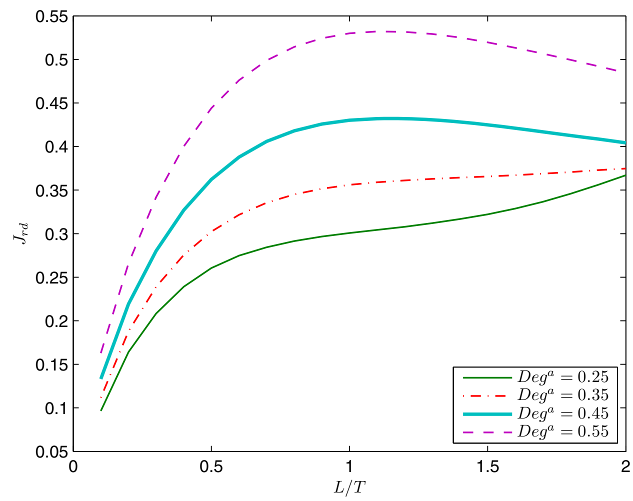

Fig. 4 Combined index J_{rd} for each degradation level tuning

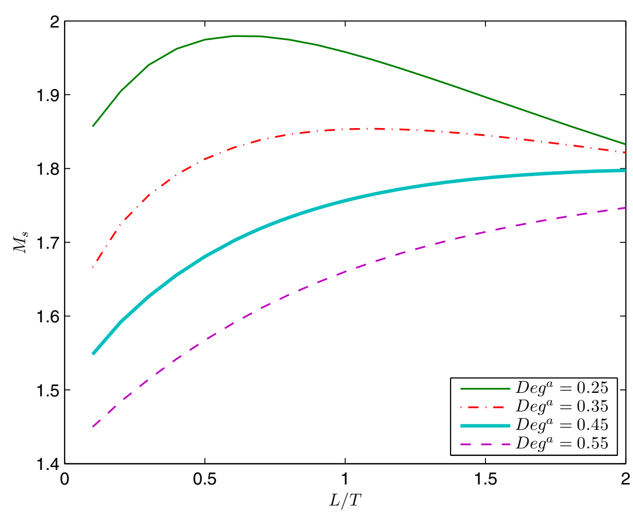

Fig. 5 Achieved robustness M_s for each degradation level tuning

4.2 PID tuning for an arbitrary performance degradation

Analogously to the tuning presented in Sect. 2.4, for fixed values of M_s^d, we want to give here a formulation that allows us to specify any value for the allowed degradation. Because the approach is based on the information provided by the fixed degradation levels, the range of validity is within $Deg^a \in [0.25, 0.55]$.

It is important to emphasize that, this extension is possible due to the simplicity and uniformity of the PID tuning parameters (7), which have the same expression for each one of the fixed degradation levels.

The aim is to provide a generic formulation to give completeness, as much as possible. Therefore, because each controller parameter has the same form, we look for a general equation, rewritten (7) as,

$$\begin{aligned} K_p K &= a_1(Deg^a)\tau^{b_1(Deg^a)} + c_1(Deg^a) \\ \frac{T_i}{T} &= a_2(Deg^a)\tau^{b_2(Deg^a)} + c_2(Deg^a) \\ \frac{T_d}{T} &= a_3(Deg^a)\tau^{b_3(Deg^a)} + c_3(Deg^a), \end{aligned} \tag{12}$$

where the constants are expressed as functions of Deg^a. Then, from Table 2, the constants a_i, b_i and c_i with their respective Deg^a value are fitted to a second-order polynomial as

$$\begin{aligned} a_1 &= 0.6425(Deg^a)^2 - 1.6940(Deg^a) + 1.2620 \\ b_1 &= 0.0500(Deg^a)^2 - 0.1132(Deg^a) - 0.9024 \\ c_1 &= -0.8425(Deg^a)^2 + 1.1650(Deg^a) - 0.0359 \\ a_2 &= -1.5650(Deg^a)^2 + 1.4530(Deg^a) + 0.5499 \\ b_2 &= 0.9525(Deg^a)^2 - 0.9609(Deg^a) + 0.8236 \\ c_2 &= 1.0930(Deg^a)^2 - 1.2830(Deg^a) + 0.7026 \\ a_3 &= 0.4550(Deg^a)^2 - 0.3128(Deg^a) + 0.3292 \\ b_3 &= -0.7025(Deg^a)^2 + 0.3467(Deg^a) + 0.8339 \\ c_3 &= -0.2425(Deg^a)^2 + 0.2255(Deg^a) - 0.0561. \end{aligned} \tag{13}$$

Specifically, controller parameters (12) jointly with the resulting constants (13), provide the PID controller tuning that choosing an arbitrary degradation value for the system performance increases the robustness, as much as possible, for the prescribed degree of optimality.

5 Comparative simulation examples

5.1 Example 1

To evaluate the proposed tuning, we will consider the following fourth-order controlled process

$$P_1(s) = \frac{1}{\prod_{n=0}^{3}(\sigma^n s + 1)} \tag{14}$$

with $\sigma = 0.50$ taken from [12]. Using a two-point identification procedure [1], an FOPDT model was obtained as: $K = 1.0$, $T = 1.247$ and $L = 0.691$.

Table 3 Particular process—PID controller parameters for P_1 ($\sigma = 0.50$)

Tuning	Deg^a	K_p	T_i	T_d
Proposed (levels)	0.25	1.723	1.257	0.189
	0.35	1.568	1.240	0.197
	0.45	1.418	1.216	0.207
	0.55	1.269	1.182	0.221
$\kappa - \tau$ ($M_s^d = 2.0$)	–	1.719	1.151	0.285
Proposed (generic)	$Deg^{\kappa-\tau}$	1.595	1.243	0.195

From the FOPDT model and using tuning (7) for each fixed degradation level of Table 2, the PID parameters can be obtained. In addition, the proposed generic tuning (12) and (13), can be used for an arbitrary value of the allowed degradation. As a specific case, the kappa–tau tuning rule ($\kappa - \tau$) [11] for $M_s = 2.0$ is considered here. Therefore, assigning $Deg^a = Deg^{\kappa-\tau}$ is setting the same degradation value for both tunings, therefore allowing for a fair comparison, to analyze the resulting robustness for each case. The parameters are shown in Table 3.

Table 4 gives the performance and robustness values provided by each tuning. On the other hand to evaluate the manipulated input usage, the total variation of the control effort $u(t)$ is defined, for a discrete signal as the sum of the size of its increments $TV_u = \sum_{k=1}^{\infty} |u_{k+1} - u_k|$. This quantity can be computed for changes in the set point (TV_{ur}) or load disturbance (TV_{ud}) and should be as small as possible. It provides a measure of the *smoothness of the control signal*. This will provide a more global and complete comparison framework.

In addition, in Fig. 6, the control system's and controller's output are shown for each allowed degradation level, whereas in Fig. 7, it is possible to see the comparison between the $\kappa - \tau$ tuning and the proposed settings with the specific value of degradation. To be more realistic, it is considered that the controllers operate at 70 % of their operating regime.

From the two approaches of the proposal, it can be concluded that the levels version has a good accuracy with respect to the selected value of the allowed performance degradation, providing at the same time an increase in the robustness and a reduction in the control effort.

Concerning the proposed tuning for an arbitrary value of Deg^a, from Table 4, it is possible to see that compared to the $\kappa - \tau$ tuning, the achieved performance is practically the same (because they have the same degradation value—$Deg^a = Deg^{\kappa-\tau}$); however, in the proposed tuning the robustness is much better ($M_s = 1.8474$ against $M_s = 2.0626$). So, concluding, for the same performance, the proposed tuning provides greater robustness and smoother control signal; therefore, being a better option to tune the controller.

Table 4 Particular process P_1 ($\sigma = 0.50$)—controller evaluation

Tuning	Deg^a	Deg^r	M_s^r	J_r	J_d	J_{rd}	TV_r	TV_d
Proposed (levels)	0.25	0.2508	1.9780	1.4229	0.7568	0.3024	1.9845	1.4192
	0.35	0.3575	1.8217	1.4421	0.8208	0.3609	1.6617	1.3415
	0.45	0.4657	1.6923	1.4755	0.8954	0.4392	1.3928	1.2771
	0.55	0.5645	1.5800	1.5283	0.9871	0.5438	1.1693	1.2271
$\kappa - \tau$ ($M_s^d = 2.0$)	–	0.3325	2.0626	1.4597	0.7366	0.3189	1.7461	1.3104
Proposed (generic)	$Deg^{\kappa-\tau}$	0.3379	1.8474	1.4378	0.8086	0.3491	1.7151	1.3544

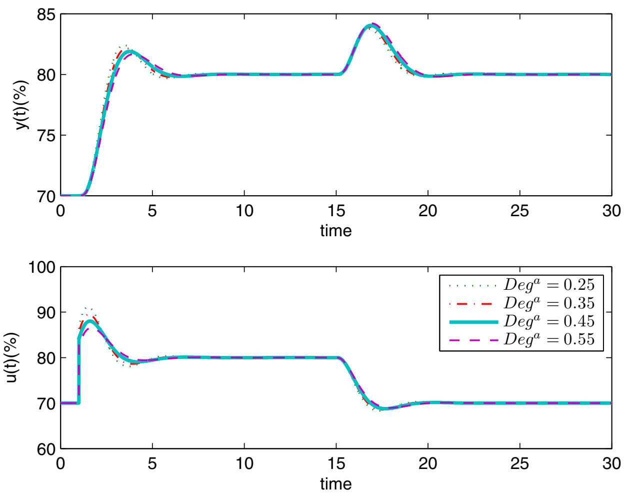

Fig. 6 Particular process P_1—the proposed method ($\sigma = 0.50$)

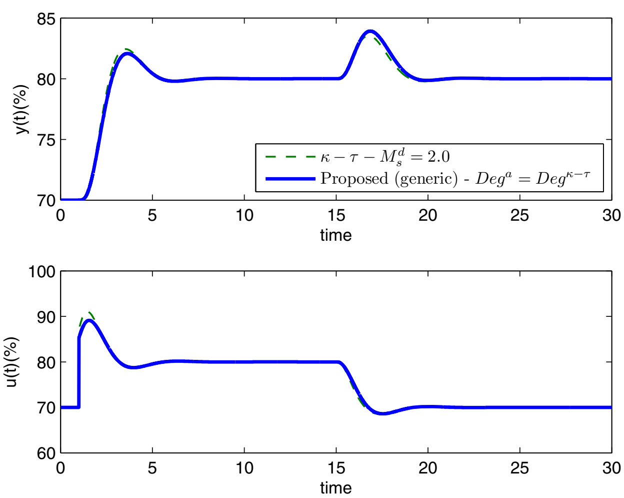

Fig. 7 Particular process P_1—control system responses ($\sigma = 0.50$)

5.2 Example 2

To add completeness to the comparison, a case study example is provided. We consider the isothermal continuous stirred tank reactor (CSTR), as the one in Fig. 8, where the isothermal series/parallel Van de Vusse reaction [24,33] takes place.

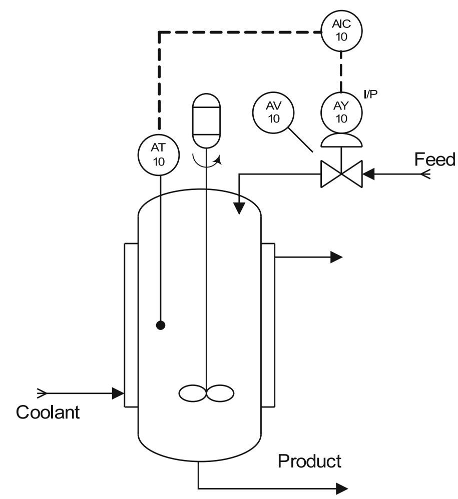

Fig. 8 CSTR system

The reaction can be described by the following scheme:

$$A \xrightarrow{k_1} B \xrightarrow{k_2} C$$
$$2A \xrightarrow{k_3} D. \tag{15}$$

Doing a mass balance, the system can be described by the following model:

$$\frac{\mathrm{d}C_A(t)}{\mathrm{d}t} = \frac{F_r(t)}{V}[C_{Ai} - C_A(t)] - k_1 C_A(t) - k_3 C_A^2(t)$$
$$\frac{\mathrm{d}C_B(t)}{\mathrm{d}t} = -\frac{F_r(t)}{V} C_B(t) + k_1 C_A(t) - k_2 C_B(t), \tag{16}$$

where F_r is the feed flow rate of product A, V is the reactor volume which is kept constant during the operation, C_A and C_B are the reactant concentrations in the reactor, and k_i ($i = 1, 2, 3$) are the reaction rate constants for the three reactions.

In this case, the variables of interest are: the concentration of B in the reactor (C_B as the controlled variable), the flow through the reactor (F_r as the manipulated variable), and the concentration C_{Ai} of A in the feed flow (whose variation can be considered as the disturbance). The kinetic parameters are chosen to be $k_1 = 5/6\ \text{min}^{-1}$, $k_2 = 5/3\ \text{min}^{-1}$, and $k_3 = 1/6\,\text{l}\,\text{mol}^{-1}\ \text{min}^{-1}$. Also, it is assumed that the nominal

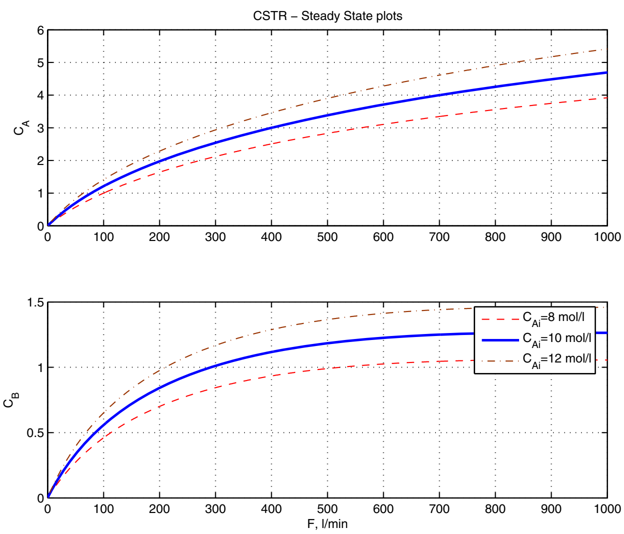

Fig. 9 Example 2—steady-state characterization for the reactor

concentration of A in the feed (C_{Ai}) is $10\,\mathrm{mol\,l^{-1}}$ and the volume $V = 700\,\mathrm{l}$.

Using (16) and the parameter values, the characterization of the steady state for the process can be obtained as shown in Fig. 9, for three concentrations of C_{Ai}, where the non-linearity of the system is easy to see.

Initially, the system is at the steady state (therefore, the operational point) with $C_{Ao} = 2.9175\,\mathrm{mol\,l^{-1}}$ and $C_{Bo} = 1.10\,\mathrm{mol\,l^{-1}}$. From this, the measurement range for C_B from 0 to 1.5714 mol/l and the capacity for the control valve with a maximum flow of 634.1719 l/ min (variation range of the flow from 0 to 634.1719 l/ min) can be selected [7]. The signals (y, u, r) will be in the percentage (0–100 %).

The sensor-transmitter element takes the form

$$y(t)_{\%} = \left(\frac{100}{1.5714}\right) C_B(t) \tag{17}$$

and the control valve with a linear flow characteristic,

$$F_r(t) = \left(\frac{634.1719}{100}\right) u(t)_{\%}. \tag{18}$$

Figure 10 shows the steady-state characterization, taking into account elements represented by (17) and (18). This is called *set actuator-process-sensor*, and from this it is clear that for the selected steady state, $r_o = 70\,\%$ and $u_o = 60\,\%$.

It is assumed that changes in the set point would be not bigger than 10 % and the possible disturbance in C_{Ai} can vary around ±10 %. In Fig. 11, the process output (including the sensor and the control valve) and also the FOPDT model for a step change in the process input ($y_u(t)$)can be seen.

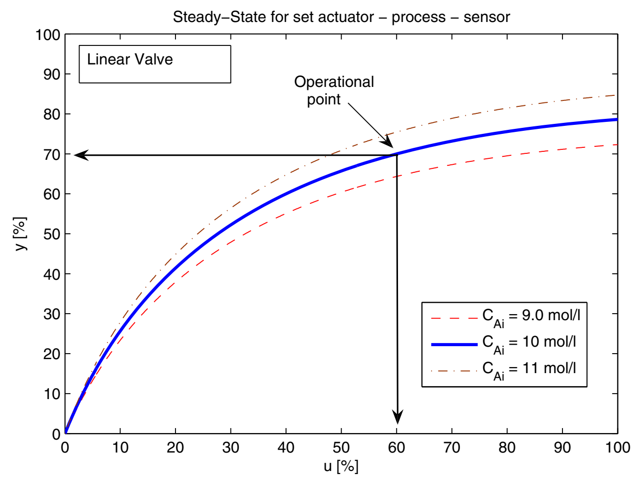

Fig. 10 Example 2—steady-state characterization for the set actuator-process-sensor

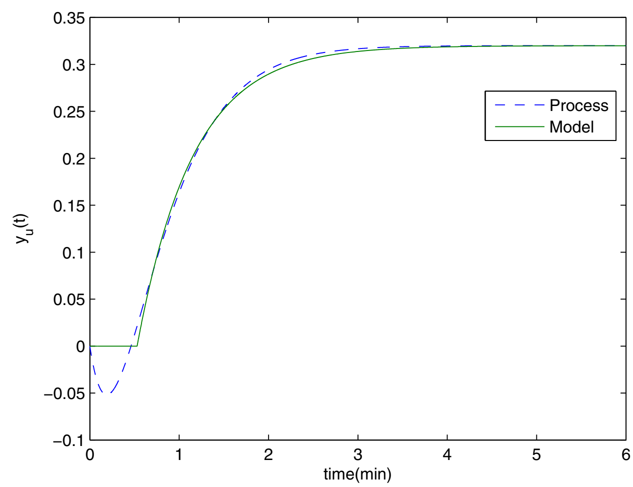

Fig. 11 Example 2—reaction curve for the process and the FOPDT model

Using the identification method [1], the determined FOPDT model is

$$P_2(s) \approx \frac{0.3199\mathrm{e}^{-0.5289s}}{0.6238s + 1}. \tag{19}$$

From (19), using the tuning formulae (7) for each fixed degradation level in Table 2, the PID controller parameters can be computed. Table 5 shows the results.

The process outputs of the closed-loop system are shown in Fig. 12, first for a set-point step change of −10 %, followed by a disturbance of +10 % and finally a new change in the set point of +5 %, all these situations using the proposed tuning. Also, the control signal ($u(t)$) can be seen. It appears that, as expected, the control signal is smoother for higher values of Deg^a.

Table 6 shows the resulting M_s values for each tuning case, where it can be seen that on increasing the allowed perfor-

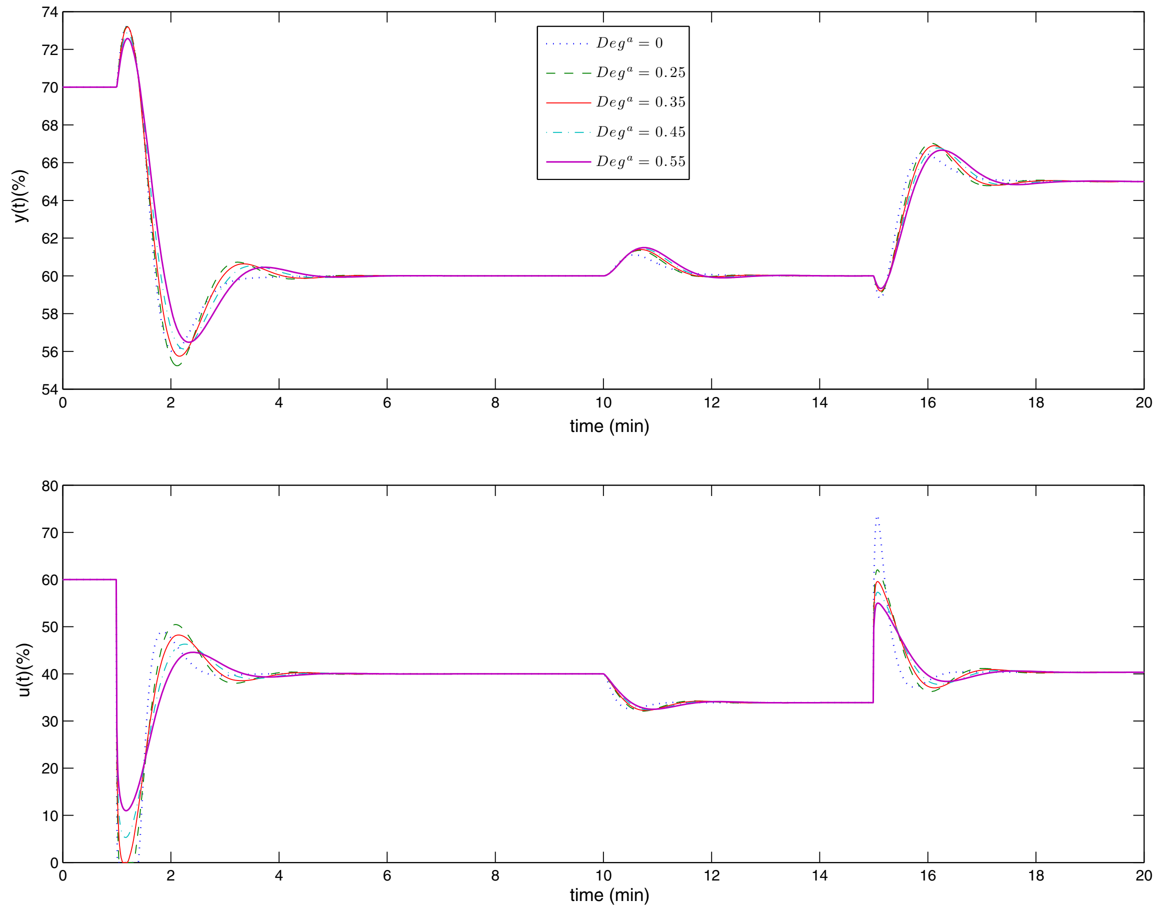

Fig. 12 Example 2—process output for the non-linear control system operating in both servo and regulation modes

Table 5 Example 2—PID controller parameters for P_2

Tuning	Deg^a	K_p	T_i	T_d
Proposed	0.00	4.642	0.798	0.181
	0.25	3.837	0.738	0.142
	0.35	3.566	0.731	0.145
	0.45	3.297	0.719	0.150
	0.55	3.012	0.700	0.157

Table 6 Example 2—robustness increase

Tuning	Deg^a	M_s^r
Proposed	0.00	2.497
	0.25	1.971
	0.35	1.849
	0.45	1.740
	0.55	1.637

mance degradation (meaning the performance is decreasing), an improvement of the robustness of the control system can be achieved. If fact, just allowing a performance degradation of 25 %, it is possible to obtain the standard minimum control system robustness of $M_s = 2.0$.

6 Conclusions

The control system's *trade-off* between performance and robustness can be studied from two points of view: as shown in Sect. 2.4, selecting a desirable value for robustness and facing the resulting performance degradation.

In this paper, we formulated the problem from the other side, selecting an allowed performance degradation to get a higher robustness, with respect to the case with zero degradation. The proposal is presented for some degradation levels (qualitative classification) and also for generic specific values of degradation within the range 25–55 %.

The results are presented as autotuning formulae, maintaining the same simplicity shown before for other proposed PID tuning approaches. The examples show the accuracy and the benefits of the contribution where the achieved increase in the robustness of the control system can be highlighted.

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