Construction and Building Materials 127 (2016) 884–895

Contents lists available at ScienceDirect

Construction and Building Materials

journal homepage: www.elsevier.com/locate/conbuildmat

Reliability analysis of rammed earth structures

Ehsan Kianfar, Vahab Toufigh *

Department of Civil Engineering, Sharif University of Technology, Tehran, Iran

HIGHLIGHTS

- Reliability analysis for rammed earth structures is performed.
- Uncertainty is included in parameters contributed to resistance and loads.
- FORM, SORM and Monte Carlo Sampling method were used and compared.
- Random variables with most impact on the reliability index are specified.
- Recommendations are given for minimum compressive strength and wall thickness.

ARTICLE INFO

Article history:
Received 23 July 2016
Received in revised form 26 September 2016
Accepted 6 October 2016
Available online 15 October 2016

Keywords:
Rammed earth
Reliability
Earthen materials
FORM
SORM
Monte Carlo Sampling
Sensitivity analysis

ABSTRACT

Rammed earth (RE) structures are widely used for more sustainable and environment-friendly buildings. Due to lack of design standards, the engineering decisions often rely on rule-of-thumb method which may lead to quite conservative or unsafe designs. In this study, load and resistance parameters were treated as random variables in reliability analysis. The reliability index and failure probability of RE structures were evaluated using First-Order-Reliability-Method (FORM) and then compared with Second-Order-Reliability-Method (SORM) and Monte Carlo Sampling method. The analysis was performed based on the different a) load combinations, b) wall geometry, c) material type (unstabilized or cement stabilized) and d) mechanical properties of the materials. Based on the results, the RE wall under moderate loading conditions require smaller wall thickness than recommended wall thickness by various guidelines such as New Mexico-USA, New Zealand and Zimbabwe Codes. However, larger wall thickness is needed under severe loadings conditions, especially when unstabilized materials are used. The compressive strength of unstabilized materials under severe loading conditions should be more than minimum recommended. The sensitivity analysis was performed by calculating different importance and sensitivity vectors. The results show that the compressive strength and the environmental loads factors are the most important random variables that contribute to reliability of the structures. The recommended wall thickness and compressive strength for different conditions are presented.

1. Introduction

Nowadays, construction with earthen materials has become more widespread due to their advantages such as low construction cost, low embodied energy and recyclability of materials. It is estimated that near to one-third of the world population live in the houses made of natural soils like earth block (adobe) masonry, cob, and rammed earth (RE) [1,2]. Rammed earth houses have been used in all around the world for decades. They are built in many countries such as Australia, China, India and many parts of Africa and Europe. The rammed earth materials are cheap, tough, and green. These structures commonly used where skilled labors are not available, using modern technologies is not possible, and/or due to impassable roads the cost of transportation is relatively high. Earthen materials have great impact on reducing the environmental effects of industrial constructions and also have economical superiority by using in-site raw materials. However, they need continuous maintenance because of susceptibility to erosion, physical degradation and cracks under low tensile and shear stresses. Due to high mass, low ductility and low tensile and shear strength they are susceptible to high damages in areas with high or moderate seismic risk [2–4].

The rammed earth houses are typically supported by bearing walls and are generally low rise (single or two-story), although higher rammed earth structures have been built. The rammed earth structures are made of a mixture of soil (local earthen

---

* Corresponding author.
E-mail address: toufigh@sharif.edu (V. Toufigh).

http://dx.doi.org/10.1016/j.conbuildmat.2016.10.052
0950-0618/© 2016 Elsevier Ltd. All rights reserved.

materials) containing a binder. The mixture of soil is prepared in near its optimum moisture content to maximize its dry density and is formed in temporary formworks in layers about 10 to 15 cm thick. The earthen material is then compacted by using manual or pneumatic rammers to have layers about 6 to 10 cm. The layers are increased up to the desired level. The soil composition varies greatly and is ideally sandy-clay gravel. Rammed earth structures are divided to two groups: Stabilized and unstabilized rammed earth. When the binder is just clay, this material is referred as "Unstabilized Rammed Earth" (URE) and when the binder is cement or lime, is referred as "Stabilized Rammed Earth" (SRE). The stabilization of soil improves mechanical properties and durability of the structure; however, it has detrimental environmental impact and increases the embodied energy [1,5,6].

The suitability of soils is typically checked based on the particle size distribution (percentages of clay, sand, and gravel) and Atterberg limits. The minimum compressive strength of the suitable soils are specified between 1 to 2 MPa in different guidelines [7–10], although higher values especially for stabilized materials have been gained. Because of variety of soil composition in different locations and not sufficient researches on the behavior of rammed earth structures, there is no comprehensive design and construction provisions for them with the same approach as for the steel and concrete structures. There are only some guidelines and handbooks which give mostly some recommendations and advices in this regard. Hence, the design of rammed earth structures are traditionally based on "rule of thumb" method and it may lead to large safety factors and increase in cost of structures. On other hand, it would result in unsafe shelters in area with severe environmental loads such as heavy snow loads, storms or ground motions [6,11–14].

The objective of this research is to perform reliability analysis on URE and cement stabilized rammed earth (CSRE) structures under dead, live, and relatively severe and moderate wind and snow loads. The results of probabilistic approach dealing with uncertainties is used to check the general recommendations on the wall thickness and compressive strength of materials. By performing sensitivity analysis, the random variable parameters which have most effect on the reliability index and strength of the buildings are determined. The recommendations about minimum compressive strength of materials and minimum wall thickness subject to common loads are presented.

2. Resistance random variables

In classical deterministic analysis, uncertainties are not included in structural design, and parameters are considered by their worst cases. Whereas, resistance parameters, applied loads, and the probability of the occurrence of them consist of randomness. To apply reliability concepts in analysis of the structures, it is essential to define the random variables and limit states precisely. Random variables describe the uncertainty and are modeled with given distribution functions and distribution parameters. Limit-state function defines the event for which the probability is considered [15].

In this section, the random variables which represent the uncertainties of the resistance of the URE and CSRE structures are discussed. The applicable probability distribution and required parameters for different resistance random variables are summarized in Table 1.

Table 1
Resistance random variables parameters.

Random Variables	Distribution	Mean Value	COV (%)
Compressive strength of URE walls (MPa)	Lognormal	1,1.5,2, and 2.5	35
Compressive strength of CSRE walls (MPa)	Lognormal	1,1.5,2, and 2.5	22
Humidity effect of External walls	Uniform	0.75	19.24
Humidity effect of Internal walls	Uniform	0.95	3.03
Smoothness factor of External walls	Normal	0.9	5
Smoothness factor of Internal walls	Normal	0.95	2
Erosion factor of URE walls	Gumbel	0.016	25
Erosion factor of CSRE walls	Gumbel	0.005	20

2.1. Compressive strength

The resistance of a RE walls supporting gravity loads and noncyclic transient loads is governed by the material compressive strength. It is impractical to have an estimation for the compressive strength based on the earthen materials composition without testing. The recommended values for unconfined compressive strength of RE material is varied between 1 to 2 MPa as per different guidelines [4,7,8]. Due to the influence of the applied compaction energy, variation of the moisture content of the materials with respect to the optimum moisture content, curing method of the materials, and using different soil compositions, the coefficient of variation (COV) of the compressive strength is relatively high especially for unstabilized soils. By brief review on the past researches [2,5,12,16] and also the tests performed by the authors, the COV for URE and CSRE materials were 35% and 22%, respectively. The proper probability distribution to model compressive strength which avoids negative possible values is Lognormal [17]. Here, four different mean values of 1, 1.5, 2 and 2.5 MPa were considered to assess the effect of compressive strength on the performance of rammed earth structures.

2.2. Wall thickness

A minimum wall thickness is one of the main influence parameters provided in the guidelines and handbooks. The minimum recommended wall thickness varies for different design standards. A summary of some recommendations are shown in Table 2. Different thicknesses were used in this study to evaluate the performance of the walls under different conditions. The internal wall thicknesses were 125, 200 and 250 mm and the external wall thicknesses were 250, 300 and 400 mm for external walls. In addition, internal walls made of URE materials with 2 MPa compressive strength were analyzed for 150 mm thick walls under moderate loads.

For most cases the surface of the wall is not smooth, and the wall thickness just after removing the formworks and during the lifetime may not be exactly equal to the designed thickness. The reason is due to large particle size grains and not heavy interlock between ingredients which causes the wall thickness be smaller than expected. Therefore, based on engineering judgment, to consider the smoothness of walls in reliability analysis, a reduction factor with normal probability distribution was used. The mean and COV were 0.9 and 5%, respectively.

Table 2
Recommended wall thickness based on different standards.

Reference	Thickness of Wall (mm)	
	Internal	External
Standards Australia [8]	125	200
New Mexico Code [18]	300	450
Standards New Zealand [7]	250	250
Zimbabwe Code [19]	300	300

2.3. Erosion

The rammed earth walls are prone to high erosion, and environmental effects are followed by reduction in wall thickness during the lifetime of the structure. Accelerated erosion test (AET) is used to evaluate the materials for erosions effect. Based on AET results, unstabilized materials could not pass the tests and the water were penetrated in the full depth of the samples [5]. However, the real structures showed different results, since the condition of the tests is dissimilar to the in-situ condition. Bui et al. [20], exposed the RE walls to natural weathering for 20 years, the mean erosion depth was approximately 0.5% of wall thickness for stabilized and approximately 1.6% for unstabilized materials. In this case, Gumbel probability distribution was used to consider the erosion effect. The above mentioned values were considered as the mean values of the distribution with COV of 25% for URE and 20% for CSRE materials. The effect of erosion was considered by multiplying a reduction factor to the thickness of the rammed earth walls.

2.4. Humidity

The resistance of the rammed earth members are quite sensitive to water content of the materials and humidity of the air [15,21–23]. The behavior of walls are complex under different humidity levels. The walls lose their strength due to rainfalls and increase in humidity of the air. Bui et al. [20] showed that the compressive strength of URE walls is not significantly decreased by slight increase in moisture content. They also showed that the CSRE samples are less sensitive to moisture content. Based on the studies by Champiré et al. [22] on URE samples, the compressive strength decrease smaller than 50% for relative humidity up to 75%. Due to lack of proper data in this regard, a uniform probability distribution between 0.5 and 1.0 for URE external walls was used. The same probability distribution between 0.8 and 1.0 was used for CSRE external walls. For internal walls which are not very susceptible to change in humidity, a uniform distribution between 0.9 and 1.0 was considered.

3. Load random variables

In deterministic approach, the minimum probable capacity of structure shall be greater than the demand of the possible acting loads considering their maximum probability. In contrast, in reliability analysis, the loads are modeled with their probabilistic data and characteristics to limit the failure probability to a desired value.

In this study, dead load as a permanent load and live, snow and wind loads as the transient loads were applied to the rammed earth walls. The height of stories (walls) was considered 3 m and the walls was supposed to be unreinforced. This type of structure is usually constructed in areas with low seismic risk. Therefore, the earthquake loads were not considered in this study. Random variables contributed to the applied loads are presented In Table 3, with the applicable probability distribution and modeling parameters.

3.1. Dead load

The dead load was considered based on the loads of a timber floor and tiled roof [23]. The mean value and COV of dead load were 1.5 kN/m^2 and 7%, respectively. The used mean value allows for the permanent loads acting on a common timber floor. It is difficult to give a constant value for the density of materials just considering their aggregates. The mean value of density of the rammed earth materials was supposed 1900 kg/m^3 with 7% COV. This value was approximately the average value of different available researches [2,5,12,14,16]. Normal distribution was used for dead load and density of materials (GBJ68-84 [24]).

3.2. Live load

Live load was considered as distributed load applied on the loading area of the walls. The Gumbel probability distribution was chosen for the live load and roof live load of the structures based on GBJ68-84 [24]. The maximum live load and COV during the lifetime of the structure were 2 kN/m^2 and 29%, respectively. This live load is generally used for residential areas of buildings [25]. The maximum roof live load and COV on the tiled timber floors were 1 kN/m^2 and 29%, respectively. The roofs were supposed to be categorized in “ordinary flat, pitched, and curved roofs” as per ASCE 7 [25] and Iranian Loading Code [26].

3.3. Snow loads

Snow loads are noticeable transient gravity loads acting on the structures especially in the mountainous regions. The weight of snow load is dependent on snow on ground load, exposure coefficient, roof slope factor and thermal condition factor. To simulate a relatively severe condition, the maximum considered snow load on ground was 3 kN/m^2 according to Iranian Loading Code [6]. This value is equivalent to the maximum load for 2% probability of exceedance in 50 years of structure lifetime. For moderate snow load, half of this value was used (1.5 kN/m^2). As suggested by Ellingwood and Rosowsky [27], the lognormal distribution is appropriate to model snow load factors as random variables. The mean value for snow exposure, thermal condition factor and slope factor were considered 1.2, 1.1 and 1.1, respectively. The COV of all snow load random variables were assumed to be 10% [27]. It was supposed that the occurrence of snow in annual occurrence time-series was described by binomial distribution and was estimated to be 60 per year [28].

Table 3
Loads random variables parameters.

Random Variables	Distribution	Maximum Value[1]	Mean Value	COV (%)
Dead Load (kN/m^2)	Normal	–	1.5	7
Density (kg/m^3)	Normal	–	1900	7
Live Load (kN/m^2)	Gumbel	2	–	29
Roof Live Load (kN/m^2)	Gumbel	1	–	29
Snow Ce Factor	Lognormal	–	1.2	10
Snow Cs Factor	Lognormal	–	1.1	10
Snow Ct Factor	Lognormal	–	1.1	10
Snow Ground Load (kN/m^2)	Lognormal	3	–	10
Wind Speed (km/h)	Gumbel	130	–	50

[1] The maximum values are obtained based on 2% probability of exceedance in 50 years lifetime of structures.

3.4. Wind load

Several parameters affect the load applied on the structures due to wind and storms. Some of these parameters are wind speed, wind direction, and geometry of the structure. In this research, wind speed was considered as random variable equal to 130 km/h. It is the maximum wind speed in Iranian Loading Code [26]. The specified value is based on maximum considered speed with 2% probability of exceedance in 50 years of structure lifetime. For moderate wind load condition, approximately 70% of wind speed for severe load condition was considered (65 km/h). This resulted in half of the applied wind load on the structure under sever condition. Several studies have been done on the probability distribution to describe wind speed. Different wind distribution such as Gumbel, Gamma, or Rayleigh could be assumed for different regions considering the various wind regimes [29,30]. In this study, the Gumbel distribution was chosen. The coefficient of variation of 50% was selected for wind speed. It is relatively quite high; however, higher COV were reported for wind speed [31]. The elastic analysis presented by Ciancio and Augarde [13] was used for wind load analysis of walls of 3 m high with wall span of 4 or 5 m.

Here, the occurrence of wind in annual occurrence time-series was described by binomial distribution and was 20 per year as per data gathered for northern stations of Iran. The occurrence rate of snow and wind load acting concurrently was estimated to be 5 per year [28].

3.5. Correlation factor

For ideal rammed earth construction, the maximum dry density is achieved by compacting the earthen materials at its optimum moisture content to have a high compressive strength. Most investigations show that there is correlation between the unconfined compressive strength and dry density of samples. The correlation factor between density and compressive strength of both URE and CSRE walls was considered 0.8 based on data extracted from past researches [2,12].

4. Analysis

4.1. Analysis method

The probabilistic analysis is described by a vector, $\mathbf{x} = [x_1, x_2, \ldots, x_n]$ to represent the random variables, and the performance of the structure is shown by the limit state function, $g(\mathbf{x})$. The limit state function is defined in terms of resistance (R) and demand (S) of the structures. The failure probability based on the defined limit state function can be calculated by computing the following integration [32]:

$$p_f = P[g(\mathbf{x} \leqslant 0)] = P[R \leqslant S] = \int_{g(\mathbf{x} \leqslant 0)} f_x(\mathbf{x}) d\mathbf{x} \quad (1)$$

where $f_x(\mathbf{x})$ is the joint probability density function of the random variables $\mathbf{x} = [x_1, x_2, \ldots, x_n]$ and the integral is calculated over the failure domain $\Omega \equiv \{g(\mathbf{x}) \leqslant 0\}$.

Finding the joint probability density function of random variables is difficult and almost impractical due to difficulty of calculation of joint probability multiple integral. Hence, some approximate methods such as First-Order-Second-Moment (FOSM), First-Order-Reliability-Method (FORM), Second-Order-Reliability-Method (SORM) and numerical integration methods for example Monte Carlo Sampling were proposed and used.

The Monte Carlo Sampling method is most accurate, however, is time consuming and expensive. This method is compared with other methods to check and verify the validity of them. When the nonlinearity of the limit state function is high, the FOSM is inaccurate. As a result, the FORM and SORM are extensively used in reliability analysis.

In FORM the random variables, $\mathbf{x}$, are transformed into standard normal space, $\mathbf{u}$, and the other steps of calculation is done in this space. The limit state function is presented in this space by $G(\mathbf{u}) \leqslant 0$. The integral in Eq. (1) is calculated over the failure region defined in standard normal space. The design point, $\mathbf{u}^*$ in the standard normal space is the point located on the limit state function $G(\mathbf{u}) \leqslant 0$ with the maximum probability density. In other words, the design point is an approximation to the limit state surface with nearest distance to origin in the failure domain. The design point can be calculated by satisfying following equation:

$$\min \|\mathbf{u}\| \text{subject to} G(\mathbf{u}) = 0 \quad (2)$$

The probability approximated at design point is:

$$P[g(\mathbf{x} \leqslant 0)] \approx \Phi(-\beta) \quad (3)$$

where β is the reliability index and is equivalent to the distance from the origin to the calculated design point and Φ is the standard normal cumulative density function. The limit state in FORM is approximated by linearizing at the design point and can be shown as follows:

$$G(\mathbf{u}) \cong \nabla G(\mathbf{u}^*)(\mathbf{u} - \mathbf{u}^*) = \|\nabla G(\mathbf{u}^*)\|(\beta - \alpha\mathbf{u}) \quad (4)$$

where $\nabla G(\mathbf{u}^*)$ is the gradient vector at the design point and $\boldsymbol{\alpha}$ is the unit negative gradient vector at the design point pointing toward the failure domain and is calculated by:

$$\boldsymbol{\alpha} = -\nabla G(\mathbf{u}^*)/\|\nabla G(\mathbf{u}^*)\| \quad (5)$$

Iterations are required to calculate the values of $G(\mathbf{u})$ and $\partial G/\partial \mathbf{u}$ at the trial points. The improved Hasofer-Lind-Rackwitz-Fiesler (iHLRF) algorithm [33] was used to select the direction vector and step size.

Generally, the FORM gives the reasonable approximation for the linear limit state functions or when the nonlinearity is not high. In the SORM, the limit state function includes the second term of Taylor series as the approximation at the design point. Thus, the probability of failure can be approximately calculated by the content of the paraboloid which is tangent to the limit state at the design point. Because of the symmetry of the standard space p_{f2} (the probability content of the paraboloid) can be defined by β and the set of curvatures $\boldsymbol{\kappa} = \kappa(\kappa_1, \kappa_1, \ldots, \kappa_n)$:

$$p_{f2} \cong p_{f2}(\beta, \kappa_1, \kappa_1, \ldots, \kappa_n) \quad (6)$$

Breitung [34] suggested a simple approximation as follows:

$$p_{f2} \cong \Phi(-\beta) \prod_{i=1}^{n-1} \frac{1}{\sqrt{1 + \Psi(\beta)\kappa_i}} \quad (7)$$

A useful approximation of SORM is conducted by fitting to the principal curvatures at the design point and is called the curvature-fitting SORM. The last two trial points in the search for the design point in FORM are used to compute the first principal curvature of the limit-state function and corrected probability of failure is calculated according to Eq. (7) [35].

For reliability analysis of the rammed earth structures, the random variables were categorized in two groups: 1) the resistance dependent variables and 2) load variables. The first group included compressive strength, wall thickness, erosion factor and humidity factor and the dead, live, snow and wind loads were classified in the latter group. The analysis were performed for four distinct values of compressive strength 1, 1.5, 2 and 2.5 MPa as mentioned before. The internal and external walls were analyzed separately.

The structures were considered single or 2-story of 3 m high with wall span of 4 or 5 m. Both the cement stabilized and unstabilized materials were considered in the analysis.

4.2. Sensitivity measure

The results obtained from a reliability model are sensitive to the random variables. The importance of random variables and the sensitivities of the reliability analysis to the random variables can be extracted by the results of FORM [35].

Considering that $\mathbf{u}$ is presented in standard normal space, the mean of $\mathbf{u}$ is zero and its covariance matrix is identity matrix. According to Eq. (4), for the linearized limit state function, the variance of the limit state function can be calculated as follows:

$$Var[G(\mathbf{u})] = \|\nabla G(\mathbf{u}^*)\|^2(\alpha_1^2 + \alpha_2^2 + \ldots + \alpha_n^2) = \|\nabla G(\mathbf{u}^*)\|^2 \tag{8}$$

The contribution of random variable $\mathbf{u}_i$ to the uncertainty is assumed to be proportioned to $\boldsymbol{\alpha}_n^2$. The result is referred as “Alpha Importance Vector” which is acceptable for the uncorrelated random variables. The “Gamma Importance Vector” uses the same method and is applicable for correlated random variables. It measures the importance of random variables by using the following equation:

$$\gamma = \frac{\boldsymbol{\alpha}\mathbf{J}_{\mathbf{u},\mathbf{x}}\widehat{D}}{\|\boldsymbol{\alpha}\mathbf{J}_{\mathbf{u},\mathbf{x}}\widehat{\mathbf{D}}\|} \tag{9}$$

where $\widehat{D}$ is the diagonal matrix of standard deviation of random variables ($\mathbf{x}$) in their defined space and $\mathbf{J}_{\mathbf{u},\mathbf{x}}$ is the Jacobian of the transformation.

For both Alpha and Gamma importance vectors, the elements with negative sign are resistance (capacity) type and the elements influenced by loads have positive sign. The sensitivity analysis was performed using the computer program Rt [36].

The two other useful sensitivity measures are “Delta” and “Eta” sensitivity vectors. Delta sensitivity vector is the gradient of reliability index (β) with respect to the mean value of random variables (μ) in standard deviation matrix of random variables ($\mathbf{D_x}$). This vector shows that the change in mean values of which random variables have the most effect in reliability index (failure probability) of the structures. This vector can be calculated as follows:

$$\delta_i = \frac{\partial\beta}{\partial\mu_i}\sigma_i(\equiv \boldsymbol{\delta} = \nabla_\mu\beta^T\mathbf{D}_x) \tag{10}$$

where σ_i is the standard deviation of random variable x_i. The sign of elements in the vector are opposite to the “Alpha” and “Gamma” importance vectors and is negative for load type variables and positive for resistance type variables.

The Eta sensitivity vector indicates the relatively importance of random variables uncertainty (the standard deviation of the random variables) in the calculation of the reliability index of structures and is calculated as follows:

$$\eta_i = \frac{\partial\beta}{\partial\sigma_i}\sigma_i(\equiv \boldsymbol{\eta} = \nabla_\sigma\beta^T\mathbf{D}_x) \tag{11}$$

4.3. Limit state functions

The limit state functions which presents the limit between failure and safe domains were defined in terms of resistance and demand of the structures as stipulated by Eq. (1). The demand on the structure was the maximum applied load (or stress) on the walls due to applicable loads, which were Dead Load (D), Live Load (L), Snow Load (S) and Wind Load (W). The resistance of structures can be calculated based on following equation:

$$R = (C_{s,x}H_y)[t(1 - Sm_y)(1 - E_x)] \tag{12}$$

where C_s, H, t, Sm and E were the compressive strength of materials, the humidity factor, wall thickness, the smoothness and erosion effect, respectively. The suffix, x, was used for the stabilized and unstabilzied materials, and the suffix, y, was used to distinct between external and internal walls.

Here, four different limit state groups were defined. In the first limit state group, which presented by “DL“, the demand on the structure was calculated for dead and live loads only (D + L). The second group assessed the demand based on dead, live and snow loads simultaneously (D + L + S) considering the defined occurrence rate. This limit state group was shown by “DLS”. The resistance was calculated by applying Eq. (12). These two groups were used for internal walls only. The reason was that the sustained gravity loads on the external walls considering the loading area of walls are normally smaller than internal walls while their thickness are normally equal to or greater than them.

The third and fourth limit state groups were defined for the case of external walls subjected to wind load. In the third group the applied load was due to the dead, live, snow and wind load (D + L + S + W) considering the appropriate occurrence rate. In the fourth group the applied gravity loads was only due to the permanent dead load without any additional gravity load. The transient load in this group was wind load with the considered occurrence rate (D + W). The symbol for third and fourth limit state groups were “DLSW” and “DW”, respectively. The former limit state used for external walls was a representation for maximum applied load and the later was a representation for overturning.

The limit states were named based on following rule: LS-THK-SP-ST-MAT. Where LS denoted the applicable loads and limit state group, in which “D”, “L”, “S”, and “W” stand for dead, live, snow and wind load, respectively. THK represented wall thickness and varied between 125, 150, 200 and 250 mm for internal walls and 250, 300 and 400 mm for external walls. SP specified the span of the wall is 4 or 5 m. ST indicated the number of stories, 1 for single story and 2 for two-story buildings. The height of stories were considered 3 m. Finally, MAT denoted the type of material. “URE” and “SRE” stand for unstabilzied rammed earth material and cement stabilized rammed earth material, respectively. For example the limit state “DLSW-250-4-1-SRE” represented 4 m span external wall of a single story rammed earth building with stabilized materials and 250 mm thickness and subjected to dead, live, snow and wind load.

4.4. Target reliability and failure probability

Reliability analysis prohibits the failure probability of system from exceeding the target probability or keep the reliability index larger than target reliability index (β_T). Since now, no reliability analysis has been performed for rammed earth structures. Hence, there is not any proposed limit for the target reliability of rammed earth structures. Based on Chinese unified standard for reliability design of building structures GB 50068 [37], for second class structures with brittle behavior, the target reliability shall be 3.7. This class includes houses, offices, etc., which are not categorized in important structures (first class) nor temporary ones (third class). In this study, the target reliability of 3.7 was selected which is equivalent to failure probability of 0.01078%. ISO 2394 [38] and Stewart and Lawrence [39] recommend target reliability $\beta_T = 3.8$ (equivalent to failure probability of 0.0072%) for unreinforced masonry structures which is close to considered value in this research.

5. Results and discussion

5.1. Verification of analysis method

To verify the results obtained by FORM, the reliability index and failure probability for the applicable limit states were derived using SORM and Monte Carlo Sampling, for 200 mm thick internal and 300 mm thick external URE walls. Some of the results are shown in Fig. 1. The results show that the difference between these methods are negligible. The reason is that the limit state functions are almost linear. It indicates that the linearized hyperplane to the functions at the design point is a good estimation of the function. Therefore, the FORM was used to perform the reliability analysis with acceptable approximation.

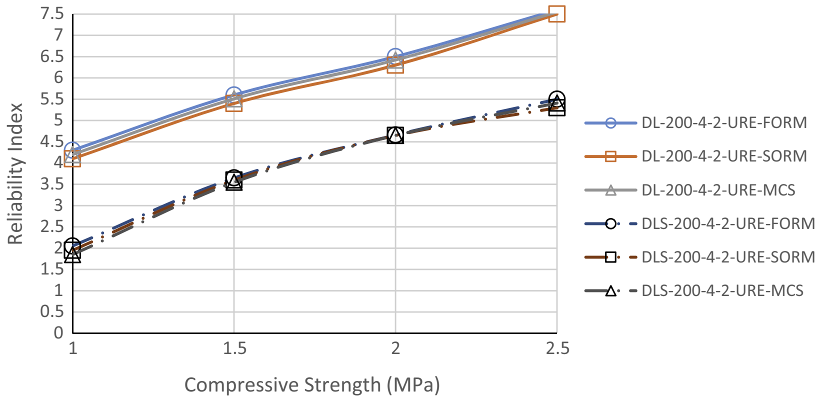
5.2. Reliability analysis of internal walls

The reliability indices were calculated for different limit states presented in previous section by using the FORM. The target reliability index was 3.7 (equivalent to 0.01078% failure probability). In the analysis of the walls, it was assumed that the slenderness effect of the walls are controlled and it does not have effect on the assessment of the walls.

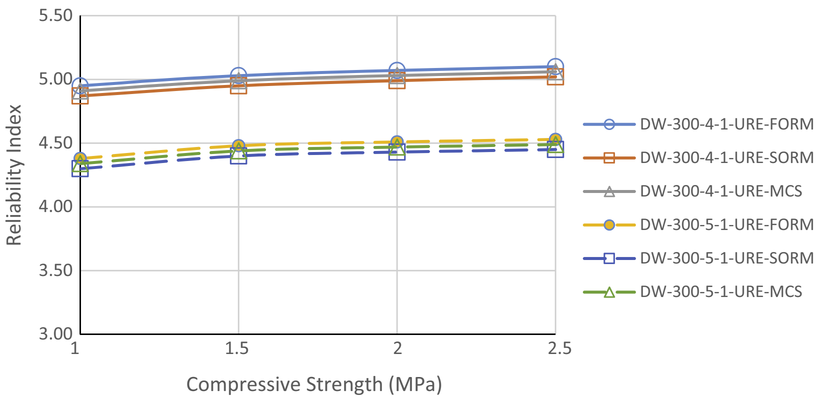
The reliability analysis of the internal walls on the first limit states group (Limit State “DL”), shows that for almost all cases the failure probability is quite smaller than the target value(the reliability index is a great deal more than the target reliability index). The results for URE walls supporting two-story buildings are shown in Fig. 2. The target reliability ($\beta_T = 3.7$) is also depicted on the graph. This quite small failure probability proves that the walls could support the normal permanent dead load and transient functional live load value with acceptable safety margin. The only concern is for 125 mm thick URE walls with 1 MPa compressive strength, supporting two-story buildings. Thus, it is recommended to use 200 mm thick walls for URE walls with 1 MPa compressive strength in two-story buildings or to use higher strength materials in this condition. For other cases under normal operation, 125 mm thick walls as recommended by Standards Australia [8] are safe.

The results for walls subjected to snow loads in addition to the dead and live load (“DLS” limit states) prove that, the reliability index is directly related to the compressive strength and wall thickness. The index is decreased by increasing the span and number of stories as expected. The same results were observed for “DL” limit states. The reliability index against the compressive strength for internal walls and “DLS” limit states are displayed in Fig. 3 for URE structures supporting two-story buildings. As can be seen, for URE structures with 1 and 1.5 MPa compressive strength, the reliability index is smaller than the target value in most cases. Using thick walls (200 mm or more) and materials with higher compressive strength (2 MPa as recommended by Burroughs [9,10]) enhance the capacity of structure to the acceptable limit.

For CSRE structures as shown in the Fig. 4, the reliability index do not pass the target value for the 125 mm thick walls with 1 MPa strength supporting two-story buildings. Considering that usually the compressive strength of the rammed earth structures stabilized with cement are far greater than 1 MPa, the minimum wall thickness is deemed adequate. It is also concluded that using thick internal walls especially when the strength of materials is high, is followed by high safety factor and uneconomical design.



(a) Internal walls



(b) External walls

Fig. 1. Reliability index based on FORM, SORM and Monte Carlo Sampling (MCS) methods.

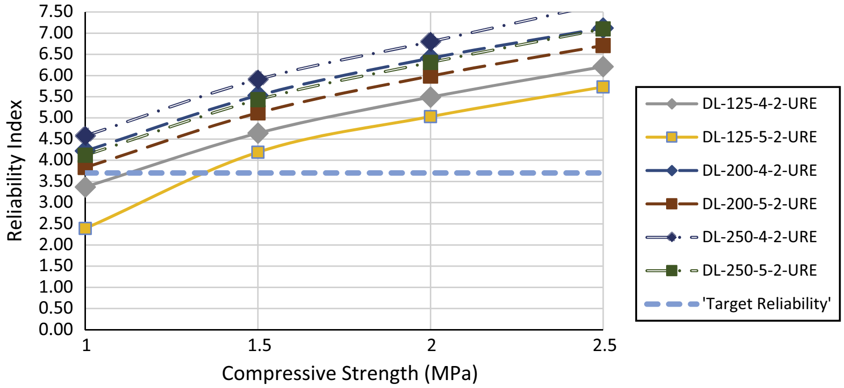


Fig. 2. Reliability index for internal walls ("DL" limit states, URE structures).

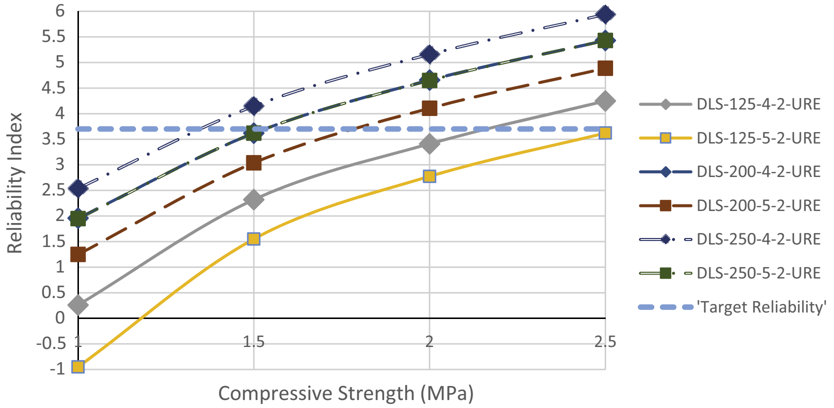


Fig. 3. Reliability index for internal walls ("DLS" limit states, URE structures).

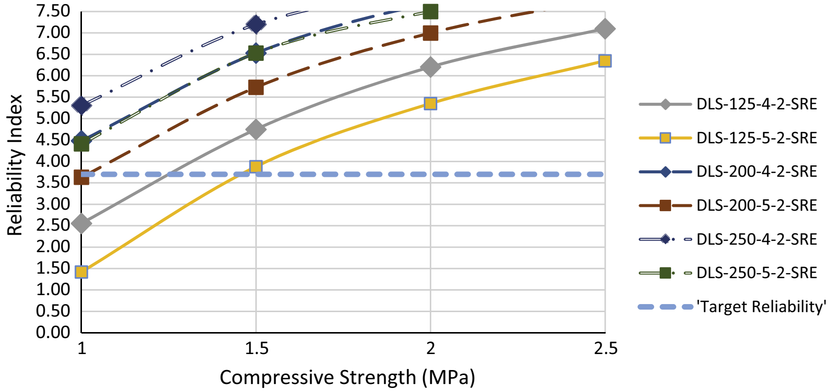


Fig. 4. Reliability index for internal walls ("DLS" limit states, CSRE structures).

For moderate environmental conditions, which was assumed to apply loads equal to half of the severe condition (snow weight on ground to be half of the severe condition and wind speed about 70% of considered severe speed), the URE internal walls of 150 mm thick behaved safe during lifetime of the rammed earth structures. 125 mm URE internal walls did not pass the target required reliability index of 3.7 for 5 m walls supporting 2-story buildings. The reliability index of URE walls of 125 and 150 mm thick, regarding 2 MPa compressive strength of materials, are presented in Table 4.

5.3. Reliability analysis of external walls

The reliability indices for "DLSW" limit state demonstrate that CSRE materials could pass the target value for all considered compressive strength and wall thickness. The results for two-story buildings with 250 mm and 300 mm thick walls are depicted in Fig. 5. The URE materials could also support single-story buildings in all cases. The results of reliability analysis of two-story buildings for URE structures are shown in Fig. 6. The same recommendations stated for internal walls made of URE materials is also applicable here.

Table 4
Reliability index for moderate loading condition of URE Internal walls.

Wall thickness	No of Stories	Span (m)	Snow Condition	Wind Condition	Load Combination	
					Dead + Live	Dead + Live + Snow
125	1	4	Moderate	Moderate	>7.5	6.17
	1	5	Moderate	Moderate	7.39	5.60
	2	4	Severe	Moderate	5.49	4.19
	2	5	Severe	Moderate	5.02	3.60[*]
150	1	4	Moderate	Moderate	>7.5	6.62
	1	5	Moderate	Moderate	>7.5	6.06
	2	4	Severe	Moderate	5.85	4.65
	2	5	Severe	Moderate	5.44	4.08

[*] Below the target reliability index of 3.7.

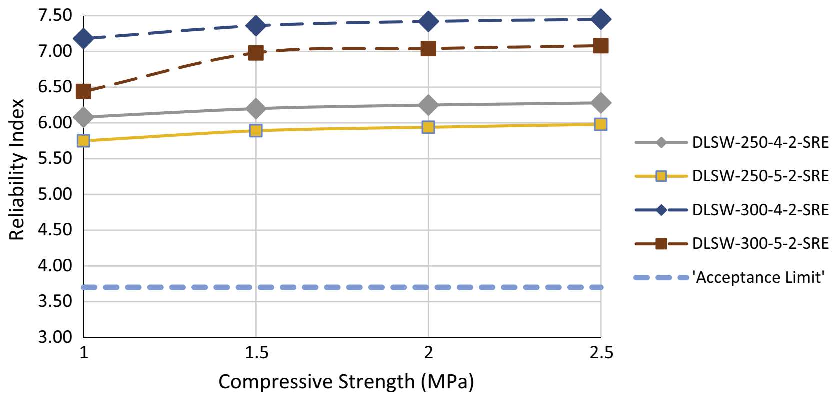


Fig. 5. Reliability index for external walls ("DLSW" limit states, CSRE structures).

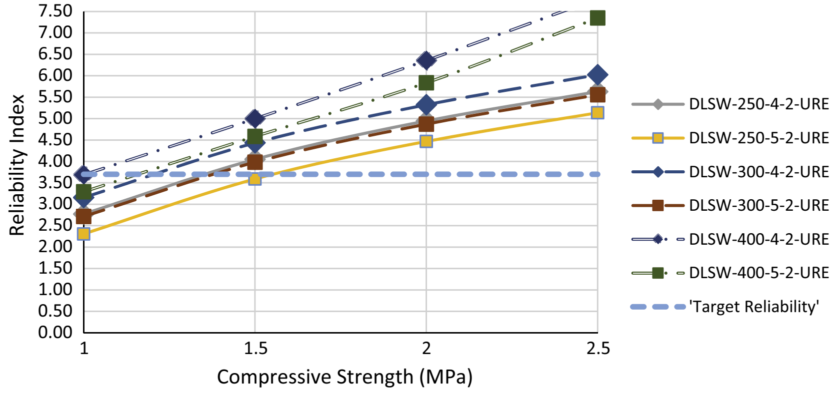


Fig. 6. Reliability index for external walls ("DLSW" limit states, URE structures).

The results for analysis of external walls for the limit state "DW" are plotted in Figs. 7 and 8 for URE and CSRE structures, respectively. The reliability index is relatively constant for different compressive strength. The reliability index is also close for the same wall thickness regardless of the material type (URE or SRE). Here, the weight and applied vertical loads have positive influence on the resistance of structure and overturning has governed the design. In this case the weight and dead loads are the major effective parameters. The reliability analysis does not satisfy for 250 mm thick walls for 5 m span; however, 300 mm external walls exhibited well in this limit state group. The results of 400 mm thick walls illustrate unnecessary safety margin for this severe loading condition. For moderate environmental condition the 250 mm thick external walls pass the 3.7 reliability index limit. The results of reliability analysis for moderate conditions of 250 mm external walls are presented in Table 5.

5.4. Sensitivity analysis

The sensitivity analysis was performed based on the data extracted from FORM analysis. Alpha and Gamma importance vectors and Delta and Eta sensitivity vectors were then evaluated. The results of Alpha and Gamma importance vectors for limit states "DLS" of unstabilized materials is shown in Fig. 9. The only correlated random variables were "density" and "compressive strength". Therefore, it is logical that the results based on Alpha and Gamma importance vector to be close. The elements of importance vector with negative sign contribute to resistance and those with positive sign contribute to load.

The importance vectors calculated for internal walls indicate that the compressive strength is, by far more, the most important uncertain parameter. The results are shown in Fig. 10. The Smoothness effect and humidity are the other most important factors

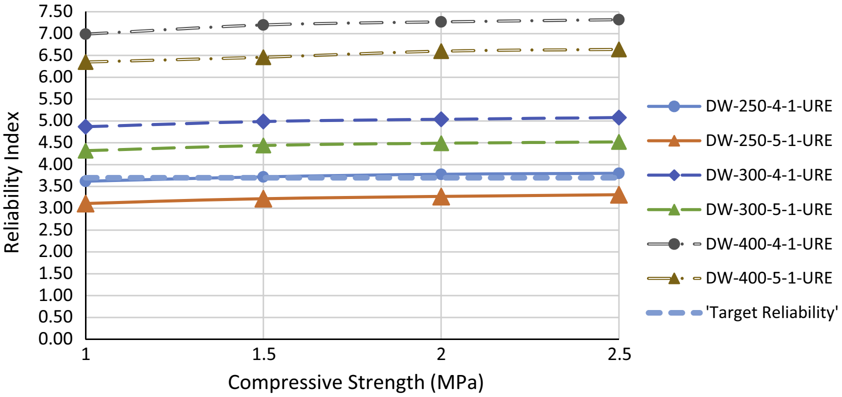


Fig. 7. Reliability index for external walls ("DW" limit states, URE structures).

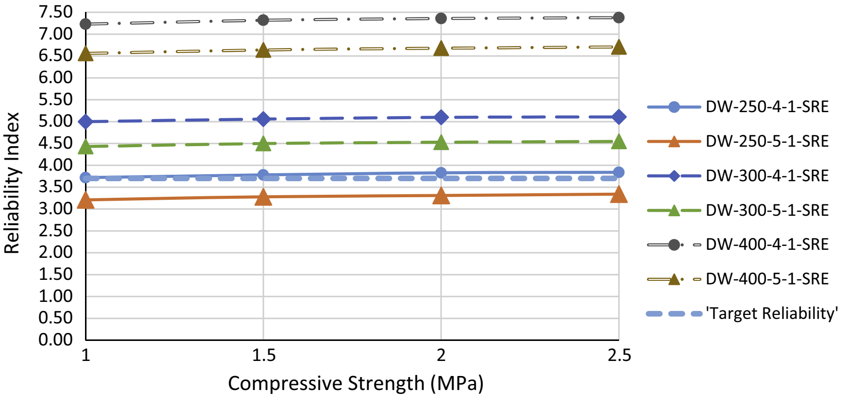


Fig. 8. Reliability index for external walls ("DW" limit states, CSRE structures).

Table 5
Reliability index for moderate loading conditions of 250 mm external walls.

Material Type	Span (m)	Snow Condition	Wind Condition	Load Combination	
				Dead + Live + Snow + Wind	Dead + Wind
URE	4	Moderate	Moderate	5.54	5.21
	5	Moderate	Moderate	5.11	4.68
	4	Severe	Moderate	4.87	5.21
	5	Severe	Moderate	4.39	4.68
SRE	4	Moderate	Moderate	>7.5	5.32
	5	Moderate	Moderate	>7.5	4.79
	4	Severe	Moderate	>7.5	5.32
	5	Severe	Moderate	>7.5	4.79

related to resistance. Live and snow load factors (in applicable limit states) are the most important variables of load group.

The calculated Gamma importance vectors for external walls are shown in Fig. 11. For "DLSW" limit states, compressive strength and humidity factor are the most important random variables. However, for "DW" limit state, it can be observed that wind speed is the most important random variable. By a far distance the density is the other most important uncertain parameter. It shall be emphasized that in this limit state group the importance vectors for density and dead load are negative. It points out that they are contributed to the resistance of structure in "DW" limit states; while in other limit state groups they are acting as load.

The results of Delta sensitivity vector is depicted on Fig. 12 for external walls. The reliability index is most sensitive to the change of the mean value of the compressive strength and the humidity factor. The results for Eta sensitivity vector is plotted on Fig. 13. The uncertainties of data provided for compressive strength, wind speed and humidity factor have the most effect on the reliability index and failure probability of the structure for both CSRE and URE materials.

It can be concluded that the reliability of the structure is quite sensitive to the compressive strength of materials, wind speed and humidity factor. By providing more realistic data about these variables, the reliability index of the rammed earth structures can be calculated more efficiently.

6. Recommendations

Based on the results of reliability analysis of the RE structures under different load combinations, some recommendations are given regarding minimum wall thickness in Table 6. The recommendations are presented based on type of material (URE or CSRE),

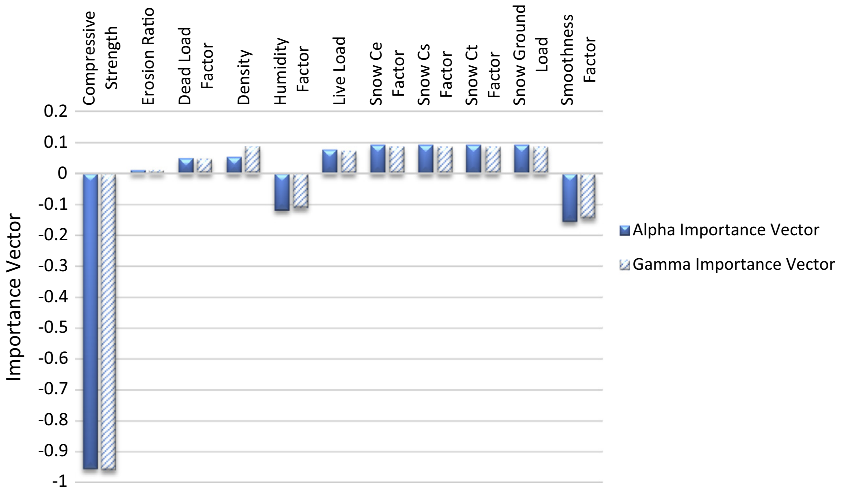


Fig. 9. Alpha and Gamma importance vectors (“DLS” limit states, URE structures).

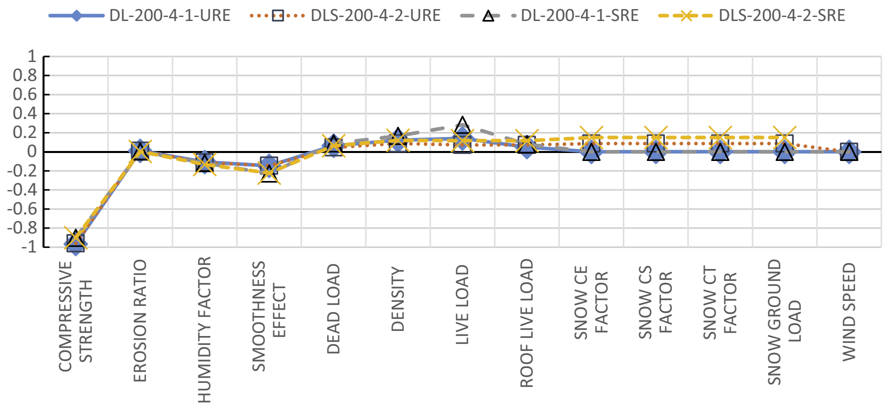


Fig. 10. Gamma importance vector for internal walls.

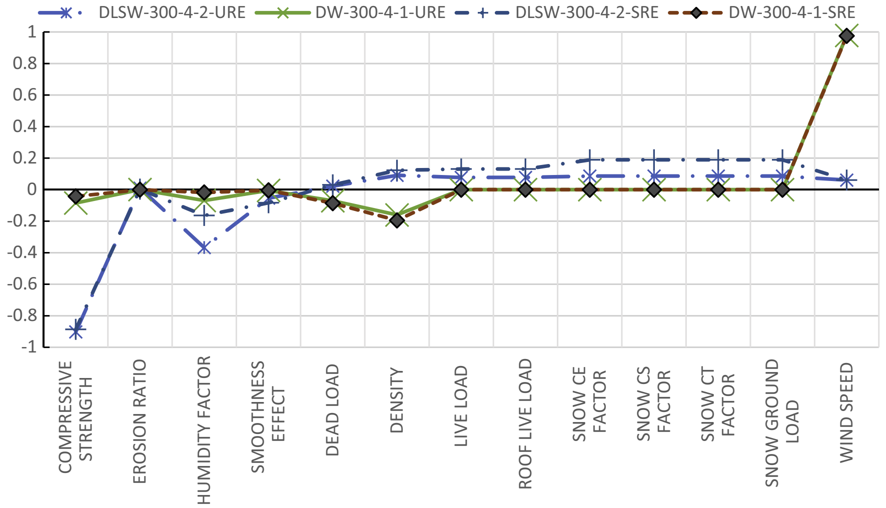


Fig. 11. Gamma importance vector for external walls.

number of stories and severity of environmental loads (moderate or severe). Considering the reliability index under different limit states, the minimum compressive strength for unstabilzied material is recommended to be at least 2 MPa. The recommendations about the wall thickness are given assuming 2 MPa compressive strength for URE and CSRE structures. The effect of slenderness and construction limitations shall be taken into account in addition to the following recommendations.

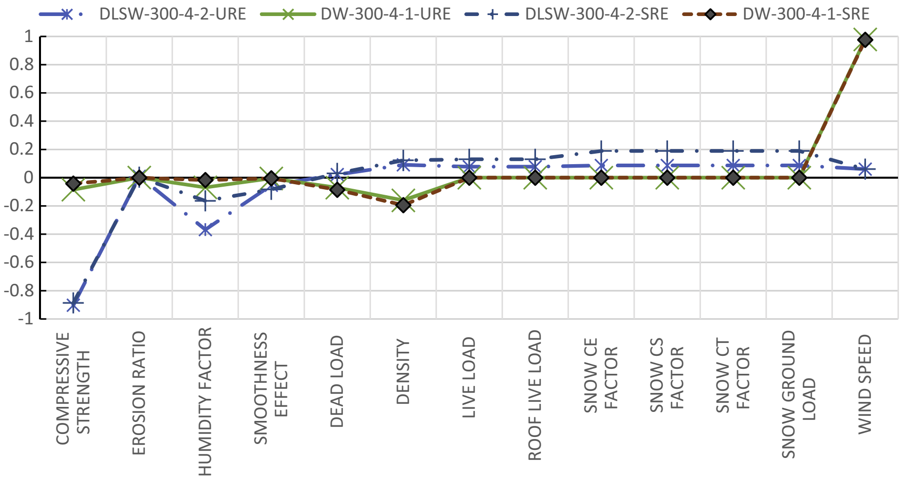


Fig. 12. Delta sensitivity vector for external walls.

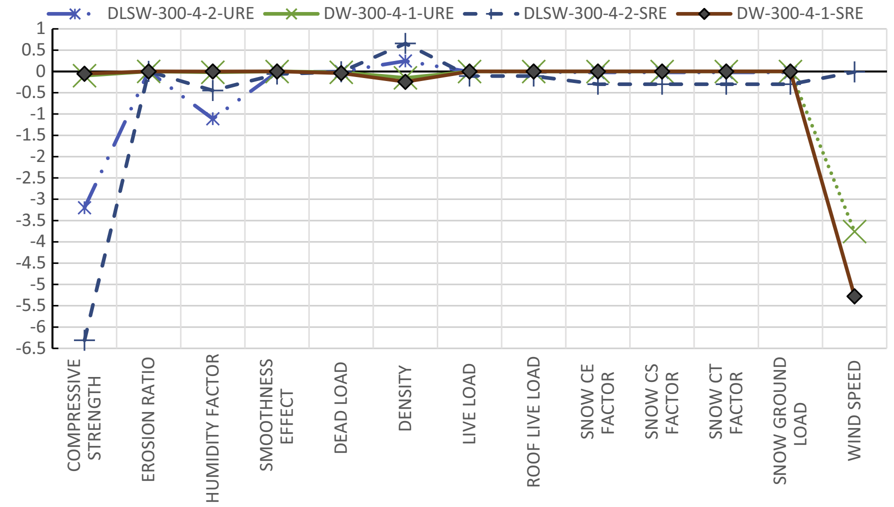


Fig. 13. Eta sensitivity vector for external walls.

Table 6
Recommended minimum wall thickness.

Type of Material	Wall Location	No. of Stories	Recommended Wall Thickness (mm)		
			Moderate Snow and Wind Load	Moderate Wind Load and Severe Snow Load	Severe Snow and Wind Load
URE	Internal	1	125	125	125
		2	150	200	200
CSRE	Internal	1	125	125	125
		2	125	125	125
URE	External	1	250	250	300
		2	250	250	300
SRE	External	1	250	250	300
		2	250	250	300

7. Conclusions

In this study, the reliability index and failure probability of the RE structures subjected to permanent and transient loads were calculated based on reliability analysis. The uncertain effective parameters on the strength of the structure and loads were modeled as random variables with proper probability distributions. Different limit states based on the applied load combinations were used in reliability analysis. The random variables with most effect on the reliability index and failure probability of such structures were found by sensitivity analysis. Finally recommendations about the minimum wall thickness for different conditions were presented. The following conclusions could be derived from the results of analysis:

The comparison between FORM, SORM and Monte Carlo Sampling, indicates that the limit state function are almost linear and the analysis by FORM which approximated the limit state by a linearized hyperplane at the design point can be used for analysis with good precision.

The reliability index of the structures are usually highly dependent on the wall thickness and the compressive strength of the materials.

The SRE structures (due to their low erosion probability, less susceptibility to loss of strength due to change in relative humidity, and less variance in compressive strength) have greater reliability index than URE structures under similar conditions.

The recommended values in codes and guidelines for the wall thickness are very conservative and uneconomical under moderate environmental loads. While on regions with heavy snow loads or high speed winds the presented values are unsafe.

The reliability analysis of external walls subjected to high speed wind loads shows that the failure probability is almost independent on the compressive strength and material type. The wall thickness and applied gravity load govern the design in this situation.

For walls subject to dead, live and snow loads, the compressive strength is the most important random variable based on Gamma Importance vector as a result of sensitivity analysis. The humidity factor for external walls has also great effect on reliability index. Sensitivity analysis of external walls subject to high speed wind load shows that the wind speed is the most important random variable. The results of reliability analysis is also sensitive to density of materials and applied gravity load.

The reliability index is most sensitive to the mean value of compressive strength, wind speed and humidity factor. The uncertainties in data provided for these items have also the most influences on the failure probability of structures based on Delta and Eta sensitivity vectors.

Funding

This research did not receive any specific grant from funding agencies in the public, commercial, or not-for-profit sectors.

References

[1] B.V. Venkatarama Reddy, P. Prasanna Kumar, Embodied energy in cement stabilised rammed earth walls, Energy Build. 42 (3) (2010) 380–385.

[2] Lorenzo Miccoli, Urs Müller, Patrick Fontana, Mechanical behaviour of earthen materials: a comparison between earth block masonry, rammed earth and cob, Constr. Build. Mater. 61 (2014) 327–339.

[3] Cheah, Jing Siong John, et al, Evaluating shear test methods for stabilised rammed earth, in: Proceedings of Institution of Civil Engineers, Construction Materials 165.6, 2012, pp. 325–334.

[4] B.V. Venkatarama Reddy, Georg Leuzinger, V.S. Sreeram, Low embodied energy cement stabilised rammed earth building-A case study, Energy Build. 68 (2014) 541–546.

[5] Daniela Ciancio, Paul Jaquin, Peter Walker, Advances on the assessment of soil suitability for rammed earth, Constr. Build. Mater. 42 (2013) 40–47.

[6] T.-T. Bui et al., Failure of rammed earth walls: from observations to quantifications, Constr. Build. Mater. 51 (2014) 295–302.

[7] Standards New Zealand, "4298: 1998." Materials and workmanship for earth buildings. Standards New Zealand.

[8] Standards Australia, P. Walker, Standards Australia: The Australian Earth Building Handbook, Sydney: Standards Australia International, 2002.

[9] Steve Burroughs, Soil property criteria for rammed earth stabilization, J. Mater. Civil Eng. 20 (3) (2008) 264–273.

[10] Steve Burroughs, Recommendations for the selection, stabilization, and compaction of soil for rammed Earth wall construction, J. Green Build. 5 (1) (2010) 101–114.

[11] Rowland Keable, Joe Martin, Vasilios Maniatidis, Rammed Earth: Design and Construction Guidelines, BRE Bookshop, Watford, 2005.

[12] B.V. Venkatarama Reddy, P. Prasanna Kumar, Cement stabilised rammed earth. Part A: compaction characteristics and physical properties of compacted cement stabilised soils, Mater. Struct. 44 (3) (2011) 681–693.

[13] D. Ciancio, C. Augarde, Capacity of unreinforced rammed earth walls subject to lateral wind force: elastic analysis versus ultimate strength analysis, Mater. Struct. 46 (9) (2013) 1569–1585.

[14] Daniela Ciancio, Boulter Michael, Stabilised rammed earth: a case study in Western Australia, in: Proceedings of the Institution of Civil Engineers-Engineering Sustainability, 165, Thomas Telford Ltd, 2012. No. 2.

[15] Mahsuli, Mojtaba, Probabilistic models, methods, and software for evaluating risk to civil infrastructure, Electronic Theses and Dissertations (ETDs) 2008+ (2012).

[16] Tripura Deb Dulal, Konjengbam Darunkumar Singh, Characteristic properties of cement-stabilized rammed earth blocks, J. Mater. Civil Eng. 27 (7) (2014) 04014214.

[17] M.H. Faber, John Dalsgaard Sørensen, Reliability Based Code Calibration, The Joint Committee on Structural Safety, Zurich, Switzerland, 2002.

[18] New Mexico Code, J.M. Tibbets, Emphasis on rammed earth-The rational, Interaméricas Adobe Builder 9 (2001) 4–33.

[19] Zimbabwe Code, Standards Association Zimbabwe Standard (SAZS) 724:2001: Standard Code of Practice for Rammed Earth Structures, Standards Association of Zimbawbe, Harare, 2001.

[20] Q.B. Bui et al., Durability of rammed earth walls exposed for 20 years to natural weathering, Build. Environ. 44 (5) (2009) 912–919.

[21] Quoc-Bao Bui et al., Effect of moisture content on the mechanical characteristics of rammed earth, Constr. Build. Mater. 54 (2014) 163–169.

[22] Pierre Gerard et al., A unified failure criterion for unstabilized rammed earth materials upon varying relative humidity conditions, Constr. Build. Mater. 95 (2015) 437–447.

[23] Florian Champiré et al., Impact of relative humidity on the mechanical behavior of compacted earth as a building material, Constr. Build. Mater. 110 (2016) 70–78.

[24] GJB 68–84. The uniform standard for building structures. 1985 (in Chinese).

[25] American Society of Civil Engineers. Minimum design loads for buildings and other structures. Vol. 7. Amer Society of Civil Engineers, 2010.

[26] Iranian Loading Code, National Building Regulations. Building and Housing Research Center, 2013, (in Persian).

[27] Bruce Ellingwood, David Rosowsky, Combining snow and earthquake loads for limit states design, J. Struct. Eng. 122 (11) (1996) 1364–1368.

[28] Jafari, Mostafa. Thunder and storm fluctuations in the Caspian region over the last half century, 2009, pp. 583–598.

[29] Eugene C. Morgan et al., Probability distributions for offshore wind speeds, Energy Convers. Manage. 52 (1) (2011) 15–26.

[30] Nurulkamal Masseran et al., The probability distribution model of wind speed over East Malaysia, Res. J. Appl. Sci. Eng. Technol. 6 (10) (2013) 1774–1779.

[31] Bruce R. Ellingwood, Paulos Beraki Tekie, Wind load statistics for probability-based structural design, J. Struct. Eng. 125 (4) (1999) 453–463.

[32] Gregory B. Baecher, John T. Christian, Reliability and Statistics in Geotechnical Engineering, John Wiley & Sons, 2005.

[33] Y. Zhang, A. Der Kiureghian, Two improved algorithms for reliability analysis, Reliability and Optimization of Structural Systems, Springer, US, 1995, pp. 297–304.

[34] Karl Breitung, Asymptotic approximations for multinormal integrals, J. Eng. Mech. 110 (3) (1984) 357–366.

[35] Der Kiureghian, Armen, First-and second-order reliability methods, Engineering design reliability handbook, 2005, 14–1.

[36] M. Mahsuli, T. Haukaas, Computer program for multimodel reliability and optimization analysis, J. Comput. Civil Eng. 27 (1) (2012) 87–98.

[37] GB 50068, Unified standard for reliability design of building structures: Standards China, 2001, (in Chinese).

[38] ISO 2394, ISO. 2394. General Principles on Reliability for Structures. Zurich: ISO (1998).

[39] Mark G. Stewart, Stephen Lawrence, Structural reliability of masonry walls in flexure, Masonry Int. 15 (2) (2002) 48–52.